Time allowed

- Perusal time: **10 minutes**
- Working time: **3 hours**

Examination materials provided:

- Paper One – Question book
- Paper One – Resource book
- Paper One – Response book

Equipment allowed

- QSA-approved equipment
- ruler graduated in millimetres
- protractor
- non-programmable calculator
- graphics/graphing calculator

Calculators with computer algebra system (CAS) functionality are not allowed.

Directions

You may write in this book during perusal time.

Paper One has **six** questions. Attempt **all** questions.

Assessment

Assessment standards are at the end of this book.

After the examination

Take this book when you leave the examination room.
Planning space
Question 1

A cricketer batted 23 times during a season. The individual scores (runs) were plotted as the stem-and-leaf plot below:

```
0 0 2 3 3 4 6
1 1
2 2 5 9
3 4 6
4 1 1 6 6 6 7 9
5 4 4 9
14 7
```

(Key: 3 4 means 34)

a. List (in ascending order) the cricketer’s scores.

b. For this cricketer, calculate the measures of central tendency:
   i. mean
   ii. mode
   iii. median
Question 2

The scores for an entrance test for an academy were recorded for a group of 25 applicants. For Paper One of the test, no extra resources were permitted. For Paper Two of the test, applicants had access to resources such as texts, formula sheets and software programs. The marks for each paper in ascending order were:

Paper One

32 36 39 40 42 44 44 45 48 50 51 52 56 57 59 60 64 68 70 72 78 84 88 96

Paper Two

25 35 46 48 54 55 57 61 62 64 65 66 66 67 67 68 69 69 71 71 72 74 85

a. State the five-number summary for each paper. (KP)

b. Construct side-by-side boxplots (box-and-whisker plots) to compare the results. (KP)

c. What conclusions can be made? Justify your conclusions by referring to each boxplot. (MP)

Question 3

a. Three employees, Anu, Bob and Charles, work for the same company. They respectively handle 40%, 35% and 25% of the company’s orders. It has been found that they make mistakes in 5%, 4% and 3% of their orders respectively.

i. Use a tree diagram to display this information.

ii. Calculate the probability that an order handled by the company has a mistake. (KP)

b. Jenny and Craig play against each other on a Playstation. The probability that Jenny wins any game is 0.45. If they play 10 games, what is the probability that Jenny wins five or more games? (KP)

c. The distribution of heights of a certain type of plant is normally distributed with a mean of 72 cm and a standard deviation of 6 cm. What is the probability that a randomly selected plant is less than 64 cm? (KP)

d. In a particular competition, achievement awards are allocated as:

- High Distinctions to the top 15% of performances
- Distinctions to the next 25%
- Credits to the next 30% and
- Participations to the remainder.

If the scores were normally distributed with a mean of 75 and a standard deviation of 12, find the minimum score a competitor has to achieve to be awarded a Credit. (MP)
Question 4

a. Solve the quadratic equation \( x^2 + 4x - 12 = 0 \) either by using the formula or by completing the square.

b. State the domain and range for each function:
   i. \( \{(1, 3), (2, 4), (3, 5), (4, 6)\} \)
   ii. \( y \)

   \[
   \begin{array}{c}
   y \\
   3 \\
   x
   \end{array}
   \]

c. Given that \( f(x) = x^2 - x \) and \( h(x) = 2x + 1 \), find:
   i. \( f(-2) \)
   ii. \( f(h(x)) \)
   iii. \( x \) where \( h(x) = 0 \)
   iv. the inverse function, \( h^{-1}(x) \).

d. Use algebra to solve the following simultaneous equations:
   
   \[
   \begin{align*}
   y &= x^2 - x - 12 \\
   4x &= 12 + y
   \end{align*}
   \]

e. The length of a closed box is three times its height. The box is 4 cm wide and the total surface area is 88 cm\(^2\). Find the dimensions of the box.
Question 5

a. Find the derivative of \( f(x) = x^2 \) from first principles.  

b. Find derivatives for the following functions:
   
i. \( y = 3x^2 + 6x - 5 \)
   
ii. \( y = (4x^3 - x)^7 \)
   
iii. \( y = e^x \sin x \)
   
iv. \( y = \ln (x^2 + 4) \)
   
v. \( y = \frac{x^2 + 2}{3x - 4} \) 

(KP)

c. A water tank is being filled at a variable rate. The height of the water, \( h \) cm, at any time, \( t \) minutes can be described by
   
   \[ h(t) = 2t^2 - 4t + 50 \]
   
   Find
   
i. the initial height of water in the tank
   
ii. the instantaneous rate at which the height is changing at 4 minutes
   
iii. the average rate at which the height has changed over the first 4 minutes.

(KP)

d. It is claimed that the tangent to the curve \( y = x^3 - 2x^2 - 4x + 3 \) at the point where \( x = 1 \) passes through the point (8,3). Evaluate the validity of this claim.

(MP)

e. A piece of wire 6 m long is cut into two unequal parts. One part is used to form a rectangle that has a length three times its width. The other part of the wire is used to form a square.
   
i. Construct a function that will give the sum of the areas of the rectangle and the square in terms of the width of the rectangle.
   
ii. Find the length of each part of the wire when the sum of the areas is a minimum.

(MP)
**Question 6**

a. State, with the aid of a labelled diagram, the cosine rule.  
   
   (KP)

b. Ship Q is 22.4 km from port on a bearing of 056T and ship R is 34.5 km from the same port (P) on a bearing of 119T. Calculate the distance between the two ships.  
   
   (KP)

c. A guy-wire 42 m long is attached part-way up a vertical radio mast. The guy-wire makes an angle of 38° with the horizontal ground. A 96 m guy-wire is secured to the same point on the ground and attached to the top of the mast. Using trigonometry, find the height of the mast to the nearest metre.  
   
   (MP)

d. For the periodic function \( y = 4\sin 2x + 1 \),
   
   i. find the amplitude, period and vertical shift  
      
   ii. draw a neat sketch of the curve for one period.  
      
   (KP)

e. The water height in an estuary fluctuates according to the tide. Flooding will occur at a certain point when the water height reaches 1.5 m. One day the high tide of 1.6 m occurred at 3:00 am and the following low tide at 9:30 am. Assume that the following high tide was also 1.6 m and the height of the tide is modelled by the equation:

   \[
   h = 1 + 0.6 \sin \frac{2\pi}{13}(t + 0.25)
   \]

   where \( h \) is the height in metres and \( t \) is the time in hours after midnight.

   Find the time when the next flooding would be expected to occur. (To graph this function, your graphics/graphing calculator should be set to radians.)  
   
   (MP)

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**End of Paper One**
### Assessment standards from the 2006 Senior External Syllabus for Mathematics B

|----------------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|
| Knowledge and procedures   | The overall quality of a candidate’s achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, **consistently demonstrates:**  
  • accurate recall, selection and use of definitions and rules  
  • accurate use of technology  
  • recall and selection of procedures and their accurate and proficient use  
  • effective transfer and application of mathematical procedures. | The overall quality of a candidate’s achievement across a range within the contexts of Application, Technology and Complexity, and across topics, **generally demonstrates:**  
  • accurate recall, selection and use of definitions and rules  
  • accurate use of technology  
  • recall and selection of procedures and their accurate use. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, **generally demonstrates:**  
  • accurate recall and use of basic definitions and rules  
  • use of technology  
  • accurate recall, selection and use of basic procedures. | The overall quality of a candidate’s achievement in the contexts of Application, Technology and Complexity, **sometimes demonstrates:**  
  • accurate recall and use of some definitions, and rules  
  • use of technology  
  • use of basic procedures. | The overall quality of a candidate’s achievement **rarely demonstrates** knowledge and use of procedures. |
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<td>Modelling and problem solving</td>
<td>The overall quality of a candidate’s achievement across the full range within each context, and across topics, generally demonstrates mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems ... and sometimes demonstrates mathematical thinking which includes: • suitable synthesis of procedures and strategies to solve problems • initiative and insight in exploring the problem • identifying strengths and limitations of models.</td>
<td>The overall quality of a candidate’s achievement across a range within each context, and across topics, generally demonstrates mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations and identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems ... and sometimes demonstrates mathematical thinking which includes: • suitable synthesis of procedures and strategies.</td>
<td>The overall quality of a candidate’s achievement demonstrates mathematical thinking which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures required to solve problems.</td>
<td>The overall quality of a candidate’s achievement demonstrates mathematical thinking which includes: • following basic procedures and/or using strategies.</td>
<td>The overall quality of a candidate’s achievement rarely demonstrates mathematical thinking which includes following basic procedures and/or using strategies.</td>
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### Communication and justification

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<td>The overall quality of a candidate's achievement sometimes demonstrates evidence of the basic conventions of language and mathematics and occasional use of mathematical reasoning.</td>
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