Time allowed

- Perusal time: 10 minutes
- Working time: 3 hours

Examination materials provided:

- Paper Two — Question book
- Paper Two — Response book

Equipment allowed

- QSA-approved equipment
- ruler graduated in millimetres
- protractor
- non-programmable calculator
- graphing calculator

Not allowed: calculators with computer algebra system (CAS) functionality.

Directions

You may write in this book during perusal time.

Paper Two has four questions. Attempt all questions.

Assessment

Assessment standards are at the end of this book.

After the examination session

Take this book when you leave.
Planning space
Paper Two has **four** questions. Attempt **all** questions.

Paper Two assesses the following criteria:

- Knowledge and procedures (KP) as indicated
- Modelling and problem solving (MP) as indicated
- Communication and justification (CJ) in all questions.

Write your responses in the response book.

Show full working where necessary to meet the standards for each criterion. Show intermediate results indicating the accurate and appropriate use of mathematical terms and symbols. Simply listing the keystrokes used on a graphing calculator does not constitute a complete response.

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**Question 1**

**a.** For the function \( y = 3x^2 - x^3 \), use algebra and calculus to find:

i. the coordinates of the intercepts

ii. the coordinates of the stationary points

iii. the nature of stationary points

iv. the coordinates of the point of inflection.

Use algebra or your graphing calculator to find:

v. the intersections of \( y = 3x^2 - x^3 \) and the line \( y = 2 \). (KP)

**b.** A family wishes to build a rectangular poultry enclosure in their backyard using an existing fence as one side of the enclosure. They will use 50 metres of mesh fencing and their backyard has an area of 375 \( \text{m}^2 \). Council regulations limit such enclosures to occupy no more than 80% of the backyard.

Determine whether the maximum area which can be enclosed using the 50 metres of mesh fencing meets council regulations. (MP)

**c.** The graph of \( y = ax^3 + bx^2 + cx + d \) touches the \( x \)-axis at \( x = -2 \). The graph also cuts the \( y \)-axis at \( y = 5 \) with a gradient of 3. Find \( a, b, c \) and \( d \). (MP)

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**Question 2**

**a.** Determine

i. \( \int (4x^3 + 3x^2 - x) \, dx \) (KP)

ii. \( \int 6e^{3x} \, dx \) (KP)

iii. \( \int \sin 4x \, dx \) (KP)

iv. \( \int \left( \frac{4}{2x + 3} \right) \, dx \)

v. \( \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos x) \, dx \) Show full working, leaving the response as a surd. (KP)
b. The velocity \( v \) in metres per second of a body after time \( t \) is given by \( v = 3t^2 - 4t + 1 \). If the body is initially 5 metres to the right of a fixed origin, find:

i. the times when the body is not moving

ii. the equation representing the displacement \( s \) at any time \( t \)

iii. the distance travelled between \( t = 2 \) and \( t = 3 \).

(KP)

c. The local council wishes to reduce the mosquito population by spraying a large shallow lagoon (see diagram). Two roads, 420 metres apart, form two of its sides. Equally spaced measurements of the width of the lagoon are shown on the diagram. The amount of spray used depends on the surface area. Use the trapezoidal rule to find the area of the lagoon.

(KP)

\[ \frac{dp}{dt} = 1200e^{0.3t} \text{ where } t \text{ is the time in months.} \]

A particular nest had a population of 5000 fire ants after one month. Determine how long it will take the nest to reach the viable stage (i.e. when the population has reached 100000).

(MP)

e. Calculate the area bounded by the \( x \)-axis and the curves \( y = \sqrt{x} \) and \( x + 2y = 15 \).

(MP)

Question 3

a. The sum of $2500 is invested for 3 years at an annual interest rate of \( x \)% compounded quarterly. The investment has then grown to $2989.05. Find \( x \), the annual interest rate.

(KP)

b. Find the present value of an annuity if payments are $500 per month, the annuity runs for 6 years and the interest rate is 7.5% p.a. compounded monthly.

(KP)

c. An amount of $6000 is paid into a pension fund every 3 months. The interest rate of the fund is 8% p.a. compounded quarterly. How much will the fund be worth after 20 years?

(KP)
d. Hercules and Delilah take out a mortgage of $180000 to be repaid over 25 years. Monthly repayments are made and interest is 6% p.a. compounded monthly.

i. Find the monthly repayments.

ii. Monthly repayments are made for 5 years. How much is still owing on the loan?

iii. At this point (i.e. after 5 years of repayments) Hercules and Delilah inherit $20000 which they use to reduce their outstanding mortgage. If they were to continue making the same repayments, how much sooner would their mortgage be paid out? (KP)

Question 4

a. Show working to prove:

i. \((\frac{5}{2})^\frac{3}{4} = 2\frac{1}{7}\)

ii. \(7^0 \times 5^1 \times 3^{-1} \times 2^{-2} = \frac{5}{12}\)

iii. \(\log_3 8 = 1.893\)

iv. \(\log_3 81 - \log_3 27 = 1\) (KP)

b. Solve:

i. \((2^x - 1)(2^x - 8) = 0\)

ii. \(16^x = 128\)

iii. \(\log_3 x = -2\)

iv. \(\log_{10} x - \log_{10} (x - 1) = 1\) (KP)

c. The average number, \(N\), of people per square kilometre who live at a distance \(x\) km from the centre of a city may be modelled by the equation

\[ N = A \times 10^{-bx} \]

The average numbers of people per square kilometre at distances 2 km and 8 km were 12500 and 4000 respectively.

i. Find \(A\) and \(b\). (KP)

ii. Discuss the strengths and limitations of using the model to determine the number of people per square kilometre of the city. (MP)

End of Paper Two
### Assessment standards from the 2006 senior external syllabus for Mathematics B

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<td>Knowledge and procedures</td>
<td>The overall quality of a candidate’s achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, <strong>consistently demonstrates:</strong></td>
<td>The overall quality of a candidate’s achievement across a range within the contexts of Application, Technology and Complexity, and across topics, <strong>generally demonstrates:</strong></td>
<td>The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, <strong>generally demonstrates:</strong></td>
<td>The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, <strong>sometimes demonstrates:</strong></td>
<td>The overall quality of a candidate’s achievement <strong>rarely demonstrates</strong> knowledge and use of procedures.</td>
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<td>• accurate recall, selection and use of definitions and rules</td>
<td>• accurate recall, selection and use of definitions and rules</td>
<td>• accurate recall and use of basic definitions and rules</td>
<td>• accurate recall and use of some definitions, and rules</td>
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<td>• recall and selection of procedures and their accurate and proficient use</td>
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<td>• effective transfer and application of mathematical procedures.</td>
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### Modelling and problem solving

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<td>The overall quality of a candidate's achievement across the full range within each context, and across topics, <strong>generally demonstrates</strong> mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems ...and <strong>sometimes demonstrates</strong> mathematical thinking which includes: • suitable synthesis of procedures and strategies to solve problems • initiative and insight in exploring the problem • identifying strengths and limitations of models.</td>
<td>The overall quality of a candidate’s achievement across a range within each context, and across topics, <strong>generally demonstrates</strong> mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations and identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems ...and <strong>sometimes demonstrates</strong> mathematical thinking which includes: • suitable synthesis of procedures and strategies.</td>
<td>The overall quality of a candidate's achievement <strong>demonstrates</strong> mathematical thinking which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures required to solve problems.</td>
<td>The overall quality of a candidate's achievement <strong>sometimes demonstrates</strong> mathematical thinking which includes: • following basic procedures and/or using strategies.</td>
<td>The overall quality of a candidate's achievement <strong>rarely demonstrates</strong> mathematical thinking which includes following basic procedures and/or using strategies.</td>
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<td><strong>Criterion</strong></td>
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| Communication and justification | The overall quality of a candidate’s achievement across the full range within each context **consistently demonstrates**:
• accurate use of mathematical terms and symbols
• accurate use of language
• organisation of information into various forms suitable for a given use
• use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or propositions
• justification of procedures
• recognition of the effects of assumptions
• evaluation of the validity of arguments. | The overall quality of a candidate’s achievement across a range within each context **generally demonstrates**:
• accurate use of mathematical terms and symbols
• accurate use of language
• organisation of information into various forms suitable for a given use
• use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or propositions
• justification of procedures. | The overall quality of a candidate’s achievement in all contexts **generally demonstrates**:
• accurate use of basic mathematical terms and symbols
• accurate use of language
• organisation of information into various forms suitable for a given use
• use of mathematical reasoning to develop simple logical arguments. | The overall quality of a candidate’s achievement **sometimes demonstrates** evidence of the basic conventions of language and mathematics and occasional use of mathematical reasoning. | The overall quality of a candidate’s achievement **rarely demonstrates** use of the basic conventions of language and mathematics. |