Time allowed

• Perusal time: **10 minutes**
• Working time: **3 hours**

Examination materials provided:

• Paper One — Question book
• Paper One — Resource book
• Paper One — Response book

Equipment allowed

• QSA-approved equipment
• ruler graduated in millimetres
• protractor
• non-programmable calculator
• graphing calculator

Not allowed: calculators with computer algebra system (CAS) functionality.

Directions

You may write in this book during perusal time.

Paper One has **five** questions. Attempt **all** questions.

Assessment

Assessment standards are at the end of this book.

After the examination session

Take this book when you leave.
Planning space
Paper One has five questions. Attempt all questions.

Paper One assesses the following criteria:

- Knowledge and procedures (KP) as indicated
- Modelling and problem solving (MP) as indicated
- Communication and justification (CJ) in all questions.

Write your responses in the response book.

Show full working where necessary to meet the standards for each criterion. Show intermediate results indicating the accurate and appropriate use of mathematical terms and symbols. Simply listing the keystrokes used on a graphing calculator does not constitute a complete response.

**Question 1**

a. An intake of 30 cadets answered a 20-question entrance examination. The cadets’ scores (out of 20) were recorded in the frequency distribution table below.

<table>
<thead>
<tr>
<th>Score</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>9</td>
<td>8</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

i. Construct a histogram using the data in the table.  

 ii. Explain what the shape of the distribution of data tells you about the cadets’ scores.

 iii. Calculate the mean for the entrance examination.

b. Construct a dataset of eight scores where the range is 9, the median is 6.5, the mode is 6 and the interquartile range is 3.

Justify all procedures by showing the mathematical reasoning involved with each step.

**Question 2**

a. A sample of 600 people was surveyed. The votes for their preferred free-to-air TV channel are shown in the table below.

<table>
<thead>
<tr>
<th>TV channel</th>
<th>Number of votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>63</td>
</tr>
<tr>
<td>7</td>
<td>214</td>
</tr>
<tr>
<td>9</td>
<td>195</td>
</tr>
<tr>
<td>10</td>
<td>128</td>
</tr>
</tbody>
</table>

i. Calculate the relative frequency for each TV channel.

ii. If 2,500,000 people across Queensland were television viewers, how many preferred Channel 9?
b. A multiple choice test has 10 questions, each question having five possible choices. Only one of the choices is correct for any question. Calculate the probability of guessing exactly four questions correctly.

(KP)

c. The heights of adult males in Australia are normally distributed with a mean of 174 cm and a standard deviation of 11 cm. (The height is measured to the nearest cm and so is a discrete variable.) What percentage of adult males will have a height greater than 180 cm?

(KP)

Question 3

a. State which of the following relations are functions.
   i. \{ (1, -1), (2, -1), (3, 0), (4, 0) \}
   ii. \( y > 2x + 1 \)
   iii. 

(KP)

b. Each of the following is a function where \( x \) is a real number. State the largest possible domain for each.
   i. \( f(x) = \sqrt{9 - x^2} \)
   ii. \( y = \frac{x}{x - 2} \)

(KP)

c. The relationship between height and weight is not linear. However, for young fit adults, this relationship approximates a linear model.

One young fit adult has a height of 160 cm and weight of 68 kg and another has a height of 180 cm and a weight of 83 kg. A fitness centre claims that a young fit adult of height 172 cm should weigh 72 kg.

Can this claim be justified? Discuss the effects of any assumptions made in reaching this conclusion.

(MP)

d. Given that \( f(x) = x + 1 \) and \( g(x) = 2x^2 \), find:
   i. \( g(-1) \).
   ii. the inverse function, \( f^{-1}(x) \).
   iii. the composite function \( g(f(x)) \).

(KP)

e. A rectangular swimming pool is 20 metres long and 7 metres wide. A concrete path of uniform width surrounding the pool has an area of 160 m\(^2\). Find the width of the path.

(MP)
Question 4

a.
   i. State, with the aid of a labelled diagram, the sine rule.
   ii. A plane flies from Boulia to Aramac, a distance of 650 km, at a bearing of 080°T. The plane then leaves Aramac at a bearing of 120°T to arrive at Charleville. It is known that Charleville is 900 km from Boulia. Find the bearing (to the nearest degree) of Boulia from Charleville. (KP)

b.
   i. Convert 135° to radians.
   ii. State the Pythagorean identity.
   iii. Solve the equation for $0 \leq \theta \leq 2\pi$:
       \[2 \sin^2 \theta + \cos \theta = 1\]
       (KP)

c. For the periodic function $y = 3 \sin(2\theta + \pi) - 1$:
   i. find the amplitude, period, phase and vertical shift, and
   ii. draw a neat sketch of the curve for one period.
   (KP)
   iii. The daily temperature in a Queensland town during November can be approximated to:
       \[T = 24 + 6 \cos \left(\frac{\pi t}{12}\right),\]
       where $T$ is the temperature in degrees Celsius and $t$ is the number of hours after 12 midday. Find the times during the day when the temperature is above 26°C. (MP)
Question 5

a. Find the derivative of \( f(x) = x^2 + 2x - 3 \) from first principles. (KP)

b. Find the derivatives for the following functions:
   
i. \( y = 4x^{-3} \)
   
ii. \( y = (3x^2 - 5x)^4 \)
   
iii. \( y = \ln(x^2 + 2) \)
   
iv. \( y = e^{2x} \cos x \)
   
v. \( y = \frac{\sin x}{x^2} \)  
   (KP)

c. P (1, 4) lies on the curve \( y = x^3 - 4x^2 + x + 6 \). The tangent and normal at P intersect the x-axis at T and N respectively. Find the area of triangle PNT. (MP)

d. **Respond to Question 5d on page 19 of the response book.**
   
   The graph of the function \( y = f(x) \) is drawn below.
   
   By considering the behaviour of the function, in the proximity of at least the marked points A to F, produce a sketch of its derivative \( f'(x) \), on the same set of axes. Give explanations for the shape of the different parts of your sketch. (KP)

End of Paper One
Assessment standards from the 2006 senior external syllabus for Mathematics B

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<tr>
<td>Knowledge and procedures</td>
<td>The overall quality of a candidate’s achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, <strong>consistently demonstrates</strong>: • accurate recall, selection and use of definitions and rules • accurate use of technology • recall and selection of procedures and their accurate and proficient use • effective transfer and application of mathematical procedures.</td>
<td>The overall quality of a candidate’s achievement across a range within the contexts of Application, Technology and Complexity, and across topics, <strong>generally demonstrates</strong>: • accurate recall, selection and use of definitions and rules • accurate use of technology • recall and selection of procedures and their accurate use.</td>
<td>The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity <strong>generally demonstrates</strong>: • accurate recall and use of basic definitions and rules • use of technology • accurate recall, selection and use of basic procedures.</td>
<td>The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity <strong>rarely demonstrates</strong> knowledge and use of procedures.</td>
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<td>Modelling and problem solving</td>
<td>The overall quality of a candidate’s achievement across the full range within each context, and across topics, generally demonstrates mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems …and sometimes demonstrates mathematical thinking which includes: • suitable synthesis of procedures and strategies to solve problems • initiative and insight in exploring the problem • identifying strengths and limitations of models.</td>
<td>The overall quality of a candidate’s achievement across a range within each context, and across topics, generally demonstrates mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations and identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems …and sometimes demonstrates mathematical thinking which includes: • suitable synthesis of procedures and strategies.</td>
<td>The overall quality of a candidate’s achievement demonstrates mathematical thinking which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures required to solve problems.</td>
<td>The overall quality of a candidate’s achievement sometimes demonstrates mathematical thinking which includes: • following basic procedures and/or using strategies.</td>
<td>The overall quality of a candidate’s achievement rarely demonstrates mathematical thinking which includes following basic procedures and/or using strategies.</td>
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Communication and justification

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<td>achievement across each context</td>
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