Supervised assessment (2c): Topics include Periodic functions and applications, and Applied statistical analysis

This sample is intended to inform the design of assessment instruments in the senior phase of learning. It highlights the qualities of student work and the match to the syllabus standards.

Criteria assessed

- Knowledge and procedures
- Modelling and problem solving
- Communication and justification

Assessment instrument

The response presented in this sample is in response to an assessment task.

STAGE 1

2. If \( y = (5x - 4)^2 \), use the Chain Rule \( \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \) to determine whether the value of the derivative when \( x = 2 \) is greater than 100 or less than 100.

STAGE 2

7. Compare the value of the derivatives of the following functions evaluated at \( x = -3 \) and hence state which is the larger:

   A \( f(x) = x^2 (x+1)^3 \)
   B \( g(x) = \frac{x^3 + 3}{(x+2)^2} \)

8. Suppose the position of a car is given by \( s(t) = 50t - \frac{7}{t+1} \), the position being measured in kilometres. Find the velocity and acceleration of the car at \( t = 0 \) hours interpreting your results in the context of the problem.

STAGE 3

10. A model, \( f(x) = \sqrt{x^2 - 4x} \), has been proposed as a means of explaining the movement of a particle. Applying the model requires the use of the derivative of the function. Develop a mathematical argument that allows you to identify any limitations of this model, for \( x = -\pi \), 0 and \( \pi \).
12. The frequency of vibrations of a vibrating violin string is given by \( F = \frac{1}{2L} \sqrt{\frac{T}{\rho}} \) where \( L \) is the length of the string, \( T \) is the tension of the string, and \( \rho \) is its linear density. Find the rate of change of frequency with respect to the tension of the string, if \( L \) and \( \rho \) are constants, and compare this to the rate of change of frequency with respect to length of the string, if \( T \) and \( \rho \) are constants. In your comparison, discuss the possible applications of these two rates of change.

**STAGE 5**

12. It is often easy to calculate the exact value of a function at a point \( a \), but rather difficult to compute values near \( a \). For example, \( \sqrt{9} \) is quite easy to calculate while \( \sqrt{8.94} \) is rather difficult to calculate.

We can find an approximate value of the function at points near \( a \) by using the tangent line to the curve at \( x = a \). If the function \( y = f(x) \) is differentiable at \( a \), then a linear approximation, \( L(x) \), model of the function, \( f \), at point \( a \) is given by:

\[
L(n) = f(a) + f'(a)(n - a), \text{ where } n \text{ is the value we are to approximate and } a \text{ is the } x\text{-coordinate of the tangent line.}
\]

Using \( a = 8 \) as the point of the tangent line consider the case \( f(x) = \sqrt{x} \). Investigate the strengths and limitations of this linear approximation strategy for finding the approximate values for \( x = 8.05 \) and for \( x = 25 \) providing a justification for your assessment of the reasonableness of using this strategy in this particular case.
### Instrument-specific criteria and standards

Student responses have been matched to instrument-specific criteria and standards; those which best describe the student work in this sample are shown below. For more information about the syllabus dimensions and standards descriptors, see [www.qsa.qld.edu.au/1892.html#assessment](http://www.qsa.qld.edu.au/1892.html#assessment).

<table>
<thead>
<tr>
<th>Knowledge and procedures</th>
<th>Standard A</th>
<th>Standard C</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>The student work has the following characteristics:</td>
<td>The student work has the following characteristics:</td>
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<tr>
<td></td>
<td>• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations</td>
<td>• recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations</td>
</tr>
<tr>
<td></td>
<td>• application of mathematical definitions, rules and procedures in routine and non-routine simple tasks, through to routine complex tasks, in life-related and abstract situations</td>
<td>• application of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations</td>
</tr>
<tr>
<td></td>
<td>• numerical calculations, spatial sense and algebraic facility in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations</td>
<td>• numerical calculations, spatial sense and algebraic facility in routine, simple life-related or abstract situations</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Modelling and problem solving</th>
<th>Standard A</th>
<th>Standard C</th>
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<tbody>
<tr>
<td></td>
<td>The student work has the following characteristics:</td>
<td>The student work has the following characteristics:</td>
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<tr>
<td></td>
<td>• use of problem-solving strategies to interpret, clarify and analyse problems to develop responses from routine simple tasks through to non-routine complex tasks in life-related and abstract situations</td>
<td>• use of problem-solving strategies to interpret, clarify and develop responses to routine, simple problems in life-related or abstract situations</td>
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<tr>
<td></td>
<td>• investigation and evaluation of the validity of mathematical arguments including the analysis of results in the context of problems; the strengths and limitations of models, both given and developed</td>
<td>• interpretation of results in the context of routine, simple problems</td>
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<thead>
<tr>
<th>Communication and justification</th>
<th>Standard A</th>
<th>Standard C</th>
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<tr>
<td></td>
<td>The student work has the following characteristics:</td>
<td>The student work has the following characteristics:</td>
</tr>
<tr>
<td></td>
<td>• appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations</td>
<td>• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations</td>
</tr>
<tr>
<td></td>
<td>• use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language</td>
<td>• use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language</td>
</tr>
<tr>
<td></td>
<td>• coherent, concise and logical justification of procedures, decisions and results</td>
<td>• justification of procedures, decisions or results</td>
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<td></td>
<td>• justification of the reasonableness of results</td>
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</tbody>
</table>
Student response — Standard A

The annotations show the match to the instrument-specific standards.
Use of mathematical reasoning to develop a sequence within a response using mathematical language in a simple, non-routine, abstract situation.

Evaluation and investigation of the validity of mathematical arguments including the strengths and limitations of a model.

Coherent, concise and logical justification of procedures and results.
Use of mathematical reasoning to develop a sequence within a response using mathematical language in a complex, non-routine, abstract situation.

Evaluation and investigation of the validity of mathematical arguments including the strengths and limitations of a model.

Coherent, concise and logical justification of procedures and results.

Numerical calculations and algebraic facility in a non-routine, complex, abstract situation.

Possible applications:
None. None at all. Really!
Use of problem solving strategies to interpret and analyse problems in a non-routine, complex, abstract situation

Appropriate interpretation and use of mathematical terms and symbols in a non-routine, complex, abstract situation

Coherent and logical justification of procedures and results

\[
L(n) = \sqrt{n} + \sqrt{n - 2} - \sqrt{n - 4}
\]

For \( n = 25 \)

\[
L(25) = 5 + \sqrt{21} - 0 = 5 + 4.583 = 9.583
\]

Suggested solutions:

Keep going - read again

Suggestions and limitations:
Supervised assessment (2c): Topics include Periodic functions and applications, and Applied statistical analysis

- Mathematics B (2008)  Supervised assessment (2c)
Student response — Standard C

The annotations show the match to the instrument-specific standards.
Appropriate use of mathematical terminology and symbols in a simple routine situation

Coherent and logical justification of procedures and results

Numerical calculations in a complex task

\[ \text{Stage 3} \]

\[ (0.5(2x) = \sqrt{2^2 - 4x}) \]

\[ (x = 0) \text{ or } (x = 2) \]

\[ \text{Sub } x = 0 \text{ or } (x = 2) \]

\[ \text{undefined answer since the denominator} \]

\[ (x = 0) \text{ or } (x = 2) \]

\[ \text{Sub } x = -\pi \]

\[ \text{no real result} \]

\[ \text{the limitations of this model} \]
Numerical calculations in a complex task

Acknowledgments

The QSA acknowledges the contribution of Sandgate District State High School in the preparation of this document.