Time allowed

- Perusal time: 10 minutes
- Working time: 3 hours

Examination materials provided:

- Paper Two – Question book
- Paper Two – Response book

Equipment allowed

- QSA-approved equipment
- ruler graduated in millimetres
- protractor
- non-programmable calculator
- graphics/graphing calculator

Calculators with computer algebra system (CAS) functionality are not allowed.

Directions

You may write in this book during perusal time.

Paper Two has six questions. Attempt all questions.

Assessment

Assessment standards are at the end of this book.

After the examination

Take this book when you leave the examination room.
Planning space
Question 1

a. The graph below shows the height of a helium-filled weather balloon for the first 70 minutes of its flight.

![Graph of a helium-filled weather balloon height over time]

i. When was the balloon neither rising nor falling?

ii. When was \( \frac{dh}{dt} < 0 \) ?

b. Use calculus techniques to find the greatest value of \( y = x^3 - x^2 - x + 1 \) over the interval \(-2 \leq x \leq 1\).

c. An open box with a rectangular base is to be constructed from a cardboard sheet measuring 6 cm wide and 16 cm long. This is to be done by cutting a square from each corner and then bending up the sides. Find the largest possible volume of such a box.

d. P is the point of intersection of a north–south highway and an east–west highway. Two cyclists, Katerina and Hilary, are each 100 km from P, one on each highway. At 12 noon, they are cycling towards P at 30 km/h and 40 km/h respectively.

i. Find the distance of each cyclist from P as a function of \( t \), the time in hours from 12 noon.

ii. Find the straight-line distance apart of the two cyclists, \( s(t) \), at any time.

iii. Find the time at which the cyclists are closest to each other.
Question 2

a. Determine
   
i. \( \int (4x^2 + 5x - 2) \, dx \)
   
ii. \( \int \left( \frac{2}{x^3} \right) \, dx \)
   
iii. \( \int_{-1}^{2} (6x^2 - x) \, dx \) \quad \text{(Show full working)}
   
iv. \( \int \left( \sin(2x + 4) + \frac{2}{x} + e^{4x} \right) \, dx \) \quad \text{(KP)}

b. Find the physical area enclosed by the curves \( f(x) = (x + 1)^2 (x - 4) \) and \( g(x) = x - 4 \). \quad \text{(KP)}

Question 3

a. Using the trapezoidal rule with five trapezia, find the area bounded by the curve \( y = \frac{1}{x + 2} \) and the lines \( x = 1 \) and \( x = 6 \). \quad \text{(KP)}

b. Find the same bounded area by evaluating the definite integral of the equation in Question 3a above. \quad \text{(KP)}

c. A particle, initially at rest at the origin, moves with a velocity of \( v = 12t - t^2 \) for the first six seconds, where \( v \) is in metres per second and \( t \) is the time in seconds. An observer wishes to know the time at which the particle had covered three-quarters of the possible displacement. Determine when this occurs. \quad \text{(MP)}

Question 4

a. Find the interest earned by an investment of $50 000 over three years at 4.5% p.a. compounded quarterly. \quad \text{(KP)}

b. How long does it take to completely pay off a home loan of $380 000 at 5.2% p.a. with monthly rests if the monthly repayments are $2250? \quad \text{(KP)}

c. On Rangi’s 18th birthday, his mother set up an annuity for him which provides $800 per month for 5 years, with the first payment to be on his 21st birthday. If money is worth 9% p.a. compounded monthly, how much must be deposited on his 18th birthday to purchase this annuity? \quad \text{(KP)}
Question 5

a. Simplify:

i. \[ \frac{(3c^2)^2 \times \sqrt[3]{c}}{3c^5} \] (Give your response with a positive index)

ii. \[ \frac{1}{2} \log_3 16 + 2 \log_3 5 \] (KP)

b. Solve:

i. \[ 2^{2x-1} = 32 \]

ii. \[ \log_3 x = 4 \] (KP)

c. The heights of plants of a certain species increase exponentially for the first five years. This growth may be modelled by the equation:

\[ H = A \times 10^{kt} \] where \( H \) is the height in centimetres after \( t \) years.

A particular plant of this species, when measured at two years old, had reached a height of 56.6 cm. At five years old, the plant was 84.9 cm high.

i. Find \( k \).

ii. Find \( A \).  

(KP)

iii. After eight years, the height of the plant was 154.8 cm. Discuss the limitations of extending the model to eight years.  

(MP)

Question 6

The rate of growth of a population of a particular strain of bacteria is proportional to the number of bacteria present. A scientist notes that the population increases from 2000 to 10 000 in four hours. How long would it have taken the initial population to have grown to 6000?  

(MP)

End of Paper Two
|-------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|
| Knowledge and procedures      | The overall quality of a candidate’s achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, **consistently demonstrates**:  
  • accurate recall, selection and use of definitions and rules  
  • accurate use of technology  
  • recall and selection of procedures and their accurate and proficient use  
  • effective transfer and application of mathematical procedures. | The overall quality of a candidate’s achievement across a range within the contexts of Application, Technology and Complexity, and across topics, **generally demonstrates**:  
  • accurate recall, selection and use of definitions and rules  
  • accurate use of technology  
  • recall and selection of procedures and their accurate use. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, and across topics, **generally demonstrates**:  
  • accurate recall and use of basic definitions and rules  
  • use of technology  
  • accurate recall, selection and use of basic procedures. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, and across topics, **sometimes demonstrates**:  
  • accurate recall and use of some definitions, and rules  
  • use of technology  
  • use of basic procedures. | The overall quality of a candidate’s achievement **rarely demonstrates** knowledge and use of procedures. |
## Modelling and problem solving

|----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| **The overall quality**    | of a candidate’s achievement across the full range within each context, and across topics, **generally demonstrates** mathematical thinking which includes:  
  - interpreting, clarifying and analysing a range of situations identifying assumptions and variables  
  - selecting and using effective strategies  
  - selecting suitable procedures required to solve a range of problems ...and **sometimes demonstrates** mathematical thinking which includes:  
  - suitable synthesis of procedures and strategies to solve problems  
  - initiative and insight in exploring the problem  
  - identifying strengths and limitations of models. | **The overall quality** of a candidate’s achievement across a range within each context, and across topics, **generally demonstrates** mathematical thinking which includes:  
  - interpreting, clarifying and analysing a range of situations identifying assumptions and variables  
  - selecting and using effective strategies  
  - selecting suitable procedures required to solve a range of problems ...and **sometimes demonstrates** mathematical thinking which includes:  
  - suitable synthesis of procedures and strategies. | **The overall quality** of a candidate’s achievement **demonstrates** mathematical thinking which includes:  
  - interpreting and clarifying a range of situations  
  - selecting strategies and/or procedures required to solve problems. | **The overall quality** of a candidate’s achievement **sometimes demonstrates** mathematical thinking which includes:  
  - following basic procedures and/or using strategies. | **The overall quality** of a candidate’s achievement **rarely demonstrates** mathematical thinking which includes following basic procedures and/or using strategies. |
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