Time allowed

- Perusal time: 10 minutes
- Working time: 3 hours

Examination materials provided:

- Paper Two — Question book
- Paper Two — Response book

Equipment allowed

- QSA-approved equipment
- ruler graduated in millimetres
- protractor
- non-programmable calculator
- graphing calculator

Not allowed: calculators with computer algebra system (CAS) functionality.

Directions

You may write in this book during perusal time.

Paper Two has five questions. Attempt all questions.

Assessment

Assessment standards are at the end of this book.

After the examination session

Take this book when you leave.
Planning space
Paper Two has **five** questions. Attempt **all** questions.

Paper Two assesses the following criteria:

- Knowledge and procedures (KP) as indicated
- Modelling and problem solving (MP) as indicated
- Communication and justification (CJ) in all questions.

Write your responses in the response book.

Show full working where necessary to meet the standards for each criterion. Show intermediate results indicating the accurate and appropriate use of mathematical terms and symbols. Simply listing the keystrokes used on a graphing calculator does not constitute a complete response.

**Question 1**

a. A function, \( f \), is defined such that \( f(1) = 0 \) and \( f'(x) = 2 \).
   i. Sketch the graph of the function.
   ii. Determine the rule that defines the function.  

(b) For the above graph, state:
   i. the coordinates of the intercepts
   ii. the coordinates of the local minimum value of the function
   iii. when the function is negative
   iv. when the derivative of the function is negative.

(c) For the function \( y = x^3 - 12x + 16 \), use algebra and calculus to find:
   i. the coordinates of the stationary points
   ii. the nature of the stationary points.
d. A small business estimates that its profit from producing $x$ items can be modelled by:

$$P(x) = 0.003x^3 - 1.5x^2 + 200x - 1000.$$  

Due to limited space and capital, the number of items produced must not exceed 350 items. Explore the model and make suggestions to the business owner to ensure the profitability of the business. Give full reasoning in support of your suggestions. (MP)

e. A man on a kayak, $K$, is 3 kilometres out to sea from the nearest point, $O$, of a straight beach. His destination, $D$, is 6 kilometres along the beach from $O$. The fastest he can paddle is 4 km/h and his maximum walking speed is 5 km/h. How far from $O$ should he go ashore to reach his destination in the least possible time? Identify any assumptions used in developing the model to solve the problem. State the effects of changing one of the assumptions. (MP)

**Question 2**

a. Determine:

i. $\int (2x^3 + 6x^2 - 5x) \, dx$  

ii. $\int \frac{2}{4x + 1} \, dx$  

iii. $\int_0^2 (4e^{2x}) \, dx$  

iv. $\int \left( \sin 2x + \frac{1}{\sqrt{x}} \right) \, dx$  

b. Use calculus to find the area under the curve:

$$y = \frac{1}{x} + 1 \text{ from } x = 1 \text{ to } x = 3.$$  

(KP)

c. An object moves in a straight line from a fixed point. The acceleration $a$ m/s$^2$ of the object after time $t$ is given by $a = 4 - 2t$. The object starts at rest from the fixed point. Find the velocity and position when the acceleration is zero.  

(KP)
d. Find the shaded area. The interval heights shown are equally spaced.

\[ \text{(KP)} \]

\[ \text{Area} = \int_{\theta=0}^{\theta=\frac{3\pi}{2}} (\sin \theta - (1 - \cos \theta)) \, d\theta \]

\[ = \left[ -\cos \theta - \sin \theta + \cos \theta \right]_{0}^{\frac{3\pi}{2}} \]

\[ = -1 - 1 + 1 = -1 \]

\[ \text{units}^2 \]

\[ \text{(MP)} \]

\[ \text{Coefficient of inequality} = \frac{\text{Area between Lorenz curve and line } y = x}{\text{Area under the line } y = x} \]

\[ \text{Coefficient of inequality} = \frac{\int_{x=0}^{x=1} (0.2x^4 + 0.3x^3 + 0.5x^2) \, dx}{\frac{1}{2}} \]

\[ = \frac{\left[ 0.0667x^5 + 0.01x^4 + 0.3333x^3 \right]_{0}^{1}}{\frac{1}{2}} \]

\[ = \frac{0.0667 + 0.01 + 0.3333}{0.5} \]

\[ = 0.8334 \]

\[ \text{(KP)} \]

\[ \text{i. What proportion of total income is earned by the poorest 40\% of the population?} \]

\[ \text{ii. Calculate the coefficient of inequality.} \]

f. A student drew the graphs of the trigonometric functions \( y = \sin \theta \) and \( y = 1 - \cos \theta \), \( 0 \leq \theta \leq 2\pi \).

The student then claimed that the area enclosed by the functions was \( \left( 2 + \frac{3\pi}{2} \right) \text{ units}^2 \).

Evaluate the validity of this claim.

\[ \text{(MP)} \]
Question 3

a. Amanda inherits $8000 and invests it for three years at an annual interest rate of 7.2% p.a. compounded monthly. How much will she need to add to her investment at the end of the three years to have a total of $10000? (KP)

b. An investor bought a coastal property for $800000 which is projected to increase in value by 4.75% per year. A property bought at the same time in the central business district for $600000 is expected to increase in value by 6.25% per year. How long will it take for the central business district property to be worth more than the coastal property? (KP)

c. Find the future value of an annuity if payments are $2000 per quarter, the annuity runs for five years and the interest rate is 7.2% p.a. compounded quarterly. (KP)

d. David’s grandparents wish to establish an annuity to pay for his university education. The annuity will be set up so that David will receive $1250 at the end of each month for four years. The annuity will earn 7.5% p.a. compounded monthly. What is the cost of setting up this annuity? (KP)

e. Zelda has $3500 in an investment account which she intends to use towards the deposit on a block of land in three years time. The account earns interest at the rate of 6% p.a. compounding monthly. She also arranges to have $450 a month transferred from her pay to an annuity offering a fixed rate of 5.25% p.a. compounding monthly. Unfortunately, 12 months after starting the annuity she was forced to withdraw $1000 from the account to pay for emergency car repairs. Zelda assumed that after three years she would have saved at least $25000 from both accounts. Evaluate the validity of her assumption. (MP)

Question 4

a. Simplify, leaving each response with positive indices:
   i. $8^{2-a} \times 32^{-a}$
   ii. $(2x^2)^{\frac{2}{3}} \times \sqrt[3]{4x^3}$

b. Simplify:
   i. $\log_x 5 + \log_x \left(\frac{1}{5}\right)$
   ii. $\log_8 512$ (KP)

c. Solve:
   i. $3^{2x+5} = 27$
   ii. $5\log_{10}x = 3 + 2\log_{10}x$ (KP)
d.  
i. On the same set of axes, sketch the graphs of \( y = 2^x \) and \( y = \log_2 x \).  
ii. Explain the relationship between the graphs.

(KP)

e. A growth function is modelled by the equation \( y = 100a^x \). Given that \( \log_{10} y = 3 \) when \( x = 1 \):  
i. determine the value of \( a \)  
ii. find the value of \( y \) when \( x = 2 \).

(KP)

**Question 5**

The Registered Auto Club of Quahog collected data on the percentage, \( p \), of cars of a certain type still on the road after \( t \) years. The data for \( t \) and \( p \) are tabulated below. To establish a mathematical model the Quahog Club included the natural logarithm of \( p \) in the table. This gave rise to a linear relationship between \( t \) and \( \ln p \).

<table>
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<tr>
<th>( t )</th>
<th>0</th>
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<th>2</th>
<th>3</th>
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a. Find:  
i. the percentage of cars on the road initially  
ii. the percentage of cars on the road after 10 years.

b. Determine the linear equation linking \( \ln p \) and \( t \).

c. Use the index and logarithm laws to express this relationship as an exponential function.

(MP)

**End of Paper Two**
Assessment standards from the 2006 senior external syllabus for Mathematics B

|----------------------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|
| Knowledge and procedures         | The overall quality of a candidate’s achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, **consistently demonstrates:**  
• accurate recall, selection and use of definitions and rules  
• accurate use of technology  
• recall and selection of procedures and their accurate and proficient use  
• effective transfer and application of mathematical procedures. | The overall quality of a candidate’s achievement across a range within the contexts of Application, Technology and Complexity, and across topics, **generally demonstrates:**  
• accurate recall, selection and use of definitions and rules  
• accurate use of technology  
• recall and selection of procedures and their accurate use. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, **generally demonstrates:**  
• accurate recall and use of basic definitions and rules  
• use of technology  
• accurate recall, selection and use of basic procedures. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, **sometimes demonstrates:**  
• accurate recall and use of some definitions, and rules  
• use of technology  
• use of basic procedures. | The overall quality of a candidate’s achievement **rarely demonstrates** knowledge and use of procedures. |
|---------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Modelling and problem solving   | The overall quality of a candidate’s achievement across the full range within each context, and across topics, generally demonstrates mathematical thinking which includes:  
  - interpreting, clarifying and analysing a range of situations identifying assumptions and variables  
  - selecting and using effective strategies  
  - selecting suitable procedures required to solve a range of problems  
  ...and sometimes demonstrates mathematical thinking which includes:  
  - suitable synthesis of procedures and strategies to solve problems  
  - initiative and insight in exploring the problem  
  - identifying strengths and limitations of models. | The overall quality of a candidate’s achievement across a range within each context, and across topics, generally demonstrates mathematical thinking which includes:  
  - interpreting, clarifying and analysing a range of situations and identifying assumptions and variables  
  - selecting and using effective strategies  
  - selecting suitable procedures required to solve a range of problems  
  ...and sometimes demonstrates mathematical thinking which includes:  
  - suitable synthesis of procedures and strategies. | The overall quality of a candidate’s achievement demonstrates mathematical thinking which includes:  
  - interpreting and clarifying a range of situations  
  - selecting strategies and/or procedures required to solve problems. | The overall quality of a candidate’s achievement demonstrates mathematical thinking which includes:  
  - following basic procedures and/or using strategies. | The overall quality of a candidate’s achievement rarely demonstrates mathematical thinking which includes following basic procedures and/or using strategies. |
**Communication and justifica**

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