Statistics

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of candidates</th>
<th>VHA</th>
<th>HA</th>
<th>SA</th>
<th>LA</th>
<th>VLA</th>
</tr>
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<tbody>
<tr>
<td>2011</td>
<td>60</td>
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<td>2010</td>
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<td>2009</td>
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<td>9</td>
<td>7</td>
<td>20</td>
<td>17</td>
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<td>12</td>
<td>8</td>
<td>7</td>
<td>10</td>
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<td>2007</td>
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<td>3</td>
<td>7</td>
<td>16</td>
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</table>

General comments

The topics of the syllabus were assessed across both papers. All candidates completed both papers and it was pleasing to note that nearly every question was attempted at some level.

Candidates’ responses indicated that the objectives of each of the three criteria were being addressed. There was a distinct improvement from previous years in candidates meeting the Communication and justification and Modelling and problem solving criteria.

Responses indicated that there was very little difference between the two papers in the contexts of application, technology, initiative and complexity. There was evidence that candidates were as confident about responding to Paper Two as they were about Paper One.

Characteristics of good responses

The concepts of calculus are fundamental to Mathematics B and candidates who demonstrated understanding of the processes of differentiation and integration had a high rate of success.

Modelling and problem solving requires candidates to demonstrate mathematical thinking. Candidates who demonstrated “interpreting, clarifying and analysing” and “selecting and using effective strategies” met the basic requirements of this criterion. Examples of these included identifying a relevant rule or formula; modelling the situation with a diagram, list or table; or indicating a strategy to solve the problem.

Candidates who performed well in Knowledge and procedures were also able to demonstrate achievement in Modelling and problem solving and Communication and justification at a standard sufficient to be awarded a Sound Achievement.
Common weaknesses

Candidates were provided with opportunities to demonstrate achievement in each criterion across the full range in both papers. Unlike previous years there was no particular topic which was noticeably more poorly handled than any other.

Candidates tended to perform better on earlier questions in the paper rather than the later ones even when there was little difference in complexity between them.

Sample solutions

The following solutions are not necessarily prescriptive model responses and are not necessarily the only way of solving a problem. Other approaches and problem-solving strategies may be just as acceptable.
a) (i) Both variables are continuous. Continuous variables have an infinite number of points with no discontinuities or "gaps."
(ii) Dependent variable is Volume (depending on Time)
(iii) Domain: $\mathbb{R} : 0 \leq t \leq 8$
Range: $V \in \mathbb{R} : 4 \leq V \leq 20$
(iv) Choose 2 points on the line $(0, 4)$ and $(8, 20)$
Equation of line:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 4 = \frac{20 - 4}{8 - 0} (x - 0)$$

$$y - 4 = 2x$$

$$y = 2x + 4$$

i.e. Required equation is $V = 2t + 4$
Question 1

(b) 

(i) \[ C = 550 + 50k \]

(ii) \[ k = 2 \Rightarrow C = 550 + 50 \times 2 \]

\[ \therefore C = \$1150 \]

(c) 

\[ (\circ C, \circ F) \Rightarrow (0, 32) \text{ and } (100, 212) \]

\[ y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \]

\[ y - 32 = \frac{212 - 32}{100 - 0} (x - 0) \]

\[ y - 32 = 1.8x \]

\[ \therefore \frac{\circ F}{\circ C} = 1.8 \]

\[ 1.8 \circ C + 32 \]
Question 2

(a)

(b)

(i) \( f(x) = 2x + 1 \)
\( f(3) = 2(3) + 1 = 7 \)
\( g(x) = x^2 - 3 \)

(ii) Let \( y = 2x + 1 \)
\( y - 1 = 2x \)
\( x = \frac{y - 1}{2} \)
\( f^{-1}(x) = \frac{x - 1}{2} \)

(iii) \( f(g(-3)) = f((x-3)^2 - 3) \)
\( = f(6) \)
\( = 2 \times 6 + 1 \)
\( = 13 \)
Question 2 (cont)

(c) \[ y = x^2 - 3x - 6 \]
\[ x - y + 6 = 0 \]

\[ x - (x^2 - 3x - 6) = 0 \]
\[ x - x^2 + 3x + 6 = 0 \]
\[ x^2 - 4x - 12 = 0 \]

\[ (x - 6)(x + 2) = 0 \]

Either \( x - 6 = 0 \) or \( x + 2 = 0 \)

\( x = 6 \) or \( x = -2 \)

Integers where \( x = 6 \) : \( y = 12 \)

\( (6, 12) \)

when \( x = 2 \) : \( y = 4 \)

\( (-2, 4) \)

Solutions are \( (6, 12) \) and \( (-2, 4) \)

(d) \[ x^2 + 4x - 8 = 0 \]

\[ a = 1, \ b = 4, \ c = -8 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{-4 \pm \sqrt{16 + 32}}{2} \]

\[ = \frac{-4 \pm \sqrt{48}}{2} \]

\[ = -2 \pm 2\sqrt{3} \]

\( (or \ -5.4, \ 4.6 \ or \ 14.6) \)
Question 2 (cont)

(a) (ii)

\[ A = \frac{bh}{2} \]

\[ 24 = x(2x+2) \]

\[ 48 = x^2 + 2x \]

\[ x^2 + 2x - 48 = 0 \]

\[ (x+8)(x-6) = 0 \]

Either \( x = -8 \) or \( x = 6 \)

Ignore \( x = -8 \) as length \( \neq -ve \)

\( x = 6 \) and \( x+2 = 8 \)

Triangle is 3, 4, 5 triangle

Hypotenuse = 10 cm

\[ P = 6 + 8 + 10 = 24 \text{ cm} \]

\[ p = a + 3x + cx^2 \Rightarrow p = 2 - x + 3x^2 \]

(2) \( p = 50 \Rightarrow 3x^2 - x + 2 = 50 \]

\[ 10 \cdot 3x^2 - x - 48 = 0 \]

Solving by \( \Delta \) Calc, formula or complete sq.

\[ x = -3.837 \quad \text{or} \quad x = 4.017 \]

(Not possible)

\[ x = 4.017 \]

Dosage required is 41.7 mg/L.
Question 3

(a) (i) \( \bar{x} = \frac{\sum x}{n} \) (or by G-Calc)

\[ \bar{x} = \frac{597}{20} \]

(ii) \( \sigma = \sqrt{\frac{\sum (x-\bar{x})^2}{n}} \) (or by G-Calc)

\[ \sigma = 16.78 \]

(iii) Trial 1 Trial 2

\[ \begin{array}{cccc}
7 & 5 & 4 & 0 \\
6 & 7 & 7 & 1 \\
3 & 6 & 3 & 2 \\
4 & 1 & 3 & 2 \\
5 & 6 & 4 & 3 \\
7 & 6 & 4 & 5 \\
4 & 3 & 6 & 3 \\
6 & 2 & 3 & 8 \\
\end{array} \]

Key: 116 = 16

3.32 = 23

(iv) The leaves in Trial 2 have a trend of higher scores. There is only one one-digit score. This suggests that students have learned to improve their scores by having had another attempt at the new game.
Question 3

(b) Class A \( \bar{x} = 62, \ n = 20 \)

\[ \Rightarrow \text{Total of scores} = 62 \times 20 = 1240 \]

New class A total = 1240 + (50 + 58 + 47 + 63)

New \( \bar{x} = \frac{602}{20+4} = 59.75 \)

Class B \( \bar{x} = 59, \ n = 28 \)

\[ \Rightarrow \text{Total of scores} = 59 \times 28 = 1652 \]

New total = 1652 - (50 + 58 + 47 + 63)

New \( \bar{x} = \frac{1434}{28-4} = 59.75 \)

(c)(i) \begin{align*}
\text{B} & \quad \text{BB} \quad \text{BR} \\
\text{R} & \quad \text{BR} \quad \text{RR} \\
\text{W} & \quad \text{BW} \quad \text{WW} \\
\text{W} & \quad \text{BW} \quad \text{WW} \quad \text{WW} \quad \text{WW} \\
\text{R} & \quad \text{BR} \quad \text{RR} \quad \text{WW} \quad \text{WW} \\
\end{align*}

(ii) \( P(\text{same colour}) = P(\text{BB}) + P(\text{RR}) + P(\text{WW}) \)

\[ = \frac{6 \times 5}{21 \times 20} + \frac{7 \times 6}{21 \times 20} + \frac{8 \times 7}{21 \times 20} \]

\[ = \frac{32}{105} \ (\approx 0.305) \]
Question 3

(a) Binomial probability

\[ p = 0.65 \]
\[ q = 0.35 \]
\[ n = 10 \]

\[ P(X = 8, 9, or 10) = P(X = 8) + P(X = 9) + P(X = 10) \]
\[ = \binom{10}{8} (0.65)^8 (0.35)^2 + \binom{10}{9} (0.65)^9 (0.35)^1 + (0.65)^{10} \]
\[ = 0.1757 + 0.2725 + 0.0135 \]
\[ = 0.4617 \]

(or use tables or 6-Cell)

(b) Continuous variable

\[ Z = \frac{X - \mu}{\sigma} \]

\[ P(1000 < X < 1250) = P(\frac{1000 - 1396}{188} < Z < \frac{1250 - 1396}{188}) \]
\[ = P(-2.106 < Z < -0.777) \]
\[ = 0.4824 - 0.2814 \]
\[ = 0.201 \]
(f) The model to be used is the normal distribution, both are normally distributed.

Multiple choice questions ⇒ variable is discrete.

Maths

\[ Z = \frac{71.5 - 68}{10} = 0.35 \]

i.e. Maths score is 0.35 standard deviations above average.

Science

\[ Z = \frac{47.5 - 45}{8} = 0.3125 \]

i.e. Science score is 0.3125 standard deviations above average.

So Maths is the better performance.
(a) \[ f'(x) = 3x^2 + 1 \]
\[ f'(x+h) = 3(x+h)^2 + 1 \]
\[ = 3x^2 + 6xh + 3h^2 + 1 \]
\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
\[ = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 1 - 3x^2 - 1}{h} \]
\[ = \lim_{h \to 0} \frac{6xh + 3h^2}{h} \]
\[ = \lim_{h \to 0} \frac{6x + 3h}{1} \]
\[ = 6x \]

(b) (i) \[ y = 3x^2 + x^3 \]
\[ \frac{dy}{dx} = 6x + 3x^2 \]

(ii) \[ y = \sqrt{x^2 + 3x} \]
Let \( y = u^{1/2} \), where \( u = x^2 + 3x \)
\[ \frac{dy}{du} = \frac{1}{2} u^{-1/2} \]
\[ \frac{du}{dx} = 2x + 3 \]
\[ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \]
\[ = \frac{1}{2\sqrt{u}} 2x + 3 \]
\[ = \frac{2x + 3}{2\sqrt{x^2 + 3x}} \]
Question 4

(c)

<table>
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<tr>
<th>x</th>
<th>-10</th>
<th>-1</th>
<th>-0.1</th>
<th>-0.01</th>
<th>-0.001</th>
<th>0</th>
<th>0.001</th>
<th>0.01</th>
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<tr>
<td>tan x</td>
<td>0.065</td>
<td>1.56</td>
<td>1.003</td>
<td>1.0003</td>
<td>1.00003</td>
<td>1/1</td>
<td>1.00003</td>
<td>1.00003</td>
</tr>
</tbody>
</table>

\[ \lim_{x \to 0^-} \frac{\tan x}{x} = 1 \]

\[ \lim_{x \to 0^+} \frac{\tan x}{x} = 1 \]

\[ \therefore \lim_{x \to 0} \frac{\tan x}{x} = 1 \]

(d)

\[ y = 2x^2 - x + 5 \]

Gradient: \( \frac{dy}{dx} = 4x - 1 \)

Gradient of \( y = 3x - 2 \) is 3.

\[ 4x - 1 = 3 \]

\[ 4x = 4 \]

\[ x = 1 \]

When \( x = 1 \), \( y = 2(1)^2 - 1 + 5 = 6 \)

15. Coordinates of point is \((1, 6)\)
(iii) \[ y = x^3 e^{2x} \]

Let \( y = u \cdot v \), where \( u = x^3 \) and \( v = e^{2x} \).

\[
\frac{du}{dx} = 3x^2 \quad \text{and} \quad \frac{dv}{dx} = 2e^{2x}
\]

\[
\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}
\]

\[
= e^{2x} \cdot 3x^2 + x^3 \cdot 2e^{2x}
\]

\[
= e^{2x} (3x^2 + 2x^3)
\]

\[
= x^2 e^{2x} (3 + 2x)
\]

(iv) \[ y = \ln (\cos x) \]

Let \( y = \ln u \), where \( u = \cos x \).

\[
\frac{dy}{du} = \frac{1}{u} \quad \text{and} \quad \frac{du}{dx} = -\sin x
\]

\[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}
\]

\[
= \frac{-\sin x}{\cos x}
\]

\[
= -\tan x
\]

(v) \[ y = \frac{\sqrt{x} + 2x}{x - 4} \]

Let \( y = \frac{u}{v} \), where \( u = \sqrt{x} + 2x \) and \( v = x - 4 \).

\[
\frac{du}{dx} = \frac{1}{2\sqrt{x}} + 2 \quad \text{and} \quad \frac{dv}{dx} = 1
\]

\[
\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}
\]

\[
= \frac{(x-4)(2x+4) - (\sqrt{x}+2x)\cdot 1}{(x-4)^2}
\]

\[
= \frac{2x^2 - 8x - 8}{(x-4)^2}
\]
Question 5

(a) \[ \tan 15^\circ = \frac{140}{x} \quad \text{and} \quad \tan 7^\circ = \frac{140}{y} \]

\[ \Rightarrow x = \frac{140}{\tan 15^\circ} \quad \text{and} \quad y = \frac{140}{\tan 7^\circ} \]

i.e. \( x = 522.487 \) and \( y = 1140.209 \)

\[ \text{Speed} = \frac{\text{Distance}}{\text{Time}} \]

\[ = \frac{(1140.209 - 522.487) \text{ m}}{15 \text{ min}} \]

\[ = \frac{617.722 \text{ m}}{0.25 \text{ h}} \]

\[ = 2468 \text{ m/h} \]

Speed is approx. 2.5 km/h
Question 5

b) (i) \[ \frac{\pi}{3} \times 180^\circ = \frac{3}{3}\pi \times 180^\circ = 180^\circ \]

(ii) \[ \sin^{-1}(120^\circ) = \sin^{-1}0.8 \]

(iii) \[ y = 10 \cos \frac{\pi}{4} (x - 2) + 5 \]
- Amplitude = 10
- Period = \( \frac{2\pi}{\frac{\pi}{4}} = 8 \)
- Phase = 2
- Vertical = 5

(iv) \[ 2 \sin^2 \theta = 3 \cos \theta + 1 \]
\[ 2 (1 - \cos^2 \theta) = 3 \cos \theta + 3 \]
\[ 2 - 2 \cos^2 \theta = 3 \cos \theta + 3 \]
\[ 2 \cos^2 \theta + 3 \cos \theta + 1 = 0 \]
\[ (2 \cos \theta + 1)(\cos \theta + 1) = 0 \]
\[ \cos \theta = -\frac{1}{2} \text{ or } \cos \theta = -1 \]
\[ \cos \theta = -\frac{1}{2} \text{ or } \cos \theta = -1 \]
\[ \theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \]
Question 5

Model is: \( y = A \cos(Bx + C) + D \)

Amplitude: \( 27.4 - 9.6 \)

\[ i.e. \ A = 9.9 \]

Period: \( \frac{\pi}{B} \)

\[ 12 = \frac{\pi}{B} \Rightarrow B = \frac{\pi}{12} \]

Max in Jan & Min in July

\[ \Rightarrow \ Max \ at \ t = 1 \quad \text{Min at } t = 7 \]

\[ i.e. \ Phase = -1 \]

\[ i.e. \ C = -1 \]

Vertical: \( 27.4 + 9.6 \)

\[ = 19.5 \]

\[ \Rightarrow \ y = 9.9 \cos\left(\frac{\pi x}{12} - 1\right) + 19.5 \]

or \( y = 9.9 \cos\left(\frac{\pi x}{6} - \frac{1}{2}\right) + 19.5 \)

Assumption: Other data points fit the model.

Overall mean is average of \( 114 \) & \( 116 \), fitting few data points provided sufficient to form the model.
Question 1

(a) (i) \( f'(1) = 0 \Rightarrow (1, 0) \)
\( f'(2) = 2 \Rightarrow \text{gradient} = 2, \text{ and} \)
so is a linear function.

(ii) From (i) \( \Rightarrow (1, 0) \) and \( m = 2 \)
\[ y - y_1 = m(x - x_1) \]
\[ y - 0 = 2(x - 1) \]
\[ \therefore y = 2x - 2 \]

(b) (i) \( x \)-intercepts \(-4, 0); (1, 0); (3, 0)\)
\( y \)-intercept \((0, 2)\)
(iii) local min at \((2, -1)\)
(iii) function is negative \(\Rightarrow y < 0 \)
\(\text{i.e. part of graph below x-axis} \)
\[ x < -4 \text{ and } 1 < x < 3 \]
(iv) derivative is negative \(\Rightarrow\) negative gradient
\[-2 < x < 2\]
(c) \[ y = x^3 - 12x + 16 \]

\[
\frac{dy}{dx} = 3x^2 - 12 \]

\[ \frac{dy}{dx} = 0 \quad \text{when} \quad 3x^2 - 12 = 0 \]

\[ 3(x^2 - 4) = 0 \]

\[ 3(x-2)(x+2) = 0 \]

i.e. when \( x = 2 \) or \( x = -2 \)

when \( x = 2 \), \( y = 2^3 - 12 \times 2 + 16 \)

\[ = 0 \]

i.e. \( (2, 0) \)

when \( x = -2 \), \( y = (-2)^3 - 12 \times (-2) + 16 \)

\[ = 32 \]

i.e. \( (-2, 32) \)

(11) Consider vicinity of \( x = -2 \)

<table>
<thead>
<tr>
<th>( x &lt; -2 )</th>
<th>( x = -2 )</th>
<th>( x &gt; -2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} )</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

Max at \((-2, 32)\)

Consider vicinity of \( x = 2 \)

<table>
<thead>
<tr>
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<th>( x = 2 )</th>
<th>( x &gt; 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} )</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Min at \((2, 0)\)
Question 1

\[ P(x) = 0.0003x^3 - 1.5x^2 + 200x - 1000 \]

Sketch of \( P(x) \) has local MTK at \((9.2, 7040)\)

End point MTK at \((3.50, 13)\)

Profit of $7040 also occurs at \( x = 3.50 \)

Businesses will want to maximise profit (with least "risk")

Consider \( x = 9.2 \): A profit of over $5000 occurs between 50 and 150 items.

Managing this production allows a steady, modest profit.

Consider \( x = 350 \): A profit of over $7040 occurs between the production of 315 and 350 items. If this production (of at least 315 items) can be maintained, then this is a preferred option.

The owner should now be able to choose either \( 50 < x < 150 \) or \( x > 315 \).
Let \( x \) be the required distance from \( O \)

\[ KX = \sqrt{x^2 + 9} \quad \text{and} \quad XD = 6 - x \]

\[ \text{Time} = \frac{\text{Distance}}{\text{Speed}} \]

Time for \( KX = \frac{\sqrt{x^2 + 9}}{\frac{1}{5}} \) for \( XD = \frac{6 - x}{\frac{5}{5}} \)

Total Time:

\[ T(x) = \frac{\sqrt{x^2 + 9}}{\frac{1}{4}} + \frac{6 - x}{\frac{5}{5}} \]

\[ T'(x) = \frac{x}{2 \sqrt{x^2 + 9}} - \frac{1}{5} \]

\[ = 0 \quad \text{when} \quad \frac{x}{2 \sqrt{x^2 + 9}} = \frac{1}{5} \]

\[ \Rightarrow \frac{5x}{2 \sqrt{x^2 + 9}} = \frac{x}{\sqrt{x^2 + 9}} \]

\[ \Rightarrow x = \sqrt{31} \quad \text{or} \quad 1.809 \]

1.5 Required dist approx 1.8 km

Assumptions:
- Winds, currents etc do not affect paddling speed.
- Max walking speed is maintained.
- No time lost changing paddling to walking.

Suppose wind and current allow paddling speed to increase, then \( x \) increases and so walking distance decreases.
Question 2

(a) (i) \( \int (2x^3 + 6x - 5x) \, dx \)
\[ = \frac{2x^4}{4} + 2x^2 - \frac{5x^2}{2} + C \]

(ii) \( \int \frac{2}{4x+1} \, dx \)
\[ = \frac{1}{2} \int \frac{1}{4x+1} \, dx \]
\[ = \frac{1}{2} \ln |4x+1| + C \]

(iii) \( \int \sqrt{2} e^x \, dx \)
\[ = \left[ \frac{e^x}{\sqrt{2}} \right]_0 \]
\[ = e^x - 2^0 \]
\[ = 2e^x - 2 \quad \text{(or} \ 107.196) \]

(iv) \( \int (\sin 2x + \frac{1}{\sqrt{x}}) \, dx \)
\[ = -\frac{1}{2} \cos 2x + \frac{\sqrt{x}}{2} + C \]
\[ = -\frac{1}{2} \cos 2x + 2\sqrt{x} + C \]
(b) \[ y = \frac{1}{x} + 1 \]

\[ \text{no } x \text{-intercepts} \]

\[ \text{Area} = \int_{1}^{3} \left( \frac{1}{x} + 1 \right) \, dx \]

\[ = \left[ \ln x + x \right]_{1}^{3} \]

\[ = (\ln 3 + 3) - (\ln 1 + 1) \]

\[ = \ln 3 + 2 \approx 3.0986 \text{ units}^2 \]

(c) \( a = 4 - 2t \) \( a = 0 \) when \( 4 - 2t = 0 \)

\[ N = \frac{\text{d}v}{\text{d}t} \]

\[ = 4t - t^2 + 2 \]

\( t = 0, N = 0 \Rightarrow 0 = 0 - 0 + 2 \) \( \Rightarrow c = 2 \)

\[ 1 \Rightarrow N = 4t - t^2 + c \]

\[ = 2t^2 - \frac{t^3}{3} + c \]

\( t = 0, s = 0 \Rightarrow 0 = a \cdot 0^2 + c \) \( \Rightarrow c = 0 \)

\[ 1 \Rightarrow s = 2t^2 - \frac{t^3}{3} \]

\( t = 2 \Rightarrow N = 4 \times 2 - 2^2 \]

\[ = 4 \text{ m/s} \]

\[ s = \frac{2t^3}{3} - \frac{t^4}{3} \]

\[ = \frac{5}{3} \text{ m} \]
Question 2 cont

(a) \[ A = \frac{1}{2} \int \left( E + 2M \right) \] by using the rule

\[ h = \frac{14}{3} \]
\[ h = 2m \]
\[ E = 18 + 20 \]
\[ E = 3.3 \]
\[ M = 10 + 8 + 7 + 9 + 5 + 18 \]
\[ = 67 \]
\[ A = \frac{1}{2} \left( 33 + 2 \times 67 \right) \]
\[ = 167 \text{ m}^2 \]

(c) \[ y = 0.2x^4 + 0.3x^3 + 0.5x \]

(i) \[ 40\% \implies x = 0.4 \]
\[ y = 0.2(0.4)^4 + 0.3(0.4)^3 + 0.5(0.4) \]
\[ = 0.1048 \]

20\% of people earn 10\% of total income.

(ii) Area under heavy curve = \( \int (2x^4 + 3x^3 + 5x) \) dx
\[ = 0.2816 \]

Area under \( y = x \) = 0.5

Coefficient = \( \frac{0.5 - 0.21866}{0.5} \)
\[ = 0.4367 \]
Enclosed area $= A_1 + A_2$

$A_1 = \int_{0}^{\pi} \left[ (\sin \theta) - (1 - \cos \theta) \right] d\theta$

$= \left[ -\cos \theta - \theta + \sin \theta \right]_{0}^{\pi}$

$= \frac{2\pi}{2}$

$A_2 = \int_{\pi}^{2\pi} \left[ (1 - \cos \theta) - (\sin \theta) \right] d\theta$

$= \left[ \theta - \sin \theta + \cos \theta \right]_{\pi}^{2\pi}$

$= 2 + \frac{3\pi}{2}$

The total is not as the student claims it - it is likely that only the area by was calculated.
Question 3

(a) \[ A = P \left(1 + \frac{i}{12}\right)^{12 \times 36} \]
\[ = 8000 \left(1 + \frac{0.072}{12}\right)^{36} \]
\[ = \$9922.41 \]
Extra needed = \$10,000 - \$9922.41
\[ = \$77.59 \text{ (or } \$78 \text{ approx.)} \]

(b) \[ FV = PV \left(1 + \frac{i}{12}\right)^{12 \times n} \]
\[ FV \text{ of Coastal} = FV \text{ of CBD} \]
\[ \frac{800,000 \left(1 + 0.0475\right)^n}{600,000} = \frac{1,0625^n}{1.0475^n} \]
\[ \frac{4}{3} = (1.0143)^n \]
\[ \log \frac{4}{3} = \log (1.0143)^n \]
\[ \frac{\log 4}{\log 3} = n \log 1.0143 \]
\[ i.e. \quad n = \frac{\log 4}{\log 1.0143} \]
\[ = 20.23 \text{ years} \]
\[ \text{i.e. CBD overtakes Coastal after 21 years} \]
Question 3

(c) \( FV = \frac{R \left( (1+i)^n - 1 \right)}{i} \)

\[ = 2000 \times \left[ \frac{(1+0.025)^20 - 1}{0.025 \times 4} \right] \]

\[ = \$47,638.64 \]

(Chk TVM: \( PMT = -2000, i = 7\%, n = 20, PV = 0, FV = ? \))

(d) This will be a present value (annuity)

\[ PV = R \left[ \frac{1 - (1+i)^{-n}}{i} \right] \]

\[ = 1250 \times \left[ 1 - \left( 1 + \frac{0.075}{12} \right)^{-48} \right] \]

\[ = \$51,697.96 \]

(e) Investment #1:

\( FV = 3500 \times \left( 1 + \frac{0.06}{12} \right)^{16} \)

\[ = \$4188.38 \]

Annuity:

\( FV = 450 \times \left[ \frac{(1 + \frac{0.0525}{12})^{12} - 1}{\frac{0.0525}{12}} \right] \)

\[ = \$5531.55 \]

Balance (after \$1200 withdrawn) = \$4531.55

This accrues interest for last 2 years

\( FV = 4531.55 \times \left( 1 + \frac{0.0525}{12} \right)^{24} \)

\[ = \$50,320.42 \]
(c) cont.

The regular payments are considered to be another annuity for 12 years.

\[ FV = 450 \times \left( \frac{1 + \frac{0.0525 \times 12}{12}}{\frac{0.0525}{12}} - 1 \right) \]

\[ = $11,361.02 \]

Total available for deposit:

\[ = 4188.38 + 5032.42 + 11361.02 \]

\[ = $20,582.00 \]

Peter did not have $25,000.
Question 4

(a) (i) \[ \frac{2a}{2a} \times \frac{3a}{6a} = \frac{2a \times 3a}{2a \times 6a} = \frac{2a \times 3a}{2a \times 2a} = \frac{3a}{2a} \]

(ii) \[ \frac{(2x^2)^2 \times \sqrt{4x^3}}{\frac{8x^2}{8x^2} - 2} = \frac{4x^4 \times 2x^{\frac{3}{2}}}{8x^2} = \frac{8x^{4 + \frac{3}{2}}}{8x^2} = x^{\frac{23}{2}} \]

(b) (i) \[ \log_x 5 + \log_x \left(\frac{1}{2}\right) = \log_x (5 \times \frac{1}{2}) = \log_x 1 = 0 \]

(ii) \[ \log_x 5^{\frac{1}{2}} = x \Rightarrow 5^{\frac{1}{2}} = x^x \Rightarrow 5 = 2^{3x} \Rightarrow 9 = 3^{3x} = 3 \times 3 = 3 \times 3 \]
Question 14 cont

(c) (i) \[ \frac{3^x + 5}{3^{2x+5}} = \frac{27}{3^3} \]
   \[ \Rightarrow 2x + 5 = 3 \]
   \[ x = -1 \]

(ii) \[ 5 \log_{10} x = 3 + 2 \log_{10} x \]
   \[ \Rightarrow 3 \log_{10} x = 3 \]
   \[ \log_{10} x = 1 \]
   \[ x = 10 \]

(ii) They are inverse functions and reflect about \( y = x \)

(e) (i) \[ y = 100a^x \]
   \[ x = 1 \Rightarrow \log_{10} y = 3 \Rightarrow y = 10^3 \]
   \[ \Rightarrow 10^3 = 100a \]
   \[ \Rightarrow a = 10 \]
   \[ x = 1 \Rightarrow 1000 = 100a \]

(ii) \[ x = 2 \Rightarrow y = \frac{100 \times 10^2}{10} \]
Question 5

(a) (i) From table \( t = 0 \Rightarrow p = 100 \)
(ii) \( t = 10 \Rightarrow p = 52.2 \)

(b) Linear \((0, 40.6051) \; \text{and} \; (10, 3.9532)\)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.9532 - 40.6051}{10 - 0} = -0.6499
\]

Using \( y = mx + c \), we have

\[
\ln p = -0.06499t + 4.6051
\]

(c) \[
p = e^{-0.06499t + 4.6051} = e^{4.6051 - 0.06499t}
\]