External Assessment subject report

Mathematics B

October 2016

160689

Queensland Government
Queensland Curriculum & Assessment Authority

For all Queensland schools

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Contents

Introduction

Queensland is working towards a new system of senior assessment and tertiary entrance that will include:

- a model that uses school-based assessment and common external assessment
- processes that strengthen the quality and comparability of school-based assessment
- a move away from the Overall Position (OP) rank to an Australian Tertiary Admission Rank (ATAR).

In Semester 1 2016, the Queensland Curriculum and Assessment Authority (QCAA) trialled external formative assessments in five subjects to:

- provide an opportunity for schools to become familiar with the use of subject-based external assessments
- test our processes for delivering external assessments.

These assessments were:

- aligned to existing syllabuses
- an alternative to a task already being undertaken at participating schools
- developed in consultation with subject experts from schools, subject associations and universities
- administered under secure conditions and graded externally.

The trial involved:

- approximately 19 000 students from 249 schools
- five Year 11 subjects:
  - Chemistry
  - English
  - Geography
  - Mathematics B
  - Modern History.

In addition, more than 400 teachers took part in the online marking operation.

This report provides information on the *External Assessment Trial: Mathematics B* assessment specifications, the sample responses and the performance characteristics of students.

The trial was conducted using the current syllabus, with Year 11 students and in a formative context. Commentaries and sample responses should be viewed in this context.

Electronic versions of the assessment are available online.

Claude Jones
Director, Assessment and Reporting Division

Overall commentary

The *External Assessment Trial: Mathematics B* was a *supervised assessment* developed by the Queensland Curriculum and Assessment Authority (QCAA) and conducted under supervised conditions. The assessment was completed by 5528 students across 95 participating schools on Wednesday 1 June 2016.

The *supervised assessment* was devised from identified subject matter from the *Mathematics B Senior Syllabus 2008 (amended 2014)* and consisted of 24 items grouped into 14 questions. Questions were ordered according to topic: Applied statistical analysis (Questions 1, 2 and 3); Introduction to functions (Questions 4, 5, 6, 7, 8, 9 and 10); and Periodic functions and applications (Questions 11, 12, 13 and 14). The *Mark distribution table* that was provided with the supervised assessment indicated the criteria assessed and the complexity (Simple and Complex) and initiative (Routine and Non-Routine) for each question.

- Questions 1, 4, 5, 6, 8, 11, 12, 13 assessed *Knowledge and procedures*.
- Questions 2, 3, 7, 9, 10, 14 assessed *Modelling and problem solving*.
- Every question assessed *Communication and justification.*

Figures 1 to 6 on the following pages indicate student performance overall, and according to gender. In terms of student performance, students generally demonstrated a stronger performance in *Modelling and problem solving*.

Note that the statistics in this report may be subject to rounding, resulting in totals not equal to 100 per cent.

Knowledge and procedures

Questions 1, 4, 5, 6, 8, 11, 12 and 13 assessed *Knowledge and procedures*. These questions provided opportunities for students to demonstrate their capability in dealing with tasks that were Simple Routine, Simple Non-Routine and Complex Routine.

Figures 1 and 2 show the statewide distribution of grades overall and with gender breakdown for *Knowledge and procedures*. The majority of the cohort (65.7%) achieved a C or better.

Figure 1: Knowledge and procedures

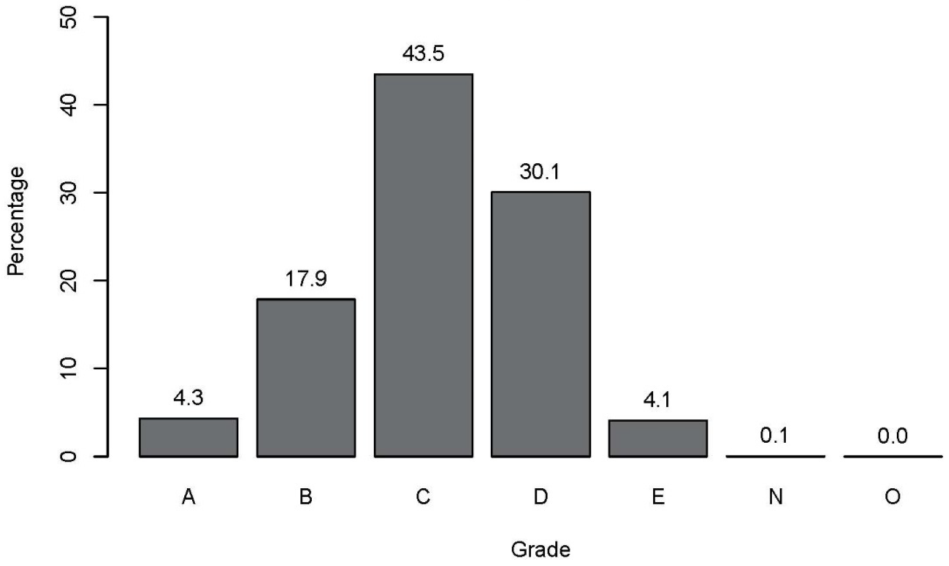
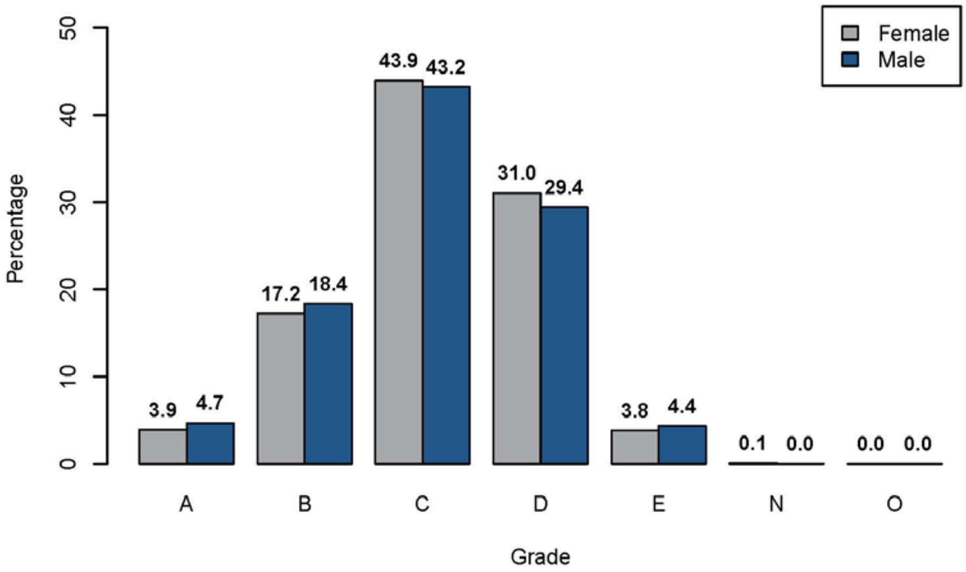


Figure 2: Knowledge and procedures by gender



Modelling and problem solving

Questions 2, 3, 7, 9, 10 and 14 assessed *Modelling and problem solving*. These questions provided opportunities for students to demonstrate their capability in dealing with tasks ranging from Simple Routine through to Complex Non-Routine.

Figures 3 and 4 show the statewide distribution of grades overall and with gender breakdown for *Modelling and problem solving*. Most of the cohort (81.4%) achieved a C or better

Figure 3: Modelling and problem solving

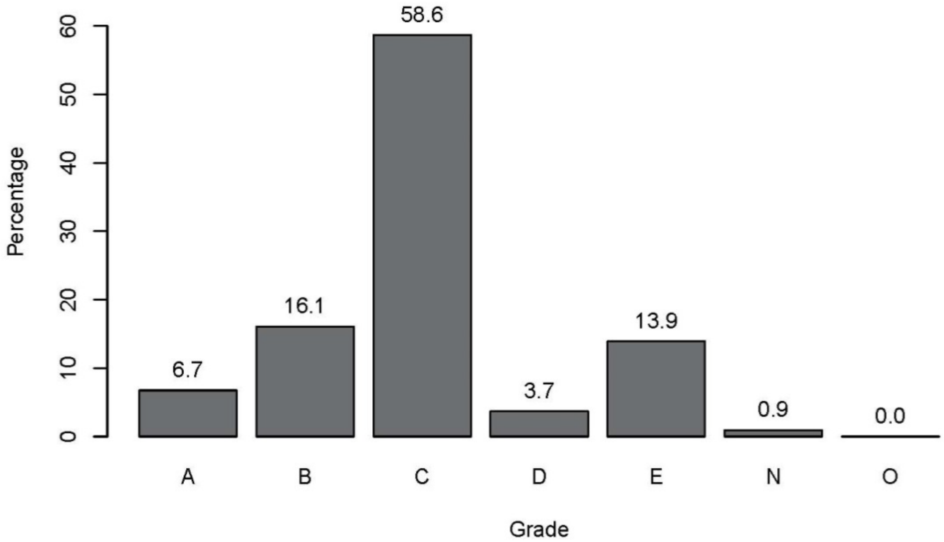
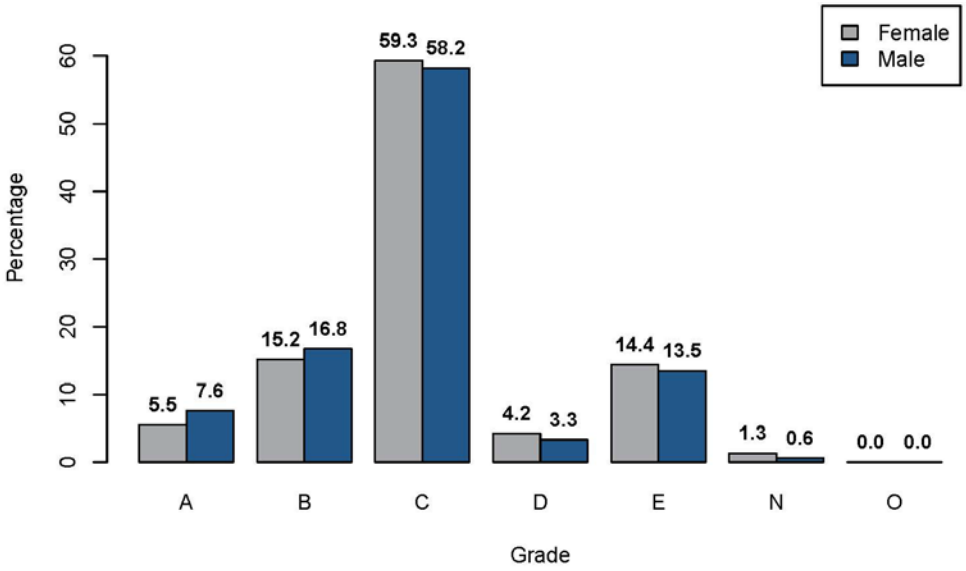


Figure 4: Modelling and problem solving by gender



Communication and justification

Communication and justification was assessed across all five groups of questions. Each group of questions was chosen to ensure a variety of objectives were assessed across each group. Cues were provided within questions to provide some guidance to students.

Figures 5 and 6 show the statewide distribution of grades overall and with gender breakdown for *Communication and justification*. Most of the cohort (70.9%) achieved a C or better.

Figure 5: Communication and justification

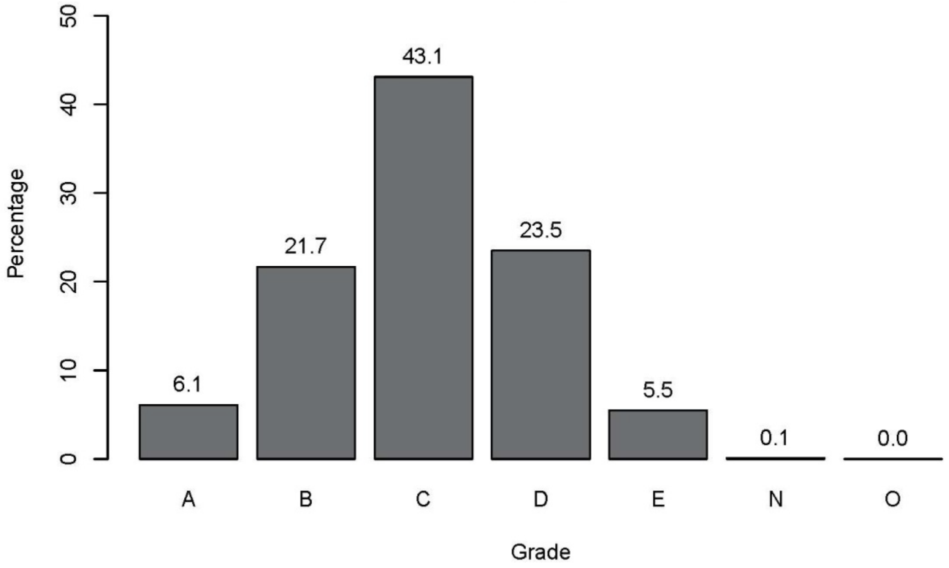
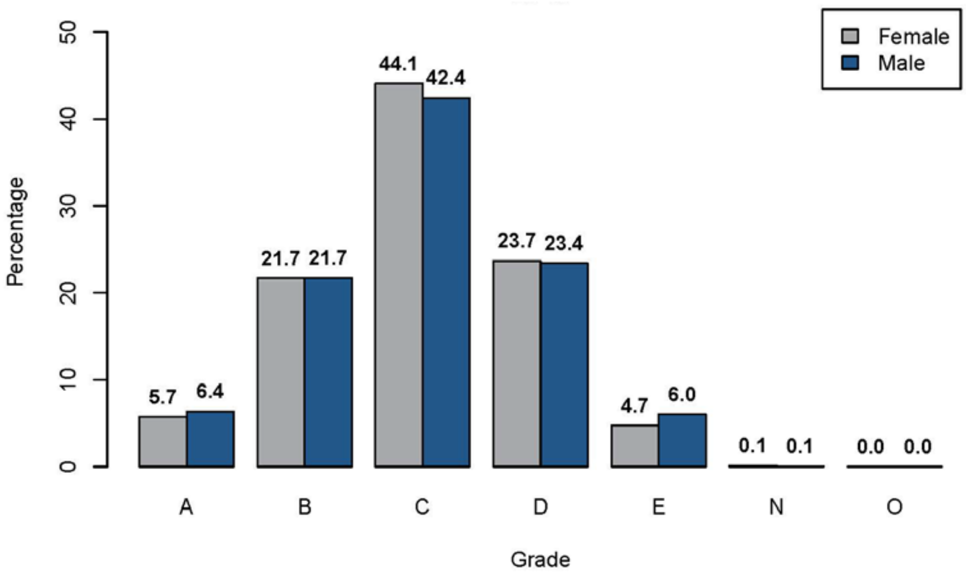


Figure 6: Communication and justification by gender



Sample responses and commentaries

This section provides commentary on sample responses to the 14 questions on the *supervised test*. It highlights the strengths in particular questions and discusses aspects where students had less success and need to improve. A sample response is provided which models an appropriate response for each question. They have not been corrected for grammar, spelling or accuracy. Responses provided are a sample of responses only, and are not necessarily exemplary responses.

Question 1

Question 1 was a *Knowledge and procedures* question in three parts that included Simple Routine and Simple Non-Routine opportunities.

Sample response

Question 1

In a small town there are 204 houses. The number of bedrooms in each house is recorded in the table below.

Number of bedrooms	1	2	3	4	5	6
Number of houses	39	59	48	32	16	10

a) Circle whether the data in the table is:

discrete or continuous.

You may use the statistical modes on your calculator for parts b) and c).

b) Write down the mean number of bedrooms per house.

Give your answer correct to 3 decimal places.

$\bar{x} = 2.789$ (bedrooms/house)

c) Determine the percentage of houses with more bedrooms than one standard deviation above the mean.

Clearly justify your answer.

$\sigma x = 1.386$

$\therefore \bar{x} + \sigma x = 2.789 + 1.386$

$= 4.175$

$\therefore$ % houses who have more than 4.175 bedroom

~~approx~~. $\therefore$ houses that have 5 or 6 bedrooms

$= 16 + 10$

$= 26.$

$\frac{26}{204} \times 100 = 12.745\%.$

Simple Routine; Simple Non-Routine — 1, 1 mark; 3 marks

Most students were able to engage with the simple routine aspects of this question with 93% of students achieving some success and 23% scoring four or more out of the possible five marks. Students were required to recall definitions to identify whether a variable was discrete or continuous. The question was designed for students to use their calculator to determine the mean and standard deviation of a 2-variable dataset. Students then needed to interpret the summary statistics to calculate a percentage related to the context.

Students achieved success with identifying whether a dataset was discrete or continuous and using their calculator to find the mean of a dataset. Some students had limited success with 1c, which required them to calculate and then use the standard deviation in a non-routine situation.

Question 2

Question 2 was a Simple Routine *Modelling and problem solving* question.

Sample response

Question 2

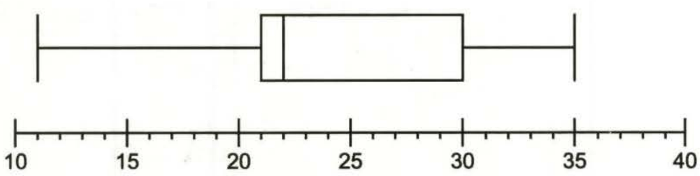
A farmer has collected samples of two different types of strawberries to help her determine which type is the best to grow. The weight in grams of the samples of each type is recorded below.

Type A

stem	leaf							
1	1	2	5	6	6	7	8	9
2	1	1	3	4	5	6		
3	2	4	5					

Key: 2 | 3 represents 23 grams

Type B



Assuming that the farmer wants the heavier strawberries, suggest which type of strawberries she should grow.

Clearly justify your answer.

Refer to different statistical measures when making your justification.

Type A

Mode = 16, 21

Median = 21

Mean = 21.47

X_{max} = 11

X_{min} = 35

Q_1 = 16

Q_3 = 25.5

Type B.

Median = 22

X_{max} = 35

X_{min} = 11

Q_1 = 21

Q_3 = 30

∴ The farmer should choose Type B strawberries as median is much greater than Type A so is Q_1 and Q_3. This states the weight are much higher than the others

Simple Routine MAP

Question 2 required students to use graphical displays and/or summary statistics, calculated either by hand or using technology, to compare datasets and explore possible relationships. They were required to interpret their results in the context of the problem to give an explicit recommendation. A model response required students to choose an appropriate strategy to compare data sets (five-number summary or box plots for both data sets) and make a recommendation using at least two statistical measures in their analysis.

Almost 50% of students achieved success calculating the five-number summary and/or using a statistical measure to make the correct recommendation. However, although the cue prompted use of different statistical measures when making a justification, some students had limited success in interpreting their results to make a recommendation to the farmer.

Question 3

Question 3 was a Simple Non-Routine *Modelling and problem solving* question.

Sample response

Question 3

A dataset has the following known measures of centre and spread:

Measure	mode	range	median	interquartile range
Value	12	19	9	8

Determine the new value of each of these measures if every member of the dataset is increased by 2.

Clearly justify your answer.

new table

measure	mode	range	median	~~IQR~~
value	14	21	11	10

every member of dataset is increased by 2.

∴ the mode would increase by 2 (as it is still the most common)

∴ mode = 14

∴ range would remain the same as all numbers are increased by 2 ∴ range = 19

Median - middle number would increase by 2 - as all numbers do.

∴ median = 11

IQR = will remain the same as every number is increased - not one that will change the Q_1, med, + Q_2.

∴ IQR = 8.

new values

mode = 14
range = 19
median = 11
IQR = 8

Simple Non-Routine MAP

Question 3 required students to identify the effects of altering the dataset by a given value.
A model response demonstrated a problem solving strategy that used a logical approach to creating and modifying the dataset, correctly showing the effect on all four measures and stating all the new values.

Over 50% of students were able to determine the effects on the median and mode when the dataset was altered, however many students had less success when clearly justifying answers. The most common misconception was assuming the increase by 2 in all members of the dataset would affect the range and inter-quartile range.

Question 4

Question 4 was a Simple Routine *Knowledge and procedures* question.

Sample response

Question 4

a) Identify which of the relations are functions.

Clearly justify your answers.

Relation 2
Relation 4

as these both have only one output for every input. That is, for every x value there is only one y value.

b) State whether each function identified in part a) is discrete or continuous.

2: Discrete

4: continuous

c) Write down the domain and range for Relation 1.

domain: $-1 \leqslant x \leqslant 5$

range: $-1.5 \leqslant y \leqslant 1.5$

Simple Routine — 2, 2, 2 marks

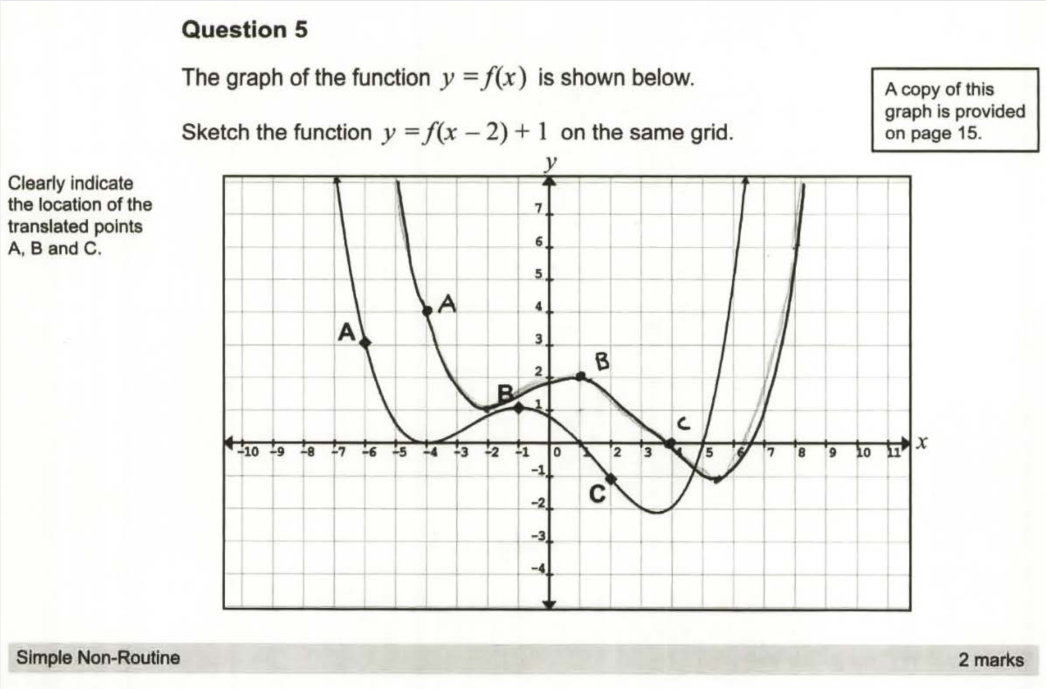
Students were required to recall and apply definitions to identify whether a relation was a function, whether the functions were discrete or continuous, and to find the domain and range of a graphical representation.

Overall this question was well answered with 71% of students achieving at least half of the marks, and 23% achieving full marks. Students achieved success with determining which relations were functions and stating whether they were discrete or continuous. Question 4c proved challenging, requiring students to recall definitions and translate information from graphical to numerical form.

Question 5

Question 5 was Simple Non-Routine *Knowledge and procedures* question.

Sample response



This question asked students to translate a function given in graphical form. Students were required to recall and apply mathematical rules and procedures to translate points A, B and C, either identified as co-ordinates or transformed points on the graph. Students then had to use their spatial sense to sketch the translated function.

Overall, students achieved some success with this question with 59% achieving at least half of the marks, and 34% achieving full marks. Students achieved success when consistently applying a translation to the identified points. Sketching the correct shape of the translated function was the most demanding aspect of this question.

Question 6

Question 6 was a *Knowledge and procedures* question in two parts that provided Simple-Routine and Complex-Routine opportunities.

Sample response

Question 6

Given that $f(x) = 2x^2 - 1$ and $g(x) = 3x + 5$ answer the following questions:

a) calculate $f(-2)$

Clearly justify your answer.

$$f(-2) = 2 \times (-2)^2 - 1$$
$$= (2 \times 4) - 1$$
$$= 8 - 1$$
$$= \underline{7}$$

b) solve for x if $f(g(x)) = 127$

Use algebraic methods.

$$f(g(x)) = 2(3x+5)^2 - 1$$
$$127 = 2(3x+5)^2 - 1$$
$$= 2((3x)^2 + (2 \times 3x \times 5) + (5)^2) - 1$$
$$= 2(9x^2 + 30x + 25) - 1$$
$$= 18x^2 + 60x + 50 - 1$$
$$127 = 18x^2 + 60x + 49$$
$$0 = 18x^2 + 60x + 49 - 127$$
$$0 = 18x^2 + 60x + (-78) \rightarrow \text{let } x. \quad a = 18,\ b = 60,\ c = -78.$$

use quadratic formula to solve for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-60 \pm \sqrt{60^2 - (4 \times 18 \times -78)}}{2 \times 18}$$
$$= \frac{-60 \pm \sqrt{3600 - (-5616)}}{36}$$
$$= \frac{-60 \pm \sqrt{9216}}{36}$$
$$= \frac{-60 \pm 96}{36}$$

$$x = \frac{-60 + 96}{36} \text{ OR } x = \frac{-60 - 96}{36}$$

$$\underline{x = 1} \text{ OR } \underline{x = -\frac{13}{3} \text{ OR } -4.\dot{3}\dot{3}}$$

Simple Routine; Complex Routine	2 marks; 4 marks

In Question 6a, students were required to select and apply rules for using function notation and then perform a numerical calculation. In Question 6b, students were required to recall, select and apply the mathematical definition for a composite function, use algebraic facility to rearrange the equation, identify this equation as a quadratic and apply mathematical rules and procedures to solve the quadratic.

There was mixed success with this question with only 11% of students receiving full marks. Students understood how to apply rules for using function notation but omitted brackets which often resulted in calculation errors. Finding the composite function proved to be difficult, however when the required quadratic equation was produced students were able to apply appropriate procedures and solve it correctly.

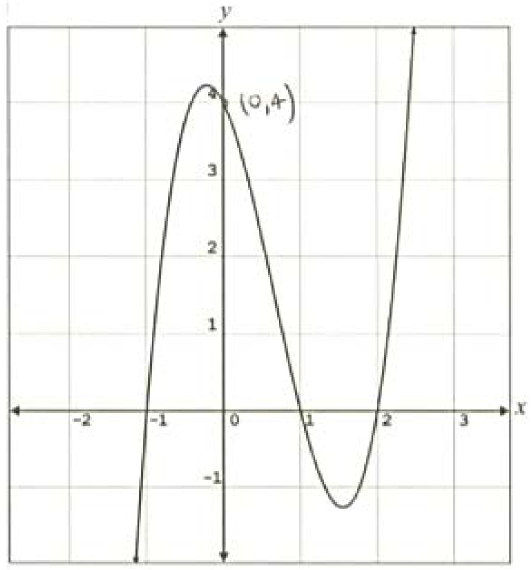
Question 7

Question 7 was a Simple Routine *Modelling and problem solving* question.

Sample response

Question 7

A sketch of the cubic function $f(x) = a(x-1)(x-b)(x+c)$ is shown below.



Determine the values of a, b and c.

Use algebraic methods.

Show mathematical reasoning.

x-intercepts are at −1, 1, 2.

$\therefore (x+1), (x-1), (x-2)$

$\therefore b = 2$ and $c = 1$

$(x-b) \rightarrow (x-2)$

$(x+c) \rightarrow (x+1)$

$\boxed{\begin{array}{l} \therefore a = 2 \\ b = 2 \\ c = 1 \end{array}}$

~~For a subs. the point (1,0)~~

$\cancel{f(x) = a(x-1)(x-2)(x+1)}$

$\cancel{0 = a(1-1)(1-2)(1+1)}$

$\cancel{= a(0 \times -1 \times 2)}$

$\cancel{a = 0}$

For a subs point (0,4)

$f(x) = a(x-1)(x-2)(x+1)$

$4 = a(0-1)(0-2)(0+1)$

$4 = a(-1 \times -2 \times 1)$

$4 = 2a$

$\therefore a = 2$

Simple Routine MAP

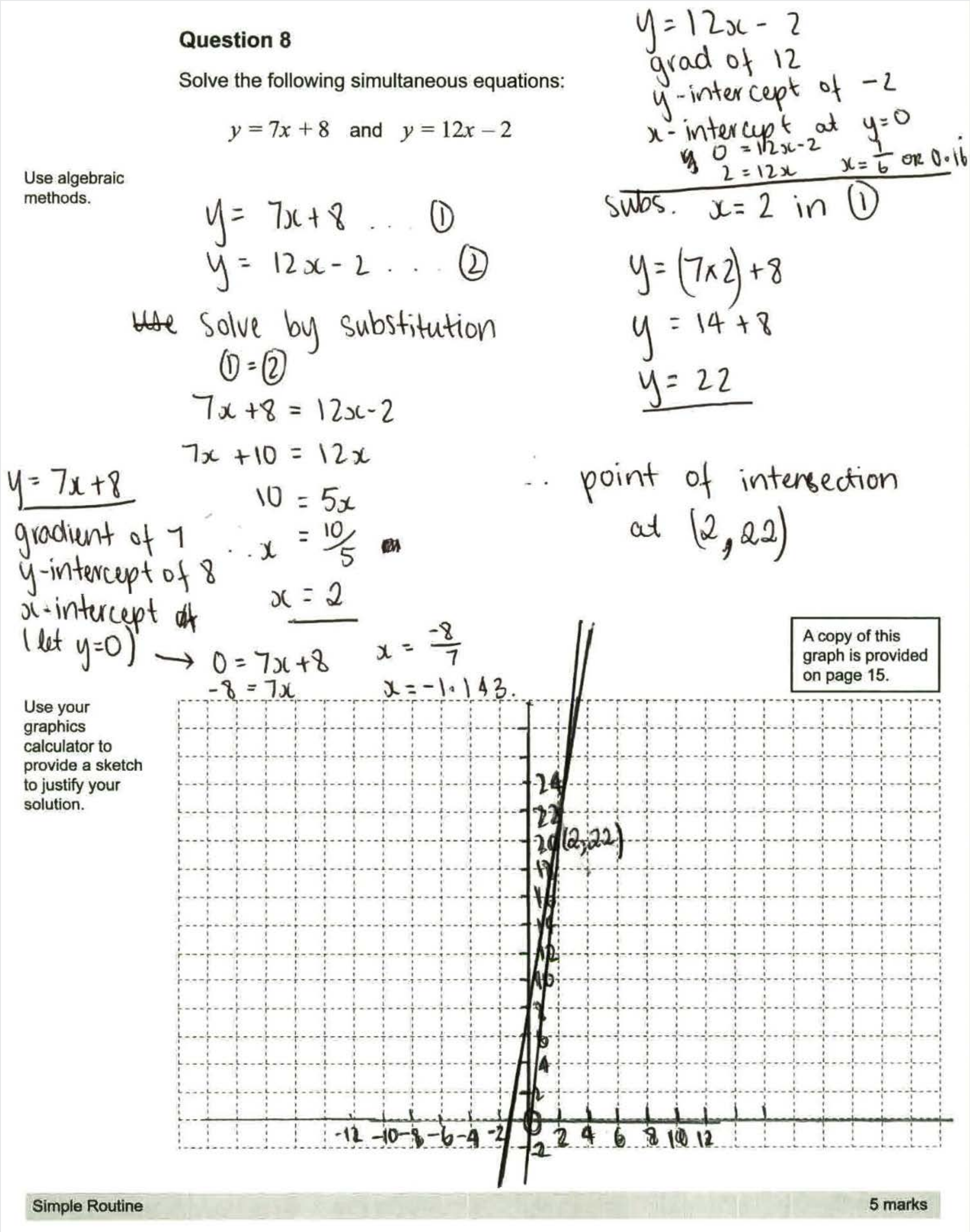
Question 7 required students to identify the critical points of a third-degree polynomial and determine the unknown parameters for a given model. A model response was awarded when a student used an appropriate strategy to correctly determine parameter values (identifying the zeros from the graph and using algebraic methods to solve for the remaining parameter).

Students achieved success with identifying the zeros on the graph. However, students had limited success with using these values to solve for the remaining parameter value. Students found this question challenging, with approximately 20% achieving either a model or a partial response.

Question 8

Question 8 was a Simple Routine *Knowledge and procedures* question.

Sample response



Question 8

Solve the following simultaneous equations:

$$y = 7x + 8 \quad \text{and} \quad y = 12x - 2$$

Use algebraic methods.

$y = 7x + 8 \ldots (1)$

$y = 12x - 2 \ldots (2)$

~~use~~ Solve by substitution

$(1) = (2)$

$7x + 8 = 12x - 2$

$7x + 10 = 12x$

$10 = 5x$

$\therefore x = \frac{10}{5}$

$x = 2$

subs. $x = 2$ in (1)

$y = (7 \times 2) + 8$

$y = 14 + 8$

$y = 22$

$\therefore$ point of intersection at $(2, 22)$

$y = 12x - 2$
grad of 12
y-intercept of -2
x-intercept at $y = 0$
$0 = 12x - 2$
$2 = 12x$
$x = \frac{1}{6}$ or 0.16

$y = 7x + 8$
gradient of 7
y-intercept of 8
x-intercept at (let $y = 0$) $\rightarrow$ $0 = 7x + 8$
$-8 = 7x$
$x = \frac{-8}{7}$
$x = -1.143$

Use your graphics calculator to provide a sketch to justify your solution.

A copy of this graph is provided on page 15.

Simple Routine 5 marks

The first section of this question required students to demonstrate their ability to select and apply a mathematical procedure, and to demonstrate their algebraic facility, by solving simultaneous equations using algebraic methods. The second section asked them to use their graphics calculator to justify their solution, and then sketch both equations, accurately showing the lines and the point of intersection.

The use of algebraic methods to solve the simultaneous equations was done well by many students. However, the point of intersection identified on the sketch was often different to the simultaneous solution and in many cases the sketch was not attempted. Full marks were achieved by 25% of students.

Question 9

Question 9 was a Complex Routine *Modelling and problem solving* question.

Sample response

Question 9

Entry tickets for the local show cost $32 for adults and $14 for children.
A group of 13 people pays $254 altogether.

Determine how many adults and how many children are in this group.

Use algebraic methods.

Identify all variables.

Develop appropriate equations to model this situation.

Justify the reasonableness of your result.

Let the number of children = x
Let the number of adults = y

this will be solved using sim. equations

$x14 + y32 = 254$ — ① (equation 1)

$x + y = 13$ — ② (equation 2)

② × 14 → $x14 + y14 = 182$ — ③ (equation 3)

$14x + 32y = 254$

① − ③ → $-14x + 14y = 182$

$18y = 72$

$y = \frac{72}{18}$

$y = 4$ — ④ (equation 4)

Substitute ④ into ② → $x + 4 = 13$

$x = 9$

∴ there were 9 children and 4 adults in this group

CHECK ⇒ $(14 \times 9) + (32 \times 4)$

$= 254$ ✓

as equation ① states

$9 + 4 = 13$ ✓ as equation ② states

Complex Routine MAP

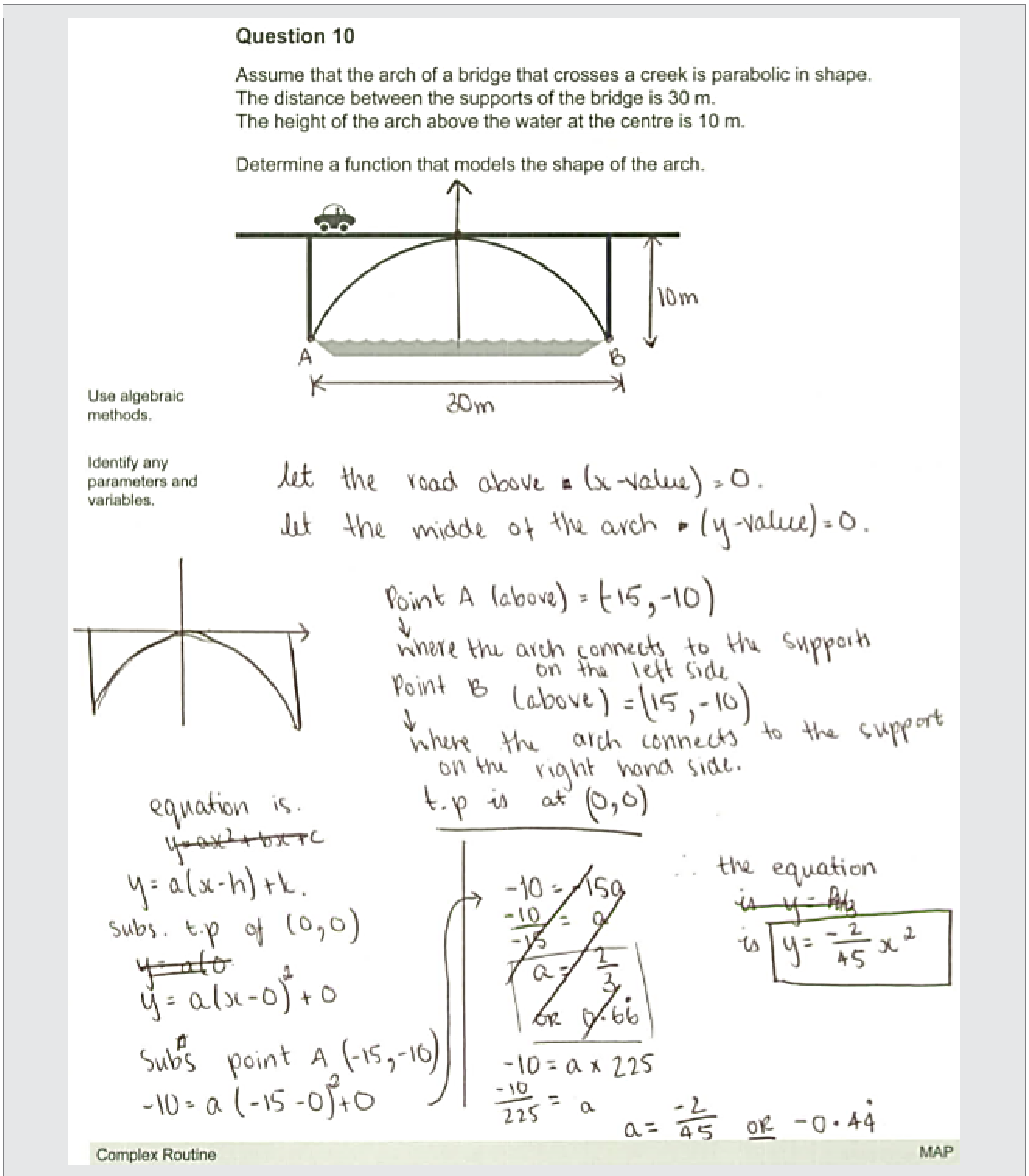
Question 9 required students to model a real-world problem by identifying variables and determining the value of the parameters. They were then required to algebraically solve the simultaneous equations and check the reasonableness of results through technology or other means. A model response required students to mathematise the problem by correctly identifying the variables, synthesising a model (simultaneous equations), solving using algebraic methods, and justifying the reasonableness of their results.

Students did not follow the cue to use algebraic methods to solve the problem. The most frequent problem solving strategy used was ‘guess and check’. Most students did not verify their solution.

Question 10

Question 10 was a Complex Routine *Modelling and problem solving* question.

Sample response



Question 10

Assume that the arch of a bridge that crosses a creek is parabolic in shape.
The distance between the supports of the bridge is 30 m.
The height of the arch above the water at the centre is 10 m.

Determine a function that models the shape of the arch.

Use algebraic methods.

Identify any parameters and variables.

let the road above (x-value) = 0.
let the midde of the arch (y-value) = 0.

Point A (above) = (-15, -10)
where the arch connects to the support on the left side
Point B (above) = (15, -10)
where the arch connects to the support on the right hand side.
t.p is at (0,0)

equation is.
~~y = ax² + bx + c~~
$y = a(x-h) + k$.
subs. t.p of (0,0)
~~y = a(0~~
$y = a(x-0)^2 + 0$
Subs point A (-15,-10)
$-10 = a(-15-0)^2 + 0$

~~$-10 = -15a$~~
~~$\frac{-10}{-15} = a$~~
~~$a = \frac{2}{3}$ or $0.\dot{6}6$~~
$-10 = a \times 225$
$\frac{-10}{225} = a$
$a = \frac{-2}{45}$ or $-0.4\dot{4}$

∴ the equation ~~is y = Ax~~ is $\boxed{y = \frac{-2}{45}x^2}$

Complex Routine MAP

Question 10 required students to represent a real-world situation using a parabolic model. They were required to use algebraic methods to generate a model.

Students achieved success with translating the given information to identify relevant coordinates. They found it more challenging to use a chosen form of a parabolic model to determine the values of parameters. Approximately 20% of the cohort achieved either a partial or a model response for this question.

Question 11

Question 11 was a Simple Routine *Knowledge and procedures* question.

Sample response

Question 11

Given that π radians $\equiv 180$ degrees, complete the following:

a) convert $\frac{4\pi}{3}$ radians to degrees

$$\frac{4\cancel{\pi}}{\cancel{3}} \times \frac{\overset{60°}{\cancel{180°}}}{\cancel{\pi}} = 240°$$

b) convert $72°$ to radians

Give your answer in terms of π.

$$72° \times \frac{\pi}{180°} = \frac{72\pi}{180}$$
$$= \frac{2\pi}{5}$$

c) calculate the acute angle θ given $\cos\theta = 0.5$

Use your graphics calculator.

Give your answer in degrees.

$$\cos\theta = 0{\cdot}5$$
$$\theta = 60°$$

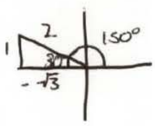
d) solve $x = \sin\left(\frac{\pi}{15}\right)$

Use your graphics calculator.

Give your answer correct to 4 decimal places.

$$x = \sin \cancel{2}12°$$
$$\cancel{x = 0{\cdot}21°}$$
$$x = 0{\cdot}2079$$

e) determine the exact value of $\tan\left(\frac{5\pi}{6}\right)$



$$\cancel{\tan(150°) = \frac{1}{-\sqrt{3}}}$$
$$\tan(150°) = -\frac{1}{\sqrt{3}}$$

Simple Routine | **1, 1, 1, 1, 2 marks**

Students were required to recall and use definitions to convert radians to degrees and degrees to radians. They had to use their calculator appropriately to determine the angle for a given trigonometric ratio and the trigonometric ratio for a given angle.

Approximately 47% of students received more than half the marks for this Simple Routine question. Errors in this question were mainly due to students not using cues that indicated the level of accuracy or the angle measure required. (Level of accuracy errors were identified as errors in communication.)

Question 12

Question 12 was a *Knowledge and procedures* question in two parts that provided Simple Routine and Non-routine opportunities.

Sample response

Question 12

a) Solve for x given that $\sqrt{2}\cos x + 6 = 5$ over the domain $0 \le x \le \pi$

Use algebraic methods.

Give answer/s in radians.

$\sqrt{2}\cos x = -1 \quad (-6)$

$\cos x = -1/\sqrt{2}$

$\cancel{\cos = opp/hyp = 1/\sqrt{2}}$

$x = \cos^{-1}(-1/\sqrt{2})$

$x = 135$

$135 \times \pi/180$

$= 135\pi/180$

$x = 3\pi/4$

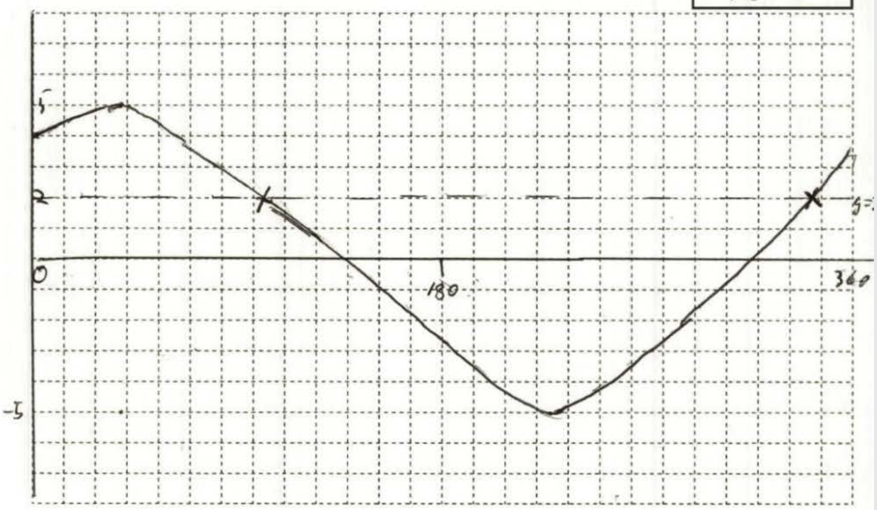
b) Determine how many solutions there are for the equation:

$3\sin x + 4\cos x = 2$ over the domain $0 \le x \le 2\pi$

There are 2 solutions as demonstrated by the graph

A copy of this graph is provided on page 16.

Use your graphics calculator to provide a sketch to justify your solution.



Simple Routine; Simple Non-Routine | 4 marks; 3 marks

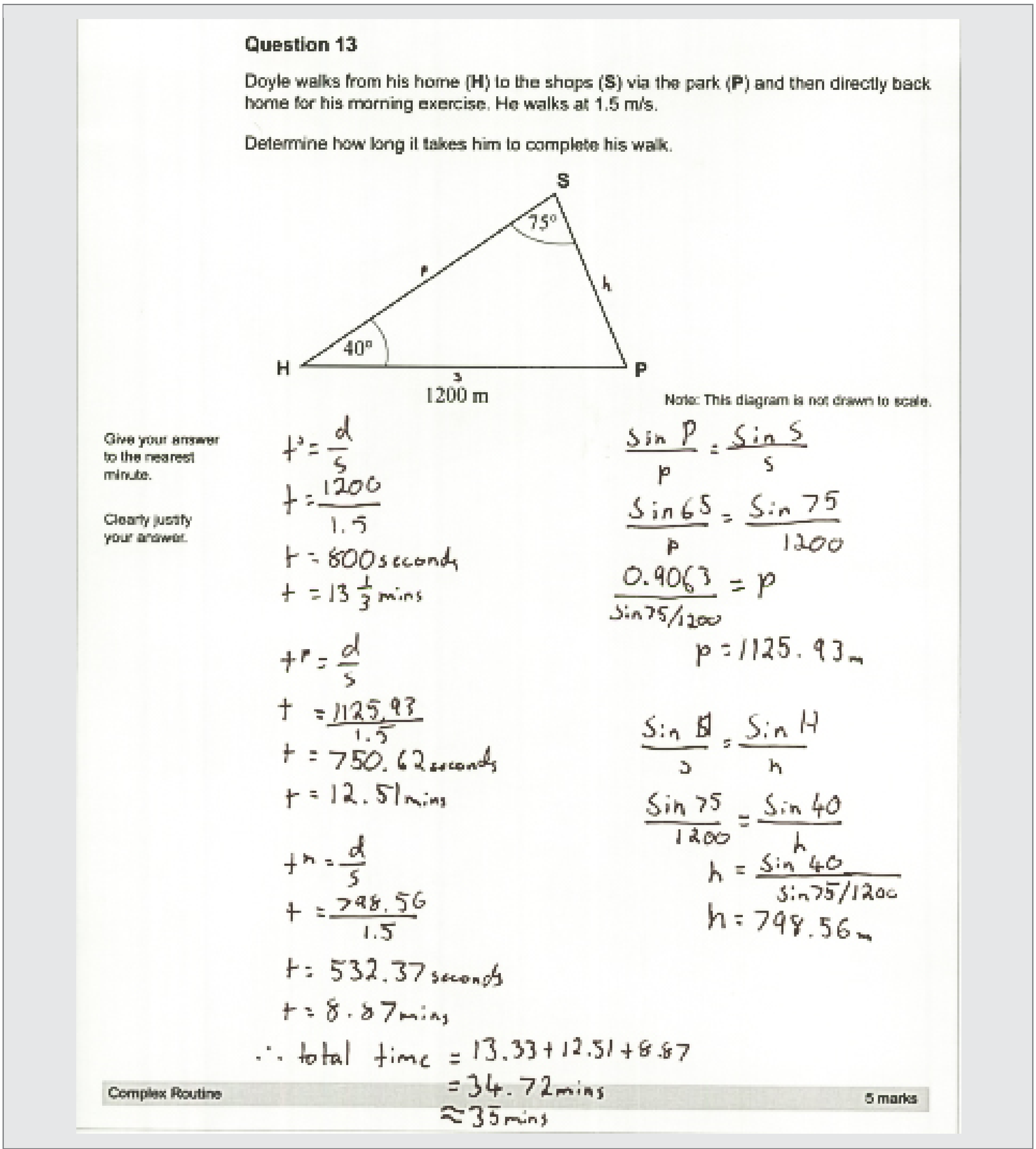
In Question 12a, students were required to solve algebraically for an exact value in radians over a given domain. In Question 12b, students were required to use a graphics calculator to determine the number of solutions for a trigonometric equation over a given domain.

Students achieved success when using algebraic facility to rearrange the trigonometric equation in Question 12a. They had limited success with recalling the exact value of the required trigonometric ratio and using the concept of the unit circle to identify which quadrant the solution was in. Question 12b proved very challenging for a number of students. They were required to use a graphics calculator to find graphical solutions to the equation and state the number of solutions. They were prompted to include a sketch of the graph. About 21% of students did not attempt this question.

Question 13

Question 13 was a Complex Routine *Knowledge and procedures* question.

Sample response



Question 13

Doyle walks from his home (**H**) to the shops (**S**) via the park (**P**) and then directly back home for his morning exercise. He walks at 1.5 m/s.

Determine how long it takes him to complete his walk.

Note: This diagram is not drawn to scale.

Give your answer to the nearest minute.

Clearly justify your answer.

$t^s = \frac{d}{s}$

$t = \frac{1200}{1.5}$

$t = 800$ seconds

$t = 13\frac{1}{3}$ mins

$t^p = \frac{d}{s}$

$t = \frac{1125.93}{1.5}$

$t = 750.62$ seconds

$t = 12.51$ mins

$t^h = \frac{d}{s}$

$t = \frac{798.56}{1.5}$

$t = 532.37$ seconds

$t = 8.87$ mins

$\therefore$ total time $= 13.33 + 12.51 + 8.87$

$= 34.72$ mins

≈ 35 mins

$\frac{\sin P}{p} = \frac{\sin S}{s}$

$\frac{\sin 65}{p} = \frac{\sin 75}{1200}$

$\frac{0.9063}{\sin 75/1200} = p$

$p = 1125.93$ m

$\frac{\sin S}{s} = \frac{\sin H}{h}$

$\frac{\sin 75}{1200} = \frac{\sin 40}{h}$

$h = \frac{\sin 40}{\sin 75/1200}$

$h = 798.56$ m

Complex Routine

5 marks

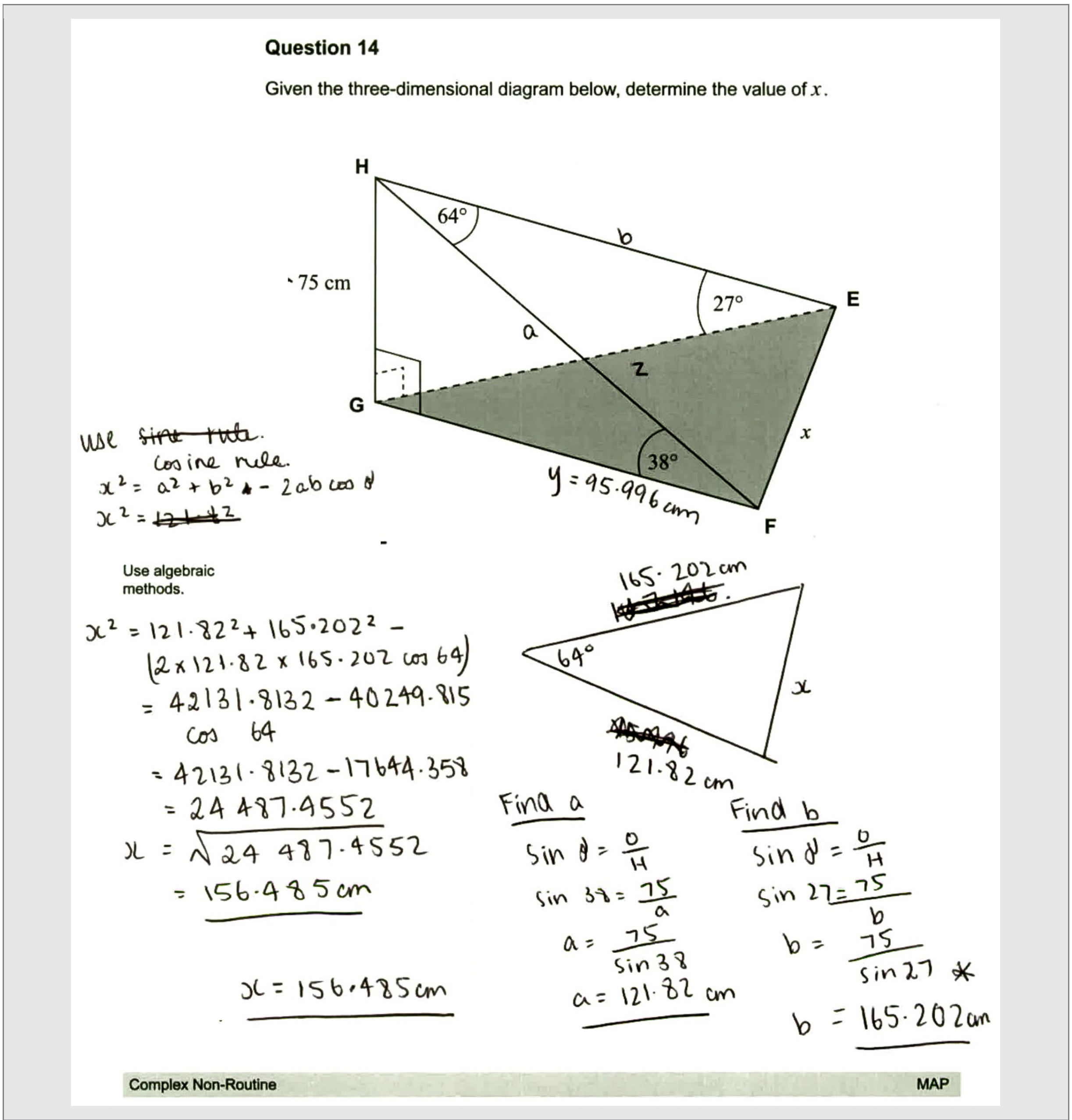
Students were required to recall, select and apply the appropriate trigonometric rules (sine rule and cosine rule) in a two-dimensional context to find the unknown distances and determine the total distance walked. The time required to complete the walk was then calculated using the given walking speed.

Overall, this question was done well with 42% of students achieving full marks. In 15% of cases students were penalised one mark as they incorrectly determined the time taken to complete the walk due to multiplying, instead of dividing by the given walking speed.

Question 14

Question 14 was a Complex Non-Routine *Modelling and problem solving* question.

Sample response



Question 14

Given the three-dimensional diagram below, determine the value of x.

use ~~sine rule~~ cosine rule.

$x^2 = a^2 + b^2 - 2ab\cos\theta$

$x^2 = \cancel{121 + 2}$

Use algebraic methods.

$x^2 = 121.82^2 + 165.202^2 - (2 \times 121.82 \times 165.202 \cos 64)$

$= 42131.8132 - 40249.815 \cos 64$

$= 42131.8132 - 17644.358$

$= 24487.4552$

$x = \sqrt{24487.4552}$

$= 156.485\text{ cm}$

$x = 156.485\text{ cm}$

Find a

$\sin\theta = \frac{O}{H}$

$\sin 38 = \frac{75}{a}$

$a = \frac{75}{\sin 38}$

$a = 121.82\text{ cm}$

Find b

$\sin\theta = \frac{O}{H}$

$\sin 27 = \frac{75}{b}$

$b = \frac{75}{\sin 27}$ *

$b = 165.202\text{ cm}$

Complex Non-Routine MAP

Students were required to interpret a three-dimensional diagram, recall and apply trigonometric ratios and use the cosine rule to determine unknown lengths.

They achieved success with the application of trigonometric ratios to determine a side length of a triangle. Students often incorrectly assumed a 90° angle for the base triangle. Approximately 30% of the cohort achieved either a model or a partial response.

Recommendations and guidance

The following should be considered when preparing students for future external examinations.

- Students need to pay careful attention to the explicit directions given in questions, such as ‘showing mathematical reasoning’, ‘using algebraic methods’, ‘justifying the reasonableness of results’, ‘giving an answer to 4 decimal places’ and ‘using different statistical measures when making justifications’.
- Annotations on particular questions were used in this paper as a means of identifying where graphics calculator use was required. It is important that all students have access to a graphics calculator without computer algebra system functionality (non-CAS) and have practised with the mathematical capabilities of this technology.
- Students must communicate how they used the calculator to find a solution with an adequate written record, for example, transferring a sketch of the graphical method they used to solve an equation to their paper.
- Students should be reminded to check their calculations, if time permits. Minor errors were often not penalised, but major errors were — for example, solving for a linear equation in Question 6b when finding the solutions to a quadratic equation was required.
- Students should also consistently practise checking the reasonableness of their results as many questions explicitly required this. They should be aware that there are often a variety of methods and checks that can be used. For example:
  - recognising a negative length for a distance is not appropriate
  - recognising when a point of intersection of two sketched linear equations is different to the result found by solving these equations simultaneously
  - understanding that an answer may be incorrect if it has decimals (for example, finding the number of adults and children in Question 9).

Appendix 1: Instrument-specific standards matrix

	A	B	C	D	E
	The student work has the following characteristics:				
Knowledge and procedures	• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations	• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations	• recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations	• use of stated rules and procedures in simple situations	• statements of relevant mathematical facts
	• application of mathematical definitions, rules and procedures in routine and non-routine simple tasks, through to routine complex tasks, in life-related and abstract situations	• application of mathematical definitions, rules and procedures in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations	• application of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations		
	• numerical calculations, spatial sense and algebraic facility in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations	• numerical calculations, spatial sense and algebraic facility in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations	• numerical calculations, spatial sense and algebraic facility in routine, simple life-related or abstract situations	• numerical sense, spatial sense and/or algebraic facility in routine or simple tasks	
	• appropriate selection and accurate use of technology	• appropriate selection and accurate use of technology	• selection and use of technology	• use of technology	• use of technology
Modelling and problem solving	• use of problem solving strategies to interpret, clarify and analyse problems to develop responses from routine simple tasks through to non-routine complex tasks in life-related and abstract situations	• use of problem solving strategies to interpret, clarify and analyse problems to develop responses to routine and non-routine simple through to routine complex tasks in life-related or abstract situations	• use of problem solving strategies to interpret, clarify and develop responses to routine, simple problems in life-related or abstract situations	• evidence of simple problem solving strategies in the context of problems	• evidence of simple mathematical procedures
		• identification of parameters and/or variables			
	• use of data to synthesise mathematical models in simple through to complex situations	• use of data to synthesise mathematical models in simple situations	• use of mathematical models to represent routine, simple situations		
		• interpretation of results in the context of simple through to complex problems and mathematical models	• interpretation of results in the context of routine, simple problems		
Communication and justification	• appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations	• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple or complex and from routine through to non-routine, in life-related or abstract situations	• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations	• use of mathematical terminology, symbols or conventions in simple or routine situations	• use of mathematical terminology, symbols or conventions
	• organisation and presentation of information in a variety of representations	• organisation and presentation of information in a variety of representations	• organisation and presentation of information	• presentation of information	• presentation of information
	• analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine	• analysis and translation of information from one representation to another in life-related or abstract situations, simple or complex, and from routine through to non-routine	• translation of information from one representation to another in simple routine situations		
	• use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language	• use of mathematical reasoning to develop coherent and logical sequences within a response in simple or complex and in life-related or abstract situations using everyday and/or mathematical language	• use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language		
	• coherent, concise and logical justification of procedures, decisions and results	• coherent and logical justification of procedures, decisions and results	• justification of procedures, decisions or results		
	• justification of the reasonableness of results				

Appendix 2: Glossary of terms

Term	Definition
calculate	determine or find (e.g. a number, answer) by using mathematical processes; ascertain/determine from given facts, figures or information
convert	to change into something of different form or properties; transmute; transform[1]
determine	establish, conclude or ascertain after consideration, observation, investigation or calculation; obtain the only possible answer; decide or come to a resolution
develop	elaborate, expand or enlarge in detail; add detail and fullness to; cause to become more complex or intricate
identify	distinguish; locate, recognise and name; establish or indicate what something is; provide an answer from a number of possibilities; recognise and state a distinguishing factor or feature
justify	give reasons or evidence to support an answer, response or conclusion; show or prove how an argument, statement or conclusion is right or reasonable
show	provide the relevant reasoning to support a response
sketch	a simple or preliminary drawing, diagram or graph, that provides the essential features without all the details
solve	find an answer to, explanation for, or means of dealing with (e.g. a problem); work out the answer or solution to (e.g. a mathematical problem)
state	a specific sentence or assertion
use	operate or put into effect; apply knowledge or rules to put theory into practice; • algebraic methods — use a methodology involving algebra to develop responses • graphical methods — use technology appropriately to apply a methodology that involves graph-based diagrams and plots to develop responses

Source:
[1] Macquarie Dictionary online, Macquarie Dictionary Sixth Edition, www.macquariedictionary.com.au