Statistics

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of candidates</th>
<th>Level of achievement</th>
</tr>
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<tbody>
<tr>
<td></td>
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<td>VHA</td>
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<td>2010</td>
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<td>6</td>
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<td>2009</td>
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<td>2008</td>
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<td>2006</td>
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General comments

The topics of the syllabus are assessed across both papers. Candidates should complete both papers to ensure sufficient achievement has been demonstrated in each criterion to meet the exit standards for a Sound Achievement or higher.

Candidates generally performed slightly better in Knowledge and procedures (KP) in Paper One while performances in Modelling and problem solving (MP) showed no discernible difference between Paper One and Paper Two. The standard achieved in Communication and justification (CJ), which includes organising and presenting information, was consistent across both papers. The candidates who performed poorly in Communication and justification were often those whose organisation and presentation was poor — indicators included incorrect spelling, grammatical and notational errors, lack of use of units, diagrams and tables not used, poor use of pencil, ruler, etc.

Some candidates did not attempt all Modelling and problem solving questions. As each criterion contributes equally to the awarding of a level of achievement, candidates are encouraged to attempt all questions and demonstrate at least some mathematical thinking.

Characteristics of good responses

Modelling and problem solving requires candidates to demonstrate mathematical thinking. High quality responses demonstrated this across the full range. Candidates are encouraged to attempt each Modelling and problem solving question to demonstrate mathematical thinking which includes interpreting and clarifying a range of situations as well as selecting strategies or procedures required to solve the question.

In Communication and justification, candidates need to justify their solutions by using mathematical reasoning. The higher quality responses included the effects of assumptions and evaluation of the validity of arguments.
Common weaknesses

Candidates who performed poorly in calculus also performed poorly in nearly all other topics. Candidates who showed a grasp of differentiation and integration invariably had a high rate of success.

The questions relating to Finance were answered better this year than previous years. Candidates who performed poorly had difficulty distinguishing between present value (PV) and future value (FV).

Candidates should use their graphing calculator to assist them to obtain a result. Full working and explanation is still required. Simply listing the keystrokes on a graphing calculator does not demonstrate or explain how the problem is solved.

Sample solutions

The following solutions are not necessarily prescriptive model responses and are not necessarily the only way of solving a problem. Other approaches and problem-solving strategies may be just as acceptable.
(a) (i) Cadet's exam scores

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
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<tr>
<td>11</td>
<td>3</td>
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<tr>
<td>12</td>
<td>1</td>
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<td>19</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
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(ii) The shape of the histogram is right-skewed; i.e. the frequency becomes less (=tail) as scores increase. In this case all students scored more than 50% (100%).

(iii) One cadet scored 100% (100) and appears to be an outlier.

The positive skew (=right skew) is reasonable as we would expect most/all intake cadets to "pass" the entrance exam.

\[ \bar{x} = \frac{\sum fx}{\sum f} \]

\[ = \frac{2 \times 11 + 6 \times 15 + \ldots + 1 \times 20}{30} \]

\[ = 13 \frac{1}{3} \text{ or } 13.3 \]
(a) There is a number of possible datasets; e.g.

- 8 scores and median = 6.5 \(\Rightarrow\) 4^th \& 5^th scores are 6, 7

- Mode = 6 \(\Rightarrow\) 2^nd score in 6 (2^nd score may also be 6)

- Min = 3 \(\Rightarrow\) Q_3 - Q_1 = 3

One possibility is Q_3 = 9 and Q_1 = 6, p.e.

- This set: 6^th score is 7, 7^th score is 7

- Range = 9 \(\Rightarrow\) Max - Min = 9

One possibility is Max = 13 and Min = 4

(b) Data set is:

4 6 6 6 7 7 11 13
(a) (i) TV Channel | No. of Votes | Rel. freq
--- | --- | ---
2 | 63 | $\frac{63}{600} = 0.105$
7 | 214 | $\frac{214}{600} = 0.356$
9 | 195 | $\frac{195}{600} = 0.325$
10 | 128 | $\frac{128}{600} = 0.213$

(ii) Expected No. of Channel 9 viewers = $\frac{195 \times 2,500,000}{600} = 812,500$

(b) Binomial experiment:
$m = 10$, $p = \frac{1}{5}$, $q = \frac{4}{5}$

$P(\text{exactly 4}) = \binom{10}{4} \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^6$

$= \frac{210 \times (0.015625) \times (0.4096)}{\text{(or from table)}}$

$= 0.0881$

(c) Discrete : lower limit = 180.5

$N = 174$, $\sigma = 11$

$Z = \frac{x - \mu}{\sigma}$

$= \frac{180.5 - 174}{11}$

$= 0.5909 \Rightarrow p = 0.2237$

$\therefore P(X > 180.5) = 0.5 - 0.2237 = 0.2763$ (or by G-Calc $0.27729$)
Question 3

(a) (i) and (iii) are functions

(b) (i) \( x \in \mathbb{R}, -3 \leq x \leq 3 \)
(ii) \( x \in \mathbb{R}, x \neq 2 \)

(c) Let the height and weight be \( x \) cm and \( y \) kg.

If \((165, 68)\) and \((180, 73)\) fit the model

\[
\begin{align*}
    y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \\
    y - 68 &= \frac{73 - 68}{180 - 165} (x - 165)
\end{align*}
\]

\[
y = \frac{5}{15} (x - 165) + 68
\]

i.e. \( y = 0.33x - 5.2 \) is the model.

i.e. weight = 0.33 x height - 5.2

Test the 3rd adult: weight = 0.33 x 172 - 5.2

= 77 kg

But the centre claims 72 kg is a 5 kg (or 6.5%) discrepancy.

i.e. Claim is not valid.

Effect of assumptions:

- Linear model gives a "ball-park" figure.
- Assuming initial measurements of \((165, 68)\) and \((180, 73)\) as perfect fit would affect the resultant relationship.
- Assuming each height has only one acceptable weight does not cater for real-life diversity.
- The limits of the approximate linear model are unknown.
(d) \( f(x) = x + 1 \)  
\( g(x) = x^2 \)

(i) \( g(-1) = 2x(-1) \)  
\( = 2 \)

(ii) Let \( y = x + 1 \)  
\( \therefore y = y - 1 \)  
\( \Rightarrow f^{-1}(y) = y - 1 \)

(iii) \( g(f(x)) = 2(x + 1)^2 \)  
\( = 2x^2 + 4x + 2 \)

(e)

Let the width of the path be \( x \) metres.

Area of pool = 2 \( \times \) 3 = 6 \( \times \) 3 = 18 \( m^2 \)

Total area of pool/path = \((20 + 2x) \times (7 + 2x)\)

\( = 140 + 54x + 4x^2 \)

\( \therefore \) Area of path = \((140 + 54x + 4x^2) - 140 = 160 \)

\( 2x^2 + 27x = 80 \)

\( (x - 5)(x + 16) = 0 \)

\( x = 5 \) or \( x = -16 \)

Ignore \( x = -16 \) as width can't be negative.

\( \therefore \) Width of path = 2.5 \( \text{m} \)
(a) \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

(b) \[ \begin{align*}
\frac{a}{\sin A} &= \frac{c}{\sin C} \\
900 &= \frac{650}{\sin 140^\circ} \\
\Rightarrow \quad \alpha &= 27.66^\circ
\end{align*} \]

\( \theta = 360^\circ - (60^\circ + \alpha) \)
\( = 272^\circ \) (nearest degree)

is bearing of Bandia from Charleville = 272^\circ
(i) \[ 180^\circ = \pi \] 
\[ 135^\circ = x \] 

\[ x = \frac{\pi x \sqrt{2}}{180} \] 
\[ = \frac{3\pi}{4} \quad \text{(or \( 216^\circ \))} \]

(ii) Pythagorean Identity: \( \sin^2 \theta + \cos^2 \theta = 1 \)

(iii) \( 2 \sin^2 \theta + \cos^2 \theta = 1 \) 
\[ \Rightarrow 2(1 - \cos^2 \theta) + \cos^2 \theta = 1 \] 
\[ 2 \cos^2 \theta - \cos^2 \theta - 1 = 0 \] 

Let \( \cos \theta = a \) 
\[ \Rightarrow 2a^2 - a - 1 = 0 \] 
\[ (2a + 1)(a - 1) = 0 \] 
\[ a = -\frac{1}{2} \quad \text{or} \quad a = 1 \] 

i.e. \( \cos \theta = -\frac{1}{2} \) or \( \cos \theta = 1 \) 
\[ \Rightarrow \theta = \frac{\pi}{3}, \frac{4\pi}{3} \] 
\[ \therefore \theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \]
(c) $y = 3 \sin (2\theta + \pi) - 1$

(i) Amplitude = 3

Period = $\frac{2\pi}{2}$

Phase Shift = $-\frac{\pi}{2}$

Vertical Shift = -1
(c)(iii) \[ T = 24 + 6 \cos \left(\frac{\pi t}{12}\right) \]

When \( T = 26 \):
\[ 26 = 24 + 6 \cos \left(\frac{\pi t}{12}\right) \]
\[ \cos \left(\frac{\pi t}{12}\right) = \frac{1}{3} \]
\[ \frac{\pi t}{12} = 1.23095 \] \( \Rightarrow \) \( t = 1.23095 \times 12 \]
\[ t = 15.36 \]

Temp above 26°C:
- From midday to 7:18 am and 7:18 am to midday
- From 7:18 am until 4:42 pm
(a) \[ f(x) = x^2 + 2x - 3 \]

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

\[ = \lim_{h \to 0} \frac{(x^2 + 2hx + 2x + 2h - 3) - (x^2 + 2x - 3)}{h} \]

\[ = \lim_{h \to 0} \frac{2hx + 2h}{h} \]

\[ = \lim_{h \to 0} \frac{h(2x + 2)}{h} \]

\[ = 2x + 2 \]

(b) (i) \[ y = 4x^{-3} \quad \Rightarrow \quad \frac{dy}{dx} = -12x^{-4} \]

(ii) \[ y = (3x^2 - 5x)^4 \quad \Rightarrow \quad \frac{dy}{dx} = 4(3x^2 - 5x)^3(6x - 5) \]

(iii) \[ y = \ln(x^2 + 2) \quad \Rightarrow \quad \frac{dy}{dx} = \frac{2x}{x^2 + 2} \]

(iv) \[ y = e^{2x} \cos x \quad \text{Product Rule:} \quad \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} + \frac{dy}{dx} \]

\[ \frac{dy}{dx} = -2e^x \sin x + 2e^x \cos x \]

(v) \[ y = \frac{x^2}{x^2} \quad \text{Quotient Rule:} \quad \frac{dy}{dx} = \frac{\frac{dy}{dx} \frac{du}{dx} - \frac{dy}{dx} \frac{du}{dx}}{u^2} \]

\[ \frac{dy}{dx} = \frac{x^2 \cos x - 2x \sin x}{x^4} \]

\[ = \frac{x \cos x - 2 \sin x}{x^3} \]
(c) \( P(1, 4) \). Find \( y = x^3 - 4x^2 + x + 6 \)
\[
\frac{dy}{dx} = 3x^2 - 8x + 1.
\]
When \( x = 1 \),
\[
\frac{dy}{dx} = 3 - 8 + 1 = -4.
\]

Thus, gradient of tangent at \( P(1, 4) \) is \(-4\).

Equation of tangent:
\[
y - y_1 = m(x - x_1)
\]
\[
y - 4 = -4(x - 1)
\]
\[
y = -4x + 8.
\]

Normal:
\[
m = \frac{1}{4} \quad \text{(as gradient of tangent = -4)}
\]

Equation of normal:
\[
y - y_1 = m(x - x_1)
\]
\[
y - 4 = \frac{1}{4}(x - 1)
\]
\[
y = \frac{1}{4}x + 3\frac{3}{4}.
\]

Let the roots:

Tangent:
\[
y = 0 \Rightarrow -4x + 8 = 0
\]
\[
x = 2
\]

Normal:
\[
y = 0 \Rightarrow \frac{1}{4}x + 3\frac{3}{4} = 0
\]
\[
x = -15
\]

\[\text{Area} = \frac{1}{2} \times 17 \times 4 = 34 \text{ units}^2\]
Graph of derivative is a parabola. (as $f(x)$ cubic)

$f'(x) = 0$ at $C$ and $E$ (turning points)

i.e. $C = (-3, 0)$ and $E = (2, 0)$

From $A$ to $C$, gradient is positive and decreasing

From $C$ to $D$, gradient is negative and increasing

$D$ is point of change of concavity, so $D$ is the M.I. of the gradient function

From $D$ to $E$, gradient is negative (i.e. $C$ to $E$ below)

From $E$ to $F$, gradient is positive (i.e. $F$ increasing)
Paper Two

(a) \( y = 3x^2 - e^x \)

(i) y-intercept when \( x = 0 \)

\[ y = 0 \]

x-intercept when \( y = 0 \)

\[ 3x^2 - e^x = 0 \]

\[ x(3x - e) = 0 \]

\[ \Rightarrow x = 0 \text{ or } x = \frac{e}{3} \]

(ii) Stationary points when \( \frac{dy}{dx} = 0 \)

\[ \frac{dy}{dx} = 6x - e^x \]

\[ 6x = 0 \text{ when } 6x - e^x = 0 \]

\[ 3x(2-x) = 0 \]

\[ x = 0 \text{ or } x = 2 \]

\( (0, 0) \text{ and } (2, 4) \)

(iii) Nature of st. pt. by sign of \( \frac{d^2y}{dx^2} \)

\[ \frac{d^2y}{dx^2} = 6 - 6x \]

\[ x = 0 \]

\[ \frac{d^2y}{dx^2} > 0 \quad \Rightarrow (0, 0) \text{ MIN} \]

\[ x = 2 \]

\[ \frac{d^2y}{dx^2} < 0 \quad \Rightarrow (2, 4) \text{ MAX} \]

(iv) Pt of inflection when \( \frac{d^2y}{dx^2} = 0 \)

\[ 6 - 6x = 0 \]

\[ x = 1 \]

\[ \text{when } x = 1, \quad y = 3(1^2) - 1 \]

\[ y = 2 \]

\( (1, 2) \text{ is pt of inflection} \)

(v) \( b^2 < 5c \Rightarrow \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \)

\( (-73, 2), (1, \frac{1}{2}), (1, \frac{3}{2}), (1, \frac{5}{2}) \)
Let \( x \) be the length of either adjacent to \( x \) in the rectangle of length \( 50 \) cm.

\[
A(x) = (50 - x) 
\]

\[
= 50x - 2x^2 
\]

\[
A'(x) = 50 - 4x 
\]

\[
= 0 \quad \text{when} \quad x = 12.5 \,
\]

\[
\text{Max Area} = \frac{50 - 2(12.5)}{12.5} \times \frac{1}{2} \times 12.5 
\]

\[
= 312.5 \text{ cm}^2 
\]

Council reguals: Limit \( \leq \frac{312.5 \times 100}{375} \%
\]

\[
\geq 83.3\% 
\]

i.e. \( \text{does not meet regulations} \)
(c) \[ y = ax^3 + bx^2 + cx + d \] passes through \((0,5)\)

\[ \frac{dy}{dx} = 3ax^2 + 2bx + c \]

\[ a(0,5), \quad m = \frac{3}{5} \Rightarrow \]

\[ \frac{dy}{dx} = 3a(0)^2 + 2b(0) + c \]

\[ x = 0 \Rightarrow 3 = 0 + 0 + c \]

\[ c = 3 \]

ie. \[ y = ax^3 + bx^2 + 3x + 5 \]

Turning point when \(x = 2\), \( \frac{dy}{dx} = 0 \).

\[ 0 = 3ax^2 + 2bx + c \]

\[ x = 2 \Rightarrow 0 = 3a(2)^2 + 2b(2) + c \]

\[ 12a + 4b + c = 0 \]

\(\text{Eqn. (1)}\):

\[ 0 = ax^3 + bx^2 + 3x + 5 \]

\[ -8a + 4b + 1 = 0 \]

\(\text{Eqn. (2)}\):

Solving \((1)\) and \((2)\) \[ a = -\frac{1}{5}, \quad b = -\frac{3}{5} \]

ie. \[ a = -\frac{1}{5}, \quad b = -\frac{3}{5} \]

\[ c = 3 \]

\[ d = 5 \]
Question 2

(i) \( \int (4x^3 + 3x^2 - x) \, dx \)
\[ = x^4 + x^3 - \frac{x^2}{2} + C \]

(ii) \( \int 6x^2 \, dx \)
\[ = \frac{6}{3} x^3 + C \]
\[ = 2x^3 + C \]

(iii) \( \int \sin(4x) \, dx \)
\[ = -\frac{1}{4} \cos(4x) + C \]

(iv) \( \int \frac{4}{2x + 3} \, dx \)
\[ = 2 \int \frac{2}{2x + 3} \, dx \]
\[ = 2 \ln(2x + 3) + C \]

(v) \( \int \frac{\cos x}{x} \, dx \)
\[ = \left[ \sin x \right]_{\frac{\pi}{2}}^{\pi} \]
\[ = \sin \pi - \sin \frac{\pi}{2} \]
\[ = 0 - \frac{1}{\sqrt{2}} \]
\[ = \frac{\sqrt{2} - 1}{\sqrt{2}} \text{ or } \frac{2 - \sqrt{2}}{2} \]
(d)(i) \( N = 3t^2 - 4t + 1 \)

At rest when \( N = 0 \)

\[ 3t^2 - 4t + 1 = 0 \]

\[ t = \frac{4}{6} \text{ or } t = 1 \]

(ii) \( S = \int (3t^2 - 4t + 1) \, dt \)

\[ S = t^3 - 2t^2 + t + c \]

\[ t = 0, S = 5 \rightarrow 5 = 0^3 - 2(0)^2 + 0 + c \]

\[ S = t^3 - 2t^2 + t + 5 \]

(iii) \( t = 2 \rightarrow S = 2^3 - 2(2)^2 + 2 + 5 \)

\[ = 7 \text{ m} \]

\( t = 3 \rightarrow S = 3^3 - 2(3)^2 + 3 + 5 \)

\[ = 17 \text{ m} \]

From (i), we know there is no change of direction after \( t = 1 \); so the body is travelling in one dir’n

Distance = 17 - 7

= 10 \text{ m/s}^2

(c) Width of one strip = \( \frac{40}{5} \)

\[ E = 60 + 40 \]

\[ = 100 \text{ m} \]

\[ M = 100 + 140 + 160 + 120 + 80 \]

\[ = 600 \text{ m} \]

\[ A = \frac{h}{2} (E + 2M) \]

\[ = \frac{20}{2} (100 + 600) \]

\[ = 1500 \text{ m}^2 \]
(d) \[ \frac{dp}{dt} = 1200 e^{0.3t} \]

\[ p = \frac{1200}{0.3} e^{0.3t} + C \]

\[ p = 4000 e^{0.3t} + C \]

\[ t = 0, p = 500 \Rightarrow (500) = 4000 e^{0.3 \cdot 0} + C \]

\[ C = -399.435 \]

\[ p = 4000 e^{0.3t} - 399.435 \]

\[ p = 100,000 \Rightarrow 100,000 = 4000 e^{0.3t} - 399.435 \]

\[ e^{0.3t} = 25.1 \]

\[ \ln 25.1 = 0.3t \]

\[ t = 10.74 \]

i.e. It will take 11 months to be viable.
(c) 

\[ y = \sqrt{x} \]

\[ x + 2y = 15 \]

\[ \Rightarrow y = \frac{15 - x}{2} \]

\[ x + 2y = 15 \]

(0, 7.5)

(9, 3)

By \( g \)-calc

\[ y = \sqrt{x} \]

Total Area = Area I + Area II

Area I = \[ \int_{0}^{9} \sqrt{x} \, dx \]

= \[ \left[ \frac{2x^{3/2}}{3} \right]_{0}^{9} \]

= \[ \frac{18}{2} \text{ units}^{2} \]

Area II = \[ \int_{9}^{15} (15 - \frac{x}{2}) \, dx \]

= \[ \left[ 15x - \frac{x^2}{4} \right]_{9}^{15} \]

= \[ 56.25 - 4.76 \]

= \[ 51.49 \text{ units}^{2} \]

Total Area = \[ 18 + 9 \]

= \[ 27 \text{ units}^{2} \]
(a) \[ P = \$2500 \quad n = 3 \times 4 \quad i = \frac{4}{400} \]

\[ A = 2989.85 \]

\[ P = A \left(1 + i\right)^n \]

\[ 2989.85 = 2500 \left(1 + \frac{4}{400}\right)^3 \]

\[ 1.01562 = \left(1 + \frac{4}{400}\right)^3 \]

\[ \frac{x}{400} = 1.015 - 1 \]

\[ x = 1015 \times 400 \]

\[ = 6\% \text{ p.a.} \]

(b) \[ PV = \frac{R \left( 1 - (1+i)^{-n} \right)}{i} \]

\[ = 500 \left(1 - \left(1 + \frac{4}{12}\right)^{-72}\right) \]

\[ = \$25918.26 \]

(c) \[ FV = \frac{P \left[ (1+i)^n - 1 \right]}{i} \]

\[ = 600 \times \left(\left(1 + \frac{0.05}{12}\right)^{80} - 1\right) \]

\[ = \$1162631.75 \]
(d) \[ PV = R \left[ \frac{1 - (1+i)^{-n}}{i} \right] \]

(i) \[ 180,000 = R \left[ \frac{1 - (1+0.06)^{-300}}{0.06} \right] \]

\[ 180,000 = R \times 155,206.864 \]
\[ \Rightarrow R = \frac{180,000}{155,206.864} = 11.691.742523 \]

\[ \text{i.e. Monthly repayments are } \$11,691.75 \]

(ii) Payments made for 5 years:
\[ PV = R \left[ \frac{1 - (1+i)^{-n}}{i} \right] \]
\[ = 1159.75 \left[ 1 - (1+0.06)^{-240} \right] \]
\[ = 1159.75 \times 139.5807297 \]
\[ \approx \$161,877.75 \]

Amount owing: \$161,877.75

(iii) New debt now = \$161,877.75 - \$20,000

\[ PV = R \left[ \frac{1 - (1+i)^{-n}}{i} \right] \]
\[ 141,877.75 = 1159.74 \left[ 1 - (1+0.06)^{-m} \right] \]
\[ 141,877.75 = 1159.74 \left[ 1 - (1+0.05)^{-m} \right] \]
\[ 1.221,335.5646 = \left[ \frac{1}{1.05^m} \right] \]
\[ 1.611678226 = \left[ \frac{1}{1.005^m} \right] \]
\[ \log 1.005 = \log 1.611678226 \]
\[ m = 18.9163 \]
Question 3

(a) can't be paid off after 190 months instead of 20 years

i.e. 50 months early

{i.e. 4 years 2 months sooner}
(a) (i) \( \left( \frac{4.9}{7} \right)^{\frac{1}{2}} \)

\[ = \sqrt{\frac{4.9}{7}} \]
\[ = \frac{7}{\sqrt{7}} \]
\[ = \frac{7}{3} \]
\[ = 2 \frac{1}{3} \]

(ii) \( 7 \times 5 \times 3^{-1} \times 2^{-2} \)

\[ = 1 \times 5 \times \frac{1}{3} \times \frac{1}{4} \]
\[ = \frac{5}{12} \]

(iii) \( \log_3 8 \)

\[ = \log_3 8 \]
\[ = \frac{\log 8}{\log 3} \]
\[ = 0.9031 \]
\[ = 0.4771 \]
\[ = 1.893 \]

(iv) \( \log_3 81 - \log_3 27 \)

\[ = \log_3 \frac{81}{27} \]
\[ = \log_3 3 \]
\[ = 1 \]
(i) \( (2^x - 1) (2^x - 8) = 0 \)

Either \( 2^x - 1 = 0 \) or \( 2^x - 8 = 0 \)

\( 2^x = 1 \) \( \Rightarrow \) \( x = 0 \) or \( 2^x = 8 \) \( \Rightarrow \) \( x = 3 \)

(ii) \( \frac{16}{x^2} = 128 \)

\( 2^{\frac{4}{x}} = 2 \)

\( \frac{4}{x} = 7 \)

\( x = \frac{4}{7} \) (or \( 1\frac{3}{7} \))

(iii) \( \log_3 x = -2 \)

\( 3^{-2} = x \)

\( x = \frac{1}{9} \)

(iv) \( \log_{10} x - \log_{10} (x-1) = 1 \)

\( \log_{10} \left( \frac{x}{x-1} \right) = 1 \)

\( \frac{x}{x-1} = 10^1 \)

\( x = 10x - 10 \)

\( 9x = 10 \)

\( x = \frac{10}{9} \)
(c) \[ N = A \cdot 10^{-2b} \]

\[ x = 2 \Rightarrow 12500 = A \cdot 10^{-2 \cdot 2} \]

\[ x = 8 \Rightarrow 4000 = A \cdot 10^{-2 \cdot 8} \]

\[ \frac{12500}{4000} = \frac{A \cdot 10^{-2 \cdot 2}}{A \cdot 10^{-2 \cdot 8}} \]

\[ b = \frac{\log(12500/4000)}{\log(10)} \]

\[ b = 0.082475 \]

Part b = 0.082475, so \[ A \cdot 10^{-2 \cdot 8} = 10 \cdot 10^{-0.082475 \cdot 8} \]

\[ 12500 = A \cdot 10^{-0.082475 \cdot 8} \]

\[ A = 18275 \]

\[ N = 18275 \cdot 10^{-0.082475x} \]

Strengths:

- The mathematical model gives good predictions from 2 to 8 km (i.e., within the restricted domain, \( 2 \leq x \leq 8 \)).
- The exponential function is an appropriate model for growth/decay.

Limitations:

- \( x = 0 \Rightarrow N = 18275 \), a prediction of 18275 people living at the edge of city, likely very inaccurate.
- \( x = 8 \Rightarrow N = 4000 \) is the outer limit of the city, implying no population beyond 8 km.
- This model gives a current snapshot but may be unreliable in the future or for a different city.