Time allowed

- Perusal time: 10 minutes
- Working time: 3 hours

Examination materials provided

- Paper Two — Question book
- Paper Two — Response book

Equipment allowed

- QCAA-approved equipment
- ruler graduated in millimetres
- protractor
- graphics calculator
- additional calculator

Equipment not allowed

- Calculators with computer algebra system (CAS) functionality

Directions

You may write in this book during perusal time.

Paper Two has five questions. Attempt all questions.

Assessment

Paper Two assesses the following assessment criteria:

- Knowledge and procedures (KP)
- Modelling and problem solving (MP)
- Communication and justification (CJ)

Assessment standards are at the end of this book.

After the examination session

Take this book when you leave.
Planning space
Paper Two has five questions. Attempt all questions.

Each question assesses Knowledge and procedures (KP), Modelling and problem solving (MP) or a combination of both. Communication and justification (CJ) will be assessed by an overall judgment of your responses to all questions.

Write your responses in the response book. Show full working where necessary to meet the standards for each criterion.

Question 1

a. Given that \( y = x^3 - 3x + 2 \), use algebraic methods to find the greatest value of the function in the domain \( -2 \leq x \leq 2 \).

b. Shown below is the graph of the derivative of the function \( y = f(x) \).

\[
\begin{align*}
\text{Sketch the function } y &= f(x) \text{, given that } y = 0 \text{ when } x = 0. \text{ Justify your response.} \\
\end{align*}
\]

\[
\begin{align*}
d\frac{H}{dt} > 0 \quad \text{and} \quad d\frac{^2H}{dt^2} < 0 \text{ for } t > 0, \text{ and comment on the significance of these results.}
\end{align*}
\]

\[
\begin{align*}
v. \text{ A scientist claims that a typical pine tree does not grow more than } 20 \text{ metres in height. Examine the validity of this claim according to the given model.}
\end{align*}
\]

\[
\begin{align*}
\text{i. Calculate the height of a typical adult pine tree when } t = 4 \text{ years.} \\
\text{ii. Calculate the average growth rate of a typical pine tree across } 4 \leq t \leq 6 \text{ years.}
\end{align*}
\]

\[
\begin{align*}
\text{iii. Show that } d\frac{H}{dt} > 0 \text{ and } d\frac{^2H}{dt^2} < 0 \text{ for } t > 0, \text{ and comment on the significance of these results.}
\end{align*}
\]
Question 2

a. Showing full working, determine:
   i. \( \int (x^2 - 7x + 6) \, dx \)
   ii. \( \int 3 \sin(4x) \, dx \)
   iii. \( \int_0^1 5e^{3x^2 + 2} \, dx \).  

(KP)

b. Shown below is the graph of the intersecting curves \( y = \sqrt{x + 3} \) and \( y = x^2 - 2 \).

i. Use your graphics calculator to determine the co-ordinates of the points of intersection, A and B.

ii. Use algebraic methods to calculate the bounded area.

(KP)

c. The cross-section of a metal pipe is shown below. The outer radius of the pipe is 5 cm and the inner radius is 2 cm.

The liquid that passes through the pipe has a temperature of 120 °C. Within the metal of the pipe the temperature \( T \), in degrees Celsius, changes according to

\[
\frac{dT}{dx} = -\frac{15}{x} \degree C/cm
\]

If \( x \) is the distance from the centre of the pipe to a point in the metal, state the limitation that exists for the value of \( x \). Use calculus methods to find the temperature of the outer surface of the pipe.

(MP)
**Question 3**

a. An open tank contains water to a depth of $x$ metres (m). The volume (m$^3$) of water in the tank is given by

$$ V = 4x(18-x)^2 $$

Water is pumped from the tank at the rate of 0.25 m$^3$/minute. Given that $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$, calculate the rate at which the depth of the water is changing in mm/minute when $x = 4$ metres.

(KP)

b. A particle oscillates about the origin. At time $t$ seconds, its displacement is $x$ cm and its velocity is given by

$$ \frac{dx}{dt} = 5 \cos t + 3 $$

i. Initially, the particle has a displacement of 4 cm. Determine the displacement of the particle when $t = \frac{\pi}{2}$ seconds.

ii. Calculate the acceleration of the particle when $t = \frac{\pi}{2}$ seconds.

(KP)

c. The normal to the curve $y = a\sqrt{x} + \frac{b}{\sqrt{x}}$ at the point where $x = 4$ has equation $y = -4x + 22$.

Find the values of the constants $a$ and $b$.

(MP)

**Question 4**

a. Simplify:

i. $\frac{(4x^2)^3 \times \sqrt[3]{x}}{2x^5}$

(KP)

ii. $\frac{\log_x 9}{2} + 2\log_x 5$.

b. Solve for $x$:

i. $(3^x - 1)(3^x - 27) = 0$

(KP)

ii. $\log_e (x + 2) - \log_e (x - 2) = \log_e 5$. 

**Question 4 continues**
c. Scientists measuring the level of carbon dioxide in the atmosphere at a site high on Mauna Loa Mountain in Hawaii recorded the following information.

<table>
<thead>
<tr>
<th>T Years since 1995</th>
<th>C Carbon dioxide level in parts per million</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>359</td>
</tr>
<tr>
<td>19</td>
<td>401</td>
</tr>
</tbody>
</table>

i. The relationship between $C$ and $T$ can be modelled by $\log_e C = aT + b$. Calculate the values of the constants $a$ and $b$.  

ii. Express $C$ as an exponential function in terms of $T$.  

iii. Identify the strengths and limitations in extending this model to predict the carbon dioxide level in 2035.  

Question 5  

a. At what annual rate of interest, compounding monthly, would $5500 have to be invested to amount to $10000 over six years?  

b. Sue and Tony borrow $320000 from their bank to purchase a house at 5.04% p.a. compounding monthly over 25 years.  

i. Calculate their monthly repayment on the loan correct to the nearest dollar.  

ii. Calculate how much of the loan is still owed after five years.  

iii. After five years Sue and Tony sought advice from their bank. The bank suggested they pay half of their current repayment fortnightly instead of monthly. Use annuities to determine the effect on the loan that this suggestion would have.  

End of Paper Two
**Assessment standards from the Mathematics B Senior External Syllabus 2006**

|---------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|
| Knowledge and procedures  | The overall quality of a candidate’s achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, **consistently demonstrates:**  
• accurate recall, selection and use of definitions and rules  
• accurate use of technology  
• recall and selection of procedures and their accurate and proficient use  
• effective transfer and application of mathematical procedures. | The overall quality of a candidate’s achievement across a range within the contexts of Application, Technology and Complexity, and across topics, **generally demonstrates:**  
• accurate recall, selection and use of definitions and rules  
• accurate use of technology  
• recall and selection of procedures and their accurate use. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity **generally demonstrates:**  
• accurate recall and use of basic definitions and rules  
• use of technology  
• accurate recall, selection and use of basic procedures. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity **sometimes demonstrates:**  
• accurate recall and use of some definitions and rules  
• use of technology  
• use of basic procedures. | The overall quality of a candidate’s achievement **rarely demonstrates** knowledge and use of procedures. |
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<tbody>
<tr>
<td>Modelling and problem solving</td>
<td>The overall quality of a candidate’s achievement across the full range within each context, and across topics, generally demonstrates mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems ... and sometimes demonstrates mathematical thinking which includes: • suitable synthesis of procedures and strategies to solve problems • initiative and insight in exploring the problem • identifying strengths and limitations of models.</td>
<td>The overall quality of a candidate’s achievement across a range within each context, and across topics, generally demonstrates mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations and identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems ... and sometimes demonstrates mathematical thinking which includes: • suitable synthesis of procedures and strategies.</td>
<td>The overall quality of a candidate’s achievement demonstrates mathematical thinking which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures required to solve problems.</td>
<td>The overall quality of a candidate’s achievement sometimes demonstrates mathematical thinking which includes: • following basic procedures and/or using strategies.</td>
<td>The overall quality of a candidate’s achievement rarely demonstrates mathematical thinking which includes following basic procedures and/or using strategies.</td>
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<td>Communication and</td>
<td>The overall quality of a candidate’s achievement across the full range within each context <strong>consistently demonstrates:</strong></td>
<td>The overall quality of a candidate’s achievement across a range within each context <strong>generally demonstrates:</strong></td>
<td>The overall quality of a candidate’s achievement in all contexts <strong>sometimes demonstrates:</strong></td>
<td>The overall quality of a candidate’s achievement</td>
<td>The overall quality of a candidate’s achievement <strong>rarely demonstrates</strong> use of the basic conventions of language and mathematics.</td>
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<td>justification</td>
<td>• accurate use of mathematical terms and symbols</td>
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<td>• accurate use of language</td>
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<td>• organisation of information into various forms suitable for a given use</td>
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<td>• use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or propositions</td>
<td>• use of mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or propositions</td>
<td>• use of mathematical reasoning to develop simple logical arguments.</td>
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<td>• justification of procedures</td>
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<td>• evaluation of the validity of arguments</td>
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