Mathematics C (2008)
Sample assessment instrument

Supervised assessment
March 2010
**Purposes of assessment**

The purposes of assessment are to:

- promote, assist and improve student learning
- inform programs of teaching and learning
- provide information for those people — students, parents, teachers — who need to know about the progress and achievements of individual students to help them achieve to the best of their abilities
- provide information for the issuing of certificates of achievement
- provide information to those people who need to know how well groups of students are achieving (school authorities, the State Minister for Education and Training and the Arts, the Federal Minister for Education).

It is common practice to label assessment as being formative, diagnostic or summative, according to the major purpose of the assessment.

The major purpose of formative assessment is to help students attain higher levels of performance. The major purpose of diagnostic assessment is to determine the nature of students' learning, and then provide the appropriate feedback or intervention. The major purpose of summative assessment is to indicate the achievement status or standards achieved by students at a particular point in their schooling. It is geared towards reporting and certification.

**Syllabus requirements**

Teachers should ensure that assessment instruments are consistent with the requirements, techniques and conditions of the Mathematics C (2008) syllabus.

**Assessment instruments**

High-quality assessment instruments:

- have construct validity (the instruments actually assess what they were designed to assess)
- have face validity (they appear to assess what you believe they are intended to assess)
- give students clear and definite instructions
- are written in language suited to the reading capabilities of the students for whom the instruments are intended
- are clearly presented through appropriate choice of layout, cues, visual design, format and choice of words
- are used under clear, definite and specified conditions that are appropriate for all the students whose achievements are being assessed
- have clear criteria for making judgments about achievements (these criteria are shared with students before they are assessed)
- are used under conditions that allow optimal participation for all
- are inclusive of students’ diverse backgrounds
- allow students to demonstrate the breadth and depth of their achievements
- only involve the reproduction of gender, socioeconomic, ethnic or other cultural factors if careful consideration has determined that such reproduction is necessary.
Mathematics C (2008)

Sample assessment instrument

Supervised assessment

Compiled by the Queensland Studies Authority
March 2010

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About this assessment instrument

This sample demonstrates:

• construction of a supervised assessment instrument
• construction of the associated instrument-specific criteria sheet

This sample assessment instrument is intended to be a guide to help teachers plan and develop assessment instruments for individual school settings.
PART A

Question 1 (K & P)

Solve the following for the unknown matrix $X$ (Show all working)

(a) \[ 3X - 4 \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \]

(b) \[ \begin{bmatrix} 4 & -1 \\ 3 & 5 \end{bmatrix} X + 2 \begin{bmatrix} 0 & 2 \\ -2 & -2 \end{bmatrix} X = \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix} \]

Question 2 (K & P)

(a) Using your graphics calculator find the inverse of:
\[ \begin{bmatrix} 1 & -2 & 0 & 0 \\ 3 & 1 & -4 & 1 \\ 1 & -3 & 2 & 0 \\ 3 & 1 & 2 & -4 \end{bmatrix} \]

(b) Use matrix methods to solve the following set of simultaneous equations:
\[
\begin{align*}
3x + 2y - 3z &= 7 \\
2x - y - z &= 4 \\
3x + 4y + z &= 1 
\end{align*}
\]

(c) Find the real values of $k$ for which the matrix product
\[ \begin{bmatrix} k & k \\ 4 & 2k \end{bmatrix} \begin{bmatrix} 2 & k \\ 4 & 3 \end{bmatrix} \]

is singular.

Question 3 (K & P)

Let \[ A = \begin{bmatrix} i & 2i & -i \\ 0 & 1 & 3i \end{bmatrix} \] and \[ B = \begin{bmatrix} 2 & -i \\ -3 & i \\ 2i & 0 \end{bmatrix} \]. Show that \((AB)^T = B^T A^T\).

Question 4 (M & P)

Suppose the financial advisor of a university’s endowment fund must invest exactly $1.3$ million in three types of securities. These are AAA bonds paying a dividend of 7%, BB stocks paying a dividend of 9%, and CC fixed-term deposits paying a dividend of 8%. The advisor has been told that the amount invested in AAA bonds must be twice the amount invested in CC fixed-term deposits.

How much money should be invested in each form of investment so that the endowment fund has an annual income of $105,000?$
<table>
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<tr>
<th>Standard A</th>
<th>Standard B</th>
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PART B

Question 5 (K & P)

(a) How many 3-digit numbers can be formed from the set \{0,3,4,7,8\}?
(b) In how many ways can 8 boys and 5 girls stand in a line if all the girls must stand next to each other and all the boys must stand next to each other.

Question 6 (K & P)

(a) How many four (4) letter codes can be made from the letters of the word COMPLEX if:
   (i) no letter can be used more than once?
   (ii) the letters can be reused?
   (iii) the letter codes must start with CO
   (iv) the letter codes do not start with CO.

(b) How many different ways can the letters of the word COOLANGATTA be arranged?

(c) Find the number of ways 7 people can be arranged around a circular table if:
   (i) there are no restrictions
   (ii) 3 particular people must always sit together
   (iii) 3 particular people must be separated.

Question 7 (K & P)

Calculate how many ways 3 student representative council members can be selected from 15 senior students if:
   (a) 1 particular student is included in every selection
   (b) 2 of the students are excluded from every selection
   (c) 1 is always included, 2 are always excluded

Question 8 (M & P)

The digits 1,2,3,4,5,6,7,8,9 are to be used to form single matrices. Without writing out all the possibilities, find:

(a) how many different square matrices are possible if all the digits are used?
(b) how many different matrices are possible if all the digits are used?
(c) how many different square matrices are possible if not all of the nine digits are used and no elements are repeated?
### Knowledge and procedures

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- evidence of simple mathematical procedures |

### Modelling and problem solving

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PART C

Question 9 (K&P)

(a) Find the values of $z$ that satisfy
$$z^4 = -16i$$

(b) By finding the solutions of $z^5 = 1$, show that
$$\cos \frac{2\pi}{5} + \cos \frac{4\pi}{5} = -\frac{1}{2}$$

Question 10 (K&P)

Given that the complex polynomial $P(z) = 2z^3 + 3z^2 + 8z - 5$ has a rational zero, solve $P(z) = 0$ for all values of $z$.

Question 11 (K & P)

Find the values of $z$ which satisfy $z^2 - (5 - i)z + (8 - i) = 0$

Question 12 (M & P)

Use de Moivres Theorem and the binomial expansion to show that
$$\cos 4\theta = 8\cos^4 \theta - 8\cos^2 \theta + 1$$
$$\sin 4\theta = 4\cos \theta (\sin \theta - 2\sin^3 \theta)$$
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