Purposes of assessment

The purposes of assessment are to:

- promote, assist and improve student learning
- inform programs of teaching and learning
- provide information for those people — students, parents, teachers — who need to know about the progress and achievements of individual students to help them achieve to the best of their abilities
- provide information for the issuing of certificates of achievement
- provide information to those people who need to know how well groups of students are achieving (school authorities, the State Minister for Education and Training and Training and the Arts, the Federal Minister for Education).

It is common practice to label assessment as being formative, diagnostic or summative, according to the major purpose of the assessment.

The major purpose of formative assessment is to help students attain higher levels of performance. The major purpose of diagnostic assessment is to determine the nature of students' learning, and then provide the appropriate feedback or intervention. The major purpose of summative assessment is to indicate the achievement status or standards achieved by students at a particular point in their schooling. It is geared towards reporting and certification.

Syllabus requirements

Teachers should ensure that assessment instruments are consistent with the requirements, techniques and conditions of the Mathematics C syllabus and the implementation year 2008.

Assessment instruments

High-quality assessment instruments:

- have construct validity (the instruments actually assess what they were designed to assess)
- have face validity (they appear to assess what you believe they are intended to assess)
- give students clear and definite instructions
- are written in language suited to the reading capabilities of the students for whom the instruments are intended
- are clearly presented through appropriate choice of layout, cues, visual design, format and choice of words
- are used under clear, definite and specified conditions that are appropriate for all the students whose achievements are being assessed
- have clear criteria for making judgments about achievements (these criteria are shared with students before they are assessed)
- are used under conditions that allow optimal participation for all
- are inclusive of students’ diverse backgrounds
- allow students to demonstrate the breadth and depth of their achievements
- only involve the reproduction of gender, socioeconomic, ethnic or other cultural factors if careful consideration has determined that such reproduction is necessary.

2 Assessment instruments are the actual tools used by schools and the QSA to gather information about student achievement, for example, recorded observation of a game of volleyball, write-up of a field trip to the local water catchment and storage area, a test of number facts, the Senior External Examination in Chinese, the 2006 QCS Test, the 2008 Year 4 English comparable assessment task.
About this assessment instrument

The purpose of this document is to inform assessment practices of teachers in schools. For this reason, the assessment instrument is not presented in a way that would allow its immediate application in a school context. In particular, the assessment technique is presented in isolation from other information relevant to the implementation of the assessment. For further information about those aspects of the assessment not explained in this document, please refer to the assessment section of the syllabus.

This sample provides opportunities for students to demonstrate the ability to:

- recall, access, select and apply mathematical definitions, rules and procedures
- apply problem-solving strategies and procedures to identify problems to be solved and interpret, clarify and analyse problems
- modify mathematical models as appropriate
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models
- interpret and use appropriate mathematical terminology, symbols and conventions
- develop coherent, concise and logical sequences within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions or propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results.

This sample assessment instrument is intended to be a guide to help teachers plan and develop assessment instruments for individual school settings.
Assessment instrument

The student work presented in this sample is in response to an assessment item which is a subset or part of an assessment instrument.

**Question**

How long will it take to break into a line of traffic at a “T” intersection?
Instrument-specific criteria and standards

Schools draw instrument-specific criteria and standards from the syllabus dimensions and exit standards. Schools will make judgments about the match of qualities of student responses with the standards descriptors that are specific to the particular assessment instrument. While all syllabus exit descriptors might not be assessed in a single assessment instrument, across the course of study, opportunities to demonstrate all the syllabus dimensions and standards descriptors must be provided.

The assessment instrument presented in this document provides opportunities for the demonstration of the following criteria:

- Knowledge and Procedures
- Modelling and Problem Solving
- Communication and Justification

This document provides information about how the qualities of student work match the relevant instrument-specific criteria and standards at standards A and C. The standard A and C descriptors are presented below. The complete set of instrument-specific criteria and standards is on pages 17–18.

<table>
<thead>
<tr>
<th>Standard A</th>
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The student’s work has the following characteristics:

• use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language
• justification of procedures, decisions or results

Key

Differences or additional requirements for demonstrating the standard.

Differences in complexity of task requirements for each standard.
## Sample student responses: Standard A

<table>
<thead>
<tr>
<th>Standard descriptors</th>
<th>Student response A</th>
</tr>
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**Use of mathematical reasoning to develop sequences within a response in a simple routine situation using everyday or mathematical language**

**Question?**

For this problem, a probabilistic model would be most suited.

As with any probabilistic model, an appropriate random variable must be established. An appropriate random variable would be the size of the gap between motor vehicles, this gap will be measured in seconds, rather than meters, as each motor vehicle has different braking, handling and accelerating capabilities.

Before a probabilistic model can be established, several assumptions must be made. These assumptions are that:

- The car at the ‘$T$’ intersection requires 6 seconds to break into the line of traffic
- On average, cars are 4 seconds apart
- There are no accidents which cause the gaps between cars to decrease
- The car turns left out of the intersection so that it only has to work for a gap in one line of traffic as shown in the diagram below:

```
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  |               |
  |               |
  |               |
  |               |
  |               |
  ---------------

  \[ \text{Journey of the car at the 'T' intersection} \]
```

Consider a rectangular distribution:

On average, cars are 4 seconds apart, therefore the mean gap between cars is 4 seconds, or the expected value between cars is 4 seconds.

\[ \mu = E(x) = 4 \text{ seconds} \]
Sample student responses: Standard A

Application of mathematical definitions, rules and procedures in routine, simple life-related situations

For the car to break into the line of traffic, the gap must be bigger than 6 seconds (5 seconds to get out and a one second gap to be left for safety).

Therefore, $P(x > 6) =$ probability car will get out.

Assuming that cars cannot get closer than 1 second apart, to allow for safe breaking, the maximum distance they are apart can be calculated.

$E(x) = \frac{b^2-a^2}{2(b-a)} = \frac{(b-a)(b-a)}{2(b-a)} = \frac{(b+a)}{2}$

Therefore, $E(x) = \frac{a+b}{2}$

$4 = \frac{1+b}{2}$

$b = 7/6$

With the rectangular distribution, cars can be a maximum of seven seconds apart.

Therefore, $P(x > 6) = \int_{b-a}^{6} \frac{1}{b-a} \, dx$

$= \left[ \frac{x}{b-a} \right]_{6}^{b} = \frac{7}{6} - \frac{6}{b-a} = \frac{1}{6}$/
The probability that the car will get out is \( \frac{1}{6} \), therefore the car expects to get out after every six cars pass, however this is only the expected value. The car may have to wait longer.

To be 90% sure that the car will break into the line of traffic, a second probability distribution must be set up.

However, the appropriate random variable will now change. The problem asks “How long will it take...” therefore the solution required is a time.

As the car expects to wait for six cars to pass on average before it can get out, an appropriate random variable would be the time the car waits.

Assuming the car is in a built up area, it can be assumed that all approaching cars are travelling at 60 km\(^{-1}\). It will also be assumed that all cars are 4.4 meters in length.

In the sketch below, the approximate time it takes for six cars to pass has been calculated:

![Sketch](image)

The sketch is set up just as a car

As the cars are 4 meters long, and are travelling at 60 km\(^{-1}\), it will take one car: 4 seconds plus 4 meters ÷ \((60 ÷ 3.6)\) m\(^{-1}\) = 4.24 seconds.

Therefore one car takes 4.24 seconds approximately to pass the intersection.

The time it takes for 6 cars to pass will be 4.24 \(\times 6\) = 25.44 seconds.

Therefore, on average, it takes around 26 seconds for six cars to pass the intersection. So if the car waits 26 seconds, it will expect to get out.
Sample student responses: Standard A

Up to this point in the response, the student has basically followed a simple routine procedure and has met the requirements of Standard B/C descriptors depending on the quality of the response.

The descriptions that follow in this response indicate that the student has demonstrated elements of a Standard A, particularly in the general objective of “modify mathematical models as appropriate.”
Sample student responses: Standard A

Assumptions and their effects:

Firstly, it was assumed that there are no accidents or increased traffic from these factors would cause the distance between vehicles to decrease and could even cause the vehicles' average velocity to decrease. If either the average distance between vehicles or the average velocity of each vehicle were to change, the probability that the car will get out will also vary. Also, it was assumed that cars can get closer than 1 second apart, to allow for safe braking. If cars were allowed to travel closer than 1 second apart, then the probability that the car would get out would decrease, as the maximum distance apart ‘b’ would also decrease. Finally, it was assumed that cars are y metres in length; if a large truck were to pass the ‘x’ intersection, the average time the car expected to wait would increase, in turn increasing the time the car has to wait to be 90% sure that it can get out.

Strengths and Limitations:

There are some strengths and limitations of using a rectangular distribution. A strength of the rectangular distribution is that it assumes a finite interval, which means that cars cannot be 0 seconds apart. However, a limitation of the rectangular distribution is that it assumes the distance between cars is distributed evenly, whereas it is more likely that the distances are distributed normally.

Refining the model:

This model can be refined to consider another probabilistic model, such as an exponential distribution.

Consider an exponential distribution:

For this exponential distribution, all of the assumptions made before when using a rectangular distribution will be considered.

Therefore: \( E(X) = 4 \text{ seconds} \)
Refinement of mathematical models

Application of mathematical definitions, rules and procedures in non-routine complex task in a life-related situation

Sample student responses: Standard A

As with the rectangular distribution, the car required a gap in the line of traffic bigger than 6 seconds to get out.

Therefore, \( P(x > 6) \) = probability the car will get out

\[(0, \lambda)^t \]

\[ E(x) = 4 \]

the exponential distribution assumes that cars can get 0 seconds apart, but can also be an infinity of seconds apart.

Nonetheless, how does the \( E(x) \) relate to an exponential probability density function? For an exponential, \( P(x, t) \).

\[ f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x < 0 \end{cases} \]

Therefore, \( E(x) = \int_{0}^{\infty} x \cdot \lambda e^{-\lambda x} \, dx \)

\[ E(x) = -\lambda \int_{0}^{\infty} x \cdot e^{-\lambda x} \, dx \]

Integrating by parts:

\[ \int u \, v' \, dx = u \, v - \int u' \, v \, dx \]

Here, \( u = x \), \( v = e^{-\lambda x} \)

\[ v' = \frac{1}{\lambda} \]

\[ x \cdot e^{-\lambda x} \, dx = x \cdot e^{-\lambda x} \cdot \frac{-1}{\lambda} - \int \frac{-1}{\lambda} \cdot e^{-\lambda x} \, dx \]

\[ = -x \cdot e^{-\lambda x} + \frac{e^{-\lambda x}}{\lambda} \]

so \( \lambda \int_{0}^{\infty} x \cdot e^{-\lambda x} \, dx \)

\[ = \lambda \left[ \frac{-x \cdot e^{-\lambda x}}{\lambda} - \frac{e^{-\lambda x}}{\lambda^2} \right]_{0}^{\infty} \]
Application of mathematical definitions, rules and procedures in non-routine complex task in a life-related situation

When graphed, the exponential function will approach zero much faster than x will approach infinity.

\[
\lim_{x\to\infty} \left[ 0 \cdot e^{-\frac{1}{x}} \right] = \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}
\]

Therefore the relationship between the expected value and \( \lambda \) is \( \mu = \frac{1}{\lambda} \) for an exponential distribution.

As the expected value is equal to 4 seconds for this problem, \( 4 = \frac{1}{\lambda} \) so \( \lambda = \frac{1}{4} \).

Now the probability that \( X > 6 \) can be calculated.

\[
P(X > 6) = 1 - P(X \leq 6)
= 1 - \int_0^6 \lambda e^{-\lambda x} \, dx
= 1 - \left[ \frac{\lambda e^{-\lambda x}}{-\lambda} \right]_0^6
= 1 - \left[ e^{-\lambda x} \right]_0^6
= 1 - \left[ e^{-\frac{1}{4} \cdot 6} - e^{-\frac{1}{4} \cdot 0} \right]
= 1 - \left[ e^{-0.223} - 1 \right]
= 1 - 0.777
= 0.223
\]

The probability that the car will get out is 0.223. Therefore, out of every 1000 cars that pass the intersection, the car would expect to be able to break out into the line of traffic 223 times. However, this
Sample student responses: Standard A

Use of mathematical reasoning to develop coherent, concise and logical sequences within a response in a complex, life-related situation using everyday and mathematical language.

Coherent, concise and logical justification of procedures, decisions and results.

Is only the expected value, the car may have to wait longer.

To be 90% sure that the car will break into the line of traffic, a second probability distribution will need to be set up, as was the case with the rectangular distribution. The appropriate random variable will again be the time the car needs to wait to get out.

As previously calculated, it takes around 26 seconds for six cars to pass the intersection. Using the exponential distribution, the probability that the car will get is 0.223. However, since the probability is not in a fraction, it means the car expects to wait for 4 and a half cars to pass out of 6, which is not possible. Therefore, to simplify the model, the probability will be rounded down to 0.2 or 0.2161. Referring to the previous calculations, it will take $4.24 \times 5 = 21.2$ seconds for 5 cars to pass.

Therefore it takes around 22 seconds for 5 cars to pass the intersection, with the car expecting to get out to be 90% sure that the car will break into the line of traffic at the intersection; a second probability distribution will be established.

As previously calculated:

$E(x) = \frac{1}{\lambda}$

$\therefore \lambda = \frac{1}{22}$

To calculate the time, $a$, that the car must wait to be 90% sure that it will break into the line of traffic:

$0.9 = \int_{a}^{\frac{1}{22}} e^{-\frac{1}{22}x} \, dx$
Sample student responses: Standard A

Identification of assumptions and their associated effects

### Assumptions and their effects

As the same assumptions were made for both the rectangular distribution model and the exponential distribution model, the effects of these assumptions are very similar (refer to previous assumptions and their effects). The only assumption which differs from the rectangular distribution model is that the exponential distribution assumes cars can be zero seconds apart; but also an infinity of seconds apart. The effect of this assumption is that cars zero seconds apart will not be able to brake safely and will have to release their brakes, which will damage some cars. Also, cars cannot be an infinity of seconds apart due to traffic lights, intersections and roundabouts.

### Strengths and limitations

There are strengths and limitations of using an exponential distribution. An obvious limitation of the exponential distribution is that it assumes cars can be zero seconds apart, which is not possible as cars would have to drive along touching, which is
Sample student responses: Standard A

Investigation and evaluation of the validity of mathematical arguments including the strengths and limitations of a model

Highly unsafe. Another limitation of the exponential distribution is that it assumes cars can be an infinity of seconds apart; this situation will never happen as roads are only built where they are needed. Therefore, there is always at least two cars on the road that are a finite distance apart.

Validity of the model:
This probabilistic model which considers both a rectangular distribution and an exponential distribution can only be considered valid for the several assumptions made. This model cannot be considered valid for all cases. In order to extend the model for a general solution, experiments could be conducted to gather statistical data on the distance between cars. The calculations could then be repeated using this experimental data, and other distributions could be considered, such as a binomial distribution, to further refine the model.

A:
This is a very extensive deliberation covering all aspects of the topic. The student's understanding and approach to solving the problem is impressive, identifying strengths and limitations. The model is extended and generalized.

And all aspects of CED are included. Logical arguments are concise, recognising conflicting arguments.
### Instrument-specific criteria and standards

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<th>Standard D</th>
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Sample assessment instrument and student responses

Extended Modelling and Problem Solving
## Instrument-specific criteria and standards

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