Mathematics B 2008
Sample assessment instrument and student responses

Extended modelling and problem solving II
December 2009
**Purposes of assessment**

The purposes of assessment are to:

- promote, assist and improve student learning
- inform programs of teaching and learning
- provide information for those people — students, parents, teachers — who need to know about the progress and achievements of individual students to help them achieve to the best of their abilities
- provide information for the issuing of certificates of achievement
- provide information to those people who need to know how well groups of students are achieving (school authorities, the State Minister for Education and Training and Training and the Arts, the Federal Minister for Education).

It is common practice to label assessment as being formative, diagnostic or summative, according to the major purpose of the assessment.

The major purpose of formative assessment is to help students attain higher levels of performance. The major purpose of diagnostic assessment is to determine the nature of students' learning, and then provide the appropriate feedback or intervention. The major purpose of summative assessment is to indicate the achievement status or standards achieved by students at a particular point in their schooling. It is geared towards reporting and certification.

**Syllabus requirements**

Teachers should ensure that assessment instruments are consistent with the requirements, techniques and conditions of the Mathematics B syllabus and the implementation year 2008.

**Assessment instruments**

High-quality assessment instruments:

- have construct validity (the instruments actually assess what they were designed to assess)
- have face validity (they appear to assess what you believe they are intended to assess)
- give students clear and definite instructions
- are written in language suited to the reading capabilities of the students for whom the instruments are intended
- are clearly presented through appropriate choice of layout, cues, visual design, format and choice of words
- are used under clear, definite and specified conditions that are appropriate for all the students whose achievements are being assessed
- have clear criteria for making judgments about achievements (these criteria are shared with students before they are assessed)
- are used under conditions that allow optimal participation for all
- are inclusive of students’ diverse backgrounds
- allow students to demonstrate the breadth and depth of their achievements
- only involve the reproduction of gender, socioeconomic, ethnic or other cultural factors if careful consideration has determined that such reproduction is necessary.

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2 Assessment instruments are the actual tools used by schools and the QSA to gather information about student achievement, for example, recorded observation of a game of volleyball, write-up of a field trip to the local water catchment and storage area, a test of number facts, the Senior External Examination in Chinese, the 2006 QCS Test, the 2008 Year 4 English comparable assessment task.
Mathematics B 2008

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Extended modelling and problem solving II

Compiled by the Queensland Studies Authority

October 2009

About this assessment instrument

The purpose of this document is to inform assessment practices of teachers in schools. For this reason, the assessment instrument is not presented in a way that would allow its immediate application in a school context. In particular, the assessment technique is presented in isolation from other information relevant to the implementation of the assessment. For further information about those aspects of the assessment not explained in this document, please refer to the assessment section of the syllabus.

This instrument provides opportunities for students to:

- recall, access, select and apply mathematical definitions, rules and procedures
- select and use mathematical technology
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- develop coherent, concise and logical responses within a response expressed in everyday language, mathematical language or a combination of both, as required, to justify conclusions, solutions and propositions
- develop and use coherent, concise and logical supporting arguments, expressed in everyday language, mathematical language or a combination of both, when appropriate, to justify procedures, decisions and results
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- apply problem-solving strategies and procedures to identify problems to be solved, and interpret, clarify and analyse problems.

This sample assessment instrument is intended to be a guide to help teachers plan and develop assessment instruments for individual school settings.
# Assessment instrument

The student work presented in this sample is in response to an assessment task which is a type of assessment instrument involving students applying and using relevant knowledge and skills to create a response to a problem or issue.

## Functions

### Part A

You have been contacted by an advertising company to develop a distinctive signage on the edge of a building similar to the photo below. You are to use two different types of functions from the following list: cubic, quadratic, trigonometric, exponential or logarithmic as a basis for your signage. NOTE: It is a symbol and does not necessarily represent words or letters.

**Task:**
1. The wall of the building is 20 metres long and 10 metres high. Decide where to place a set of axes on the building. Use this as a basis for your functions.
2. Provide a list of functions for the wall (at least 4).
3. Your functions must be repeated to create at least four curves on the wall. Decide on a domain and range for each function.
4. Provide a computer-generated graph of all functions and what they look like on the wall.
5. Enclose the design with a border and show this border on your graph.
6. Work out the percentage of the wall that is encompassed by this border.
7. Explain the strengths and limitations of your design in relation to the functions chosen.

### Part B

Consider the function $y = (ax^2 - b)(x + c)$. NOTE: $a \neq \frac{b}{\sqrt{a}}$, and $\frac{b}{\sqrt{a}}$ is an integer.

**Task:**
8. Choose values for $a$, $b$ and $c$ such that you can satisfy the above conditions. (Your results need to be different from other students.)
9. Provide an appropriate graph of the function.
10. Find “algebraically” (i.e. using algebra and calculus) all intercepts and turning points (maximum and minimum) including justification.
11. Find the length of the line joining the turning points.
12. Find the intersection of the line joining the turning points and the curve.

**Stimulus materials (photograph) removed due to copyright restrictions**
Instrument-specific criteria and standards

Schools draw instrument-specific criteria and standards from the syllabus dimensions and exit standards. Schools will make judgments about the match of qualities of student responses with the standards descriptors that are specific to the particular assessment instrument. While all syllabus exit descriptors might not be assessed in a single assessment instrument, across the course of study, opportunities to demonstrate all the syllabus dimensions and standards descriptors must be provided.

The assessment instrument presented in this document provides opportunities for the demonstration of the following criteria:
- Knowledge and procedures
- Modelling and problem solving
- Communication and justification.

This document provides information about how the qualities of student work match the relevant instrument-specific criteria and standards at standards A and C. The standard A and C descriptors are presented below. The complete set of instrument-specific criteria and standards is on pages 34–37.

<table>
<thead>
<tr>
<th>Standard A</th>
<th>Standard C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Knowledge and procedures</strong></td>
<td><strong>The student’s work has the following characteristics:</strong></td>
</tr>
<tr>
<td></td>
<td>• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations</td>
</tr>
<tr>
<td></td>
<td>• application of mathematical definitions, rules and procedures in routine and non-routine simple tasks, through to routine complex tasks, in life-related and abstract situations</td>
</tr>
<tr>
<td></td>
<td>• numerical calculations, spatial sense and algebraic facility in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations</td>
</tr>
<tr>
<td></td>
<td>• appropriate selection and accurate use of technology.</td>
</tr>
<tr>
<td><strong>Modelling and problem solving</strong></td>
<td><strong>The student’s work has the following characteristics:</strong></td>
</tr>
<tr>
<td></td>
<td>• use of problem-solving strategies to interpret, clarify and develop responses from routine simple tasks through to non-routine complex tasks in life-related and abstract situations</td>
</tr>
<tr>
<td></td>
<td>• identification of assumptions and their associated effects, parameters and/or variables</td>
</tr>
<tr>
<td></td>
<td>• use of data to synthesise mathematical models and generation of data from mathematical models in simple through to complex situations</td>
</tr>
<tr>
<td></td>
<td>• investigation and evaluation of the validity of mathematical arguments, including the analysis of results in the context of problems; the strengths and limitations of models, both given and developed.</td>
</tr>
<tr>
<td><strong>The student’s work has the following characteristics:</strong></td>
<td>• use of problem-solving strategies to interpret, clarify and develop responses to routine, simple problems in life-related or abstract situations</td>
</tr>
<tr>
<td></td>
<td>• use of mathematical models to represent routine, simple situations and generate data</td>
</tr>
<tr>
<td></td>
<td>• interpretation of results in the context of routine, simple problems routine, simple problems.</td>
</tr>
</tbody>
</table>
The student’s work has the following characteristics:

- appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations
- organisation and presentation of information in a variety of representations
- analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine
- use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language
- coherent, concise and logical justification of procedures, decisions and results
- justification of the reasonableness of results.

The student’s work has the following characteristics:

- appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations
- organisation and presentation of information
- translation of information from one representation to another in simple routine situations
- use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language
- justification of procedures, decisions or results.

Key:

- Differences or additional requirements for demonstrating the standard
- Differences in complexity of task requirements for each standard
- Differences or additional requirements for demonstrating the standard
Sample student response: Standard A

<table>
<thead>
<tr>
<th>Standard descriptors</th>
<th>Student response A</th>
<th>Standard descriptors</th>
</tr>
</thead>
</table>

**Part A:**

**Introduction:**

A distinctive signage is required to design on the edge of a building with 20 metres long and 10 metres high by an advertising company. Different types of functions have been given in a list of cubic, quadratic, trigonometric, exponential and logarithmic. Two types of those functions are asked to be used at least 2 times to draw this signage. And other functions or relations are available as well.

For this 20 metres long and 10 metres high wall, I decided to place x-axes start from 0 to 20 and y-axes from -5 to 5 which looks like below:

![Graph Image](image)

To make an advertisement for this wall, I choose it for a skateboard advertisement. And a graph of skateboarder with letters is chosen as the distinctive signage for skateboard shop or club. For this graph, the head and part of body have been designed with curve by using quadratic, cubic and trigonometric functions and circle relations. And parts of bodies like arms or legs are drawn by using linear equations. For those letters, different types of equations and relations has been chosen due to the shape of letters. Functions that have been chosen for it include: linear, quadratic, cubic, trigonometric functions and circle relation.
Sample student response: Standard A

Use of problem-solving strategies to interpret, clarify and analyse problems to develop responses to simple non-routine tasks in a life-related situation

- **Linear**: \( y = mx + c \)
- **Quadratic**: \( y = a(x - h)^2 + k \)
- **Cubic**: \( y = a(x - h)^3 + k \)
- **Trigonometric**: \( y = a \sin(bx + c) + d \)
- **Circle**: \( r^2 = (x - h)^2 + (y - k)^2 \)

For each function, the domain has been presented in brackets "\( \{ \} \)" and the range is represented as \( \{ y : \} \).

**My designed graph is:**

![Graph with the word SKATC](image)

All of the equations that have been used for the logo are:
### Sample student response: Standard A

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solutions</th>
</tr>
</thead>
</table>
| $(x - 6)^2 + (y - 3)^2 = 0.8^2$                                          | $y = -\sqrt{1^2 - (x - 6)^2} + 3$  
$y = -\frac{x}{8} + 2.875$                                              | $\{5.4, 6.5\}$  
$y = -0.25x + 2.95$                                                     | $\{3, 5.2\}$    |
| $(x - 3)^2 + (y - 2.35)^2 = 0.15^2$                                     | $y = -1.02x + 8.76$  
$y = -\frac{13x}{15} + \frac{107}{15}$                                  | $\{6.5, 8\}$    
| $(x - 8)^2 + (y - 0.4)^2 = 0.2^2$                                       | $y = 5.75x - 35.875$  
$y = 5.75x - 28.25$                                                     | $\{6.3, 6.5\}$  
|   | $y = -\frac{7x - 28}{34} + \frac{17}{3}$                              | $\{6.3, 8\}$    
|   | $y = -\frac{4x - 32}{3}$                                              | $\{8, 9.3\}$    
|   | $y = -\frac{15x}{13} + \frac{109}{13}$                                | $\{7.7, 9\}$    
|   | $y = -\frac{3x}{19} + \frac{136}{190}$                                | $\{5.6, 7.7\}$  
|   | $y = -\frac{133x}{70} + \frac{7329}{700}$                            | $\{5.6, 6.1\}$  
|   | $y = \sqrt{0.09 - (x - 4.8)^2} + 0.3$                                 | $\{4.5, 5\}$    
|   | $y = -1.5x + 7.05$                                                    | $\{4.5, 5.7\}$  
|   | $y = 9.88(x - 9)^3 - 2$ with domain from 9 to 9.3                      |                
|   | $y = 5.9375(x - 5.7)^3 - 1.5$ with domain from 5.7 to 6.1              |                

**Application of mathematical definitions and rules in non-routine simple tasks in a life-related situation**
Sample student response: Standard A

\[ y = -\frac{5x}{33} - \frac{7}{11} \{5.7, 9\} \]
\[ y = \frac{5}{9}(x - 9)^2 - 2 \{9, 9.6\} \]
\[ y = (x - 5.7)^2 - 1.5 \{5.1, 5.7\} \]
\[ y = \frac{5x}{33} - \frac{46}{55} \{5.7, 9\} \]
\[ y = \frac{5}{9}(x - 9)^2 - 2.2 \{9, 9.6\} \]
\[ y = (x - 5.7)^2 - 1.7 \{5, 5.7\} \]
\[ (x - 5.7)^2 + (y + 1.85)^2 = 0.15^2 \]
\[ (x - 9)^2 + (y + 1.9)^2 = 0.15^2 \]
\[ y = -6(x - 5.1)^2 - 1.14 \{5, 5.1\} \]
\[ (x - 9.6)^2 + (y + 1.9)^2 = 0.1^2 \{9.6, 9.7\} \]
\[ (x - 11.5)^2 + (y - 3)^2 = 0.25 \{11, 11.5\} \]
\[ y = -2(x - 11.5)^2 + 3.5 \{11.5, 12\} \]
\[ (x - 11.5)^2 + (y - 1.9)^2 = 0.36 \{11.5, 12.6\} \]
\[ y = 0.8(x - 11.5)^2 + 1.3 \{11, 11.5\} \]
\[ x = 12.5 \{y : 1.3, 3.5\} \]
\[ y = 0.6 \sin(\pi x + 12\pi) + 1.9 \{12.5, 13.6\} \]
\[ y = 0.5 \sin(\pi x + \pi) + 3 \{12.5, 13.5\} \]
\[ y = \frac{30x}{7}(x - 14.7)^2 + 3.5\{14, 15.4\} \]
\[ y = 2.4 \{14.35, 15.05\} \]
\[ y = 3.5 \{15.7, 16.9\} \]
Design process:

During graphing, results which do not come out evenly are approximately to 2 and 3 decimals depends on different situation.

**Skater**

**Skater’s Head and Neck:**

First for the head of skater is a circle because head usually looks like a circle. And neck of the skater is part of circle to match the head. The two circles are with different radii but same centre which is placed on (6, 3) and the radius for head is 0.8 units in this circle which is 0.2 units smaller than the radius of circle that neck needed:

The original equation for a circle is \((x - h)^2 + (y - k)^2 = r^2\) where \((h, k)\) is the centre of the circle.

Hence the equation for the head is \((x - 6)^2 + (y - 3)^2 = 0.8^2\)

For the neck of skater, only a small part of half circle is needed which equation could be found by

\[ y = \sqrt{r^2 - (x - h)^2} + k \]

Therefore the equation for the neck and its domain is

\[ y = \sqrt{1^2 - (x - 6)^2} + 3 \] with domain of \{5.4, 6.5\}

**Skater’s arms:**

For the two arms, both are straight line which initial function is \(y = mx + c\). Choose some points that fit the position of arms and the hand is like a circle to make it look simple and clear.

Two end points of neck are connected by arms. (5.4, 2.2) is the right end point of his neck and (6.5, 2.13) is the left end point of his neck.

**For skater’s right arm:**
Substitute (5.4,2.2) into \( y=mx+c \) and also choose another point to get the equation for it is

\[ y=-1/8x+2.875 \text{ with domain of } \{3.5,4\} \rightarrow \text{top line} \]

Similar to the other line for right arm:

\[ y=-0.25x+2.95 \text{ with domain of } \{3,5.2\} \rightarrow \text{bottom line} \]

And for the little hand, to match the arm, the radius should be 0.15 with centre at (3,2.35)

\[ (x-3)^2+(y-2.35)^2=0.15^2 \rightarrow \text{little hand} \]

For skater’s left arm: Same process to find the equations but different points:

\[ y=-1.02x+8.76 \text{ with domain of } \{6.5,8\} \rightarrow \text{top line} \]

\[ y=-13/15x+107/15 \text{ with domain of } \{6.5,8\} \rightarrow \text{bottom line} \]

\[ (x-8)^2+(y-0.4)^2=0.22 \text{ with domain of } \{8,8.2\} \rightarrow \text{little hand} \]

Straight lines are suitable for arms though it does not bent or curve.

**Skater’s body-line:**

After lining arms and head, the body is next part to make which is very simple with two straight lines. To connect bodies together, same points between end of arms and start of body are needed.

**For skater’s body:**

Point (5.2,1.65) is the connect point of right arm and body and another point on body line is (5,0.5) hence the equation is \( y=5.75x-35.875 \) with domain of \{6.3,6.5\} → right part

Point (6.5,1.5) is the connect point of left arm and body and another point on it is (6.3,0.35), hence the equation is \( y=5.75x-28.25 \) with domain of \{5.5,2\} → left part
Sample student response: Standard A

**Skater's Legs:**

Hence, two legs are important part of this graph to show skater's "Ollie" action.

For skater's right leg which is bent need a curve to describe it and a part of circle graph is used with initial function of $y = -\sqrt{r^2 - (x - h)^2} + k$ and also straight lines are used to draw his knee.

Similar to his arms, substitute points into linear equation $y = mx + c$ and get equations for the

For skater's left leg:
- $y = -\frac{7}{34}x + \frac{28}{17}$ with domain of {6.3,8}
- $y = -\frac{4}{3}x + \frac{32}{3}$ with domain of {8,9.3}
- $y = -\frac{15}{13}x + \frac{109}{13}$ with domain of {7.7,9}
- $y = -\frac{3}{19}x + \frac{136}{190}$ with domain of {5.6,7.7}

For skater's right leg:
- $y = -\frac{133x}{70} + \frac{7329}{700}$ with domain of {5.6,6.1}
- $y = -1.5x + 7.05$ with domain of {4.5,5.7}

For the knee of skater's right leg, a part of circle is needed, substitute points which are passed through by circle with centre at (4.8,0.3) and the equation is: $y = \sqrt{0.09 - (x - 4.8)^2} + 0.3$ with domain of {4.5,5.5}

Part of half circle which is used for his knee is suitable and looks nice however the connect points between knee and body and leg has to be correct.

**Skater's Feet:**

Two feet of him are designed by using cubic function with standard form $y = a(x-h)^3 + k$ where (h,k) is the centre.

For his left foot, $y = a(x-h)^3 + k$ has centre at point (9,-2) and pass (9.3, -26/15) where his left leg lines are ended, substitute (9,-2) and (9.3, -26/15) into $y = a(x-h)^3 + k$.

Hence got the equation of his left foot is $y = 9.88(x-9)^3 - 2$ with domain from 9 to 9.3. Because only part of $y = 9.88(x-9)^3 - 2$ is required from x=9 to x=9.3.
Sample student response: Standard A

Similar to his right foot, substitute (5.7,-1.5) and (6.1,-1.12) into $a(x-h)^3+k$, so the equation for his right foot is $y=5.9375(x-5.7)^3-1.5$ with domain from 5.7 to 6.1 as from $x=5.7$ to $x=6.1$ of $y=5.9375(x-5.7)^3-1.5$. Graph is needed.

**Skateboard**

Another important part for this graph is the skateboard, because the shape of a skateboard has upward parts at both end sides and flat for the middle part with 2 wheels.

For the upward parts of the board, quadratic functions are suitable for them. The upper side of the skateboard could be divided into 3 parts: two upwards and flat board and also similar to bottom part.

To make the board stick with skater’s foot, there are two same points between his feet and board: (9,-2) and (5.7,-1.5). Hence substitute these two points into the linear function and get the equation for upper board is $y=(-5/33)x-46/55$ with domain of {5.7,9}. The upward parts of board could be build up by quadratic with turning point of (9,-2) and (5.7,-1.5) respectively.

For skater’s left upward on upper board: the graph pass through (9.6,-1.8) with turning point at (9,-2), and therefore the function for it is $y=(x-9)^2-2$ with domain of {9,9.6}

For skater’s right upward on upper board: the graph pass through (5.1,-1.34) with turning point at (5.7, -1.7) therefore the function for it is $y=(x-5.7)^2-1.5$ with domain of {5.1,5.7}

Similar to the bottom part of skateboard: same way to find the equations for bottom board:

Equation for bottom board is $y=(-5/33)x-46/55$ with domain of {5.7,9}

The left upward on bottom board is $y=5/9(x-9)^2-2.2$ with domain of (9,9.6)

The right upward on bottom board is: $y=(x-5.7)^2-1.7$ with domain of (5,5.7)

And two wheels are placed near the ends of board which means it pass through (5.5,-1.7) and (9.0,-2.2) and make the radius of both wheels 0.15. One of the wheel with centre at (5.7,-1.85) and another one with centre at (9,-2.35)

Appropriate interpretation and use of mathematical terminology, symbols and conventions in simple non-routine life-related situations
Therefore the functions for the fore wheel is $(x-5.7)^2+(y+1.85)^2=0.15^2$

And the function for the back wheel is $(x-9)^2+(y+2.35)^2=0.15^2$

$y=-6(x-5.1)^2-1.14\{5,5.1\}$ and $(x-9.6)^2+(y+1.9)^2=0.12$ with domain of $\{9.6,9.7\}$ used to combine two separated boards.

**Letters**

The range for “SKATE” is $\{11,18.4\}$ and the domain and to make it looks good, curves are used such as circle and trigonometric.

**S:** according to the shape of S which has many curves, circle relation and quadratic functions are used.

- The left up part: $(x-11.5)^2+(y-3)^2=0.5^2$ with domain of $\{11,11.5\}$
- The right up part: $y=-2(x-11.5)^2+3.5$ with domain of $\{11.5,12\}$
- The left bottom part: $(x-11.5)^2+(y-1.9)^2=0.6^2$ with domain of $\{11.5,12.6\}$
- The right bottom part: $y=0.8(x-11.5)^2+1.3$ with domain of $\{11,11.5\}$

**K:** To make K looks more interesting, it is made to have curve instead of all straight lines.

For the left part is only a straight line with function of: $x=12.5$ with range of $\{y:1.3,3.5\}$

The right parts of K are designed by using trigonometric function:

$$y=\text{asin}(bx+c)+d$$

Where $a=\text{amplitude of the graph}$

$$b=\frac{2\pi}{\text{period}}$$
c/b= shift in x direction

For the right bottom part of K:
Let the amplitude of graph is 0.6 where: a=0.6
Period of this graph equal to 2, so b=π
Lift the graph 12 units right, hence c/b=12
c=12π
(In order to make the curve of the graph match with the point where the straight line of “K” is)
Hence y=0.6sin(πx+12π)+d, let the graph pass through point(12.5,2.5)
2.5=0.6sin(12.5π+2π)+d
d≈1.900
Hence the function for right bottom part of K is y=0.6sin(πx+12π)+1.9 with domain from 12.5 to 13.6. The domain is chosen because to make “K” looks good.

Similar to y=0.6sin(πx+12π)+1.9(12.5,13.6), the top right part of K has amplitude of 0.5, period of 2 (b=π) and lift 11.5 units to right, (c=11.5π) and also pass point(12.5,2.5), so the function for bottom right part of K is : y=0.5sin(πx+11.5π)+3 with the domain from 12.5 to 13.6. The domain of this one is chosen to match the top right part of K.

A: a graph of parabola is chosen to be the shape and with a straight line. The graph of parabola pass through (14,1.3) with turning point as (14.7,3.5) therefore the equation for it is :y=-30/7(x-14.7)^2+3.5 with domain of {14,15.4}
And a straight line is placed inside of this parabola but not connected with parabola with the equation of y=2.4 with domain of {14.35,15.05}
Sample student response: Standard A

**T:** two straight lines are suitable enough for it.
The horizontal line equation is: \( y=3.5 \) with domain of \{15.7,16.9\}
The vertical line equation is: \( x=16.3 \) with domain of \{y:1.3,3.5\}

**E:** A half circle is used to make E looks pretty with 1.1 radius and centre at \( (18.4,2.4) \) with equation is \((x-18.4)^2+(y-2.4)^2=1.1^2\) with domain of \{17.3,18.4\} the domain of this graph is because only half circle is required.
And a little straight line \( y=2.4 \) with domain of \{17.3,18.5\} is used to fill inside.
A border is required to encompass the signal on the wall. This graph’s border is shown in the above picture which connect four points (4,5), (1,0), (5,-4) and (20,-1) by 3 straight lines: y = (5/3)x-(5/3) with domain of {1,4}, y = -x+1 with domain of {1,5} and y = x/5-5 with domain of {5,20}.
The above picture shows the way to calculate the encompassed area.

The area encompassed by this border is equal to:

\[
19\times 9 - \left( \frac{2\times 5}{2} + \frac{4\times 4}{2} + \frac{15\times 3}{2} \right) = 135.5 \text{ unit}^2
\]

The total area of the wall is equal to \(10\times 20 = 200 \text{ units}^2\)

Therefore to calculate the percentage of the wall that is encompassed by this border could be calculated.

The percentage of the wall that is encompassed by this border is equal to: \(\frac{135.5}{200}\times 100\% = 67.75\%\)

**Strengths:**

- Clear appearance for people to understand and attractive for people who loves skateboard.
- It could be used as logo or advertisement.
- Good advertisement for skateboard shop/ skateboard club.
- It could be a logo for skateboard brand.
- In this graph, different types of functions and relations are used to make it look good.

**Limitations:**

- The joining points between arms and body do not match quiet well when it is removed from one computer to another.
- The right hand of skater is better being half circle but is not available in this graphmatica.
- The letters which are designed in graph is for the request of using of certain functions (trigonometric)
- The curve which is used to draw his left foot doesn’t match with leg line very well.
- Because the width and height of the graph are different and cause the graph doesn’t appear as in standard square units.
Part B:

Introduction:

For a function $y = (ax^2 - b)(x + c)$ where $c \neq \frac{b}{a}$ and $\frac{b}{a}$ is an integer

The chosen value for $\frac{b}{a}$ is 2 and let $a=2$, $b=8$ and $c=1$.

Substitute $a=2$, $b=8$ and $c=1$ into $y = (ax^2 - b)(x + c)$, therefore $y = (2x^2 - 8)(x + 1)$.

Draw this diagram by Graphmatica and the graph given is:

![Graph of the function](image)

The units of $x$ axes and $y$ axes in this graph are not same this is in order to show the turning point and intersections clearly, therefore the different units causes the graph does not look like what it should be.
To calculate the intercepts on axis.

**X-intercepts** (when the graph of \( y = (2x^2 - 8)(x+1) \) cuts x-axes which means when \( y = 0 \))

When \( y = 0 \), \( 0 = (2x^2 - 8)(x+1) \)

\[ 2x^2 - 8 = 0 \text{ or } x + 1 = 0 \]

So \( x^2 = 4 \) or \( x = 0 \)

\( x = 2, x = -2, x = -1 \)

So when \( y = 0 \), \( x = 2, x = -1, x = -2 \)

Therefore there are three x-intercepts: (2,0), (-1,0) and (-2,0) this is also shown in the graph.

**Y-intercept** (when graph of \( y = (2x^2 - 8)(x+1) \) cuts y-axes which means when \( x = 0 \))

When \( x = 0 \), \( y = (0 - 8)(0 + 1) \)

\[ y = -8 \times 1 \]

\[ y = -8 \]

Hence y-intercept is (0, -8)

Application of mathematical definitions and rules in non-routine simple tasks in a life-related situation
Turning point (where when $y' = 0$)

To find the turning point of $y = (2x^2 - 8)(x+1)$, $y'$ is necessary and helpful and the turning point means when $y' = 0$. To find $y'$, the product rule is used for help.

Let $y = (2x^2 - 8)(x+1) = u \times v$

where $u = 2x^2 - 8$ and $v = (x+1)$

$\therefore u' = 4x$ and $v' = 1$

$y' = u \times v' + v \times u'$

$= (2x^2 - 8) \times 1 + (x+1) \times 4x$

$= 2x^2 - 8 + 4x^2 + 4x$

$= 6x^2 + 4x - 8$

When $y' = 0 = 6x^2 + 4x - 8$

$0 = 3x^2 + 2x - 4$

By using TI-83Plus calculator’s quadratic program, the values for $x$ in this equation are:

$x = -1.535$ or $x \approx 0.869$.  

(The calculations have been approximately to 3 decimals.)
Sample student response: Standard A

Substitute $x \approx 1.535$ and $x \approx 0.869$ into $y' = 6x^2 + 4x - 8$ to check.

For $x \approx 1.535$, $y' = 6(-1.535)^2 + 4(-1.535) - 8 \approx 0$

For $x \approx 0.869$, $y' = 6(0.869)^2 + 4(0.869) - 8 \approx 0$

Use the $y'$-nature diagram to check the nature of the turning points.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1.535</th>
<th>0</th>
<th>0.869</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y'$</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>slope</td>
<td>/</td>
<td>-</td>
<td>\</td>
<td>-</td>
<td>/</td>
</tr>
</tbody>
</table>

Substitute $x \approx -1.535$ into $y = (2x^2 - 8)(x+1)$

$y = [2(-1.535)^2 - 8][-(-1.535)+1] \approx 1.759$

So one of the turning points is A ($-1.535, 1.759$), according to the $y'$-nature diagram above, turning points is A ($-1.535, 1.759$) is local maximum.

Substitute $x \approx 0.869$ into $y = (2x^2 - 8)(x+1)$

$y = (2 \times 0.869^2 - 8)(0.869+1) = -12.129$

So the other turning point is B ($0.869, -12.129$), also according to the $y'$-nature diagram above, turning points B ($0.869, -12.129$) is local minimum.

Hence, there are three $x$ intercepts: (2,0), (-1,0) and (-2,0)

One $y$ intercept: (0,-8)
And two turning points: A (-1.535, 1.759) is local maximum
B (0.869, -12.129) is local minimum.

And all the “algebraically” has been shown is the follow diagram:

To find length of the line joining the turning points

The line joining turning points A and B is the distance between two turning points A and B.
A is (-1.535, 1.759) and B is (0.869, -12.129). The distance between two points could be calculated by Pythagorean Theorem,\(c=\sqrt{a^2+b^2}\)

\[
AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}
\]

\[
= \sqrt{(-1.535 - 0.869)^2 + (1.759 - (-12.129))^2}
\]

\[
= \sqrt{198.406}
\]

\[
\approx 14.086
\]
(The result has been approximated to 3 decimals.)
So the length of AB is 14.086 units.

To find the intersections of the curve and the line AB.

From the graph, the line joining A and B also across \( y = (2x^2-8)(x+1) \) at another point. To find that point, the equation of the straight line pass through A and B could be expressed as

\[ y = mx + c \]

Substitute two points A \((-1.535, 1.759)\) and B \((0.869, -12.129)\) into \( y = mx + c \)

\[ 1.759 = -1.535m + c \quad \text{①} \]
\[ -12.129 = 0.869m + c \quad \text{②} \]

\[ ① - ② \Rightarrow 1.759 - (-12.129) = 1.535m - 0.869m \]
\[ 13.888 = -2.404m \]
\[ m = -5.777 \]

Sub \( m = -5.777 \) into ①

\[ 1.759 = (-1.535) \times (-5.777) + C \]
\[ C = -7.109 \]
Hence the equation for the line pass through A and B is \( y = -5.777x - 7.109 \).

Intersections of the line joining the turning points and the curve means these two graphs go across each other.

\( y = (2x^2 - 8)(x+1) \) and \( y = -5.777x - 7.109 \) has three intersections: point A, point B and the third point.

By using TI-83 Plus graphic calculator, the intersections of \( y = (2x^2 - 8)(x+1) \) and \( y = -5.777x - 7.109 \) are two turning points and the third one is (-0.334, -5.180).
Sample student response: Standard A

Substitute (-0.334, -5.180) into both equations to have a check.

\[ y = (2x^2 - 8)(x+1) \]

When \( x = -0.334 \)

\[ y = [2 \times (-0.334)^2 - 8][-0.334 + 1] \]

\[ y \approx -5.179 = -5.180 \]

Hence it pass through point(-0.334, -5.180)

\[ y = -5.777x - 7.109 \]

When \( x = -0.334 \)

\[ y = -5.777 \times (-0.334) - 7.109 \]

\[ y \approx -5.179 = -5.180 \]

Hence it pass through point(-0.334, -5.180)

And hence the third intersection of \( y = (2x^2 - 8)(x+1) \) and \( y = -5.777x - 7.109 \) is (-0.334, -5.180)
### Standard C

*Note:* "[...]" indicates where the text has been abridged.

<table>
<thead>
<tr>
<th>Standard descriptors</th>
<th>Student response C</th>
<th>Standard descriptors</th>
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<tbody>
<tr>
<td><strong>Selection and use of technology</strong></td>
<td><img src="image" alt="Diagram" /></td>
<td></td>
</tr>
</tbody>
</table>
Part A

1. The wall proposed for the logo is 20m long x 10m high. It was drawn from -10 to 10 on the x-axis and 0 to 10 on the y-axis to centre the drawing. On the graphing program *Graphmatica* the grid range was set up as

   [Left: -15.16 Right: 15.16 Bottom: -2 Top: 18]

   to provide a squared off view which is realistic.

2. The logo was chosen to be placed on the wall of a surf shop and was therefore inspired by the beach. Below is a representation of it showing the different functions (and relations) used:

   ![Graph](image)

   *Why is the graph appropriate?*
The border is from 3.5m up the wall to 10m up the wall. It covers all the length of the wall. Therefore the area is $3.5 \times 10 = 35\text{m}^2 \div 200 = 0.175\text{m}^2$.

6. The logo’s area, as a percentage of the wall’s area is:

$$\frac{35}{200} \times 100 = 17.5\%$$

But this would be the area not covered by the logo.

Why didn’t you put a border around the logo but you were instructed in task 5? It is okay as long as you give an explanation.
Application of mathematical rules and definitions in routine simple life-related situations

Standard C

Wall:
\[
y = 0 \quad \{-10,10\} \\
y = 10 \quad \{-10,10\} \\
x = -10 \quad \{0,10\} \\
x = 10 \quad \{0,10\}
\]

Mountains:
\[
y = 0.75 \sin 2x + 5.85 \quad \{-10,-.7854\}
\]

Seabed:
\[
y = -0.75 \sin 2x + 4.25 \quad \{-0.7854,10\}
\]

Water/wave:
\[
y = 0.25(x-3)(x-7)(x-8) + 5.2821128 \quad \{3.5, 7.5, 252524\} \\
0.75^2 = (x+7)^2 + (y-8)^2 \\
y = 5 \quad \{-0.7854,3.5\} \\
y = 5 \quad \{7.527524,10\}
\]

Sun:
\[
1.1254314^2 = (x-3.5)^2 + (y-6.1254314)^2 \quad \{3.5,10\}
\]
Part B

1. The values chosen for a, b and c were chosen to be 4, 16 and 1 respectively. It satisfied the requirements because:
c did not equal the square root of \((b/a)\) \(I\) does not equal the square root of (16/4)\]
the square root of \((b/a)\) is an integer. \([square root of (16/4) is an integer = 4]\)

\[
\begin{array}{c}
a = 4 \\
b = 16 \\
c = 1 \\
\end{array}
\]

2. Below is a graph of the function \(y = (4x^2 - 16)(x+1)\):

3. Maximum and minimum values were found by first finding the derivative of the function.
\[
y = (4x^2 - 16)(x+1)
\]
\[
= 4x^3 + 4x^2 - 16x + 16
\]
\[
\frac{dy}{dx} = 3x^2 + 2x - 4
\]

Appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations
The max. and min. values occur when the derivative equals 0; when the gradient is 0.

X values were found

\[
\frac{dy}{dx} = 0 \quad \therefore \quad 3x^2 + 2x - 4 = 0
\]

\[x_1 = \frac{-0.8685}{4} \text{ (4 dec. places)}\]

\[x_2 = \frac{-1.3522}{4} \text{ (4 dec. places)}\]

The x-values were substituted into the equation \((4x^2-16)(x+1)\) to find the y-values of the max. and min. points

with \(x_1\) \(y = \frac{-110}{4} \text{ (4 dec. places)}\)

\[y_1 = -24.2584\]

\[y_2 = 3.5177 \text{ (4 dec. places)}\]

The x-intercepts are found when \(y=0\)

\[0 = (4x^2 - 16)(x+1)\]

\[x = -2, 2, \text{ or } -1 \quad \therefore \quad (-2,0), (2,0), (-1,0)\]

The y-intercept is found when \(x=0\)

\[y(0) = (4(0^2) - 16)(0+1)\]

\[= -16 \times 1\]

\[= -16 \quad \therefore \quad (0,-16)\]

Incomplete
## Instrument-specific criteria and standards

<table>
<thead>
<tr>
<th>Knowledge and procedures</th>
<th>Standard A</th>
<th>Standard B</th>
<th>Standard C</th>
<th>Standard D</th>
<th>Standard E</th>
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<tr>
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<td>- application of mathematical definitions, rules and procedures in routine complex tasks, in life-related situations involving exponential functions</td>
<td>- recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations</td>
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The student’s work has the following characteristics:

- recall, access, selection of mathematical definitions, rules and procedures in **routine** and **non-routine** **simple tasks through to routine complex tasks**, in life-related and abstract situations

- application of mathematical definitions, rules and procedures in routine complex tasks, in life-related situations involving exponential functions

- recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations

- use of technology.

- use of technology.
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<td>responses from <strong>routine</strong> <strong>simple tasks</strong> and <strong>non-routine complex</strong> tasks.</td>
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<td></td>
<td>- identification of assumptions and their associated</td>
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<td>effects, parameters and/or variables</td>
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<td><strong>exponential functions</strong> using everyday and</td>
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</tr>
<tr>
<td>- use of mathematical models to represent routine, <strong>simple</strong> situations and generate data</td>
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<tr>
<td>- interpretation of results, in the context of routine, <strong>simple problems</strong>.</td>
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<tr>
<td>Instrument-specific criteria and standards</td>
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<tr>
<td>------------------------------------------</td>
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<td>Communication and justification</td>
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Instrument-specific criteria and standards

- justification of the reasonableness of results