Supervised Assessment: topics covered include matrices and vectors, structures and patterns, and conics

This sample is intended to inform the design of assessment instruments in the senior phase of learning. It highlights the qualities of student work and the match to the syllabus standards.

Criteria assessed
- Knowledge and procedures
- Modelling and problem solving
- Communication and justification

Assessment instrument
The response presented in this sample is in response to assessment items.

Question One (KP)
  a) Find the dot product of the vectors \( \vec{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \) & \( \vec{u} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \)
  b) Find the angle between \( \vec{v} \) & \( \vec{u} \)

Question Two (KP)
  a) Determine the Linear Transformation Matrix that was applied to figure A to obtain as an image, figure B.

b) Find the image of a rectangle \( O(0,0), B(3,1), C(3,2), D(0,2) \) under a shear parallel to the y axis, with shear factor 3.
  c) Use the determinant to calculate the area of the image of \( OBCD \) under this transformation.
Question Three (KP)

A geometric sequence begins 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, ... 

a) Find the common ratio and the 9th term.

b) For the geometric sequence above, find the sum to infinity of the series.

Question Four (KP)

The population of Amoeba in a petri dish is modelled by the function:

$P(t) = 100 \times 1.20^t$ where $P$ is the number of Amoeba and $t$ is the time in minutes.

a) How many Amoeba are there after 6 minutes?

b) When will the petri dish contain 100000 Amoebea?

Question Five (KP)

a) Sketch the graph of the relation $|z - 2 + i| \leq 3$ on the Argand Plane.

b) Show clearly, using Autograph, the relation $|z - 2 + i| = 3$ on the Argand plane. Use vectors to illustrate at least one $z \in \mathbb{C}$, that satisfies the relationship $|z - 2 + i| = 3$

c) Print a copy OR sketch a graph of the relationship & an associated vector & state WHY the vector illustrated satisfies the relationship $|z - 2 + i| = 3$

Question Six (KP)

Find the equation of the loci of points $P(x, y)$ such that $2PA = 3PB$

One such point is shown on the Cartesian Plane above.
Due to copyright restrictions, the image has been removed.

The task provided a map of Queensland from the New South Wales border up to Townsville and West to Charters Towers, Tambo and Cunnamulla.

Take Ipswich as the origin.
Let the reference axes, \( \overrightarrow{OX}, \overrightarrow{OY} \) be parallel to the horizontal & vertical edges of the diagram respectively.

Let the co-ordinates of Roma be \( R(-12, 1) \).

a) Find the equation of the circle, with centre Ipswich, such that Roma is on the circumference of the circle.

b) Mt Morgan is also on the Circumference of the circle. Determine Mt Morgan's co-ordinates. [Approximate answers within reasonable limits are acceptable]
Question Eight (MP)

Two matrix transformations are applied to the shape as in the diagram below.

Find a single matrix transformation that would take Image 2 back to the object.

Question Nine (MP)

Above is a grid map of a city in which two spies are secretly hiding out with radio receivers, awaiting instructions from their minder. Each grid line is 1 km apart.

The minder has a transmitter which only generates decipherable signals in ratios of 2:1, for any desired distance, to within 20 km.

The transmissions are scrambled for all other positions to avoid detection.

a) Determine all the positions the minder could send a signal so that both receivers can get the signals simultaneously.

b) Use your model to determine the two positions on the grid, at the furthest distance apart from each other, that the minder could be yet still get signals to both spies simultaneously.
**Instrument-specific criteria and standards**

Student responses have been matched to instrument-specific criteria and standards; those that best describe the student work in this sample are shown below. For more information about the syllabus dimensions and standards descriptors, see www.qsa.qld.edu.au/1896.html#Assessment.

<table>
<thead>
<tr>
<th>Standard A</th>
<th>Standard C</th>
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<tbody>
<tr>
<td>Knowledge and procedures</td>
<td>The student work has the following characteristics:</td>
</tr>
<tr>
<td>• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations</td>
<td>• recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations</td>
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<tr>
<td>• application of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations</td>
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<tr>
<td>• numerical calculations, spatial sense and algebraic facility in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations</td>
<td>• numerical sense, spatial sense and algebraic facility in routine, simple life-related or abstract situations</td>
</tr>
<tr>
<td>• appropriate selection and accurate use of technology</td>
<td>• selection and use of technology</td>
</tr>
</tbody>
</table>

| Modelling and problem solving | The student work has the following characteristics: |
| • use of problem-solving strategies to interpret, clarify and analyse problems to develop responses from routine simple tasks through to non-routine complex tasks in life-related and abstract situations | • use of problem-solving strategies to interpret, clarify and develop responses to routine, simple problems in life-related or abstract situations |
| • identification of assumptions and their associated effects, parameters and/or variables | |
### Standard A

**Communication and justification**

The student work has the following characteristics:

- appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations
- organisation and presentation of information in a variety of representations
- analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine
- use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language
- coherent, concise and logical justification of procedures, decisions and results
- justification of the reasonableness of results

### Standard C

The student work has the following characteristics:

- appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations
- organisation and presentation of information
- translation of information from one representation to another in simple routine situations
- use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language
- justification of procedures, decisions or results
Student response — Standard A

The annotations show the match to the instrument-specific standards.

Comments

Algebraic facility in a routine, simple abstract situation

Appropriate interpretation and use of mathematical terminology, symbols and conventions in a routine, simple abstract situation

\[ \vec{a} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} \]

\[ \vec{a} \cdot \vec{b} = \vec{a}_1 \cdot \vec{b}_1 + \vec{a}_2 \cdot \vec{b}_2 + \vec{a}_3 \cdot \vec{b}_3 \]

\[ = -1 \cdot 2 + 3 \cdot 3 + 2 \cdot 3 \]

\[ = 14 \]

\( i \) Angle between \( \vec{a} \) and \( \vec{b} \).

\[ \vec{a} \cdot \vec{b} = \frac{\vec{a}_1 \cdot \vec{b}_1}{|\vec{a}| \cdot |\vec{b}|} \cos \theta \]

\[ \vec{a} \cdot \vec{b} = \vec{a}_1 \cdot \vec{b}_1 \]

\[ = -1 \cdot 2 + 3 \cdot 3 + 2 \cdot 3 \]

\[ = 14 \]

\[ \cos \theta = \frac{\vec{a}_1 \cdot \vec{b}_1}{|\vec{a}| \cdot |\vec{b}|} \]

\[ \cos \theta = \frac{\vec{a}_1 \cdot \vec{b}_1}{|\vec{a}| \cdot |\vec{b}|} \]

\[ \theta = \cos^{-1} \left( \frac{-1 \cdot 2 + 3 \cdot 3 + 2 \cdot 3}{\sqrt{(-1)^2 + 3^2 + 2^2} \cdot \sqrt{(-2)^2 + 5^2 + 3^2}} \right) \]

\[ \theta = \cos^{-1} \left( \frac{14}{\sqrt{14} \cdot \sqrt{38}} \right) \]

\[ \theta = 101.45^\circ \text{ or } 1.7706^\circ \]
<table>
<thead>
<tr>
<th>Comments</th>
</tr>
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<tr>
<td><strong>Application of mathematical definitions, rules and procedures in a routine, simple abstract situation</strong></td>
</tr>
<tr>
<td>Let ( \mathbf{L} = \begin{bmatrix} a &amp; b \ c &amp; d \end{bmatrix} ) be the transformation which causes ( A \to A' ).</td>
</tr>
<tr>
<td>( a = a' )</td>
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<td>( \Rightarrow \begin{bmatrix} a &amp; b \ c &amp; d \end{bmatrix} \begin{bmatrix} 1 \ 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \ \frac{5}{2} \end{bmatrix} )</td>
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<tr>
<td>( \Rightarrow a + 2b = \frac{5}{2} )</td>
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<tr>
<td>( \Rightarrow c + 2d = \frac{5}{2} )</td>
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</tr>
<tr>
<td>( \Rightarrow 2a + b = \frac{5}{2} )</td>
</tr>
<tr>
<td>( \Rightarrow 2c + d = \frac{5}{2} )</td>
</tr>
<tr>
<td>Unnecessary step</td>
</tr>
</tbody>
</table>

**Question 2**

(a) (Refer to Exam diagram)

- \( A(1, 2) \to A'(\frac{5}{2}, 5) \)
- \( B(2, 1) \to B'(5, \frac{5}{2}) \)

- Let \( \mathbf{L} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \) be the transformation which causes \( A \to A' \).

- \( a = a' \)

- \( \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ \frac{5}{2} \end{bmatrix} \)

- \( \Rightarrow a + 2b = \frac{5}{2} \)

- \( \Rightarrow c + 2d = \frac{5}{2} \)

- \( b = b' \)

- \( \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ \frac{5}{2} \end{bmatrix} \)

- \( \Rightarrow 2a + b = \frac{5}{2} \)

- \( \Rightarrow 2c + d = \frac{5}{2} \)

- \( \Rightarrow a + 2b = \frac{5}{2} \) - (iv)

- \( \Rightarrow c + 2d = \frac{5}{2} \) - (iii)

- \( \Rightarrow a + 2b + 0c + 0d = \frac{5}{2} \) - (iv)
Comments

Application of mathematical definitions, rules and procedures in a routine, simple abstract situation

- Co-efficient Matrix \( R \) = \[
\begin{bmatrix}
1 & 2 & 0 & 0 \\
0 & 0 & 1 & 2 \\
2 & 1 & 0 & 0 \\
0 & 0 & 2 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
a \\
b \\
c \\
d
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 2 \\
2 & 1 & 0 \\
3 & 2 & 0
\end{bmatrix}
\]

- \[
\begin{bmatrix}
\frac{1}{3} & 0 & \frac{2}{3} & 0 \\
0 & \frac{1}{3} & 0 & \frac{2}{3} \\
0 & 0 & \frac{1}{3} & 0 \\
0 & 0 & 0 & \frac{1}{3}
\end{bmatrix}
\]

- \[
\begin{bmatrix}
a \\
b \\
c \\
d
\end{bmatrix}
= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

- \[
\begin{bmatrix}
\frac{1}{\sqrt{3}} & 0 \\
0 & \frac{1}{\sqrt{3}}
\end{bmatrix}
\]

The transformation required to transform image A into image B is a dilation about the Origin.
b) Quadrilateral \( O(0,0), B(3,1), C(8,2), D(0,2) \) is \( \) parallel to the \( y \)-axis by a factor of \( 3 \)

\[-\text{ Shear Matrix } S_{xy} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \]

\[-Q' \rightarrow 0\]

\[-Q' = S_{xy} Q \]

\[-\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow O'(0,0)\]

\[-B' \rightarrow B \]

\[-k' = S_{xy} k \]

\[-\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \end{bmatrix} \rightarrow B'(3,10)\]

\[-P' \rightarrow P \]

\[-\xi' = S_{xy} \xi \]

\[-\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix} \rightarrow C'(3,11)\]
Coherent, concise and logical justification of procedures, decisions and results

\[ \begin{align*}
\text{Question 2 (b) cont.} \\
\therefore \quad D' \\
\therefore \quad y' = 2 \times y \\
\therefore \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} = D'(0,2)
\end{align*} \]

The image of the quadrilateral \( OBCD \) under

\[ \begin{align*}
\text{where } & \quad O'(0,0) \\
& \quad B'(3,10) \\
& \quad C'(3,11) \\
& \quad D'(0,2)
\end{align*} \]
Question 2 Cont.

(c) To find the area of $OBCD$ transformed under $E$ by using determinants we first must find the area of $OBCD$ so the determinant of the matrix which was used to transform a shape is the factor by which the shapes area has increased.

\[ \text{Area of } OBCD \]

\[ \begin{array}{c}
\text{P}(0, a) \\
\text{D}(0,0) \\
\text{E}(3, 0) \\
\text{C}(3, a) \\
\text{B}(3, 1) \\
\end{array} \]

To find area of $OBCD$ take $\triangle OEB$ from rectangle $OECB$.

\[ \text{Area of } OECB \]

\[ \begin{align*}
OE &= (3-0), (0-0) = 3 \text{ units} \\
ED/DB &= (3-3), (3-0) = 2 \text{ units}.
\end{align*} \]

\[ \text{A. of } OECB = 1 \times 1 \times 2 = 2 \text{ units}. \]
Application of mathematical definitions, rules and procedures in a routine, simple abstract situation

Appropriate interpretation and use of mathematical terminology, symbols and conventions in a routine, simple abstract situation

Analysis and translation of information from one representation to another in a routine, simple abstract situation

Coherent, concise and logical justification of procedures, decisions and results
Question 3.

\( t_n = \{87, 9, 3, 1, \frac{3}{7}, \frac{1}{2}, \ldots \} \)

a) Find \( r \) and \( t_q \).

\( r = \frac{3}{7} \times \frac{9}{2} = \frac{27}{14} \neq \frac{3}{7} \)

\( t_n = a \times r^{n-1} \)

\( \Rightarrow t_n = 27 \times \left(\frac{3}{7}\right)^{n-1} \)

\( t_q = 27 \times \left(\frac{1}{2}\right)^{\infty} = \frac{1}{243} \)

b) If \( |r| < 1 \), \( \exists S_\infty = \frac{a}{1-r} \)

\( r = \frac{3}{7}, a = 27 \)

\( S_\infty = \frac{27}{1-\left(\frac{3}{7}\right)} = 40.5 \)

The sum to infinity = 40.5.
Comments

Recall, access, selection of mathematical definitions, rules and procedures in a routine task in a life-related situation

Use of mathematical reasoning to develop coherent, concise and logical sequences within a response to a complex, life-related situation using mathematical language

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**Question 4**

\[ P(t) = 100 \times 1.20^t \]

a) \[ P(6) = 100 \times 1.20^6 \]

\[ = 298.6 \]

\[ \approx 299 \text{ amoebae} \]

b) Find when \( P = 1000 \,000 \) amoebae.

\[ 1000 \,000 = 100 \times 1.20^t \]

\[ \Rightarrow 10 \,000 = 1.20^t \]

\[ \Rightarrow t = \log_{1.20} (10 \,000) \]

\[ \approx 50.52 \text{ mins} \]

- After 50 mins and 31 seconds the number of amoebae will be 1000000 (or in the 51 minute they will have exceeded 1000000).
Question 5

a) \[ |z - 2 + i| \leq 3. \]

- Let \( z = x + iy \).

- \( z - 2 + i = (x - 2) + (y + 1)i \).

- \[ |z - 2 + i| = \sqrt{(x - 2)^2 + (y + 1)^2} \]

- \[ (x - 2)^2 + (y + 1)^2 \leq 3 \]

\( \Rightarrow \) Centre \((2, -1)\), \( r = 3 \).

Recall, access, selection of mathematical definitions, rules and procedures in a non-routine, simple task in an abstract situation.
Comments

Appropriate selection and accurate use of technology
Question 5 cont.

Part (b) i) \[
\text{Let } z = x + ai. \quad (\text{In diagram}).
\]

- Take the centre \( C(x, y) \) as the complex no. origin.
- \( z - 2 = 0 + ai \)
- \( z - 2 + i = 0 + 3i \)
- \( |z - 2 + i| = \sqrt{3^2} = 3 \)

- The complex no. satisfies \( |z - 2 + i| = 3 \)
- because when the modulus of \( z - 2 + i \) was found it equaled \( 3 \), hence it belonged to the circle \( (x - 2)^2 + (y + 1)^2 = 3 \) or \( |z - 2 + i| = 3 \), meaning that it satisfied the requirement.

\* Note that the complex nos. have been translated hence if the centre of the circle was moved to the origin then the complex nos.' actual values would apply.
Question 6

$P(x,y)$ is the bary of points s.t.

$2\overrightarrow{PA} = 3\overrightarrow{PB}$

where $A(-3,5), B(-10)$

$\therefore 4\overrightarrow{PA} = 9\overrightarrow{PB}$

$\therefore PA^2 = (x+3)^2 + (y-5)^2$

$\therefore PB^2 = (x+10)^2 + y^2$

$\therefore 4[ (x+3)^2 + (y-5)^2 ] = 9 \left[ (x+10)^2 + y^2 \right]$

$\Rightarrow 4[x^2 + 6x + 9 + y^2 - 10y + 25] = 9[x^2 + 20x + 100 + y^2]$

$\Rightarrow 4x^2 + 24x + 10y^2 - 10y + 136 = 9x^2 + 180x + 9y^2 + 90$

$\Rightarrow 4x^2 + 24x + 10y^2 - 10y + 136 = 9x^2 + 180x + 9y^2 + 90$

$\Rightarrow 127 = -5x^2 - 60x + 5y^2 + 40y$

$\therefore x^2 - \frac{60}{5}x + y^2 + 8y = \frac{127}{5}$

$\Rightarrow x^2 - \frac{6}{5}x + \frac{9}{5} + y^2 + 8y + 16 = \frac{127}{5} + \frac{9}{5} + 16$

$\Rightarrow (x - \frac{3}{5})^2 + (y+4)^2 = \frac{109}{5}$

$\Rightarrow C = (x - \frac{3}{5})^2 + (y+4)^2 = \frac{6.19}{5}$

$\therefore$ Centre $C \left( \frac{3}{5}, -4 \right)$, $r = \sqrt{\frac{6.19}{5}}$
Question 7

a) Ipswich is the origin, & Roma is at (-12, 1)

If R(-12,1) is on the circumference of a circle with a centre at Ipswich the radius of the circle is the distance between R and I.

\[ d = \sqrt{(12-0)^2 + (0-1)^2} \]
\[ = \sqrt{145} \text{ radius} \]

- Take general circle equation.

\[ (x-h)^2 + (y-k)^2 = r^2 \]

where, \( P(h,k) \) is the centre.

- If the centre = I(0,0) \( r = \sqrt{145} \)

\[ x^2 + y^2 = 145 \]
b) \[ \text{radius} = \sqrt{145} \text{ units, however on the map x is } \approx 53 \text{ mm}. \]

\[ \Rightarrow x^2 + y^2 = 53^2. \]

Also point R(-12,1) is equal to R(50mm, 4.5mm)

\[ \Rightarrow 1 \text{ unit } \approx 4.5 \text{ mm}. \]

\[ \Rightarrow \text{On the map, Mt. Morgan is } 17.5 \text{ mm to the left of the x-axis and } 47.5 \text{ mm above the x-axis.} \]

\[ \Rightarrow M(-17.5, 47.5) \]

\[ \Rightarrow \text{If } 1 \text{ unit } \approx 4.5 \text{ mm.} \]

\[ \Rightarrow \begin{align*} x &= -17.5 + 4.5 \\ &\approx -3.89 \end{align*} \]

\[ \Rightarrow \begin{align*} y &= 47.5 - 4.5 \\ &\approx 10.56 \end{align*} \]

\[ \Rightarrow M(-3.89, 10.56) \]

\[ \Rightarrow \text{Mt. Morgan is located at } (-3.89, 10.56) \]
Comments

Use of problem-solving strategies to interpret, clarify and analyse problems in a routine, complex abstract situation

Appropriate interpretation and use of mathematical terminology, symbols and conventions in a routine, complex, abstract situation

Question 8

The transformation that took the object to Image 1 was a reflection in the line y = 0.

\[ R_{x=0} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \]

The transformation that took Image 1 to Image 2 was a shear parallel to the x-axis with a factor of 3.

\[ S_{x=3} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \]

To go from the object to Image 2 in one step the following matrix should be applied:

\[ S_{x=3} \times R_{x=0} = A \]

\[ A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix} \]

To get back from Image 2 to the object in one step the inverse of A should be applied.

\[ A^{-1} = \frac{1}{-1^2 - 3^2} \begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix} = \frac{1}{-10} \begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/10 & 3/10 \\ 0 & 1 \end{bmatrix} \]

The transformation that would get Image 2 back to the object is \( A^{-1} \).
Use of problem-solving strategies to interpret, clarify and analyse problems in a non-routine, complex task in a life-related situation

Use of mathematical reasoning to develop coherent, concise and logical sequences within a response to a complex, non-routine task in a life-related situation using mathematical language

Question 1

Let $P(x, y)$ be the centre of circle $C$. Therefore, $A(0, 0)$ is on the circumference of the circle.

Let $B(2, 2) \notin C$. But $B$ is not on the circumference.

General circle: $C = (x - x_c)^2 + (y - y_c)^2 = r^2$.

For $C_2$, the equation is $r(x - x_2)^2 + (y - y_2)^2 = r^2$.

Let $A \in C_1$

+ Let $B \in C_2$

$x_2 = x_c$, $y_2 = y_c$

$\Rightarrow$ Sub $A$ into $C_1$:

$x_c^2 + y_c^2 = r^2$

$\Rightarrow 4x_c^2 + 4y_c^2 = 4r^2$ (1)

$\Rightarrow$ Sub $B$ into $C_2$:

$(x - x_2)^2 + (y - y_2)^2 = (2r)^2$

$\Rightarrow$ Take (1) from (2)

This gives one of the desired circles.

$(x - x_c)^2 - 4x_c^2 + (y - y_c)^2 - 4y_c^2 = 0$

$\Rightarrow 4x_c - x_c^2 - 4y_c + y_c^2 - 4y_c = 0$

Then $4x_c - x_c^2 - 4y_c + y_c^2 - 4y_c = 0$ (constant on both sides).
Question 9 cont

The marker could be on a circle with centre at \( P(-2,-2) \) with a radius of 4.

\[
\begin{align*}
8x_c^2 - 4x_c - 3y_c^2 - 4y_c &= -8 \\
\Rightarrow x_c^2 + \frac{3}{2}x_c + y_c^2 + \frac{3}{2}y_c &= \frac{8}{3} \\
\Rightarrow (x_c + \frac{3}{2})^2 + (y_c + \frac{3}{2})^2 &= \frac{32}{9}
\end{align*}
\]

\[
\Rightarrow \text{The marker could be on a circle with centre at } \left( -\frac{3}{2}, -\frac{3}{2} \right) \text{ of a radius of } \frac{4\sqrt{2}}{3}.
\]
Comments

Use of problem-solving strategies to interpret, clarify and analyse problems to develop a response in a non-routine complex task in a life-related situation

Use of mathematical reasoning to develop coherent, concise and logical sequences within a response to a complex, life-related situation using mathematical language

### Question 1

(b) The points A and B lie on the line $y = x$

To find the max distance, we need to find when $y = x$ intersects the circle $(x + \frac{3}{2})^2 + (y + \frac{3}{2})^2 = \left(\frac{5}{3}\right)^2$

- Sub $(x, y)$ into the circle equation

- $(x + \frac{3}{2})^2 + (x + \frac{3}{2})^2 = \left(\frac{5}{3}\right)^2$

- $2(x + \frac{3}{2})^2 = \left(\frac{5}{3}\right)^2$

- $2(x^2 + \frac{3}{2}x + \frac{9}{4}) = \frac{25}{9}$

- $2x^2 + 3x + \frac{9}{2} = \frac{25}{9}$

- $2x^2 + 3x - \frac{17}{18} = 0$

- $a = 2, b = \frac{3}{2}, c = -\frac{17}{18}$

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- $x = \frac{-\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - 4 \times 2 \times -\frac{17}{18}}}{2 \times 2}$

- $x = \frac{2}{3}$

- $x = -\frac{2}{3}$
Coherent, concise and logical justification of procedures, decisions and results.

Question 9 Cont.

- Substitute back into (2).
  - \( y = \left( \frac{a}{b} \right) \)
  - \( y = \left( \frac{2}{3} \right) \)

- The two points are \( P_1 \left( \frac{2}{3}, \frac{2}{3} \right) \) and \( P_2 \left( \frac{2}{3}, \frac{2}{3} \right) \).

Not quite!
Identification of parameters and variables in a non-routine task in a life-related situation

Use of problem-solving strategies to interpret, clarify and analyse problems to develop a response in a non-routine complex task in a life-related situation

Question 10

Minute hand = \(360^\circ/h\)
Hour hand = \(360^\circ/12h = 30^\circ/h\).

- When minute moves 1°, hour moves \(\frac{1}{12}\)°.

6. First, move 60°. (Check)
   - But, hour hand hasn’t moved 5 1/2 more.
   - Minute hand 60°. But, hour hand has moved 5° more.

- Forms a geometric sequence

\[ a = 60, \quad r = \frac{1}{12} \]

\[ S_n = \frac{60}{1 - \frac{1}{12}} = \frac{60}{\frac{11}{12}} = \frac{720}{11} = 120 \text{ minutes} \]

- \(S_0 = 64.45° \div 720^\circ/h = 30^\circ/h\) to find

- \(t = 2.648\) hours.

This is correct time since the minute hand is not equal to the hour hand.

The time is 2:36:48.

Time = 2 hours and 8 minutes. 5:30 less 2:36:48 = 2:36.48.
Student response — Standard C

The annotations show the match to the instrument-specific standards.

<table>
<thead>
<tr>
<th>Comments</th>
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<tbody>
<tr>
<td>Numerical sense, spatial sense and algebraic facility in a routine, simple, abstract situation</td>
</tr>
<tr>
<td>Appropriate interpretation and use of mathematical terminology, symbols and conventions in a routine, simple, abstract situation</td>
</tr>
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Question One:

\[
\begin{align*}
\text{a) } & \mathbf{x} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} -2 \\ -4 \\ 3 \end{bmatrix} \quad \mathbf{x} \cdot \mathbf{y} = \left( -1 \times 3 + 3 \times -4 + 2 \times 3 \right) \\
& = -14 \\
\text{b) } & |\mathbf{x}| = \sqrt{(-1)^2 + 3^2 + 2^2} = \sqrt{14} \\
& |\mathbf{y}| = \sqrt{(-2)^2 + (-4)^2 + 3^2} = \sqrt{29} \\
& \therefore \cos \theta = \frac{-4}{\sqrt{14} \cdot \sqrt{29}} \\
& = -\frac{4}{\sqrt{406}} \\
\Rightarrow & \quad \theta = \cos^{-1}\left( -\frac{4}{\sqrt{406}} \right) \\
& = 101.45^\circ \\
\end{align*}
\]
Question Two

a) \( \begin{bmatrix} 5/2 & 0 \\ 0 & 5/2 \end{bmatrix} \)

b) \( O(0,0) \rightarrow (3,1), (3,2) \rightarrow (0,2) \)
(See print out)

c)
Question Two

Part B

Shear parallel to the y axis, with a shear factor of 3

Original shape = O(0,0) A(3,1) B(3,2) C(0,2)

Image after shear = O(0,0) A'(3,3) B'(3,6) C'(0,6)

Comments

Appropriate selection and accurate use of technology
Comments

Recall, access, selection of mathematical definitions, rules and procedures in a routine task in an abstract situation

Question Three:

\[ \text{a) } a = 27, \quad r = \frac{9}{27} = \frac{1}{3} \]

9th term = \( a \times r^8 \)
\[ = 27 \times \left( \frac{1}{3} \right)^8 \]
\[ = \frac{1}{243} \]

b) \[ S_n = \frac{a(1-r^n)}{1-r} \]
\[ = \frac{27}{1-(\frac{1}{3})} = \frac{80}{3} \times \text{Calculation error} \]
\[ = \frac{27}{2/3} = 27 \times \frac{3}{2} = \frac{81}{2} \]

Question Four:

\[ P(t) = 100 \times 1.20^t \]

a) \[ P(6) = 100 \times 1.20^6 \]
\[ = 298.60 \quad \text{(299)} \]

b) \[ \text{Logarithms required!} \]

\[ 1000 \,000 \times (t) = 100 \times 1.20^t \]
\[ \sqrt[10]{1000 \,000} = 91.29 \]
Comments

Use of problem-solving strategies to interpret and develop responses to a routine, simple situation

Question Seven

Ipswich = (0,0) Roma R (−12, 1)

a) \((x-0)^2 + (y-0)^2 = 12\) \((12,0)\)

b) \((-12,13)\)

Question Eight

Image 1 = Reflection in the y axis of object

Image 2 = Shear along the x axis with a factor of 3 of Image 1

Matrix transformation = \[
\begin{pmatrix}
-1 & 0 \\
0 & 3 \\
\end{pmatrix}
\] (Answer through trial and error: okay!)