Time allowed

• Perusal time: 10 minutes
• Working time: 3 hours

Examination materials provided:

• Paper Two — Question book
• Paper Two — Response book

Equipment allowed

• QSA-approved equipment
• ruler graduated in millimetres
• protractor
• graphing calculator (additional calculator allowed)

Not allowed: calculators with computer algebra system (CAS) functionality.

Directions

You may write in this book during perusal time.
Paper Two has four questions. Attempt all questions.

Assessment

Assessment standards are at the end of this book.

After the examination session

Take this book when you leave.
Planning space
Paper Two has four questions.

Paper Two assesses the following criteria:

- Knowledge and procedures (KP) as indicated
- Modelling and problem solving (MP) as indicated
- Communication and justification (CJ) in all questions.

Attempt all questions. Write your responses in the response book.

Show full working where necessary to meet the standards for each criterion. Show intermediate results indicating the accurate and appropriate use of mathematical terms and symbols. Simply listing the keystrokes used on a graphing calculator does not constitute a complete response.

Question 1

a. For the function \( y = (x + 3)(x - 1)^2 \), use algebra and calculus to find:
   i. the coordinates of the intercepts
   ii. the coordinates of the stationary points
   iii. the nature of the stationary points
   iv. the coordinates of any points of inflection.
   v. Use parts i to iv to sketch the graph of \( y = (x + 3)(x - 1)^2 \) over a suitable domain. (KP)

b. Show full working to determine where the function \( y = x^2 - 8 \) and its derivative intersect. (KP)

c. A rectangle is inscribed inside the region bounded by the curve \( y = 18 - 2x^2 \) and the x-axis. Find the maximum possible area of the rectangle. (KP)

d. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. The area of printed material inside the margins on the poster is fixed at 384 cm\(^2\). Find the dimensions of the poster so that the total area of the poster is a minimum. (MP)

Question 2

a. Nick has $10000 cash which he plans to invest for two years. He has two options to consider.
   Option A: Invest the money at 5% p.a. simple interest.
   Option B: Invest the money at 4.75% p.a. compounded monthly.
   Nick chooses option A as he reasons that 5% is greater than 4.75%.
   Investigate the validity of Nick’s choice. (KP)

b. Two years ago Martina invested $5000 in an investment account which yields 6% p.a. interest compounding half-yearly. Today she borrows $10000 at 7.5% p.a. flat for the next four years. How much will she have to add to her investment account in three years time to ensure that this new debt is covered at its maturity date? (KP)
c. An annuity runs for 10 years and has an interest rate of 5.2% p.a. with payments of $480 per month.
   Calculate the future value of this annuity.
   (KP)

d. A young entrepreneur borrows $250000 to establish a business. The conditions of the loan are:
   • interest rate of 7.75% compounded annually
   • repayment period of 10 years
   • a fixed annual payment ($x) for the first five years
   • annual payments doubled ($2x) for the last five years.
   Find the amount of the initial fixed annual payment ($x).
   (MP)

Question 3

a. Determine:
   i. \( \int (6x^2 - 2x + 1) \, dx \)  
   (KP)
   ii. \( \int_1^2 (4e^{2x}) \, dx \)  
   (KP)
   iii. \( \int (6\cos 3x) \, dx \)  
   (KP)
   iv. \( \int \left( \frac{2}{\sqrt{x}} + 2 \sqrt{x} \right) \, dx \)  
   (KP)

b. If \( \int_1^a \left( \frac{4}{2x - 1} \right) \, dx = 1 \), find the exact value of \( a \).  
   (MP)

c. Find the physical area enclosed by the curves \( f(x) = 12 + x - x^2 \) and \( g(x) = x + 3 \).  
   (KP)

d. The velocity, \( v \) metres per second, of a body \( t \) seconds after starting from rest, is given by
   \( v = 3t^2 - t^3 \).
   Find how far the body has travelled when it next comes to rest.  
   (KP)

e. Find the area bounded by the curves \( y = 1 + e^{0.5x} \), the x-axis and the lines \( x = 1 \) and \( x = 2 \) by using the trapezoidal rule with five trapezia.  
   (KP)

f. The following information is given about a quadratic equation.
   \[ f(0) = 1, \]
   \[ f'(1) = 8, \]
   \[ \int_0^1 f(x) \, dx = 1 \]
   Find the equation for this quadratic function.  
   (MP)
Question 4

a. Simplify the following:
   
   i. \( \frac{(3x^2)^3 \times \sqrt[3]{8x^6}}{6x^4} \)
   
   ii. \( \log_{10} \sqrt[3]{10^x} \)
   
   iii. \( \log_a x^3 \times \log_a a^4 \)

b. Solve:
   
   i. \( 2^{3-x} = 16 \)
   
   ii. \( 4^x + 2^x - 2 = 0 \)
   
   iii. \( \log x + \log (x - 3) = \log 18 \)

   (KP)

c. The diameter, D, of a tree trunk when first measured was 12 cm. The diameter increases by 8% each year.
   
   i. Set up an exponential model for the above information.
   
   ii. Find the diameter of the tree 10 years after its first measurement.
   
   iii. How long after the first measurement would the diameter be predicted to reach 41 cm?
   
   iv. Identify the limitations of using this model.

   (MP)

d. The temperature, T °C, in an electric oven \( t \) minutes after being turned on was recorded in the following data table.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>1</th>
<th>3</th>
<th>5</th>
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<tbody>
<tr>
<td>Temperature (°C)</td>
<td>70</td>
<td>200</td>
<td>300</td>
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</table>

   The oven is thermostatically controlled so that the maximum temperature possible is 500 °C.

   Two students, Bernard and Lleyton, were asked to formulate and use a model to determine when the temperature would reach 350 °C.

   Bernard modelled the change in temperature over time using a quadratic function, while Lleyton used a logarithmic function.

   What effect will the different models have on their answers? How effective are each of these models for oven temperatures greater than 350 °C?

   (MP)

End of Paper Two
### Assessment standards from the Mathematics B Senior External Syllabus 2006

|----------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Knowledge and procedures   | The overall quality of a candidate's achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, **consistently demonstrates:**  
  - accurate recall, selection and use of definitions and rules  
  - accurate use of technology  
  - recall and selection of procedures and their accurate and proficient use  
  - effective transfer and application of mathematical procedures. | The overall quality of a candidate's achievement across a range within the contexts of Application, Technology and Complexity, and across topics, **generally demonstrates:**  
  - accurate recall, selection and use of definitions and rules  
  - accurate use of technology  
  - recall and selection of procedures and their accurate use. | The overall quality of a candidate's achievement in the contexts of Application, Technology and Complexity, **generally demonstrates:**  
  - accurate recall and use of basic definitions and rules  
  - use of technology  
  - accurate recall, selection and use of basic procedures. | The overall quality of a candidate's achievement in the contexts of Application, Technology and Complexity, **sometimes demonstrates:**  
  - accurate recall and use of some definitions, and rules  
  - use of technology  
  - use of basic procedures. | The overall quality of a candidate's achievement **rarely demonstrates** knowledge and use of procedures. |
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<td>Modelling and problem solving</td>
<td>The overall quality of a candidate’s achievement across the full range within each context, and across topics, <strong>generally demonstrates</strong> mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems …and <strong>sometimes demonstrates</strong> mathematical thinking which includes: • suitable synthesis of procedures and strategies to solve problems • initiative and insight in exploring the problem • identifying strengths and limitations of models.</td>
<td>The overall quality of a candidate’s achievement across a range within each context, and across topics, <strong>generally demonstrates</strong> mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations and identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems …and <strong>sometimes demonstrates</strong> mathematical thinking which includes:</td>
<td>The overall quality of a candidate’s achievement <strong>demonstrates</strong> mathematical thinking which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures required to solve problems.</td>
<td>The overall quality of a candidate’s achievement <strong>sometimes demonstrates</strong> mathematical thinking which includes: • following basic procedures and/or using strategies.</td>
<td>The overall quality of a candidate’s achievement <strong>rarely demonstrates</strong> mathematical thinking which includes following basic procedures and/or using strategies.</td>
</tr>
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</table>
| Communication and justification  | The overall quality of a candidate’s achievement across the full range within each context **consistently demonstrates:**  
  • accurate use of mathematical terms and symbols  
  • accurate use of language  
  • organisation of information into various forms suitable for a given use  
  • use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or propositions  
  • justification of procedures  
  • recognition of the effects of assumptions  
  • evaluation of the validity of arguments. | The overall quality of a candidate’s achievement across a range within each context **generally demonstrates:**  
  • accurate use of mathematical terms and symbols  
  • accurate use of language  
  • organisation of information into various forms suitable for a given use  
  • use of mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or propositions  
  • justification of procedures. | The overall quality of a candidate’s achievement in all contexts **generally demonstrates:**  
  • accurate use of basic mathematical terms and symbols  
  • accurate use of language  
  • organisation of information into various forms  
  • use of some mathematical reasoning to develop simple logical arguments. | The overall quality of a candidate’s achievement **sometimes demonstrates** evidence of the basic conventions of language and mathematics and occasional use of mathematical reasoning. | The overall quality of a candidate’s achievement **rarely demonstrates** use of the basic conventions of language and mathematics. |