Time allowed

- Perusal time: **10 minutes**
- Working time: **3 hours**

Examination materials provided:

- Paper One — Question book
- Paper One — Resource book
- Paper One — Response book

Equipment allowed

- QSA-approved equipment
- ruler graduated in millimetres
- protractor
- graphing calculator
- additional calculator

Equipment not allowed

- Calculators with computer algebra system (CAS) functionality

Directions

You may write in this book during perusal time.

Paper One has **six** questions. Attempt **all** questions.

Assessment

Paper One assesses the following assessment criteria:

- Knowledge and procedures (KP)
- Modelling and problem solving (MP)
- Communication and justification (CJ)

Assessment standards are at the end of this book.

After the examination session

Take this book when you leave.
Planning space
Paper One has six questions. Attempt all questions. Each question assesses Knowledge and procedures (KP), Modelling and problem solving (MP) or a combination of both. Communication and justification (CJ) will be assessed by an overall judgment of your responses to all questions. Write your responses in the response book. Show full working where necessary to meet the standards for each criterion.

**Question 1**

a. The speeds of 80 cars travelling in a 50 km/h speed zone were measured and then recorded in the following table.

<table>
<thead>
<tr>
<th>Speed (km/h)</th>
<th>Number of cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 to &lt; 45</td>
<td>16</td>
</tr>
<tr>
<td>45 to &lt; 50</td>
<td>32</td>
</tr>
<tr>
<td>50 to &lt; 55</td>
<td>18</td>
</tr>
<tr>
<td>55 to &lt; 60</td>
<td>10</td>
</tr>
<tr>
<td>60 to &lt; 65</td>
<td>4</td>
</tr>
</tbody>
</table>

i. Construct a histogram using the data in the table.

ii. Using a calculator, estimate the mean and standard deviation correct to one decimal place. (KP)

b. Two sprinters achieved the following training times in seconds over 100 metres.

<table>
<thead>
<tr>
<th>Sprinter A</th>
<th>12.3</th>
<th>12.4</th>
<th>12.5</th>
<th>12.3</th>
<th>12.4</th>
<th>12.8</th>
<th>13.1</th>
<th>12.1</th>
<th>12.7</th>
<th>12.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprinter B</td>
<td>12.1</td>
<td>12.0</td>
<td>12.0</td>
<td>16.8</td>
<td>12.1</td>
<td>12.2</td>
<td>12.4</td>
<td>12.1</td>
<td>12.3</td>
<td>12.4</td>
</tr>
</tbody>
</table>

i. Draw back-to-back box-and-whisker plots for the sprinters’ training times. (KP)

ii. Their coach believes that Sprinter A is a more consistent sprinter. Analyse the box plots to evaluate the validity of the coach’s opinion. (MP)
Question 2

a. Five graphed relations are shown below.

i. Identify which of the relations are functions.

ii. State whether each function identified is discrete, continuous or discontinuous.

iii. Find the domain and range of the relation graphed in D above.

b. A person delivering junk mail is paid a base salary of $32 a week and an additional 1 cent per leaflet delivered. The maximum number of leaflets available to be delivered in each week is 1000.

i. Identify the independent and dependent variables in this relationship.

ii. Is the independent variable discrete or continuous? Explain your response.

iii. Determine an equation that models the relationship. State what each variable represents.

(KP)

c. Describe the transformations required to convert \( y = |x| \) to produce \( y = 2|x - 1| + 1 \).

(KP)

d. Today, Jenny is five times the age of her daughter. In four years time, Jenny will be three times her daughter’s age.

Using algebraic methods, determine how many more birthdays from today Jenny will celebrate before she is twice the age of her daughter.

(MP)
Question 3

a. Convert 225° into radians, leaving your response in terms of $\pi$.

b. Using the Pythagorean identity,

i. show that $2\cos^2 \theta + \sin \theta = 2 + \sin \theta - 2\sin^2 \theta$

ii. solve $2\cos^2 \theta + \sin \theta = 1$ for $\theta$, given that $0 \leq \theta \leq 2\pi$.

(c) For the periodic function $y = 12 - 6\cos(2x + \pi)$,

i. determine the amplitude, period, phase and vertical shift

ii. find, using a graphics calculator, the values of $x$ for which $y = 9$ given that $0 \leq x \leq \pi$.

d. The points A, B and C stand in the same horizontal plane. C is located on a bearing N 25° E of A and N 55° W of B. The distance between A and B is 100 metres and A is due west of B.

At point C stands a tower. From A, the top of the tower has an angle of elevation of 34°.

Calculate the height of the tower.
Question 4

a. Given that \( f(x) = x^2 - 4x + 2 \) and \( g(x) = x - 5 \), evaluate:
   i. \( f(g(x)) \)
   ii. \( g(f(2)) \)
   iii. \( g^{-1}(x) \).

b. Solve for \( x \): \( x^2 - 4x - 7 = 0 \).

c. Using algebraic methods, determine the points of intersection between the curve \( y = 3x^2 - 12x + 14 \) and the straight line \( y = 6x - 1 \).

d. At \( x = 2 \), the point \( P \) lies on the curve \( y = (x - 1)^2 \). The tangent and normal at \( P \) meet the \( y \)-axis at \( B \) and \( A \) respectively.

Calculate the area of \( \Delta ABP \).
Question 5

a. Show from first principles that the derivative of \( f(x) = x^2 + 6 \) is \( 2x \).  

(KP)

b. Determine \( \frac{dy}{dx} \) for each of the following:

i. \( y = 5x^4 + 3x - 1 \)

ii. \( y = (7x + 2)^6 \)

iii. \( y = e^x \cos(3x) \)

iv. \( y = \log_e(x^2 + 5) \)

v. \( y = \frac{e^{2x}}{x^3 - 4} \) 

(KP)

c. At time \( t \) seconds the displacement, \( D \) metres, of a particle is given by 

\[ D = 3(t - 4)^2 + 2. \]

i. Calculate the average velocity of the particle between \( t = 4 \) seconds and \( t = 8 \) seconds.

ii. Calculate the instantaneous rate of change of the displacement of the particle when \( t = 6 \) seconds. 

(KP)

d. A rectangular enclosure, to be attached to an existing structure, is made using 700 m of wire. The dimensions of the enclosure are shown below.

![Diagram of the rectangular enclosure](image)

Use calculus methods to determine the maximum area of the enclosure. 

(MP)
Question 6

a. A box contains seven blue balls and five red balls. Two balls are drawn in succession from the box. If the first ball is **not replaced** before the second drawing, what is the probability of obtaining a blue ball and a red ball in that order? (KP)

b. In a game where balls are thrown at a target, the probability of hitting the target is 0.4. The discrete random variable, $X$, represents the number of successes in hitting the target. If a person has 12 attempts at the target, determine $P(X > 8)$, correct to four decimal places. (KP)

c. Using the normal distribution,
   i. for the standard normal variable, $z$, find $P(-1.52 \leq z \leq 2.156)$, correct to three decimal places.
   
   ii. the random variable, $X$, is normally distributed with $\mu = 200$ and $\sigma = 25$. Find the value of $k$ if $P(X > k) = 0.05$. (KP)

d. The quarterly electricity consumption by BBC Manufacturing is normally distributed with a mean of 55000 kilowatt hours (kWh) and a standard deviation of 5000 kWh. Electricity is supplied at a cost of 12.5 cents per kWh for the first 50000 kWh and 15 cents per kWh thereafter.

   Find the probability that the electricity bill for BBC Manufacturing for the next quarter will be between $6125 and $7450. Identify the variables and comment on the effect of assumptions. (MP)

**End of Paper One**
### Assessment standards from the Mathematics B Senior External Syllabus 2006

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<tbody>
<tr>
<td><strong>Knowledge and procedures</strong></td>
<td>The overall quality of a candidate’s achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, <strong>consistently demonstrates</strong>: • accurate recall, selection and use of definitions and rules • accurate use of technology • recall and selection of procedures and their accurate and proficient use • effective transfer and application of mathematical procedures.</td>
<td>The overall quality of a candidate’s achievement across a range within the contexts of Application, Technology and Complexity, and across topics, <strong>generally demonstrates</strong>: • accurate recall, selection and use of definitions and rules • accurate use of technology • recall and selection of procedures and their accurate use.</td>
<td>The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity <strong>generally demonstrates</strong>: • accurate recall and use of basic definitions and rules • use of technology • accurate recall, selection and use of basic procedures.</td>
<td>The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity <strong>sometimes demonstrates</strong>: • accurate recall and use of some definitions and rules • use of technology • use of basic procedures.</td>
<td>The overall quality of a candidate’s achievement rarely demonstrates knowledge and use of procedures.</td>
</tr>
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<td>---------------------------------</td>
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</tbody>
</table>
| Modelling and problem solving   | The overall quality of a candidate’s achievement across the full range within each context, and across topics, generally demonstrates mathematical thinking which includes:  
  • interpreting, clarifying and analysing a range of situations identifying assumptions and variables  
  • selecting and using effective strategies  
  • selecting suitable procedures required to solve a range of problems  
  ...and **sometimes** demonstrates mathematical thinking which includes:  
  • suitable synthesis of procedures and strategies to solve problems  
  • initiative and insight in exploring the problem  
  • identifying strengths and limitations of models.                                                                                                                                                                                                                      | The overall quality of a candidate’s achievement across a range within each context, and across topics, generally demonstrates mathematical thinking which includes:  
  • interpreting, clarifying and analysing a range of situations identifying assumptions and variables  
  • selecting and using effective strategies  
  • selecting suitable procedures required to solve a range of problems  
  ...and **sometimes** demonstrates mathematical thinking which includes:  
  • suitable synthesis of procedures and strategies.                                                                                                                                                                                                                           | The overall quality of a candidate’s achievement demonstrates mathematical thinking which includes:  
  • interpreting and clarifying a range of situations  
  • selecting strategies and/or procedures required to solve problems.                                                                                                                                                                                                  | The overall quality of a candidate’s achievement sometimes demonstrates mathematical thinking which includes:  
  • following basic procedures and/or using strategies.                                                                                                                                                                                                                           | The overall quality of a candidate’s achievement rarely demonstrates mathematical thinking which includes following basic procedures and/or using strategies.                                                                                                               |
The overall quality of a candidate’s achievement across the full range within each context consistently demonstrates:

- accurate use of mathematical terms and symbols
- accurate use of language
- organisation of information into various forms suitable for a given use
- use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or propositions
- justification of procedures
- recognition of the effects of assumptions
- evaluation of the validity of arguments.

The overall quality of a candidate’s achievement in all contexts generally demonstrates:

- accurate use of basic mathematical terms and symbols
- accurate use of language
- organisation of information into various forms suitable for a given use
- use of mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or propositions
- justification of procedures.

The overall quality of a candidate’s achievement sometimes demonstrates:

- accurate use of basic conventions of language and mathematics and occasional use of mathematical reasoning.

The overall quality of a candidate’s achievement rarely demonstrates use of the basic conventions of language and mathematics.