Time allowed

- Perusal time: 10 minutes
- Working time: 3 hours

Examination materials provided:

- Paper One — Question book
- Paper One — Resource book
- Paper One — Response book

Equipment allowed

- QSA-approved equipment
- ruler graduated in millimetres
- protractor
- graphing calculator (additional calculator allowed)

Not allowed: calculators with computer algebra system (CAS) functionality.

Directions

You may write in this book during perusal time.

Paper One has six questions. Attempt all questions.

Assessment

Assessment standards are at the end of this book.

After the examination session

Take this book when you leave.
Planning space
Paper One has **six** questions.

Paper One assesses the following criteria:

- Knowledge and procedures (KP) as indicated
- Modelling and problem solving (MP) as indicated
- Communication and justification (CJ) in all questions.

Attempt **all** questions. Write your responses in the response book.

Show full working where necessary to meet the standards for each criterion. Show intermediate results indicating the accurate and appropriate use of mathematical terms and symbols. Simply listing the keystrokes used on a graphing calculator does not constitute a complete response.

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**Question 1**

a. Use trigonometric ratios to calculate values for $x$ and $\theta$.

i. 

![Diagram](image)

$\theta = 36^\circ$

$x = 5.3$ m

ii. 

![Diagram](image)

$\theta = ?$

$24.5$ mm

$18.2$ mm

(KP)

b. 

i. State the cosine rule for any triangle, ABC.

ii. Calculate the largest angle in the triangle with sides $6.2$ m, $7.4$ m and $11.8$ m.

(KP)

c. Two identical buildings each $40$ m tall are erected on a straight road running from east to west. From a point due south of the most easterly building, the angle of elevation to the top of that building is $21^\circ$. From the same point, the angle of elevation to the top of the other building is $15^\circ$.

Find the distance between the buildings. Identify any assumptions used in the calculation of this distance.

(MP)
Question 2

a. Convert the following:
   i. \(225°\) to radians
   ii. \(\frac{7\pi}{12}\) radians to degrees

b. Find the exact value of:
   i. \(\tan 60°\)
   ii. \(\cos \frac{5\pi}{4}\)

c. Given that \(2\sin \theta = -1\), find \(\theta\) where \(0 \leq \theta \leq 2\pi\).

d. For the function \(y = 1 + 3\sin 2x\), state the:
   i. period
   ii. amplitude
   iii. vertical shift.
   iv. Use parts i to iii to sketch the function \(y = 1 + 3\sin 2x\) for one period.

e. Alice, Bob and Carol are very good mathematics students. They were asked to provide a model for the sketch below.

![Sketch](attachment:image.png)

Alice’s model was \(y = -2\sin(2x) + 1\).

Bob stated that his model of \(y = 2\sin\left(x - \frac{\pi}{2}\right) + 1\) was correct.

Carol insisted that her model of \(y = 2\cos\left(2x + \frac{\pi}{2}\right) + 1\) fitted the sketch.

Show full working to investigate the validity of the models provided by Alice, Bob and Carol.
Question 3

a. Identify which of the following are not functions.

i. \( y = 2x^2 + \sqrt{2} \)

ii. \{ (0, -1), (1, 0), (1, 1), (2, 2) \}

iii. 

iv. \( y^2 = x + 3 \)

b. Given that \( f(x) = 2x^2 - 32 \), find:

i. \( f(-2) \)

ii. \( x, \text{ if } f(x) = 0 \)

(KP)

c. Given that \( f(x) = x^2 - 2x \) and \( h(x) = 3x + 4 \), find:

i. \( f(h(x)) \)

ii. \( h^{-1}(x) \)

(KP)

d. State the domain and range for the function \( y = 4 - x^2 \). Support your decisions with a sketch.

(KP)

e. Solve the equation \( x^2 - 6x + 5 = 0 \) by the method of completing the square.

(KP)

f. A straight line has a gradient of \(-3\). The triangle that the line forms with the positive axes has an area of 9 square units.

Select and use suitable strategies and procedures to find the equation of the line.

(MP)

g. A parabola has the equation \( y = ax^2 + bx \). The points \((-2, 14)\) and \((3, 9)\) lie on the parabola.

Find the equation of this parabola.

(MP)
Question 4

a. Find the derivative of \( f(x) = x^2 + 3x + 4 \) from first principles. 

\( \text{(KP)} \)

b. Find derivatives for the following functions:

i. \( y = 3x^3 + 2x^2 + 6x + 1 \)

ii. \( y = (2 \sin x)^5 \)

iii. \( y = e^x \cos x \)

iv. \( y = \ln(x^2 + 2x) \)

v. \( y = \frac{2x^2 + 4x}{3 - x} \) 

\( \text{(KP)} \)

c. In an experiment a liquid is cooled for five minutes so that its temperature, \( T \) °C, after \( t \) minutes is given by the formula 

\[ T = t^2 - 11t + 60 \]

i. Find the average rate of cooling during the five minutes of the experiment.

ii. Find its rate of cooling at the end of the five minutes. 

\( \text{(KP)} \)

d. Find the equation of the tangent to the curve \( y = x^3 + x^2 + x + 1 \) at \( (1, 4) \). 

\( \text{(KP)} \)

Question 5

a. The numbers of words per sentence (for the first twenty sentences) in a newspaper article were:

<table>
<thead>
<tr>
<th>12</th>
<th>28</th>
<th>32</th>
<th>18</th>
<th>17</th>
<th>16</th>
<th>28</th>
<th>25</th>
<th>24</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>28</td>
<td>16</td>
<td>25</td>
<td>17</td>
<td>23</td>
<td>24</td>
<td>24</td>
<td>12</td>
<td>27</td>
</tr>
</tbody>
</table>

The numbers of words per sentence (for the first twenty sentences) in an article in a scientific magazine were:

<table>
<thead>
<tr>
<th>26</th>
<th>37</th>
<th>31</th>
<th>28</th>
<th>17</th>
<th>30</th>
<th>40</th>
<th>31</th>
<th>28</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>22</td>
<td>32</td>
<td>40</td>
<td>24</td>
<td>18</td>
<td>22</td>
<td>21</td>
<td>17</td>
<td>18</td>
</tr>
</tbody>
</table>

i. Construct side-by-side boxplots to show a comparison between the two datasets.

ii. Comment on differences between the two sources. Justify your comments by referring to the boxplots.

\( \text{(KP)} \)
b. The degree of skewness, $S$, of a dataset may be defined as:

$$S = 3 \times \frac{\text{mean} - \text{median}}{\text{standard deviation}}$$

i. Determine the degree of skewness for the following dataset.

<table>
<thead>
<tr>
<th>Number</th>
<th>42</th>
<th>31</th>
<th>36</th>
<th>41</th>
<th>39</th>
<th>32</th>
<th>24</th>
<th>42</th>
<th>41</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of families</td>
<td>0.18</td>
<td>0.14</td>
<td>0.36</td>
<td>0.22</td>
<td>0.06</td>
<td>0.03</td>
<td>0.01</td>
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</tr>
</tbody>
</table>

ii. Use an appropriate class interval to draw a histogram which displays this data.

iii. What can be concluded about the value for $S$ and the symmetry of the histogram?

(MP)

Question 6

a. Data about the number of children in each Australian family is shown below. The small number of families with 7 or more children has been ignored.

<table>
<thead>
<tr>
<th>Number of children</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of families</td>
<td>0.18</td>
<td>0.14</td>
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<td>0.22</td>
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<td>0.01</td>
</tr>
</tbody>
</table>

Calculate the mean number of children in an Australian family.

(KP)

b. A company manufactures 10000 pens per day, of which 1000 are found to be defective. Pens are sold in boxes of 12.

i. List the characteristics of the binomial distribution which apply to this situation.

ii. Find the probability that a box of pens chosen at random has exactly two defective pens.

(KP)

c. A normal distribution has a mean of 50 and a standard deviation of 8. Find the probability that a value taken at random from the population lies between 50 and 54.

(KP)

d. Stanines are used to scale test scores on a nine-point standard scale. Stanine 1 is the lowest and stanine 9 is the highest.

To obtain stanines a normal distribution is divided into nine symmetrically placed intervals. Except for the first and ninth intervals (which are the tails of the distribution), each interval has a width of 0.5 standard deviations.

Find the percentage of scores in stanine 7.

(MP)

End of Paper One
### Assessment standards from the Mathematics B Senior External Syllabus 2006

|-------------------------------|----------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Knowledge and procedures      | The overall quality of a candidate’s achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, **consistently demonstrates:**  
   • accurate recall, selection and use of definitions and rules  
   • accurate use of technology  
   • recall and selection of procedures and their accurate and proficient use  
   • effective transfer and application of mathematical procedures. | The overall quality of a candidate’s achievement across a range within the contexts of Application, Technology and Complexity, and across topics, **generally demonstrates:**  
   • accurate recall, selection and use of definitions and rules  
   • accurate use of technology  
   • recall and selection of procedures and their accurate use. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, **generally demonstrates:**  
   • accurate recall and use of basic definitions and rules  
   • use of technology  
   • accurate recall, selection and use of basic procedures. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, **sometimes demonstrates:**  
   • accurate recall and use of some definitions, and rules  
   • use of technology  
   • use of basic procedures. | The overall quality of a candidate’s achievement **rarely demonstrates** knowledge and use of procedures. |
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<tbody>
<tr>
<td>Modelling and problem solving</td>
<td>The overall quality of a candidate’s achievement within each context, and across topics, generally demonstrates mathematical thinking which includes: interpreting, clarifying and analysing a range of situations and identifying assumptions and variables, selecting and using effective strategies, selecting suitable procedures required to solve a range of problems, sometimes demonstrates synthesis of procedures and strategies to solve problems.</td>
<td>The overall quality of a candidate’s achievement within each context, and across topics, generally demonstrates mathematical thinking which includes: interpreting, clarifying and analysing a range of situations and identifying assumptions and variables, selecting and using effective strategies, selecting suitable procedures required to solve a range of problems, sometimes demonstrates synthesis of procedures and strategies to solve problems.</td>
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<tr>
<td>Communication and justification</td>
<td>The overall quality of a candidate’s achievement across the full range within each context <strong>consistently demonstrates:</strong> • accurate use of mathematical terms and symbols • accurate use of language • organisation of information into various forms suitable for a given use • use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or propositions • justification of procedures • recognition of the effects of assumptions • evaluation of the validity of arguments.</td>
<td>The overall quality of a candidate’s achievement across a range within each context <strong>generally demonstrates:</strong> • accurate use of mathematical terms and symbols • accurate use of language • organisation of information into various forms suitable for a given use • use of mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or propositions • justification of procedures.</td>
<td>The overall quality of a candidate’s achievement in all contexts <strong>generally demonstrates:</strong> • accurate use of basic mathematical terms and symbols • accurate use of language • organisation of information into various forms • use of some mathematical reasoning to develop simple logical arguments.</td>
<td>The overall quality of a candidate’s achievement <strong>sometimes demonstrates</strong> evidence of the basic conventions of language and mathematics and occasional use of mathematical reasoning.</td>
<td>The overall quality of a candidate’s achievement <strong>rarely demonstrates</strong> use of the basic conventions of language and mathematics.</td>
</tr>
</tbody>
</table>