Mathematics literature review

Senior syllabus redevelopment

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Executive summary

A review of the literature associated with the enactment of mathematics syllabuses of top-performing nations reveals distinct attributes. In the reform/redesign of Queensland senior mathematics syllabuses, these attributes could provide insights to strengthen the competitiveness of our graduates. These attributes include:

- maintaining beliefs and perceptions of mathematics as a hierarchical body of disciplined knowledge and modelling this logically in curriculum documents
- linking basic skills and knowledge with problem-solving and reasoning across all levels
- encouraging an orientation of mastery learning rather than enacting a spiral syllabus
- using technology purposefully
- balancing intrinsic and extrinsic motivational factors for teachers and students
- promoting and implementing a range of pedagogies with teachers carefully monitoring the classroom discourse
- developing and supporting teachers with advanced mathematical expertise
- developing and maintaining high expectations of students in terms of levels of effort, ability and abstraction
- creating explicit and comprehensive syllabus documents that connect mathematical content with pedagogy and student readiness
- utilising external examinations for formative and summative assessment purposes
- developing 21st century skills through the learning of mathematics and not as an add-on
- developing and promoting creativity in the teaching and learning contexts of mathematical problem-solving.

Hong Kong and Singapore have excellent mathematics systems with quality components that align to produce students who learn mastery. These include logical national mathematics frameworks, mathematically rich problem-based textbooks, challenging external assessment and highly qualified teachers who focus on teaching to mastery. This level of organisation, rationality and support is not nearly as evident in Australian including Queensland mathematics teaching and learning.

Chapter 1: Significant emerging trends

Introduction

The effectiveness of mathematics systems and curriculums has been the focus of national academic improvement agendas for decades. This literature review examines the effective mathematical curriculums and systems across nations that produce students with advanced mathematical competencies. The fundamental motivation for examining mathematics proficiency in this paper is articulated by the Organisation for Economic Cooperation and Development (OECD 2014b, p. 6):

> Proficiency in mathematics is a strong predictor of positive outcomes for young adults, influencing their ability to participate in post-secondary education and their expected future earnings.

Essentially, this means mathematical competence correlates highly with senior education engagement and consequently, future financial prospects for those students as young adults. Therefore, students who are mathematically competent look forward to a brighter financial future than those who are mathematically incompetent. This literature review examines effective mathematical curriculums across Australian states against those of high performing nations across the world. The Australian states examined in this review include Queensland, New South Wales, Victoria, and Western Australia. The high performing nations included in this review are Singapore and Hong Kong. The senior mathematics curriculum of the State of California in the United States is also examined, as it provides an alternative model to the Asian systems.

Nation performance is determined and ranked by interpreting student assessment data gathered from various international mathematics assessment tools measuring knowledge and application. The mathematical assessment tools used to gather data for this paper were the OECD Programme for International Student Assessment (PISA) (OECD, 2015).

Eastern Asian countries were at the forefront of the 2014 international mathematics rankings. The top ranked nations were Shanghai (China), Singapore, Hong Kong (China), Chinese Taipei, Korea, Macao (China), Japan, followed by Liechtenstein. While Hong Kong's mean scores were very high, considerable variance was recorded. Therefore, some students in some Hong Kong schools performed poorly, making Hong Kong's ranking questionable compared to other nations with less varying scores. The nations with relatively high average scores and the least variance were Iceland and Finland (McGaw, 2006) but these nations were not included in the 2014 top nations due to the dominance of the East Asian systems in educating the elite of mathematics students. The first English-speaking country was Canada ranked at number 13, and Australia was ranked 19th.

The Shanghai (China) models of mathematics curriculums and systems would be most effective for this review as their data exceeded any other nation. However, difficulties involving language translation and accessing documents made this too difficult in the permitted timeframe. Subsequently, the mathematical teaching and learning systems and curriculums enacted in Singapore and Hong Kong are the chosen exemplars for building optimal mathematical competencies amongst students. The mathematical successes in these Eastern Asian nations can guide the curriculum reform of other nations like Canada, Australia and the United States.

A factor to consider when learning from East Asian education systems is that cultural factors contribute to the success of their students. Nonetheless, cultural factors alone cannot explain the relatively recent rise of standards of mathematics learning in these nations. Cultural factors contributing to student academic achievements typically encompass gender, age, socio-economic status, residential location, parent education levels and tangible learning tools (Jensen, 2012; Otsuka, 1996). Otsuka (1996) claims the East Asian nations can attribute their mathematical achievement to a relatively homogenous population of highly motivated students with a culture that rewards delayed gratification. Thus, the argument goes, attempting to learn from them is doomed to failure since our society is different. This argument is defeatist and does not credit the structural factors that contribute to the success of these nations. Jensen (2012) has noted that popular stereotypes about Asian education are strong in some countries. But the evidence challenges these beliefs. High performance in education systems in East Asia comes from effective education strategies that focus on implementation and well-designed programs that continuously improve learning and teaching. Neither cultural difference nor Confucian values can explain how, in just five years, Hong Kong moved from 17th to 2nd in the international assessment of Grade 4 students' reading literacy. Instead, education reforms created rapid changes in reading literacy. Success cannot be explained by rote learning either. PISA assesses meta-cognitive content knowledge and problem-solving abilities. These skills are not conducive to rote learning. In fact, rote learning in preparation for PISA assessment would lead to lower scores. Consequently, we need to reflect upon what it is that we can learn from high performing nations that meet the needs of our students and is feasible. Syllabus review processes attempt to identify such characteristics.

There have been considerable volumes of material written on the state of Queensland mathematics education. Reports by Menkens (Education and Innovation Committee, 2013) and, Matters and Masters (2014) have revealed societal connections with academic achievements. A major motive behind these inquiries and reports was the recognition that quality mathematical knowledge is necessary for the economic wellbeing of the nation, especially for Queensland. There was a perception in the community that Queensland was lagging behind other states at the top end of mathematics knowledge and skills, a finding recognised for primary and middle years mathematics earlier (Masters, 2009). Reports by Menkens (2013) and Matters and Masters (2014) focused on assessment and the role of assessment in shaping senior mathematics learning. The relationship between assessment and teaching was clearly recognised. A critical concern of Menkens (2013) was that the methods of assessment were too highly focused on higher order capabilities such as analysis, synthesis and the application and communication of mathematical knowledge at the expense of fundamental lower order skills. It is not the intention to revisit the detail in those reports, but rather to put a somewhat different and comparative summary of the literature together.

A matter of concern was the relatively low level of students attempting the most abstract level of mathematics with Mathematics C counting for only 9% of OP-eligible (overall position eligible) students and Mathematics B about 27% (Kennedy, Lyons & Quinn, 2014). The enrolments in Mathematics A are approximately 49% and are on an upward trend in terms of enrolment proportion. This is set against a trend of decreasing percentages of OP-eligible students, which sits at about 54% as of 2013 (Matters & Masters, 2014, p. 5). In this review of the literature and syllabuses, we examine different views on the nature of knowledge and how that impacts on syllabus design. Additionally, we visit factors that shape student motivation and engagement, and draw from these insights that inform syllabus design.

Limitations

In undertaking an analysis of international and domestic syllabuses, it must be noted that limited access and the timeframe set restrict the review to examination of the intended curriculum. In order to describe the enacted curriculum we would need to access additional documents including tests, textbooks, other resources and consult broadly with key stakeholders. Such stakeholders would include curriculum and assessment bodies, heads of departments, teachers, university academics who teach our students, as well as industry bodies who employ them. This detail and depth is well beyond the brief of this review. We start by examining the different views on the nature of mathematical knowledge.

Views on the nature of mathematics and its learning

The most effective way to reflect on the presented data is through a theoretical lens. Analysing data against empirical theories allows for greater objectivity surrounding systemic and focused actions for constructive system reform. In this review, two quite distinctive images of the nature of mathematics and its learning have been drawn from literature and used for critical data analysis. The way mathematics is viewed has considerable impact on how the curriculum is organised and ultimately how the mathematics is taught. Since the way mathematics is viewed differs from state to state, and more so from country to country, this theoretical lens is especially relevant.

Bernstein (1996, 2000) distinguishes between *mundane* and *esoteric* knowledge. He argues the knowledge transmitted through schooling is a privileged form of esoteric knowledge which, when acquired, confers privilege on the acquirers. Esoteric knowledge has a privileged status and teachers are afforded status to assist with the acquisition process, which tends to be cumulative in nature. The inability to access deep knowledge of esoteric forms is associated with the perpetuation of social inequality. Therefore, esoteric knowledge acts as a gateway to symbolic mastery, which is typically associated with tertiary entrance to prestigious academic study and subsequently, higher paying jobs. This is particularly the case with mathematics-dependent courses and careers. The word *vertical* has been used to describe the nature of esoteric knowledge, since it generally comes from a 'higher place' (i.e. a research institute or university) and is taught sequentially. For example, whole-number computations precede fractions, which precede algebra, which precedes calculus. The advanced mathematics syllabuses (i.e. Mathematics B and C in Queensland, Specialist Mathematics in the Australian Curriculum) tend to emphasise the hierarchical nature of mathematical knowledge and are viewed as essentially esoteric. Muller and Taylor (1995) use the term *discipline knowledge* to describe Bernstein's esoteric knowledge.

In contrast to esoteric knowledge is what Bernstein refers to as the *mundane* knowledge of *everyday life*. Mundane, or everyday knowledge, is acquired through everyday activities in a segmental fashion and is not logically, coherently or cumulatively connected. Furthermore, a special or privileged status is not conferred on this knowledge, or on those who acquire this knowledge, because it is usually accessible to everyone, and does not require specialist teachers or instructors to assist with the acquisition process. To a much lesser extent, the hierarchical distinction is not made between these different types of knowledge or the languages that lead to knowledge growth or progression. The term *horizontal* can be used to describe everyday knowledge in part because knowledge diffuses from the outside community into the classroom, and also because there is much less emphasis placed on the sequence of learning.

Australian state mathematics syllabuses present both esoteric and mundane courses. In every Australian state syllabus document, the most advanced mathematics courses offered are esoteric in nature (i.e. Mathematics B and C or Specialist Mathematics). Generally, lower levels of abstraction are associated with horizontal depictions of mathematics and across nations. The most mundane mathematics, the mathematics that are most closely aligned to direct application, tend to be the least demanding, such as Mathematics A and Prevocational Mathematics in Queensland, or Essential Mathematics in the Australian Curriculum[1].

The syllabus documents of Singapore and Hong Kong are at the extreme end of depicting mathematics as essentially esoteric and their higher levels of secondary education mathematics are equivalent to tertiary level mathematics in Australia. There are other international models that warrant brief mention. Germany's mathematical educational system differs again. In Germany's instance, students at Year 5 are tested and the pathway of the student towards an academic school (Gymnasium) or trades-orientated school (Hauptschule and Realschule) is essentially established (Frankfurt International School, 2015). Approximately 30% of German students attend academic pathways compared to approximately 56% attending trades-orientated schools. As well as differing in curriculum, these academic and trades schools are physically separate. This German example offers another way of viewing primary and secondary schooling that recognises trades-associated mathematics as essentially mundane and academic mathematics as esoteric. Similar differentiation selection processes operate in California, but the basis is subject selection rather than school streaming.

In Australia, and Queensland in particular, the lower, more applied mathematics is set in authentic contexts directly related to the everyday lives of Australian students, such as budgeting, choosing a mobile phone plan, buying a car or paying tax. These mathematics courses are frequently taught contextually and require student investigation skills. It needs to be noted that advanced mathematics such as Mathematics B and C still has real-life contexts, but most schools (including Queensland schools) use textbooks that teach the concepts esoterically and then apply these in *authentic* contexts. Generally, each chapter starts with a series of worked examples, the students practice computation methods and the chapter ends with students applying the recently learned concepts in *authentic* word problems. In most cases, these word problems would not normally be encountered in the daily lives of students. Standard texts do not attempt to teach through the contextual

[1] Neither Hong Kong nor Singapore offer such courses.

situation. However, a rare example of a text that attempts to teach through the contextual situation is the Mary Barnes series *Investigating Change: An Introduction to Calculus for Australian Schools* (1991). Frequently the textbook becomes the de-facto syllabus, especially if the syllabus documents offer flexibility. In summary, within Australia, all states recommend teaching in authentic problem-based settings consistent with a horizontal nature of mathematics, but in practice this is most applied in the lower stream levels of mathematics such as Mathematics A and Prevocational Mathematics.

Situating senior mathematics

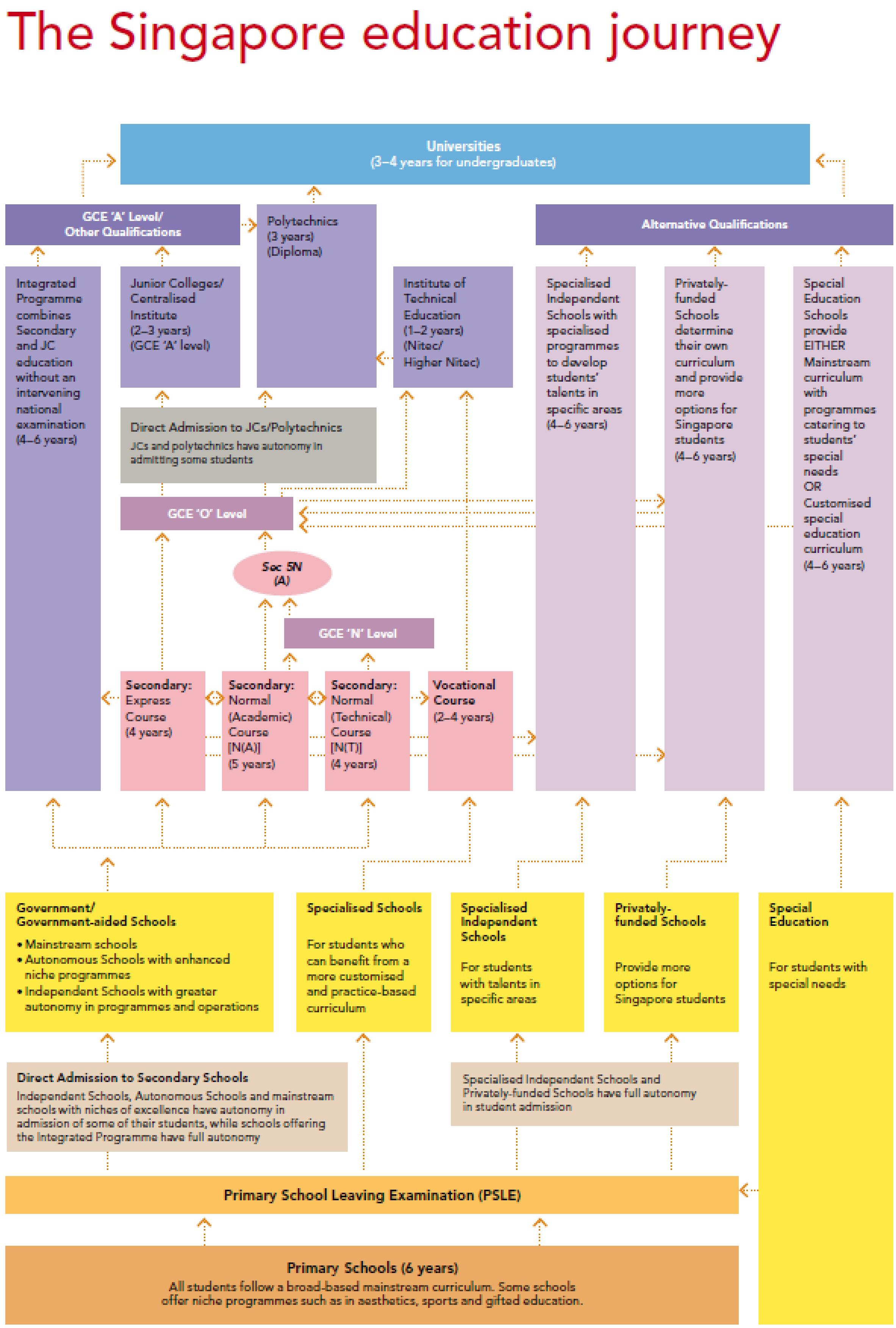
The discussion above looks at differences in viewing the nature of mathematics and its depiction in senior syllabuses. It is important to situate senior mathematics in the context of all school mathematics. The Australian Curriculum’s P–10 Mathematics has a much stronger focus on the hierarchical nature of mathematics than earlier junior Queensland mathematics curriculums such as *Essential Learnings* (QCAA, 2007). Recent revisions of mathematics syllabuses have seen all states move towards commonalty with the Australian Curriculum.

The Australian Curriculum: Mathematics (ACARA, 2012) contains content strands including *number and algebra*, *measurement and geometry*, and *probability and statistics*. It also has proficiency strands including *understanding*, *fluency*, *problem-solving* and *reasoning*. Clearly, while all the strands intertwine to develop in students the capability to tackle real world problems and problems of complexity, the foundation is basic content and processes. If students lack basic knowledge and processes, they will be unable to apply them in problem settings. The Australian Academy of Science (AAS) recognised the intertwined relationship between conceptual understanding, procedural fluency, problem-solving and reasoning (2015). AAS (2015, p. 17) states:

> Mathematics is a hierarchical subject, where new knowledge is on earlier learning in a highly connected way … Mathematics has a highly connected web of concepts and skills, so these have to be firmly consolidated to provide a basis for new learning.

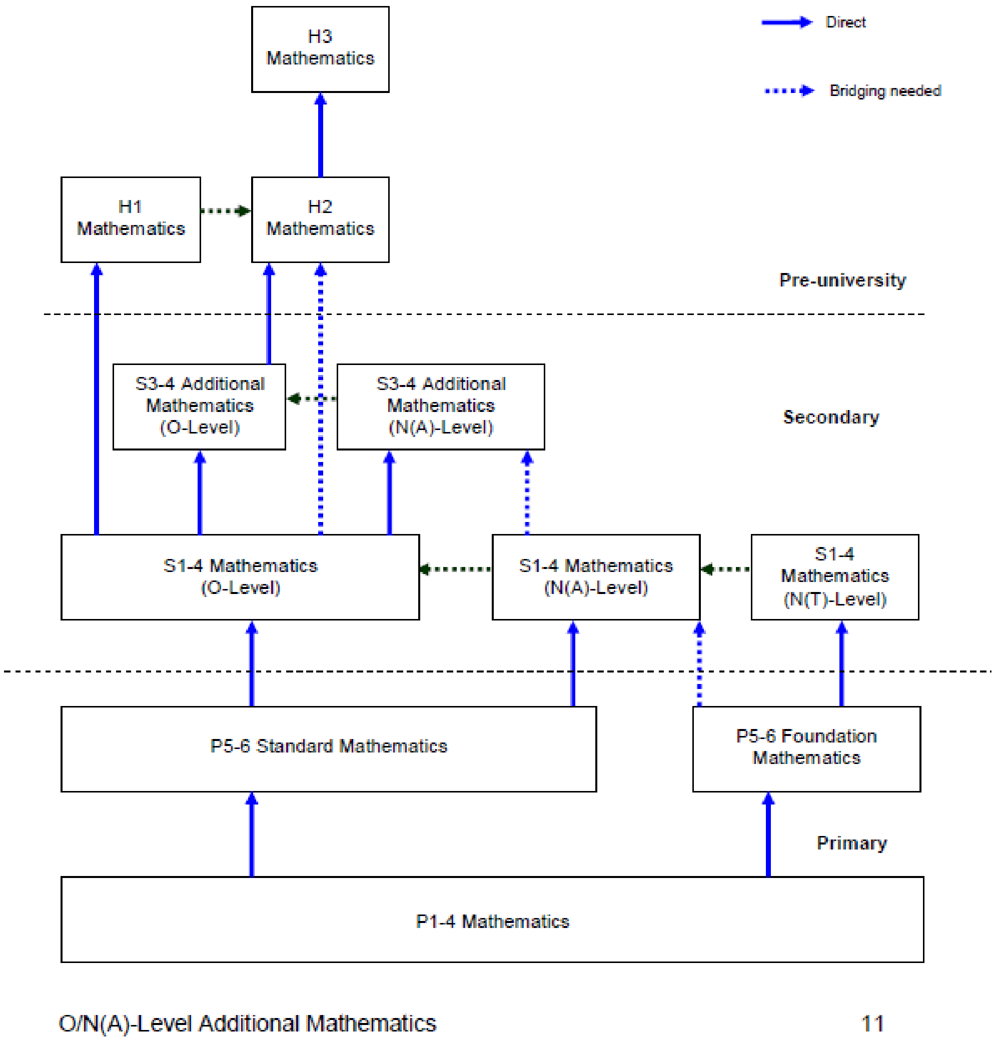
There is a discernible trend towards greater recognition of the esoteric nature of mathematics in Australian educational research. Two further considerations inform us about the nature of mathematics syllabuses. The first is the amount of structure in showing the connections between various school mathematics courses including pathways from primary to secondary. High performing nations such as Singapore have plotted the relationship in detail as illustrated in Figure 1 below. There is a great deal of rationality and transparency associated with study pathway.

Figure 1: The Singapore education journey (Ministry of Education, Singapore 2008, p. 3)



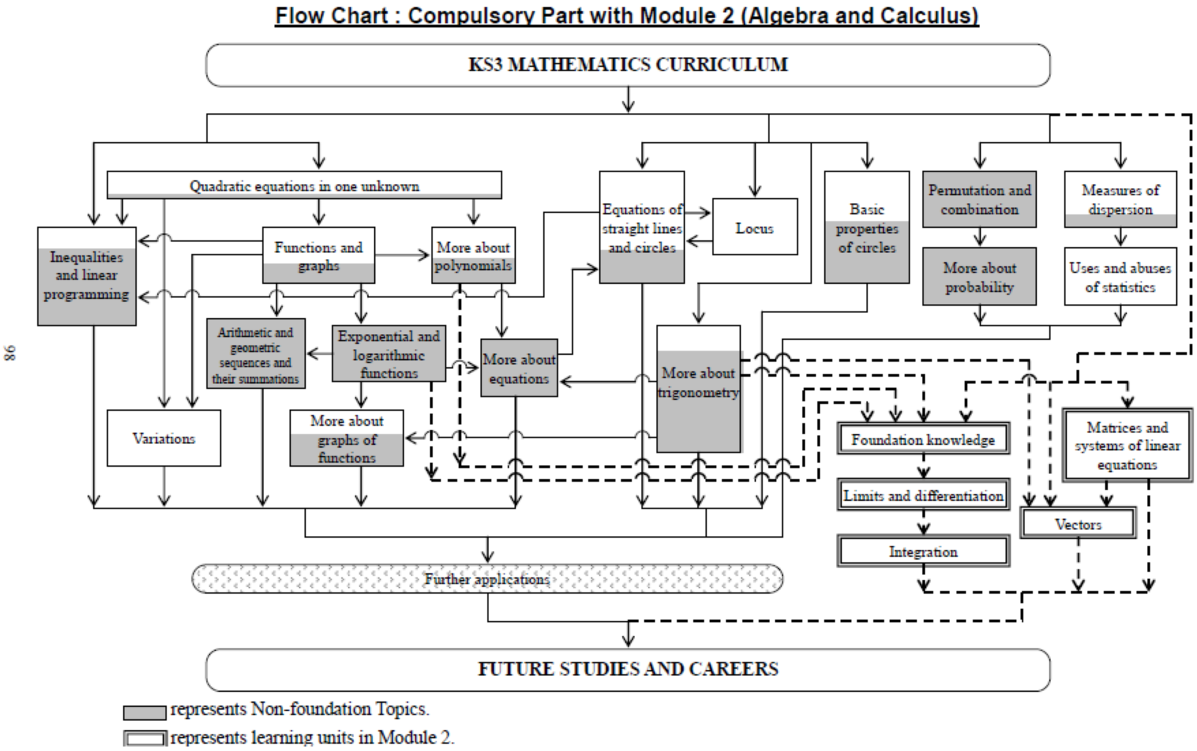
At Year 6, external examinations determine student's placement in secondary school courses that suits their learning pace and aptitude. The systematic approach to planning in mathematics pathways is illustrated in Figure 2 below. The additional mathematics S3 level contains most of what Mathematics B students would study.

Figure 2: Singapore pathways for school mathematics learning (Singapore Ministry of Education, 2012, p. 11)



The second factor is the detail and relationships between concepts. Historically NSW and Victoria have had explicit syllabus documents. These documents tell the teachers what to teach in what sequence and, in the case of NSW, there was also considerable detail on how to teach it (e.g. Board of Studies NSW, 2003). Similarly, what we see in the syllabus documents of top-performing international competitors such as Singapore and Hong Kong is great detail of specifics on what is to be taught in what sequence. The hierarchical and connected nature of the key mathematical concepts is explicitly displayed. Figure 3 below illustrates how the Hong Kong educational system shows the connections between concepts in a course equivalent to Mathematics B, but at a higher level of abstraction.

Figure 3: Hong Kong flow chart illustrating the relationships between key concepts (HKEAA-CDC, 2007)



This degree of organisation contrasts with the current Queensland syllabus documents that treat content as discrete units, do not mandate the sequence, have flexibility in topics taught, give little specificity to the content, and do not explicitly make the links between units of work. It could be argued that this lack of specificity gives teachers greater professional flexibility. It could also be argued that the lack of specificity is symptomatic of a lack of guidance and/or expectation.

In short, the highest performing nations have mathematics syllabuses that more strongly reflect a view of mathematics that is essentially vertical. We see differences in abstraction and problem-solving noted in Trends in International Mathematics and Science Study (TIMSS) results from early high school whereas East Asian students are significantly advanced compared to Australia and indeed almost all Western students. The East Asian nations have much higher proportions of students in the top bands (Thomson, Hillman, Schmid & Munene, 2012; Thomson, Wernert, Underwood & Nicholas, 2007; U.S. Department of Education, 2008).

Not all authors accept the validity of such assessment types and point to test tainting and possibly sampling errors as well as different curriculum focuses, including questioning formats (Malatt, 2000). All of these variables could influence the validity of the tests; however, in the case of TIMSS, over half a million students were sampled in 2008 and Martin and Mullis (2009) reported that, overall, the test authors and examiners had made considerable steps to ensure the collection of high-quality, comparative data consistent with the recommendations to increase validity articulated by Wolf (1998). Bracey (2014) argued that the knowledge forms tested by TIMSS and PISA are largely irrelevant to the overall prosperity of a nation and thus international test outcomes should not be a concern to educators. Nonetheless, the Austrialian press and government spokespeople regularly cite the importance of such indicators and recent curriclum commentry echoes such concerns (e.g. AAMT, 2015; AAS, 2015).

Unfortunately, we do not have multinational data to make valid comparisons at senior high school levels. However, the advanced senior high school syllabus in Singapore and Hong Kong are more demanding — in terms of calculus in particular — than comparable Queensland syllabuses.

The relationship between foundation knowledge and problem-solving

The relationship between foundation knowledge and problem-solving is worth exploring in greater detail, as it is a key difference between top-performing nations and those that struggle. Basic facts and processes were emphasised in past Western curriculums, however recent decades have seen the role of basic facts and processes somewhat deemphasised. Muller (2000) points out that for several decades there has been a struggle for a new mathematics curriculum and for the most part (in the West) it has been dominated by a strong constructivist alliance. The constructivist alliance and associated reform movements challenged the emphasis on esoteric mathematics, building from the basics up, and this continues to be the case, particularly in how mathematics is developed. This reduced emphasis on basic facts and processes became a factor identified by Menkens (2013) as a major concern among Queensland teachers.

Part of the reason for the reduced emphasis on basic skills and manual computation is that computers and calculators can very quickly carry out procedural steps, a fact recognised by the UK’s Cockcroft Report (1982) and the Australian Association of Mathematics Teachers (AAMT, 1996 & 2014), and has been echoed by the National Council of Teachers of Mathematics (NCTM, 2014). A further reason to deemphasise basic facts and processes is that they were considered menial tasks, and spending time on them took time away from developing deeper understanding (Torbeyns, Verschaffel & Ghesquiere, 2005), and contributed to cognitive passivity (Ruthven, 2001) as well as having the potential to disconnect students from understanding (Marshall, 2003). Hiebert (2003) sums up the

Western curriculum reform movement motivations and suggests that we consider ‘alternative instructional programs designed with more ambitious learning goals in mind’ (Hiebert, 2003, p. 18). Ambitious learning goals include an emphasis on studying mathematics that is relevant to student lives (e.g. Nardi & Steward, 2003; Sealey & Noyes, 2010) and demands deeper understandings of the application of mathematical content (AAS, 2015; NCTM, 2014). AAS (2015, p. 4) stated the goals as:

> We must aim to develop mathematical capabilities that are perceived by learners as powerful and genuinely useful in the present and future, through learning experiences students generally find engaging and that offer opportunities for exploration, explanation and creativity.

In tandem with this desire for depth and authenticity, the AAS (2015) expressed concern that problem-solving, the development of reasoning and capacity to tackle problems of complexity were a matter of concern in Australian curriculum development. In effect, there is a gap here. We want deep learning and creativity, but many do not see the link between these and fluency with basic facts and processes. A lack of basic skills and processes has been seen to be a source of failure, especially in those who view mathematics as essentially vertical and esoteric.

Basic facts and processes are the building blocks of mathematical processes. Cognitive load theorists explain the critical importance of basic facts and processes, in terms of the brain’s capability, is to use long-term memory as a reservoir that frees up short-term memory to solve problems (Kirschner, Sweller & Clark, 2006; Owen & Sweller, 1989). Good problem solvers are those with an extensive range of mathematical facts and techniques and can recognise a broad range of problem structures. This information is stored in long-term memory. When faced with a new problem, good problem solvers can draw on this information and apply it to the new problem. Subsequently, short-term memory is freed up to focus on the peculiarities of the new problem. In contrast, students lacking such a reservoir of information in long-term memory find themselves overwhelmed by the multiple steps that short-term memory is trying to process. Hattie (2009) used a similar argument describing the conditions for developing higher order understandings from basic fact knowledge. OECD (2014) supports this relationship and notes that large proportions of Australian 15-year-olds lack basic problem-solving skills and this is related to understanding prerequisite knowledge.

Queensland student's basic knowledge

NAPLAN[2] testing gives us ample evidence that very significant portions of our secondary students have great difficulty with basic computation and knowledge. If we go back to Year 5 and 7, we can see fundamental errors in upper primary and lower secondary school. In 2014, approximately 30% of Year 5 students could not do a subtraction problem with renaming:

> $145 takeaway $79

The Year 7 fail rate was 20% for the same question (QCAA, 2014b). This is Year 3 standard of whole number computation. What we see here in this little bit of data (and more is available) is the spiral curriculum unsuccessfully at work. Fundamental misconceptions identified in Year 5 were improved minimally by Year 7 (QCAA, 2015).

The situation is much more graphic with respect to fraction operations. Year 7 success is on par with guessing probability for the operation:

> Find half way between $\frac{1}{3}$ and $\frac{3}{5}$

This task requires converting the fractions to common denominators and counting to the middle, or taking the average of $\frac{5}{15}$ and $\frac{9}{15}$. Since it was multiple-choice format, students could substitute rather than work out the solution.

Furthermore, 2015 data revealed the Year 9 NAPLAN national success rate for the following question was 71% (QCAA, 2015):

> Round 46.718 to 2 decimal places

The success rate for the following simple algebra problem was 66%:

> Jane has $5 more than Ben. Jane correctly writes this fact using *j* for her money and *b* for Ben's. Which of these could be Jane's equation?

Students could substitute and give the correct option among four given equations.
A further illustration for the multiple-choice question:

> Simplify: $\frac{2^3 \times 5^2 \times 3^4}{3 \times 3^3 \times 5 \times 2^2}$

The Year 9 Queensland success rate was 26%, which is about guessing probability for this multiple-choice question.

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[2] National Assessment Program — Literacy and Numeracy (ACARA, 2013)

A 2015 Year 9 basic algebraic manipulation question was:

> Calculate the value of x in: $\frac{x}{2} = \frac{3}{11}$

The Queensland success rate given multiple-choice options was 16%.

Multiply both sides by 2, and x is $\frac{6}{11}$**.** This is a one-step problem and is simple procedure. Such results indicate there is a great deal of learning to take place in order to prepare students to study senior mathematics in the remaining year and a half. By coincidence, this is exactly the percentage of students who undertake advanced mathematics in senior years (Mathematics B).

Such lack of basic facts and processes is considered a critical failing of the US school system (Klein, 2005). Generally, the trend is for Queensland average success rates to be a few percentage points behind the top states such ACT, NSW and Victoria. Masters (2009) used Year 7 2008 NAPLAN data and Year 8 2007 TIMSS data to indicate that Queensland trailed NSW, Victoria and ACT. In addition, on the Year 9 2008 NAPLAN, Queensland trailed WA and SA as well. On the PISA for 15-year-old students, Queensland trailed SA, NSW, WA and Victoria. Masters (2009) noted that there had been an absolute decline in numeracy levels in the prior several years in government schools. Recent reforms may have likely mitigated that deficit to some degree. What is concerning is the extent of knowledge across the nation in comparison to top-performing nations and the impact that this lack of fluency with basic facts and processes has upon students' subsequent subject selection.

The recently released Australian Curriculum: Mathematics (ACARA, 2012, p. 6) recognises the critical nature of basic knowledge by allocating an entire proficiency strand, *fluency,* to describe how content is explored and developed:

> Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly use facts, and when they can manipulate expressions and equations to find solutions.

The forms of data above indicate very serious challenges for students wishing to progress to the study of more advanced mathematics and have implications for curriculum structure in primary, middle and senior schools. Long-term consequences of these low success rates in Queensland leads to/creates a deficit of students who are eligible to undertake higher levels of senior mathematics study and subsequently advanced university study said to underpin the knowledge economy.

Limited data on Senior Mathematics standards

Limited data comparing senior mathematics students across Australian states exists. Arnold and Sidhu (in press) examined the scores of students from all states who undertook engineering at the University of NSW between 2007 and 2014. The study was a statistical analysis and all students had to have completed Year 12 in their state in the previous year as well as have an ATAR (Australian Tertiary Admissions Rank) score. Queensland students made up 35% of the cohort and NSW made up 25%, with the remainder coming from Victoria, SA, ACT, Tasmania and WA. Table 1 summarises their data.

Table 1: Statistical analysis of mathematics scores at UNSW engineering course (Arnold and Sidhu, in press, p. 6)

State or territory	Average mark	Standard deviation
ACT	64.0	17.8
NSW	66.5	17.3
Queensland	57.0	16.9
SA & NT	71.1	14.3
Tasmania	69.3	14.8
Victoria	68.8	15.3
WA	66.6	20.3

ANOVA (the analysis of variance) statistics revealed that only Queensland students had statistically different scores at the 5% level. Similar results were replicated in physics. Queensland students in this course struggled and tended to occupy the lower quartile. Arnold and Sidhu noted that the Queensland students did not enter with a lower ATAR score, so it did not seem that the difference could be attributed to UNSW attracting the less able of the Queensland eligible students. Nor was the factor related to age, and the authors noted that prior to 1996 such differences in performance were not evident. The authors noted that the content/test was similar to tests and courses other Group of Eight universities used. It might be argued that the Queensland students may improve later in the course, but the point is they seem to have started behind the other states on key mathematics competency. Clearly, it would be useful to gather wider data comparing the mathematical readiness of our Year 12 students to undertake rigorous tertiary study.

In summary, the emerging Australian mathematics curriculum documents recognise the importance of fluency with basic skills and processes for problem-solving and they affirm an essentially vertical view of mathematics. However, in practice, fluency is not occurring to near the extent we would like.

Spiral or mastery curriculum

Other than the limitations associated with how the curriculum has been implemented, a further hypothesis to at least partly explain the lack of fluency is the perception of a crowded curriculum. It seems crowded because there is so much mathematics to do in such little time. An alternative hypothesis is that ineffective re-teaching is occurring. Evidence of ineffective re-teaching can be seen in the Year 11 and 12 Prevocational and Mathematics A syllabuses. Concepts that form the conceptual basis of lower level senior syllabuses are generally at a level of abstraction appropriate for junior secondary and, in some instances, primary school in top-performing nations such as Singapore and Hong Kong. Syllabus documents for Mathematics A and Prevocational Mathematics are largely based on repeating middle year concepts (whole number computation, fractions, rate, ratio, simple trigonometry and linear algebra in contextual settings) and may be spiral or taught in discrete blocks related to themes.

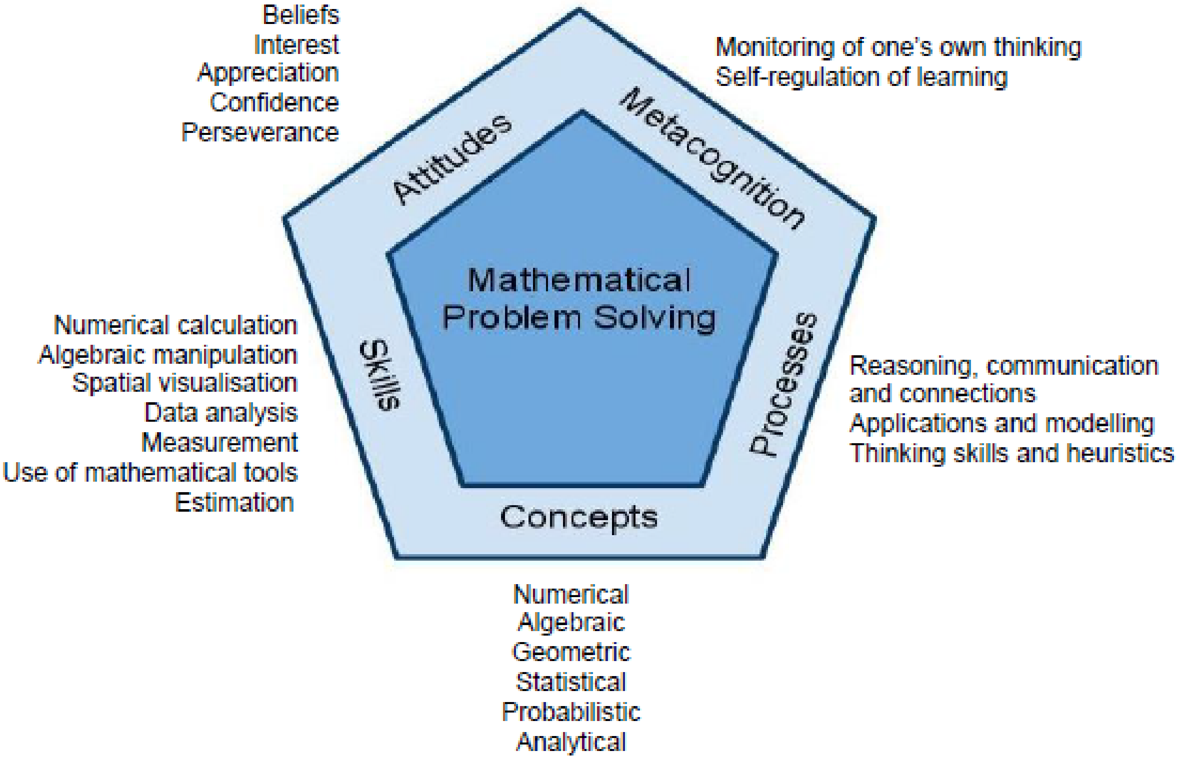
The enactment of middle school and senior B and C mathematics syllabuses is essentially spiral as noted in the Mathematics B syllabus, ‘a spiralling and integrated sequence should be developed’ (QCAA, 2014a, p. 8). Further, the choice of how the spiral is constructed is a matter of school choice: ‘The order in which the topics and items are given do not imply a teaching sequence’ (QCAA, 2014a, p. 12). Top-performing nations do not have the same challenges with lack of remediation of basic knowledge and not all have spiral curriculum.

Singapore and Hong Kong do not have spiral curriculums to nearly the same extent as observed in Australia. Mastery is demanded in syllabus documents and is enacted in textbooks (Ginsburg, Leinwand, Anstrom & Pollock, 2005). In their comparison of Singapore’s mathematics syllabus with the US syllabus, these authors noted that each year Singapore students did fewer topics and spent more time on each topic area. An example in-depth learning is seen in the Singapore text *Additional Mathematics* (Seng & Yee, 2007) used in Year 10/Year 11. There is a chapter on differentiation (25 pages) followed by a chapter on differentiation and its applications (22 pages) followed by further applications of differentiation (16 pages), followed by differentiation of trigonometrical functions (17 pages). Students study differentiation as an extended block. As a further example, there are 40 pages of teaching exercises involving indices, surds and logs, and they are not revisited until the final high stakes examination. Students are expected to remember or revise. Extended focus on related concepts affords the opportunity to develop mastery. In addition, the syllabus documents explicitly set out this intention and teachers are encouraged not to progress students until mastery has been demonstrated. Further, the necessity to gain grades through external assessment acts as an extrinsic motivating force to pressure students to attain mastery. Hattie (2009) was strong in his support of mastering material before moving onto new learning.

Top systems are explicit and connected and maintain expectations

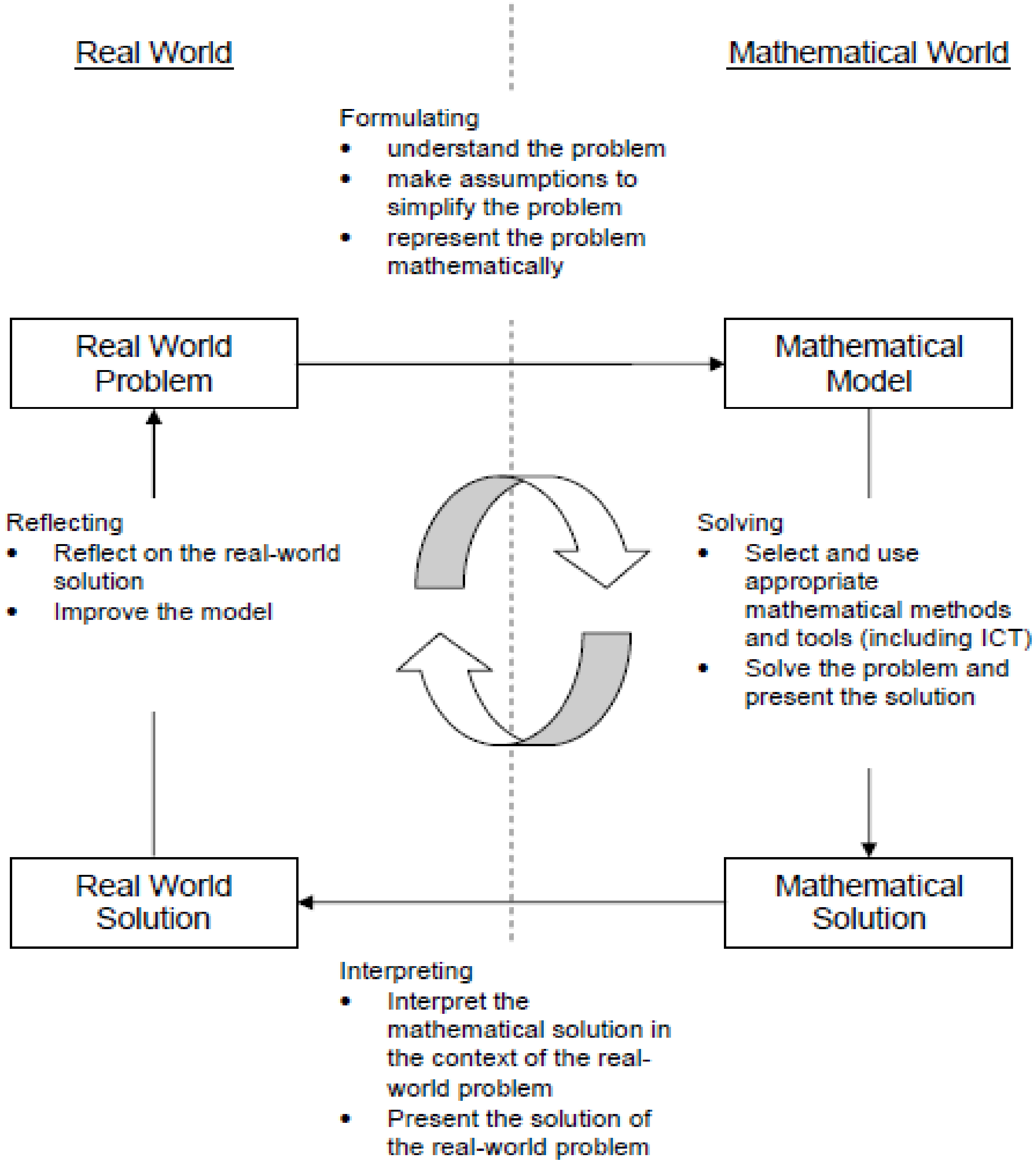
A further difference between what is expected of students and teachers is the level of specificity in syllabus documents with respect to *processes*. For example, the Singapore problem-solving model shown in Figure 4 below.

Figure 4: Singapore problem-solving model (Singapore Ministry of Education, 2012, p. 16)



Such a model considers the integration of affect, skills, mathematical concepts, reasoning and metacognition. This is a very systematic way of thinking about what is necessary to succeed in problem-solving. Each of the key terms, *skills*, *concepts*, *processes*, *attitudes* and *metacognition* have explicit meanings that are related to mathematical problem-solving. The identification of skills as a distinct component places a high priority on computation and mental arithmetic. This explicit and connected approach to teaching mathematical problem-solving is further elaborated in the Singapore processes of problem-solving model in Figure 5 below.

Figure 5: Singapore details of mathematical problem-solving cycle model (Singapore Ministry of Education, 2012, p. 18)



The interesting point in these models is the use of language, especially the word *mathematical*. The key elements of 21st century skills such as *reason*, *interpret problems*, *reflect*, and *persist* are present in these models and developed in mathematical contexts.

Use of technology

The failure of so many Australian, including Queensland students on basic tasks raises the question as to how this has come about. In seeking to answer this question, we can begin by looking at the forms of technology that make such skills obsolete.

Calculator use

The technology form that has been most commonly used in classrooms at all levels is the calculator. Of particular concern is the over-reliance on calculating devices before key number facts and processes have been learned. The *Statement on the use of calculators and computers for mathematics in Australian schools* (AAMT, 1996, p. 1) recommended that:

> all students have ready access to appropriate technology as a means to support and extend their mathematics learning; priority be given to the use of calculators and computers as natural media for mathematics learning within a technologically-rich learning environment

For early childhood learning, AAMT (1996, p. 4) noted:

> access to a basic calculator is required of ALL students as a most significant means for introducing and developing number sense

A major reason for the wide use of calculators was that it would improve attitudes, in particular student confidence (AAMT, 1996). The senior syllabus documents in Queensland, including Mathematics B and C (QCAA, 2014a), write the use of technology up as an end in itself. It is normal practice for all high school students to have calculators at hand in all mathematics lessons, and since all assessment is school-based in Queensland, there is no extrinsic motivation not to rely extensively on technology in assessment. There are few guidelines for the use of technology other than 'use it a lot'. Klein (2005) considers that the overuse of calculators is 'one of the most debilitating trends in the current state of math standards …' (p. 10). The main reason for this is that it relieves the student from the necessity of dealing with numerical calculations. This is perfectly legitimate in many circumstances, such as calculating lots of numbers or plotting functions and their derivatives. Computers are especially powerful in quickly displaying functions and carrying out complex calculations, but the overuse of technology such as calculators before fluency with basic facts and processes can be problematic.

Other information and communication technologies (ICTs)

There are multiple forms of technology that can be used to replicate traditional classroom practice, such as the *KhanAcademy.org* (2006). In contrast, *ComputerBasedMath.org* (2016) enables complex questions to be explored in a variety of contexts. Spreadsheet programs such as Microsoft Excel are very powerful. Spreadsheet competency is very useful in industry, especially those tasks related to finance and business, but how extensively and effectively this program is used in Queensland is unknown. There are many powerful function plotters including *MathsHelp Plus* (Vaughan, 1996) that plot functions, their derivatives and second derivatives as well as carry out a multitude of calculations. Further, there is considerable scope to blend the use of electronic technologies with simpler forms of expressing mathematical ideas. For example, if students begin to plot sine functions by representing a spot on the perimeter of a rotating wheel and constructing a table of ordered pairs as the wheel rotates, they are more likely to understand the links, compared to observing patterns manipulated on a computer screen (Barnes, 1991). AAMT (2015) encourages pedagogies that utilise the power of technologies including graphing calculators, online games, spreadsheets, geometry packages, statistical software and computer algebra systems to assist students to make the bridges between real-life modelling, symbolic, and visual representations of mathematical phenomenon. However inappropriate use of technology, especially overuse and premature use of calculators, has the potential to limit students' understanding of mathematics. Hattie (2009) reports a low positive effect for the use of calculators to reduce cognitive load and to support well-designed pedagogy, especially for computational use when checking work. AAMT (2015) notes teachers simply do not have the time and support to develop pedagogies to use the technologies.

The Queensland syllabus recognises the potential danger of overuse of technology. For example, the Mathematics B syllabus (QCAA, 2014a, p. 9) states:

> Complete dependence on calculator and computer technologies at the expense of students demonstrating algebraic facility may not satisfy the syllabus requirements.

The Australian Curriculum: Mathematics (ACARA, 2012) has as similar caution with respect to the use of technology. Menkens (2013) reported to some Queensland academics teaching tertiary mathematics that even Mathematics B graduates lacked basic algebraic skills. The author's testing of primary and secondary mathematics education in pre-service teachers confirm this to be the case (Norton, 2011, 2012). Overuse of technology is less the case in Victoria and NSW, where external examinations for more advanced mathematics subjects have sections where calculators are not permitted. In Victoria, for example, the Mathematics Methods and Specialist Mathematics each have a two-hour technology-active examination and a one-hour technology-free external examination. The use of technology is encouraged and recognised for its potential to enhance meaning, but the extent of use of technology such as calculators in lower and middle schools is much less, and at senior levels students are tested at least in part without calculators available. A major difference between Australia and high performing nations such as Singapore and Hong Kong is that the latter have a very strong foundation on basic skills and processes before students are introduced to time-saving technology.

Digital classrooms

An emerging field of study is the use of technology in transforming classroom discourse through the use of tools such as laptops, smart phones and computers as learning support (Chan, 2010). The main attribute capitalised upon is the use of technology to connect learners in academic discourse. The connection is with other learners and with learning resources (Chan, 2010; Wong & Looi, 2011). The technology enables a knowledgeable teacher to enact constructivist learning principles whereby students critically process and assimilate mathematics. Students may be doing so with students in different nations and with students and mentors of varying expertise. Such a learning environment helps students to develop aspects of 21st century skills and dispositions as well as potentially helping with mathematical understanding and problem-solving. However, the same warnings that apply to overuse of calculators can be applied to other technologies when they impede or interfere with students' understanding and ability to carry out basic mathematical procedures and recall facts and then apply these in increasingly sophisticated problem settings.

The perceived utility of mathematics and student engagement

Australian senior syllabuses, including Queensland's, emphasise the importance of mathematics being seen as relevant to the lives of students. QCAA's Mathematics B and C senior syllabuses state:

> Students must have the opportunity to recognise the usefulness of mathematics through its application, and the power of mathematics that comes from the capacity to abstract and generalise. Thus students' learning experiences and assessment programs must include mathematical tasks that demonstrate a balance across the range from life-related to pure abstraction. (QCAA, 2014, Mathematics B: p. 7, Mathematics C: pp. 5–6)

This is a balanced statement reflecting the utility of mathematics. It is situated at the end of 10 years of mathematics study where the utility of mathematics has been considered an integral part of justification for the study of mathematics. A relatively recent expression of *life-related* is the term *numeracy* as described by the *Melbourne Declaration of Educational Goals for Young Australians* (MCEETYA, 2008). Numeracy is described as encompassing knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations, particularly life-related contextual situations. The intent of this connection to the lives of students was to make it relevant and thus interesting. There is good reason for this attachment. Student valuing of mathematics was found to be highly correlated to achievement in mathematics across nations (OECD, 2004). By Year 8, about 50% of boys and 40% of girls reported valuing mathematics. OECD (2014) noted that the countries that had the highest mathematics scores were the same as those where students were engaged and had self-belief in their abilities.

In 1999, the Queensland Government launched the New Basics project. Grauf, (2001, p. 1) reported:

> The New Basics is a framework for curriculum, pedagogy and assessment that provides opportunities for students to develop the skills and knowledges to survive and flourish in changing economic, social and technological conditions.

The New Basics project had a transdisciplinary focus, instrumentalist orientation (Page, 2003). It is an example of an attempt to apply a mundane curriculum. Students would learn the new basics by engaging with rich tasks and supported by productive pedagogies. It was essentially an experiment in applying a mundane curriculum using constructivist pedagogies. Cooper's study related to Queensland's New Basics project noted that emphasis on relevance was in part due to teachers attempt to capitalise upon intrinsic motivational factors (Cooper, 2007). Ainley (2004) reported that there was no difference between trial schools and comparative schools on International Schools Assessments. Most students from both trial and non-trial schools received a 'below pass' grade on World Class Tests in problem-solving for 9-year-old and 13-year-old students. Ainley reported higher levels of school satisfaction for primary students in trial schools, but this difference was not evident for secondary schools. The lack of clear evidence supporting a strong gain in student outcomes (Ainley, 2004; Cooper, 2007) saw New Basics quietly dropped.

The assumption underpinning New Basics is similar to that underpinning other mundane curriculum. If mathematics is authentic, it ought to appeal to students since they may see the relevance to their current and anticipated lives. On the other hand, some suggest the relationship between attitude towards mathematics and achievement in mathematics is not strong and 'has no meaningful practical application' (Ma & Kishor, 1997, p. 39). These authors do however concede that at the elementary (primary school) level, the relationship, while not strong, may have practical implications. Ma and Kishor cautioned that measuring student attitude is particularly imprecise for younger students.

Appeal to the authenticity of mathematics as a motivating factor finds strongest expression in the least difficult and least abstract versions of mathematics syllabuses, such as Mathematics A and Prevocational Mathematics. Advanced calculus involves long and abstract manipulation of symbols and extensive use of abstract rules and thus, authenticity sees less emphasis. Complex integration, for example, might be used to find the volume of a rotated solid, but few students would sincerely believe that they would ever wish to carry out such calculations in their future lives. Similarly, only a rare student could envision using binomial expansion to calculate the theoretical odds of an event occurring. Thus, students who study the more advanced mathematics tend to do so whether it is relevant to their daily lives or not. Students studying advanced mathematics contextualise their learning in terms of their future. Generally, they undertake advanced study of mathematics for extrinsic motivational factors such as gaining entry to tertiary courses. In the process of studying advanced mathematics, many students across all nations actually find it quite satisfying.
A different situation is in existence for students undertaking the lower levels of mathematics such as Mathematics A and Prevocational Mathematics because classroom engagement can be a challenge. Perhaps one of the reasons so many students consider mathematics boring or not useful is that they cannot do it and this diminishes their expectations.

Maths is boring because I do not understand it

The strong association between intrinsic (fun/boring) and instrumental (authentic/not relevant) motivations has been discussed (OECD, 2004; Sealey & Noyes, 2010). The feedback from ‘behaviour’ to ‘boring’ and ‘understanding’ and ‘relevant’ is two-way, since if the class or individual is off task, understanding is likely to suffer, and with failure comes ‘no fun’ and therefore, ‘maths is boring’. It is well reported that dissatisfaction with the instructional discourse of schools has manifested itself in disruptive behaviours as well as low engagement associated with ‘resigned acceptance’ (Nardi & Steward, 2003, p. 346) and overt disruptive behaviour. It may be that by being able to label engagement in traditional mathematics as boring, some students were able to avoid a conflict in identity, a sentiment supported by Hannula (2002). That is, if the work could be labelled as boring, it was the subject matter that was at fault and the student need not be so confronted by their inability to understand it, or indeed to attempt to understand the mathematics. This two-way relationship between success and attitude has been well reported (e.g. Attard, 2013, Haladyna, Shaughnessy & Shaughnessy, 1983, Hannula, 2002, OECD, 2004, OECD, 2014; Pintrich, Marx & Boyle, 1993) and needs to be taken into account in curriculum design. A further consideration is that many senior students may perceive that higher levels of mathematics are not needed for their future studies and careers. The relatively low levels of enrolment in Mathematics B and C suggest this may be the case.

The problem for Australia and Queensland in particular, is that the strategy of relying upon intrinsic motivation to be found in authentic learning has not worked to engage the majority of students. Thus we find a relatively small portion of OP eligible students are enrolled in abstract mathematics such as Mathematics B. Critics might suggest that is a product of an entire educational system that strives to make mathematics authentic and engaging from prep onwards.

Top-performing nations such as Singapore and Hong Kong have much less reliance on intrinsic motivational factors. External examinations, desire to attain entry to advanced study and subsequently economic success are strong extrinsic factors that are sufficient to motivate most students. In contrast, TIMSS data indicate that 20% of Australian Year 8 students aspire to a Postgraduate degree compared to 29% as the international average (Thomson et al., 2012). Similarly, 14% of Australian Year 8 students wished to undertake university study but do not wish to continue to postgraduate study, compared to 27% for the international average. In contrast, 30% of the Australian sample considered post-secondary but not university. The international average for this statistic was 14%. It seems that most Australian children do not have extrinsic motivations to study mathematics, and as they progress through school, intrinsic motivation wanes, especially for the more applied levels of mathematics.

The role of the teacher

The research companion to the *Principles and Standards for School Mathematics* (Hiebert, 2003) described *traditional* teaching of mathematics as being dominated by teacher-directed explanation and questioning, followed by practice of similarly structured problems on pencil-and-paper assignments with an emphasis on procedures, especially computational procedures. Gregg (1995) notes that this discourse is resilient; he calls it the *school mathematics tradition*. This approach was criticised for being too passive, poor in developing conceptual ideas, and doing little creative work such as inventing procedures or analysing new problems, hence having little process relevance. Further, traditional teaching was seen as having the potential to alienate substantial proportions of students (Attard, 2013; Nardi & Steward, 2003; Noyes, 2012; Sealey & Noyes, 2010). The traditional approach was viewed as *boring* by many students and lacking relevance to their lives. Sealey and Noyes noted that different school communities were likely to have different views of what constituted relevance, with immediate practical relevance being less important for students likely to value transferrable process skills or entry to specific professions.

Western pedagogy reform

In the West, including Australia, and especially in Queensland, pedagogy is strongly influenced by social constructivist thinking. From the late 1980s, there was a change in advice given to teachers about how students were to learn. For example, teachers were advised to consider ‘alternative instructional programs designed with more ambitious learning goals in mind’ (Hiebert, 2003, p. 18). In essence, teachers were encouraged to teach for understanding, and the consensus was that drawing on social-cultural theory offered the best hope for reform. As noted above, a key attribute of the reform movement was that mathematics ought to have immediate relevance to the lives of students (e.g. Nardi & Steward, 2003; Sealey & Noyes 2010; Sfard, 2003). A further attribute of the reform approach to mathematics and its teaching was the empowerment of students in classrooms. Motivation for this drive may derive in part from the weighting of Vygotsky’s (1987) thinking on the importance of the social nature of learning, which held that conceptual understanding is a product of communication using language. Sfard (2003) argued that recognition of this principle encouraged the wider acceptance of cooperative learning and a shift from teacher-centred communication to more student articulation of their thinking and increased student-to-student discourse. Hattie (2009, p. 26) noted:

> Constructivism too often is seen in terms of student-centred inquiry learning, problem-based learning, and task-based learning, and common jargon words include ‘authentic’, ‘discovery’ and ‘intrinsically motivated learning’. The role of the constructivist teacher is claimed to be more of facilitation to provide opportunities for individual students to acquire knowledge …

He suggests, ‘These kinds of statements are almost directly opposite to the successful recipe for teaching and learning’ (p. 26). Hattie noted that sometimes deeper concepts needed more specific and direct teaching while more surface concepts might be learnt via inquiry or problem-solving. Hattie distinguished between problem-based teaching, where the teacher directed class discourse and problem-based learning that was more student guided. The first he considered effective and the latter having potentially negative effects.

Unlike early Queensland mathematics curriculums (e.g. New Basics), the Queensland senior syllabuses (QCAA, 2014a) do not make recommendations on pedagogy. It is the responsibility of the school to determine the nature of academic discourse that occurs in senior classrooms.

Eastern pedagogy reform

The literature reviewed on teaching in East Asia, including Hong Kong and Singapore, indicate that it is highly teacher centred. Zhang, Li, and Tang (2004) discuss the deferred teaching of problem-solving in East Asia by examining *two basics*. *Two basics* refers to the expectation that students learn basic skills and basic knowledge as essential parts of learning to problem solve (Lai & Murray, 2012; Gu, Huang & Marton, 2004). Asian students, including those from Singapore and Hong Kong, are expected to understand the principles and logic underpinning rules, and this is achieved through tailored pedagogy that has been described as teaching with variation (Lai & Murray, 2012; Gu, Huang & Marton, 2004) and a 'learning-questioning and learning-review instructional model' (An, 2004). Wang and Murphy (2004, p. 112), with reference to Chinese schools, describe this as follows:

> The teacher uses language to connect the well-structured activities explicitly. In this way, the student can easily organise the knowledge coherently.

Hattie (2009) describes such an approach as *visible learning*, where students are provided with clear definitions of learning tasks, students are required to master class activities, and teachers take on the role of explicit directors of learning. The product of understanding may come sometime after rules have been memorised and used in different contexts. Once understood, rules could be applied and practised until students were quick and accurate in the processes (Li, 2004).

The idea of being quick and accurate is reinforced by the Chinese tradition of high-stakes examinations that have existed possibly since the 6th century AD (Zhang et al., 2004; Huang & Leung, 2004) and demand high memory recall, precision and speed. Wang and Murphy (2004) stressed the central role of the teacher is to construct well-structured activities explicitly. In this way, the student can easily organise the knowledge coherently. The Hong Kong *Mathematic Curriculum and Assessment Guide* (Hong Kong Examinations and Assessment Authority & Curriculum Development Council, 2007, p. 107) describes *teaching as direct instruction*:

> … a very frequently used approach in the Mathematics classroom, can make a positive contribution to the learning of mathematics if it is dynamic, and well planned and organised.

Teachers are encouraged to use a wide range of pedagogies including *teaching as inquiry*:

> … where the emphasis is on the process and action undertaken by the learner … [with] extensive dialogue among students … [and] …there should sufficient 'wait time' so that students can explain their thinking. (p. 109).

In other words, the teacher is expected to manage the discourse, but give students opportunities to construct and articulate meaning. A third pedagogy suggested is *teaching as co-construction*:

> The teacher plays a central role in developing a problem-solving environment in which students feel free to talk about mathematics. (p. 110).

The paramount role of teachers in structuring academic discourse in detail is supported by the Education Consumers Foundation (2011) in their analysis of direct instruction. These descriptions of teacher's roles with respect to pedagogy are supported by an examination of Hong Kong and Singapore texts books. They tend to focus on reasoning and logic behind problem solutions to a greater degree than commonly used compared with Queensland textbooks, especially in the primary and middle school levels.

What we see in the curriculum documents and textbooks is that reforms in pedagogy in the top-performing nations have included a move to embrace constructivist learning principles, with a focus on understanding and life-related situations, but was done so in the context of strong basic skills (Hong Kong Examinations and Assessment Authority & Curriculum Development Council, 2007; Quong, 2011). However, they do so in classrooms where the teacher tightly orders academic discourse.

The Hong Kong and Singapore educational success combines a history of strong culture of examinations-focused learning with a move away from rote learning towards learning with understanding. The success of Hong Kong and Singapore students in international tests needs to be seen in the context of a highly competitive academic environment and high respect for learning based on Confucian values (e.g. An, 2004; Lafayete De Mente, 2009; Lee, 1996; Huang & Leung, 2004; Jensen, 2012). There is a further advantage in terms of academic learning time, with very high percentages of students (in the order of 80%) studying at cram schools or after-school tutor programs (e.g. Jensen, 2012; Haung, 2004; Zhao & Singh, 2010).

Assessment forms shape pedagogy

The nature of assessment will clearly have a considerable influence on classroom activity. While there is some variation, most Queensland schools enacting senior syllabuses have a blend of inquiry-based take-home assignments and written tests based on one or two units of work. Other Australian states including NSW and Victoria use school-designed assignment based assessments for the same reasons as Queensland does. Assignments offer the student an opportunity to express his or her knowledge in a variety of ways as well as providing a tool to engender intrinsic motivation. Menkens (2013) documented concern among teachers that the length of some take-home assignments meant too much time was spent learning too little at the expense of critical mathematics knowledge. While teachers may have made such comments in their submissions to the Education and Innovation Committee, there has been little published research into the extent of the problem.

The main point of difference between Queensland, NSW and Victoria is that the other states have subject-specific external examinations. The same can be said of Hong Kong and Singapore. In fact, external examinations dominate East Asian assessment processes. For example, The Hong Kong Examinations and Assessment Authority administer over 200 examinations leading to academic, professional or practical qualifications (Education Commission Working Group on the Development of Educational Services in Hong Kong, 2011). The external examinations are summative and sample all of the work studied to date, not just the past term as in the case of Queensland school-based tests. The potential problem in testing only the past few months' work is that students have less incentive to revise past semesters and thus there is less attempt to put key concepts and processes into long-term memory. Hattie (2009) considered examinations that are well aligned with the syllabus and also explicit in the desired goals important contributors to visible learning.

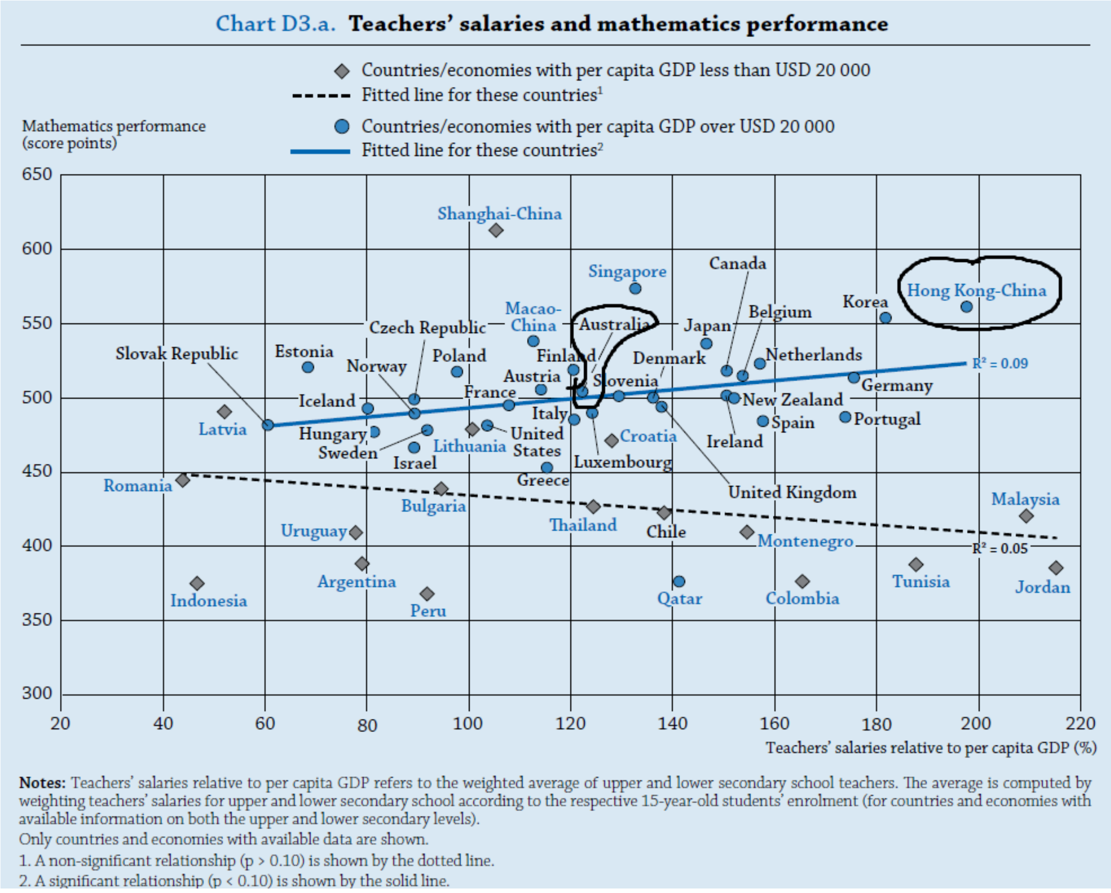
The form and use of external assessment shapes the Singapore and Hong Kong classroom discourse. Since schools are held accountable for knowledge gains, and students understand the importance of high grades for future study and careers, the tests are definably high stakes, and occur from primary school onwards. A feature of the tests is the expectation that students show working and extended answers including justification to increasingly complex multistep problems (this contrasts to the NAPLAN multiple-choice format or single solution responses). Ginsburg et al. (2005) noted that constructed-response questions are generally more suitable for demonstrating students' higher level cognitive processes in mathematics (p. xiii). Showing working and explaining solutions encourages teachers to teach modelling and problem-solving from an early age. In achieving this, Asian classrooms in which problem structures are unpacked, the discourse in classrooms are quite teacher-centred or, more accurately, teacher-directed. This means the teacher must have a very deep knowledge of the subject material in order to scaffold student learning. It can be argued that NAPLAN numeracy-style examinations that are dominated by multiple-choice formats (about 85% of questions) encourage guessing or working backwards rather than solving the problem.

Pedagogy and teacher knowledge

The data available reflecting the level of primary Queensland graduate teachers' depth of knowledge of mathematics is not encouraging and this impacts strongly on junior high and subsequently senior high school achievement (Norton, 2011, 2012). Ginsburg et al. (2005) noted that Singapore primary teachers are required to demonstrate mathematical skills superior to their Unites States counterparts and at every phase of pre- and post-service training, they receive better instruction in both mathematics content and in mathematics pedagogy. Further, in Queensland the use of out-of-field teachers in secondary mathematics has been previously reported as a matter of concern (Queensland Audit Office, 2013; Vale, 2010). Undertrained teachers are likely contributing to lack of student performance no matter what the syllabus intends. Attempting to reform mathematics teaching and learning by attending to the curriculum issues alone is tenuous, as knowledgeable teachers are necessary to deliver any syllabus effectively.

The differences in teacher quality can clearly be seen when examining the differences in mathematics achievement attained. East Asian teachers are highly qualified in terms of discipline knowledge, with Ma (1999) and Li et al. (2008) suggesting that strong basic content knowledge has been the foundation of quality mathematics teaching in China in recent decades, and Jensen (2012) suggesting the same factors are relevant more broadly across East Asia. In international comparisons of teacher knowledge, the general pattern is East Asian teachers are more mathematically competent (e.g. Burghes, 2011; Ginsburg et al., 2005). The teaching profession in Singapore and Hong Kong attracts top academic candidates with teacher salaries being more relative to the earnings of tertiary-educated workers (OECD, 2014). In Australia, teacher salaries are less than the average. Teacher salaries represent the largest single cost in formal education and have a direct impact on the attractiveness of the teaching profession. Figure 6 below clearly illustrates the relationship between salary and mathematics performance of students (OECD, 2014a, p. 457).

Figure 6: Relationship between teacher salaries and mathematics performance (OECD, 2014a)



Both the demonstrated mathematics performance and the salary enticement are evident in top-performing nations such as Hong Kong, Korea, Singapore, Japan, and to a lesser degree Netherlands, Belgium and Denmark. In Shanghai (China), all mathematics teachers are specialists, from primary school onwards. Jensen (2012) reports East Asian initial teacher education is of very high quality, with governments having the capacity (and inclination) to close down ineffective teacher education courses. The Teacher Education Ministerial Advisory Group (TEMAG) (2014) noted that not all initial teacher education programs are equipping graduates with the content knowledge, strategies and skills needed for classroom readiness. Further, Australian teacher training institutions have been less selective in enrolling potential teachers of demonstrated high academic capacity. The certification of teacher education programs is the responsibility of state-based bodies such as Queensland College of Teachers (QCT). The AAMT (2015) describes considerable diversity in Australian students' classroom experiences in learning mathematics and used the terms 'quite depressing picture' (p. 5) in which over-reliance on textbooks and focus on content as not preparing students effectively for the knowledge economy. Barber and Mourshed (2007) commented that top-performing systems were relentless in improving the quality of classroom instruction. In addition, boards that certify tertiary institutions to prepare teachers of mathematics at all levels in Singapore and Hong Kong are made up of stakeholders, including university mathematics departments. These people have deep knowledge of the discipline of mathematics. This is not the case in Queensland, where QCT does not have access to such expertise. In summary, poorly trained, not trained and/or over-worked teachers may find themselves unable to implement curriculum effectively — no matter what its specific form.

In summary

A number of theorists have argued that there needs to be a re-evaluation of the curriculum principles based on assumptions that mathematics ought to be taught in contextual settings and via student-centred mathematical discourses (e.g. Kotzee, 2012; Muller, 2000, 2009; Moore & Muller, 2002). These theorists might take heart in the reported high engagement and satisfaction of many East Asian students studying esoteric mathematics in teacher-directed classrooms (Norton & Zhang, 2013). This is not to say that constructivism is a failed theory. Constructivism is a key factor in the reform efforts of top-performing nations such as Hong Kong and Singapore, but this is attempted by building on strong foundational skills with knowledgeable teachers controlling the discourse (QECD, 2014). AAMT (2015) suggests that inquiry-based learning (consistent with constructivist thinking) has considerable potential to develop mathematical skills as well as orientation to problem-solving. However, effective use of the pedagogy requires very high levels of teacher capacity, especially as the mathematics gets increasingly complex, which possibly is a major reason why many Australian teachers rely so extensively on traditional textbooks. The use of traditional textbooks enables the teacher to abdicate much of the control of classroom discourse. The key factors that stand out in high performing nations are similar to those identified by Hattie (2009) and were summed up earlier by McGaw (2006, p. 20) as:

- focus on learning not teaching — being more explicit about standards expected of students; focusing on what they should know and are able to do; more deliberate and rigorous data monitoring in many systems (quality examinations and assessment)
- purposes of schooling — meeting the demands of knowledge economy; not losing sight of personal and social outcomes
- getting the balance right: expertise — which is domain specific; generic competencies.

At this point in time, there is evidence that many Queensland students enter senior mathematics without the cognitive skills to engage with abstract mathematics. Very little research has been undertaken to determine how this has impacted on the enactment of Mathematics B and C syllabus documents or how to remediate this situation. What evidence has been presented is not encouraging (Arnold & Sidhu, in press). Anecdotal evidence from teachers and academics cited by Menkens (2013), particularly from classroom teachers, is also not encouraging. There is a perception in the community of senior teachers that the curriculum system is not working as intended.

Analysis of the Queensland curriculum documents, considering the general literature on the teaching and learning of mathematics, reveals different balances of pedagogy and different curriculum are necessary for different cohorts of Queensland students. Those who currently enrol in Mathematics A or Prevocational Mathematics are not ready for an esoteric Singapore or Hong Kong senior syllabus. A more mundane and applied syllabus is more likely to be seen as relevant and meaningful to their lives. Hattie (2009) recognised that constructivist based teaching could be appropriate for less difficult learning. What we can learn from top-performing nations is that the role of the teacher in managing classroom discourse is critical. Queensland teachers need appropriate resources and models of assessment and pedagogy that are suitable for the diversity of clientele. They may also need additional and ongoing training, a recommendation made by Masters (2009) and described by Jensen (2012) as central to the success of East Asian nations' mathematics teaching and learning.

Chapter 2: Subjects in the group — comparison, connections and expectations

Introduction

Chapter 1 reviews the literature on major trends in mathematics teaching and learning. It used a theoretical lens focused on esoteric/mundane knowledge forms to help illuminate the different ways of viewing mathematics as well as its purposes. In this process, how curriculums were structured and presented was described. A critical factor of those embracing a vertical (hierarchical) and esoteric depiction of mathematics was a focus on basic skills as a prerequisite for problem-solving. Viewing mathematics as hierarchical was strongly associated with controlled early use of calculators, and very explicit syllabus documents that included not just what was to be taught, but also the sequence of how it was to be taught. Additionally, different nations and states used intrinsic and extrinsic motivational factors in different ways. Past Queensland syllabuses, at all levels, were characterised as being insufficiently prescriptive, most flexible and were unique in not having external subject-based examinations. At the primary school and middle year levels, the situation with regard to specificity has changed over the past few years due to the influence of the Australian Curriculum. The senior syllabuses, Mathematics A, B and C, have largely remained the same. The comparisons below are referenced from the perspective of the Queensland Mathematics A, B and C syllabuses because they are most familiar to readers.

Learning from top international competitors

There are some differences with respect to the subjects offered by the international competitors and Australia, as indicated in Table 2 below.

Table 2: International comparison of senior mathematics subjects offered

Australia	Singapore	Hong Kong	California
University studies	H3	Extended Mathematics Module 1	Advanced Placement Probability and Statistics Calculus
Specialist Mathematics[1] Mathematics C[2]	H1 and H2	Extended Mathematics Module 2	Algebra II Mathematics II *algebra component only*[3] Mathematics III *algebra component only*[3]
Mathematical Methods[1] Mathematics B[2]	N/A	Compulsory Mathematics	Algebra I Mathematics I *algebra component only*[3]
General Mathematics[1] Mathematics A[2]	N/A	N/A	N/A
Essential Mathematics[1] Prevocational Mathematics[2]	N/A	N/A	N/A
N/A[1] Functional Mathematics[2]	N/A	N/A	N/A
Key	[1] Australian Curriculum [2] Queensland [3] Equivalent to algebra components of Australian subjects only		

General mathematics courses not offered

There are two key patterns seen in the Australian and international comparison. The first is that comparison nations do not offer non-academic mathematics such as Functional Mathematics, Prevocational Mathematics or Mathematics A. Hong Kong has almost universal secondary school enrolment in the student-aged population, but there is little need for blue-collar work training since most of these jobs migrated to Mainland China. What is needed are graduates to service high value-added services such as engineers, ICT workers and business modellers (Centre on International Education Benchmarking, 2015). With this in mind, the Hong Kong government initiated reform in early 2000 and the new senior curriculum was instigated in 2009. Hong Kong and Singapore are economically and culturally similar with top end service-based economies and historical roots in Confucian thinking (An, 2004; Lafayete De Mente, 2009; Lee, 1996; Huang & Leung, 2004; Jensen, 2012).

California possesses an entirely unique system of subject structuring when compared to Australia, Hong Kong, and Singapore. Consequently, practical comparison of these are complicated. The Californian system of schooling is structured similarly to university, where students have set selections of subjects that they may choose to study at almost any point during their secondary education. What is compared Table 2 above is all of the mathematics subjects available from Year 7 to Year 12 in California. The system seems to operate under the presumption that Algebra I and II and Mathematics I, II, and III are largely junior secondary subjects, meaning that students choose if they are going to continue mathematics study in senior years. This system is likely the result of a country with an exceptionally high diversity in jobs and culture. This is significantly different to Hong Kong and Singapore's tailored schooling system that streams students into a narrow range of careers. However, though students have much greater range in the Californian senior schooling syllabus, the advanced mathematics subjects match the rigour of Hong Kong and Singapore and are equivalent to tertiary study in Australia.

The lack of any basic mathematics subjects in Hong Kong, Singapore and California is difficult to understand from an Australian perspective. Their governments have recognised that the economic viability of nations is dependent upon the quality of the workforce to innovate. In Hong Kong and Singapore, blue-collar jobs are dwindling; lower skilled jobs in Hong Kong have tended to migrate to mainland China and in the case of Singapore to Malaysia or Indonesia. The majority of children in these nations are aware that to succeed in life and to gain reasonable employment, a good education is essential (Quong, 2011). In the case of California, there is no basic mathematics offered for different reasons, and they are similar to why Germany does not offer Mathematics A type courses. Non-academic students who are not destined to pursue post-secondary university study are streamed early. In California, this occurs by student selection of subjects at the end of middle school, as it is possible to decline taking any mathematics in senior years. The only requirement for graduation is that students must complete two of the offered mathematics subjects sometime between Year 7 and 12.

Several decades ago, streaming according to intended occupations occurred in Australia, with Year 10 being the normal leaving time, and frequently the beginning of apprenticeships and trades training. This situation has changed, and as of 2012, about 60% of the population aged between 15 and 24 are in full-time study, with 75% of males and 84% of females enrolled in the equivalent of Year 12. Further, since 1988, domestic enrolment in higher education has risen from 0.4 million to 1 million, with participation rates of about 25% for 17- to19-year-olds and 14% for 20- to 29-year-olds (Norton, 2014). As of 2013, about 30% of bachelor-degree enrolments were for arts degrees and about 18% for science degrees.

Of the science degrees, about 6% were studying engineering, about 3% architecture and building, and similar amounts for information and technology. The point of these statistics is that the main function of our secondary education system has not been to prepare students for hard sciences, but to prepare the majority to undertake a trade-related or arts-related undergraduate degree, or a soft science degree that does not require high levels of abstract mathematics. Thus we have a high proportion of students — very high in Queensland — who see no real need for the study of abstract mathematics (Australian Bureau of Statistics, 2013).

The statistics of current study and employment options justify the need to offer lower levels of mathematics study in senior school. The majority of high school students' perception that the study of abstract mathematics is unnecessary has been supported by nationwide employment opportunities: healthcare 12.2%, retail 10.9%, construction 8.8%, manufacturing 8.1%, food and accommodation 6.7%, education 7.8%, public administration 6.6% with professional and scientific at 7.9% and financial services at 3.6% (Parliament of Australia, 2015). A small portion, about 12%, might expect to use abstract mathematics in future study. Further, many Australian students would reasonably assume that they could learn the necessary mathematics on the job. The figures stated above regarding student intentions justify the need for Australian mathematics curriculums to offer mathematics subjects such as General Mathematics. The topics that are studied in General Mathematics and, in the Queensland context, Mathematics A and Prevocational Mathematics, require further discourse and will change as the effects of the Australian Curriculum become apparent and Australia intensifies its focus on the knowledge economy.

Despite mathematics curriculums under-preparing students for the study of abstract mathematics, the federal government wishes to move the economy towards higher innovation and value adding. This requires higher levels of education, especially in the hard sciences and mathematics (AAS, 2015). A major driver of this is the same that drove Hong Kong and Singapore to enact educational reform. There is a political/economic need for innovation and economic prosperity via a highly educated workforce (Office of the Chief Scientist, 2013; Riegle-Crumb, King, Grodsky & Muller, 2012). Inevitably, high innovation and value adding to the knowledge economy is associated with higher levels of mathematical abstraction and higher engagement of students in associated studies of science, technology, engineering and mathematics (STEM). Thus, there is a need to enhance the portion of students studying more advanced mathematics, as well as gradually lifting the standards expected of the non-academic mathematics subjects such as Mathematics A or General Mathematics.

The level of abstraction

The further difference between Australia and top-performing systems such as Hong Kong, Singapore and California is the level of abstraction that is gained in the higher levels of mathematics such as Specialist Mathematics (Mathematics C) and Mathematical Methods (Mathematics B). The Asian students studying advanced mathematics are engaging at a level normally associated with tertiary study in Australia. As noted in the literature review, it is not just about what is in the curriculum, but how it is enacted. There are multiple variables that influence the quality of pedagogy irrespective of what the curriculum suggests should be enacted.

High stakes assessment is an examination

External examinations play a dominant role in assessment of students in Hong Kong and Singapore; they shape academic discourse from primary school onwards.

Hong Kong has a reputation for developing fair and reliable external examinations. The compulsory examination part of the mathematics strand has two papers: one 2¼-hour paper and one 1¼-hour paper. Module 1 (Calculus and statistics) has a 100% paper of 2½-hours, the same demands are replicated for Module 2 (Algebra and calculus). A student studying the compulsory component and one module can expect 6 hours of discipline-specific public examinations for mathematics alone. There are about 375 hours of dedicated tuition to prepare for this. Singapore has similar high stakes externally-set examinations throughout primary school at critical transition grades, and exit examinations at the end of secondary school.

In the past Queensland has not had a discipline-specific examination. The Matters and Masters (2014) report to the Queensland Government endorses the implementation of discipline-specific external assessment to account for 50% of most key subjects. This brings Queensland more into line with the top-performing Australian states such as NSW and Victoria. Assessment via external examination or some other form of assessment will be discussed later.

In summary, the two Asian systems end with higher expectations of all students who remain at school. In the main, students achieved higher levels of abstraction from the implementation of very systematic and detailed curriculum documents which include not just what to teach but also the sequence and how to teach it. This approach has been supported by a highly educated and trained teaching workforce with above-average income compared to gross domestic product (GDP). The culture is also different, with most students reflecting a respect for education and the instrumental motivation of desire for service adding jobs and external examinations at play.

Our schools have more flexibility

A potential negative in the East Asian approach is the lack of a mathematics subject for students who do not wish to study abstract mathematics. There is little in their syllabus structure for students who wish to have a minimum level of functional mathematics that might be appropriate for non-academic or non-science orientated career orientations. A further potential disadvantage is that there appears to be less flexibility at every level. This means less flexibility for students in terms of course and specific subject choice as well as less flexibility for teachers to undertake the teaching of electives. A teacher in a regional school in Australia, for example, may wish to teach Mathematics A or Prevocational Mathematics in the context of agricultural production. Such an approach may add authenticity to student learning and harness intrinsic motivational potential. Such a scenario is possible in the current Queensland and Australian curriculum, but would be more difficult (possibly impossible) to enact in Singapore and Hong Kong. Maintaining some flexibility is a worthwhile goal as it offers the potential to teach in an integrated way across subject disciplines, something considered to have engagement potential. Notably, across-discipline teaching has not been a focus for Singapore or Hong Kong, perhaps because it is difficult to show learning outcomes have been achieved (Atkinson & Mayo, 2010). Others simply say there is no evidence that a transdisciplinary approach works (Page, 2003). Obvious reasons for this are that the teacher needs to have across-discipline expertise, and that considerable planning is required.

Reacting to what we learned from top nations

Analysis of top mathematics curriculums and comparison with Queensland and Australian mathematics curriculums suggests there are two challenges. The first is to engage more students in the study of mathematics, especially higher level mathematics and the second is to increase the competency of students studying at higher levels. It would be unreasonable to contemplate replicating their systems in Australia; however there are some lessons that can be learned. These might include:

- the development of a common framework from P to 12, with much greater detail than has previously been the case
- developing explicit connections between topic areas and more defined sequencing guidelines
- greater emphasis on and orientation of the mastery of critical facts and processes at each step
- explicit instructions to teachers regarding pedagogy, including detailed models of various forms
- very clear assessment guidelines, including external discipline-based examinations — much more detail is needed for school-based assessment, including moderation practices and what constitutes a reasonable standard and how it might be graded.

Learning from other Australian states

In the past, Queensland was quite distinct in its syllabus construction and assessment protocols; however there is now greater commonality with the other states in content and processes as a result of the implementation of the F–10 Australian Curriculum. Table 3 indicates there is considerable overlap in senior curriculum structure across Australian states. Queensland, NSW, Victoria, WA and the Australian curriculums all have a specialist mathematics subject (e.g. Mathematics C), a subject with a good foundation in calculus, such as mathematical methods (e.g. Mathematics B), and some form of general mathematics that contains limited algebraic abstraction (e.g. Mathematics A). Some states also have simpler forms of mathematics, such as Prevocational Mathematics, some do not. Table 3 below illustrates this patterning.

Table 3: Summary of Australian Senior Mathematics Subjects offered

<table>
<tr><th>Queensland</th><th>NSW</th><th>Victoria</th><th>WA</th><th>Australian Curriculum</th></tr>
<tr><td>Mathematics C</td><td>Mathematics Extension 1
Mathematics Extension 2</td><td>Specialist Mathematics</td><td>Mathematics Specialist</td><td>Specialist Mathematics</td></tr>
<tr><td>Mathematics B</td><td>Mathematics</td><td>Mathematical Methods</td><td>Mathematics Methods</td><td>Mathematical Methods</td></tr>
<tr><td>Mathematics A</td><td rowspan="2">General Mathematics</td><td>General Mathematics (modified appropriately)</td><td>Mathematics Applications (between A and B)</td><td>General Mathematics</td></tr>
<tr><td>Prevocational Mathematics</td><td>N/A</td><td>Mathematics Essential</td><td>Essential Mathematics</td></tr>
<tr><td>Functional Mathematics</td><td>N/A</td><td>Foundation Mathematics</td><td>Mathematics Foundation</td><td>N/A</td></tr>
<tr><td>N/A</td><td>N/A</td><td>N/A</td><td>Mathematics Preliminary</td><td>N/A</td></tr>
</table>

General Mathematics

All of the Australian states studied have the equivalent of a Mathematics A subject. Financial mathematics and other applied mathematics domains tend to be the context to effectively teach middle years mathematics concepts. The contextual setting is designed to harness the motivational potential of authentic mathematics, which was discussed in detail in Chapter 1. The general assessment guideline is for school-based assessment (Victoria: foundation mathematics and general mathematics; WA: preliminary mathematics and foundation mathematics). NSW has a Higher School Certificate (HSC) general mathematics examination that contributes to exit grades.

The literature, including comparisons with other Australian states, suggests that there is merit in including topics and skills that are in demand in industry. This includes the use of ICT for specific purposes, such as spreadsheet applications, drawing and modelling packages as well as mathematics data presentation and analysis packages, including statistics packages. It is worth considering increasing the level of data and statistics taught in this course, as it would advantage students not intending to enrol in the hard sciences, but still needing a higher level of statistics than is currently taught in Mathematics A. Page (2003) cautions about the value in trying to predict what skills are likely to be relevant a decade in advance. The author's appendix contains further commentary on the peculiarities of the Australian Curriculum General Mathematics and its match with current Queensland practices in Mathematics A.

Mathematical Methods and Specialist Mathematics

There is some divergence in approach between the studied states as to what is taught in the equivalent of Mathematical Methods and Specialist Mathematics subjects. The two have to be considered together in part because the Australian Curriculum makes the assumption that most students taking the path of abstract mathematics will take both subjects. In reality, enrolment numbers may not support this assumption.

The contents of the new Mathematics B and C syllabuses will likely be determined to a considerable degree by the emerging Australian Mathematical Methods syllabus. If there is any flexibility, the topic of statistics, including hypothesis testing, might be considered for inclusion. This would be very useful for students not intending to take Specialist Mathematics but wishing to have some calculus knowledge and skill and wishing to have some preparation in dealing with statistics at a tertiary level.

The data presented by Arnold and Sidhu (in press) suggest detail of calculus and other advanced mathematics including those related to probability theory, geometry, matrices, vectors, complex numbers, combinatorics and mechanics is worth examining. This cannot be validly done without access to the detail of assessment, and input from experts who implement the syllabus in each state. Further, it is suggested that tertiary institutions that have a stake in the graduates be consulted.

Assessment

The comparisons with international top-performing nations and states lend support to the recommendations made by Menkens (2013) and subsequently supported in part by Matters and Masters (2014): that 50% of student grades should be determined by external assessment. The reason for this is guided by what we have learned from our top international competitors. That is, the state rather than the school sets minimum standards and, in particular, that there is appropriate focus on content and mastery, greater commonality of fundamental knowledge throughout the state and confidence in validity of the assessment. The form of external assessment favoured by top-performing systems has a broad and proportional coverage of the specified content and processes of the syllabus. The Victorian model of having some portion of this examination calculator-free has merit, as it would likely encourage students to learn and remember key basic skills necessary for more advanced problem-solving. The literature supporting this assumption is detailed in Chapter 1.

Menkens (2013) recommended that the *examination* results be used as the scaling or moderation data for schools rather than assessments taken to moderation panels. Matters and Masters (2014) used the term *external assessment* rather than examination. It is noteworthy that these authors were referring to *all major subjects*, and that subjects such as drama and art have different traditions of assessment to assess different cognitive processes. Matters and Masters (2014) suggested school-based assessment be moderated by panels, as is the case at present, but in a reformed form. Analysis of international curriculums suggest that the use of examinations as the moderating tool, such as NSW does with the HSC, is characteristic of top-performing nations. Matters and Masters (2014) suggest retaining some form of cross-curriculum capability such as those that currently exist with the Queensland Core Skills test, albeit in a reduced form. Top-performing nations in mathematics do not have such tests at present, nor are the authors aware of a plan to assess cross-curriculum capabilities.

In summary

Top-performing systems internationally have produced syllabus documents that are explicit and connected. They are explicit in what is to be taught, in what sequence, how it is to be taught and how the concepts are interconnected. External traditional examinations are used to grade students, to ensure mastery at critical stages and to assist in making standards explicit. Further, such high stakes examinations provide extrinsic motivation to encourage students to understand and remember mathematical concepts and how to apply them.

Chapter 3: Focus on 21st century skills

What are 21st century skills?

There is an increasing rate of publication relating to 21st century skills. This comes from perceptions that new focuses in education are required to equip students to benefit from new forms of learning and socialisation, and to enable them to contribute to an economy where knowledge is considered the major asset (Ananiodou & Claro, 2009). Scardamalia, Bransfford, Kozma and Quellmaiz (2012) refer to new economies as *knowledge building environments*. The verbs that are frequently associated with 21st century skills include *analyse*, *reason*, *communicate effectively*, *reflect*, *solve* and *interpret* problems in a variety of situations, as well as having a commitment to lifelong learning. Inevitably, the acquisition and application of these skills involves the use of technology. Dede (2009) reports the need for new skills comes from the evolution of technology such that increasingly, jobs require expert thinking and communication, that is, tasks that are not readily automated or done by either machines or computers. Most literature discussing 21st century skills consider that technology is central for accessing, evaluating, organising, transforming and communicating knowledge forms. Communication is also central to the process of engaging in the 21st century economy, in part because increasingly productivity occurs in teams of people collaborating (Ananiodou & Claro, 2009; Dede, 2009).

Mathematics and 21st century skills

The Australian Curriculum: Mathematics (ACARA, 2012) reflects the importance of 21st century skills by highlighting the importance of creative thinking, personal and social capability and ethical understanding in mathematics. Further, students are expected to develop ‘increasingly advanced communication, research and presentation skills to express viewpoints’ (p. 10). The kinds of generic skills that are anticipated to come from 21st century education are also articulated in the Queensland senior mathematics syllabuses, (QCAA, 2014a). The Mathematics A syllabus has the following key competencies:

> collecting, analysing and organising information; communicating ideas and information; planning and organising activities; working with others and in teams; using mathematical ideas and techniques and using technology

This is typical of mathematics curriculum global objectives that may differ slightly from state to state and across nations. The NSW and Victorian syllabuses have a focus on problem-solving and the functionality of mathematics but are less explicitly linked to broader 21st century skills.

The broad aims of Mathematics education in Singapore are to enable students to (Singapore Ministry of Education, 2012, p. 9):

- acquire and apply mathematical concepts and skills
- develop cognitive and metacognitive skills through a mathematical approach to problem-solving
- develop positive attitudes towards mathematics.

The senior secondary syllabus lists its own specific set of aims for all students (Singapore Ministry of Education, 2012, p. 10):

- acquire mathematical concepts and skills for continuous learning in mathematics and to support learning in other subjects
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem-solving
- connect ideas within mathematics and between mathematics and other subjects through applications of mathematics
- build confidence and foster interest in mathematics.

In terms of 21st century skills, the major difference between nations that demonstrate high performance in mathematics with those that do not is the degree to which mathematics is explicitly linked to thinking. As Rotherham and Willingham (2009, p. 21) pointed out, 'skills and knowledge are not separate, but intertwined.' OECD (2014, p. 33) supports this view and noted:

Research shows that training problem-solving skills out of context is not the solution.

Further, Rotherdam and Willingham (2009, p. 21) reported:

> Without better curriculum, better teaching, and better tests, the emphasis on 21st century skills will be a superficial one that will sacrifice long-term gains for the appearance of short-term progress.

Literature from the United States mathematics learning has revealed curriculum, teacher expertise and assessment have all been weak links in past educational reform efforts, a sentiment shared by other authors (e.g. Scardamalia et al., 2012). The focus on 21st century skill necessities has motivated discussion about 21st century pedagogy.

Scardamalia et al. (2012) has reported that teaching 21st century skills is not the same as job training. The skills that need to be developed are generic and transferrable, and the process of development of these skills is continual. The individual needs to have the capacity to problem solve in multiple emerging environments. Some authors (e.g. Bell, 2010; Scardamalia et al., 2012) have connected the development of 21st century skills to project-based learning, and pedagogies have become associated with constructivist learning, discovery learning and problem-based learning. Project-based learning, where students have considerable autonomy as well as responsibility to generate their own products, frequently in collaborative contexts, offers rich opportunities for 21st century skill development.
For example, in completing projects, students may search for and select information, restructure and model that information and communicate this by effectively using ICT, all key attributes of 21st century skills (Ananiadou & Clao, 2009).

The problem in linking the development of 21st century skills with constructivist pedagogy (for mathematics learning) is that those educational systems that excel in mathematics development use constructivist-based pedagogy sparingly, and those that have moved most to embrace collaborative project-based learning have declined more markedly. Those nations who retain teacher-controlled discourse have done well on international tests of mathematics irrespective of the relative poverty of resources and being in the early phases of curriculum reform. Vietnam, for instance, has not long emerged from war and sanctions. Yet, in its first attempt at PISA 2012, Vietnam ranked 17th in mathematics literacy, just behind Germany but a little ahead of Australia, UK, US, and New Zealand, to name a few much more affluent nations (OECD, 2014). Like other East Asian nations, mathemaical discourse in Vitenamese classrooms is hightly controlled by the teacher (Norton, in press). As noted in Chapter 1, increasingly educational theorists are recommending a balanced use of constructivist pedagogies (e.g. Hattie, 2009; Kirschner et al, 2006; Klein, 2005; Kotzee, 2012; Owen & Sweller, 1989). It should also be noted that mathematics is not the only subject students study, and there are ample opportunities for students to engage in dominantly constructivist pedagogies for the development of generic capability in drama, geography, history, study of religion and some sciences.

Interestingly, Menkens (2013), who summarised the multiple (288) submissions to the *Education and Assessment Committee,* dealt with the issue of 21st century skills. The terms she used *were lower and higher order skills*. Higher order skills were, in effect, key aspects of 21st century skills and included *evaluation*, *synthesis*, *analysis*, and *application of knowledge*. Menkens (2013, p. 13) reported:

some submissions to this inquiry indicate that the aim [of developing higher order thinking skills] is at the expense of teaching ‘lower order’ skills — the fundamental content knowledge for mathematics, chemistry and physics

In fact, around 80% of the submissions supported marks for test and/or state examinations and a major common theme throughout submissions supporting external examinations was concern for the degradation of key knowledge due to pedagogy and assessment forms that were too generic.

Further, a concern of many of the submissions related to the forms of communication that was expected of mathematics students. In particular, teachers criticised the overuse of prose in extended take-home projects.

Communicating mathematical thinking

In mathematics, higher order thinking has traditionally been conveyed using symbols that mathematicians understand. The most popular example of a profound idea expressed simply is E = mc^2. It is worth looking at a higher order thinking problem that might be given to Year 11 students (Figure 7 below) and considering how the solution to this might be described.

Figure 7: Chain rule problem Year 11 Mathematics B.
Taken from Barnes (1993, p. 37), ‘More about Functions and Differentiation: Unit 6’

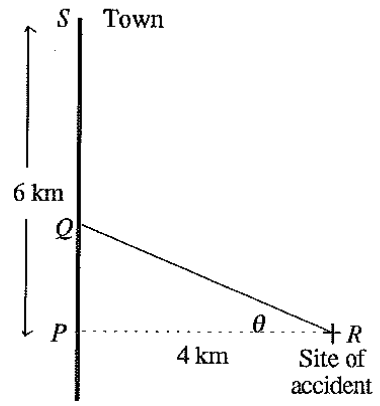


Figure 22

A member of a bushwalking party has had an accident, and two of the group plan to go to the nearest town for help. The site of the accident is 4 km from a straight road that runs to a town, as shown in figure 22. The nearest point on the road is 6 km from the town. They estimate that they can walk at 4 km/hr through the bush and jog at 8 km/hr along the road.

What direction α should they set off in to get to the town in the shortest time?

There are two ways an optimisation problem of this nature can be solved. The first is to get someone else to do it. Some call this cheating, others collaborating. The second way to arrive at a solution is to do the problem sequentially. At a minimum, the following concepts and processes are involved in providing a coherent answer:

- recognition that this is an optimisation problem requiring differentiation
- rate and its applications
- time calculations

- Pythagoras calculations including roots and square roots
- relatively high levels of algebraic manipulation
- further index notation
- differentiation involving the chain rule, including working with negative indices
- factorisation
- surd properties
- trigonometry (including appropriate use of technology)
- substitution to find the actual time taken if the angle is 30 degrees
- more time conversions.

The successful completion of such a problem by Year 11 students requires the coherent communication of their thinking on each of these concept areas, and it is almost entirely symbolic. The processes involved require a high level of mathematical conceptualisation and fluency with basic mathematical processes. The act of learning to complete such problems is an ideal opportunity to develop 21st century thinking skills including analysis, reason, making decisions, solve and interpret problems, systematic organisation and determination. The fact is, almost all study of mathematics, when appropriately taught, is an ideal medium through which to develop 21st century skills. This has been the case for several millenniums. However, mathematics, when poorly taught, has limited transferability.

The example in Figure 7 above is not trivial. It illustrates the importance of domain-specific knowledge. As McGraw (2006) points out, business leaders may speak of generic competencies as though this is all that matters, but ask them what is needed in an accountant or an engineer and they would likely say something related to domain-specific knowledge. McGaw notes (2006, p. 20):

> The psychological research literature on expertise does make clear that it is domain-specific and dependent on a deep knowledge of a domain and not on a set of generic competencies that are transferable across domains. Both are needed.

Matters and Masters (2014) recommended a focus on 21st century skills and specifically mentioned teamwork, problem-solving, creativity and verbal communications as well as managing information dynamically. These authors were considering the totality of the senior high school student's educational opportunity. Mathematics curriculum writers and teachers need to be aware that verbal expression is the focus of subjects including English and drama, and creativity is the focus of drama, art and supporting activities including debating.

Verbal expression and some teamwork may be included in a mathematics curriculum, but we need to be careful that this is not at the expense of problem-solving in the contexts of mathematics. Similarly, mathematics provides an excellent opportunity to manage information dynamically, for example the use of spreadsheets to display data in a table, graph or to carry out descriptive statistics. Further, aspects of functions can be displayed as a graph, in tabular form and in symbolic forms in various configurations. Powerful ICT technology can readily carry out the transformations and add meaning to the study of calculus if the technology is used with purpose. The use of ICTs may be different for different students and syllabus needs. Spreadsheets will likely find greater utility in finance subjects associated with General Mathematics or Mathematics A, and graphing programs such as Maths Helper Plus with calculus-orientated study associated with Mathematical Methods or Mathematics B subjects.

Hattie (2009) comments that the major shift in learning is that we are involved in a shift from an industrial to a knowledge society. Increasing amounts of work is done on conceptual objects rather than physical objects. The study of mathematics is a rich environment in which to derive one knowledge object from another, propose problems and solutions and interact in an increasingly abstract way with powerful representations of physical and abstract realities. It is an ideal medium through which to develop 21st century skills. The challenge is to do so in the context of mathematical content, and this requires high levels of teacher capacity. As Hattie (2009, p. 27) noted:

> If the students are not doing enough thinking, something is seriously wrong with the instruction.

Mathematics and creativity

Creativity is complex and multifaceted in nature. There are numerous definitions, and creative characteristics vary within and among people and across disciplines (Treffinger, Young, Selby & Shepardson, 2002). These authors list common cognitive characteristics including *generating ideas*, *digging deeper into ideas*, *openness* and *courage to explore ideas*. Most authors agree that mathematical creativity has its base in reasoning as distinct from locked in algorithmic thinking (Boesen, 2006). In addition to cognitive orientation, mathematical creativity is said to require commitment, determination, persistence, self-direction, work ethic and self-direction. Thus, creativity is not just about ideas and thinking, but the disposition to find solutions and persist with thinking tasks.

Munakata and Vaidya (2012) report that creativity is not usually associated with the study of mathematics, thus there is a perception problem. The tendency has been to associate the study of mathematics with linear processes and deductive reasoning, whereas creative thinking is generally considered non-linear and more inductive. Sriraman (2004) described mathematician's creativity as generally following four states: *preparation*, *incubation*, *illumination*, *multifaceted verification*. The *preparatory* stage includes detailed research on what other mathematicians had done on the problem, trying a variety of heuristics and using a back-and-forth approach. This was a period of prolonged hard work and was followed by a period of *incubation* where the problem was set aside from the conscious to the unconscious mind. At some time, often unexpectedly, there was a period of *illumination* where the solution manifests in the conscious mind. The final period of *verification* was one of formalising the idea, including constructing a proof. Proof is generally based on the foundations of existing mathematical conventions and facts. The relevance of this for classroom practice is that creative students are likely to need a deep understanding of mathematical concepts associated with the problem, the ability to simulate problems of complexity, and prolonged engagement to formulate solutions.

Kind and Kind (2007) analysed the literature on creativity in science education and generally found that *good* creative teaching was associated with open-ended student-orientated teaching, while *bad* traditional teaching was more teacher-orientated, closed-task oriented. Too often in traditional mathematics classes, the tasks are closed in nature and thus the student's ability to generate new ideas may not fully develop (Treffinger et al., 2002). These authors suggest that creativity is fostered best when there is a balance between *traditional* and *creative* thinking. Two critical factors are manifested: the first is that students become confident to take risks (Kind & Kind 2007). As we noted in Chapter 1, competency engenders confidence. The second factor is that students need the cognitive tools to engage with the problem, just as Sriraman (2004) and Boesen (2006) described with mathematician's creative endeavours.

An additional complication is that creativity in mathematics education has been associated with gifted students and become part of gifted and talented programs that targeted elite students (Leikin, Berman & Koichu, 2009). These authors contend that all students be exposed to challenge and all students could demonstrate mathematical creativity. Still, it was recognised that some students could progress faster and ultimately demonstrate much deeper understanding of mathematics and indeed generating their own alternative explanations. The authors contended that a two-way relationship exists, in that the more all students are taught well, the more students will be identified as mathematically creative and gifted. The general pedagogical advice is that students need to develop competency with basic facts and processes and then extend this by teaching through problems, presenting the same problem in different ways and supporting the move towards abstraction through multiple models and discourse. Students cannot formulate a mathematical argument to verify their creative efforts if they are unfamiliar with the rules and assumptions of the discipline area.

A further critical aspect of pedagogy is appropriate *challenge*. Excessive challenge can damage the confidence of any student. Too little challenge does not stimulate and extend students. While all students need cognitive tools to engage with the challenge, the most able are capable of reaching and remembering mathematical concepts and processes at a much faster rate. This has led to accelerated programs (Leikin et al., 2009).

Assessing mathematical creativity is challenging. There is little to guide the classroom teacher with respect to formal metrics. Persistent behaviour patterns, including those listed above, might be documented by the classroom teacher, and might encourage the teacher to seek out deeper challenges for students who exhibit the characteristics of mathematical creativity. The old adage: *what you test is what you get* holds true (Boesen, 2006). Students practising questions for short written tests are unlikely to create classroom environments that foster student creativity. On the other hand, such testing is likely to promote the foundation knowledge upon which mathematical creativity is based. Thus the conflicting view is, to be creative requires fluency with basic facts and processes which can be effectively developed in classrooms where the discourse is tightly controlled by the teacher, but to extend this to foster creativity, the discourse needs to become less scripted and the teacher empower student autonomy. Teachers of students with divergent needs are especially challenged. However, teachers who can differentiate a curriculum effectively can manage classroom activities that develop creativity appropriate to individual student needs.

Bibliography

AAMT (Australian Association of Mathematics Teachers). (1996). *Statement on the use of calculators and computers for Mathematics in Australian schools.* http://eric.ed.gov/?id=ED399156

AAMT. (2014). *Position paper on digital learning in school mathematics*

AAMT. (2015). *Desktop Review of Mathematics School Education Pedagogical Approaches and Learning Resources*. https://docs.education.gov.au/documents/desktop-review-mathematics-school-education-pedagogical-approaches-and-learning-resources

AAS (Australian Academy of Science). (2015). *Desktop review of mathematics school education.* Department of Education.

ACARA (Australian Curriculum Assessment and Reporting Authority). (2012). *The Australian Curriculum: Mathematics*. Available from: www.australiancurriculum.edu.au/Mathematics/Rationale

ACARA. (2013). NAPLAN (National Assessment Program — Literacy and Numeracy). www.nap.edu.au/naplan/naplan.html

ACARA. (2014). *Senior secondary Mathematics subjects*. www.australiancurriculum.edu.au/seniorsecondary/mathematics/specialist-mathematics/senior-secondary-mathematics-subjects

ACARA. (2015). *2015 NAPLAN national report: Achievement in Reading, Persuasive Writing, Language Conventions and Numeracy.* www.nap.edu.au/results-and-reports/national-reports.html

Ainley, J. (2004). *Evaluation report on the New Basics Research Program*. Australian Council for Educational Research. Department of Education and the Arts.

An, S. (2004). Capturing the way of teaching: The learning-questioning and learning-reviewing instructional model. In L. Fan, N-Y Wong, J. Cai & S. Li (eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 462–502). New Jersey: World Scientific.

Ananiadou, K. Claro, M (2009). 21st century skills and competences for new millennium learners in OECD countries. *EDU Working paper, 41*

Arnold, J. & Sidhu, (in press); *A statistical analysis of the performance of first year engineering students at UNSW Canberra and the impact of the state where they undertook Year 12 study.* http://arxiv.org/pdf/1506.07400v1.pdf

Atkinson, R. D. & Mayo, M. J. (2010). *Refueling the US innovation economy: Fresh approaches to science, technology, engineering and mathematics (STEM)*. The Information Technology & Innovation Foundation.

Attard, C. (2013). ‘If I had to pick any subject, it wouldn’t be maths’. Foundations for engagement with mathematics during middle years. *Mathematics Education Research Journal, 25*(4), 569-587.

Australian Bureau of Statistics (2013). *4125.0-Participation and Retention*. www.abs.gov.au/ausstats/abs@.nsf/Lookup/4125.0main+features2110Jan%202013

Barber, M. & Mourshed, M. (2007). *How the world’s best-performing schools come out on top*. McKinsey & Company

Barnes, J. A. (1999). Creative writing in trigonometry. *Mathematics Teacher, 92(6)*

Barnes, M. (1991). *Investigating Change: An Introduction to Calculus for Australian Schools.* Curriculum Corporation.

Barnes, M. (1993). *More about functions and differentiation: Unit 6: Investigating change.* Curriculum Corporation.

Bell, S. (2010). Project-Based Learning for the 21st Century: Skills for the Future. *The Clearning House: A Journal of Educational Strategies, Issues, and Ideas , 83*, 39–43.

Bernstein, B. (1975). *Towards a theory of educational transmissions (Class, codes and control Vol 3).* London: Routledge and Kegan Paul.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity: Theory, research, critique.* London: Taylor and Francis.

Bernstein, B. (2000). *Pedagogy, Symbolic Control and Identity* (Revised ed.). Lanham, Boulder, New York, Oxford: Rowan & Littlefield Publishers, Inc.

Board of Studies, NSW. (2003). *Mathematics Years 7-10*. Sydney, Board of Studies, NSW.

Board of Studies, Teaching & Educational Standards (BOSTES) NSW. (2012*). Syllabus Documents: Mathematics, Mathematics Extension 1, Mathematics Extension 2.* www.boardofstudies.nsw.edu.au/syllabus_hsc/mathematics-advanced.htm

Boesen, J. 2006). *Asssessing Mathematical Creativity: Comparing nationanl and teacher made tests, explaining the differences and impact*. www.gu.se/forskning/publikation/?publicationId=128530

Bracey, G. (2014). *International comparisons: More fizzle than fizz*. The Huffington Post. www.huffingtonpost.com/gerald-bracey/international-comparisons_b_149690.html

Burghes, D. (2011). *International comparative study in Mathematics training: Recommendations for initial teacher training in England.* Education Development Trust. www.cimt.plymouth.ac.uk/papers/icsmtt.pdf

California State Board of Education. (2013). *California Common Core Standards, Mathematics.* www.cde.ca.gov/be/st/ss/documents/ccssmathstandardaug2013.pdf

Center on International Education Benchmarking (2015). *Hong Kong, System and School Organisation*. www.ncee.org/programs-affiliates/center-on-international-education-benchmarking/top-performing-countries/hong-kong-overview/hong-kong-system-and-school-organization

Chan, T. W. (2010). How East Asian classrooms may change over the next 20 years. *JCAL (25.*

Chow. Y., Koh, C., Koh., Ling. & Shee, S. (2005*). College Mathematics: Syllabus C, Volume 2.* Singapore. SNP Panpac.

Cockcroft, W.H. (Chairperson). (1982). *Mathematics counts (Report of the Committee of Inquiry into Teaching of Mathematics in Schools)*. London: H.M.S.O.

ComputerBasedMath.org (2016). www.computerbasedmath.org

Cooper, R. (2007). An investigation into constructivism within an outcomes based curriculum. *Issues in Educational Research, 17,* 1–20. www.iier.org.au/iier17/cooper.html

Dede, C. (2009). *Comparing Frameworks for '21st Century Skills'.* www.watertown.k12.ma.us/dept/ed_tech/research/pdf/ChrisDede.pdf

Education Commission Working Group on the Development of Educational Services in Hong Kong, (2011). *Report on the development of education services in Hong Kong.* www.e-c.edu.hk/eng/online/annex/Report%20Dev%20Edu%20Service%20Eng.pdf

Education Consumers Foundation, (2011). *Direct Instruction: What the Research Says.* http://education-consumers.org/pdf/DI_Research.pdf

Fan, L. & Zhu, T. (2004). How have Chinese students Performed in Mathematics? A perspective from Large-Scale International Comparisons. . In L. Fan,. N-Y Wong, J. Cai & S. Li. (Eds.), *How Chinese Learn Mathematics: Perspectives from Insiders* (pp. 3–26). New Jersey, World Scientific.

Frankfurt International School. (2015). *Primary School Curriculum*. www.fis.edu/page.cfm?p=1036

Ginsburg, A., Leinwand, S., Anstrom, T. & Pollock, E. (2005). *What the United States can learn from Singapore's world-class mathematics system: And what Singapore can learn from the United States*. Washington. American Institutes for Research.

Government of Western Australia School Curriculum and Standards Authority. (2015). *Mathematics*. wace1516.scsa.wa.edu.au/syllabus-and-support-materials/mathematics

Gregg, J. (1995). The tensions and contradictions of the school mathematics tradition. *Journal for Research in Mathematics Education, 26*, pp. 442–466.

Gu, L., Huang, R. & Marton, F. (2004). Teaching with variation: A Chinese way of promoting effective mathematics learning. In L. Fan, N-Y Wong, J. Cai & S. Li, (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 309–347). New Jersey: World Scientific.

Haladyna, T., Shaughnessy, J. & Shaughnessy, J. (1983). A causal analysis of attitude toward mathematics. *Journal for Research in Mathematics Education, 14*(1), pp. 19–29.

Hannula, M. (2002). Attitudes towards mathematics: Emotions, expectations and values. *Educational Studies in Mathematics, (49)*, 25–46.

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. London: Routledge.

Hiebert, J. (2003). What research says about the NCTM standards. In J. Kilpatrick, W. Martin & D. Schifter (Eds), *A research companion to principles and standards for school mathematics* (pp. 5–23). Reston, VA: NCTM.

HKEAA-CDC (Hong Kong Examinations and Assessment Authority & Curriculum Development Council). (2007). *Mathematics Curriculum and Assessment.* Retrieved December 10 2015, from www.edb.gov.hk/en/curriculum-development/kla/ma/curr/ss-math-2007.html

Hong, K., Mei, Y. & Lim, J. (2009). *Singapore model method for learning mathematics*. Singapore. Ministry of Education.

https://www.moe.gov.sg/docs/default-source/document/education/syllabuses/sciences/files/additional-mathematics-syllabus-sec-3-to-4-express-n(a)-course.pdf

Huang, R. & Leung, K. (2004). Cracking the paradox of Chinese learners: Looking into mathematics classrooms in Hong Kong and Shanghai. In L. Fan,. N-Y Wong, J. Cai & S. Li. (Eds.), *How Chinese Learn Mathematics: Perspectives from Insiders* (pp. 348–381). New Jersey, World Scientific.

Jensen, B. (2012). *Catching up: Learning from the best school systems in East Asia*. Grattan Institute. http://grattan.edu.au/static/files/assets/23afab6b/129_report_learning_from_the_best_main.pdf

Kennedy, J., Lyons, T. & Quinn, F. (2014). The continuing decline of science and mathematics enrolments in Australian high schools. *Teaching Science , 60 (2),* pp. 34–46.

KhanAcademy.org (2006). www.khanacademy.org

Kirschner, P., Sweller, J. & Clark, R. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*(2), pp. 75–86.

Klein. (2005). *The Sate of State Math Standards*. Thomas B. Foundation. www.math.jhu.edu/~wsw/papers2/education/02-SOS-Klein-05.pdf

Kotzee, B. (2012). Expertise, fluency and social realism about professional knowledge. *Journal of Education and Work*. www.tanfonline.com/loi/cjew20

Lafayete De Mente, B. (2009). *The Chinese Mind*. Tokyo, Tuttle.

Lai, M. & Muray, S. (2012). Teaching with procedural variation: A Chinese way of promoting deep understanding of mathematics. *International Journal of Mathematics Teaching and Learning*; www.cimt.plymouth.ac.uk/journal

Lee, W.O. (1996). The cultural context of Chinese learners: Concepts of learning in the Confucian Tradition. In D.A. Watkins & J.B Briggs (Eds.), *The Chinese learner* (pp. 45–67). Hong Kong: Comparative Education Research Centre.

Leukin, R., Berman, A. & Koichu, B. (2009). *Creativity in Mathematics and Education of Gifted Students*. Rotterdam, Sense.

Li J. (2004). Chinese cultural model of learning. In L. Fan, N-Y Wong, J. Cai & S. Li. (Eds.)*, How Chinese learn mathematics: Perspectives from insiders* (pp. 124–156). New Jersey: World Scientific.

Li, Y., Zhao, D., Huong, R & Ma, Y. (2008). Mathematics preparation of elementary teachers in China: Changes and issues. *Journal of Mathematics Teacher Education, 11,* pp. 417–430.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, N.J. Lawrence Erlbaum Associates, Inc.

Ma, X. & Kishor, N. (1997). Assessing the relationship between attitude towards mathematics and achievement in mathematics*. Journal for Research in Mathematics Education, 28*(1), pp. 26–47.

Malatt, D. (2000). *TIMSS: A question of validity.* University of Melbourne. http://trove.nla.gov.au/work/153116957?versionId=166871283

Marshall, J. (2003). Math Wars: Taking sides. *Phi Delta Kappan, 85*(3), 193–200, 249. www2.ed.gov/about/bdscomm/list/mathpanel/documents/math-wars-taking-sides.pdf

Martin, M. & Mullis, V. (2009). *Overview of TIMSS advanced procedures for ensuring comparative validity.* In A. Arora, P. Foy, M. O. Martin & I V.S. Mullis (Eds.), *TIMSS Advanced 2008 Technical Report*. International Study Centre. Boston. http://timssandpirls.bc.edu/timss_advanced/downloads/TA08_Technical_Report.pdf

Masters, G. (2009). *A shared challenge: Improving literacy, numeracy and science learning in Queensland schools*. Austrlaian Council for Eucational Research.

Matters, G. & Masters, G. (2014*). Redesigning the secondary–tertiary interface: Queensland Review of Senior Assessment and Tertiary Entrance*. Melbourne. ACER. http://research.acer.edu.au/qld_review

MCEETYA (Minsterial Council on Education, Employment, Traijning and Youth Affairs). (2008). *Melbourne Declaration on Educational Goals for Young Australians*. http://apo.org.au/resource/melbourne-declaration-educational-goals-young-australians

McGaw, (2006). *An International view of Hong Kong's Educational Reform. Report on the Development of Educational Services in Hong Kong*. Speech at the Hong Kong Education Commission's 4th Reporting Session on Educational Reform. www.e-c.edu.hk/eng/reform/Prof%20Barry%20McGaw%20Speech.pdf

Menkens, R (Chair). Education and Innovation Committee. (2013). *The assessment methods used in senior mathematics, chemistry, and physics in Queensland schools: Report No. 25.* www.parliament.qld.gov.au/documents/committees/eic/2013/qldassessment/rpt-025-14oct2013.pdf

Ministry of Education: Singapore. (2008). *Education in Singapore*. *A part of Singapore's success story.* Ministry of Education.

Moore, R. & Muller, J. (2002). The growth of knowledge and the discursive gap. British *Journal of Sociology of Education, 23(4),* pp. 627–637.

Muller, J. & Taylor, N. (1995). Schooling and everyday life: Knowledges sacred and profane. *Social Epistemology, 9(3),* pp. 257–275.

Muller, J. (2000). *Reclaiming knowledge: Social theory, curriculum and education policy*. London and New York: Routledge Falmer.

Muller, J. (2009). Forms of knowledge and curriculum coherence*. Journal of Education and Work.* www.tandfonline.com/loi/cjew20

Muntakata, M. & Vaidya, A. (2011). Encouraging creativity in mathematics and science through photography. *Teaching Mathematic and its Applications. 31*, pp. 121–132.

Nardi, E. & Steward, S. (2003). Is mathematics T.I.R.E.D? A profile of quiet disaffection in the secondary mathematics classroom. *British Journal of Educational Research, 29*(3), pp. 345-367.

National Council of Teachers of Mathematics. (2014). *Executive summary: Focus in high school mathematics: Reasoning and sense making.* *www.nctm.org/Research-and-Advocacy/advocacy-toolkit*

Norton, A. (2014). *Mapping Australian higher Education 2014-2015.* Grattan Institute. http://grattan.edu.au/report/mapping-australian-higher-education-2014-15

Norton, S. & Zhang, Q, (2013). Chinese student's engagement with mathematics learning. *International Journal of Mathematics Teaching and Learning.* December 10. www.cimt.plymouth.ac.uk/journal

Norton, S. (2012). Prior study of mathematics as a predictor of pre-service teachers' success on mathematics content and pedagogical content knowledge tests. *Mathematics Teacher Education and Development, 14*(1), pp. 2–26*.*

Norton. S. (2011). How deeply and how well? How ready to teach mathematics after a one year program? *Mathematics Teacher Education and Development, 12*(1)*,* pp. 65–82.

Noyes, A. (2012). It matters which class you are in: Student-centred teaching and the enjoyment of mathematics. *Research in Mathematics Education, 14*(3), pp. 273–290.

OECD (Organisation for Economic Cooperation and Development). (2004)*. Learning for tomorrow's world: Student learning, engagement and strategies.* www.oecd.org/edu/school/programmeforinternationalstudentassessmentpisa/33918006.pdf

OECD. (2014a). *Education at a Glance 2014:OECD Indicators,* OECD Publishing. www.oecd-ilibrary.org/education/education-at-a-glance-2014_eag-2014-en

OECD. (2014b). *PISA 2012 Results in Focus: What 15-year-olds know and what they can do with what they know.* http://apo.org.au/resource/pisa-2012-results-focus-what-15-year-olds-know-and-what-they-can-do-what-they-know

OECD. (2015). *Mathematics performance (PISA) (indicator)*. https://data.oecd.org/pisa/mathematics-performance-pisa.htm

Office of the Chief Scientist, (2013). *Science, Technology, Engineering and Mathematics in the National Interest: A Strategic Approach*. Australian Government, Canberra.

Otsuka, S. (1996). Why do Asians do well at school? *Deepsouth, 2(1) (Autumn).* www.otago.ac.nz/deepsouth/vol2no1/setsuo.html

Owen, E. & Sweller, J. (1989). Should problem solving be used as a learning device in mathematics? *Journal for Research in Mathematics Education, 20*(3), pp. 322–328.

Page, J. (2003). Towards a critical appraisal of the Queensland New Basics Project. *Perspectives on Educational Leadership 13*(6), pp. 1–2.

Parliament of Australia (2015). *Snapshot of employment by industry.* www.aph.gov.au/About_Parliament/Parliamentary_Departments/Parliamentary_Library/FlagPost/2014/February/Employment-by-industry-2012-13

Pintrich, P., Marx, R. & Boyle, R. (1993). Beyond cold conceptual change: The role of motivational beliefs and classroom contextual factors in the process of conceptual change. *Review of Education Research, 63*(2), pp. 167–199.

Pirjo, A. Ee, J., Lim, S., Hautamaki, J. & Luit, J. (2010). Yong children's number sense in Finland, Hong Kong and Singapore. International Journal of Early Years Education, 12(3), pp. 195–216. http://dx.doi.org/10.1080/0966976042000268681

QCAA (Queensland Curriculum and Assessment Authority). (2007). *Essential Learnings*. www.qcaa.qld.edu.au/p-10/past-curriculum-documents/implementing-essential-learnings-standards/essential-learnings-by-year-juncture

QCAA. (2014a). *Mathematics Senior Subjects.* www.qcaa.qld.edu.au/senior/subjects/mathematics

QCAA. (2014b). *NAPLAN 2014 outcomes*. www.qcaa.qld.edu.au/publications/statistics

QCAA. (2015). *NAPLAN 2015 outcomes*. www.qcaa.qld.edu.au/publications/statistics

Queensland Audit Office. (2013). *Supply of specialist subject teachers in secondary schools* [Report to Parliament]. The State of Queensland: Queensland Audit Office. www.qao.qld.gov.au/report2-:-2013-14

Quong, T. (2011). *An analysis of educational reform at the school level in Hong Kong.* www.ied.edu.hk/apclc/roundtable2011/paper/Quong.pdf

Riegle-Crumb, C., King, B., Grodsky, E. & Muller, C. (2012). The more things change, the more they stay the same? Prior achievement fails to explain gender inequality in entry into STEM college majors over time. *American Education Research Journal, 49,* pp. 1048–1073

Rotherdam, A & Willingham, D. (2009). To work, the 21st century skills movement will require keen attention to curriculum, teacher quality, and assessment. *Educational Leadership, September,* pp. 16–21.

Ruthven, K. (2001). The English experience of a calculator-aware number curriculum. In J. Anghileri (Ed.), *Principles and practices in arithmetic teaching: Innovative approaches for the primary classroom* (pp. 165–188). Buckingham: Open University Press.

Scardamalia, Bransford, Kozma & Quellmalz. (2012). *Assessment and Teaching of 21st Century Skills: New assessments and environments for knowledge building.* Springer Netherlands.

Sealey, P. & Noyes, A. (2010). On the relevance of mathematics curriculum to young people. *The Curriculum Journal, 21*(3), pp. 239–253.

Seng & Yee. (2007). *New Syllabus: Additional Mathematics* (Vol. 8th). Shinglee Publishers.

Sfard, A. (2003). Balancing the unbalanceable: The NCTM standards in the light of theories of learning mathematics. In J. Kilpatrick, W. Martin & D. Schifter (Eds), *A research companion to principles and standards for school mathematics* (pp. 353-392). Reston: National Council of Teachers of Mathematics.

Singapore Ministry of Education. (2012). *Additional Mathematics Syllabus Sec 3 to 4 Express & N(A) Course*. www.moe.gov.sg/docs/default-source/document/education/syllabuses/sciences

Sriraman, B. (2004). The characteristics of mathematical creativity. *The Mathematics Educator, 14*(1), pp. 19–34.

TEMAG (Teacher Eduation Ministerial Advisory Group). (2014). *Action now! Classroom ready teachers*. https://docs.education.gov.au/documents/action-now-classroom-ready-teachers-report

The Government of the Hong Kong Special Administrative Region: Education Bureau. (2007). *Secondary Maths Curriculum.* Retrieved from:www.edb.gov.hk/en/curriculum-development/kla/ma/curr/ss-math-2007.html

Thomson, S., Hillman, K., Wernert, N., Schmid, M. & Munene, A. (2012). *Highlights from the TIMSS and PIRLS 2011 from Australia's perspective.* Melbourne: Australian Council for Educational Research.

Thomson, S., Wernert, N., Underwood, C. & Nicholas, M. (2007). *TIMSS 07: Taking a closer look at mathematics and science in Australia.* ACER. www.acer.edu.au/documents/TIMSS_2007-AustraliaFullReport.pdf

Thong, H., Chiang, T. & Meng, K. (2007). *College Mathematics: Syllabus C. Volume 1.* Singapore, Pan Pacific.

Torbeyns, J., Verschaffel, L. & Ghesquiere, P. (2005). Simple addition strategies in a first-grade class with multiple strategy instruction. *Cognition and Instruction, 23*(1), 121.

Trefffinger, D., Young, G., Selby, E. & Shepardson, C. (2002). *Assessing Creativity: A Guide for Educators.* National Research Centre on the Gifted and Talented. (NRC/GT). Connecticut, NRC/GT.

U.S. Department of Education. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Vale, C. (2010). Supporting 'out-of-field' teachers of secondary mathematics. *Australian Mathematics Teacher, 66*(1), pp. 17–24.

Vaughan, B (1996). *Maths Helper Plus*. *Teachers' Choice Software*. www.teacherschoice.com.au

Victorian Curriculum and Assessment Authority. (2010). *Mathematics: Victorian certificate of education study design*. www.vcaa.vic.edu.au/documents/vce/mathematics/mathsstd.pdf

Vygotsky, L. S. (1987)*. The collected worked of L.S. Vygotsky Volume 1*. New York: Plenum Press.

Wang, T. & Murphy, J. (2004). An examination of coherence in a Chinese mathematics classroom. Li J. (2004). Chinese cultural model of learning. In L. Fan,. N-Y Wong, J. Cai & S. Li. (Eds*.), How Chinese Learn Mathematics: Perspectives from Insiders* (pp. 107–124). New Jersey, World Scientific.

Wolf, M. (1998). Validity issues in international assessments. *International Journal of Educational Research, 29*(6), pp. 491–501.

Wong, L. H. & Looi, C. K. (2011). What seams do we remove in mobile-assisted seamless learning? A critical review of literature. *Computers and Education, Elsevier, 2011, 57(4),* pp. 2364–2381.

Zhang, D., Li, S. & Tang, R. (2004). The 'two basics': Mathematics teaching and learning in mainland China. In L. Fan, N-Y Wong, J. Cai & S. Li (Eds.)*, How Chinese learn mathematics: Perspectives from insiders* (pp. 189–207). New Jersey: World Scientific.

Zhao, D. & Singh, M. (2010). Why do Chinese-Australian students outperform their Australian Peers in Mathematics; A comparative case study. *International Journal of Science and Mathematics Education, 9;* pp. 69–87.