Senior External Syllabus

Mathematics B

Syllabus for the Senior External Examination

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Mathematics B Syllabus for the Senior External Examination

This syllabus should be used for the first time in the Senior External Mathematics B Examination 2007.

Queensland Studies Authority, PO Box 307, Spring Hill, Queensland 4004, Australia
Phone: (07) 3864 0299
Fax: (07) 3221 2553
Email: office@qsa.qld.edu.au
Website: www.qsa.qld.edu.au

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1. Rationale

Mathematics is an integral part of a general education. It enhances an understanding of the world and the quality of participation in society. The range of career opportunities requiring an appropriate level of mathematical competence is rapidly expanding into such areas as health, environmental science, economics and management, while remaining crucial in such fields as the physical sciences, engineering, accounting, computer science and information technology. Mathematics is essential for widespread computational and scientific literacy, for the development of a more technologically skilled work force, for the development of problem-solving skills and for the understanding and use of data and information to make well-considered decisions. It is valuable to people individually and collectively, providing important tools that can be used at personal, civic, professional and vocational levels.

At the personal level, the most obvious use of mathematics is to help make informed decisions in areas as diverse as buying and selling, home maintenance, interpreting media presentations and forward planning. The mathematics involved in these activities includes analysis, financial calculation, data description, inference, number, quantification and spatial measurement. The generic skills developed by mathematics are also constantly used at the personal level.

At the civic, professional and vocational levels, the generic skills, knowledge and application of mathematics underpin most of the significant activities in industry, trade and commerce, social and economic planning, and communication systems. In such areas, the concepts and application of functions, rates of change, total change and optimisation are very important. The knowledge and skills developed in Mathematics B are essential for all quantitative activities in the above areas. Higher-order thinking skills developed in problem solving are essential for further development in quantitative areas.

Mathematics has provided a basis for the development of technology. In recent times, the uses of mathematics have increased substantially in response to changes in technology. The more technology is developed the greater the level of mathematical skill required. Candidates should be given the opportunity to appreciate and experience the power that has been given to mathematics by this technology. Such technology should be used to encourage candidates to understand mathematical concepts, allowing them to “see” relationships and graphical displays, to search for patterns and recurrence in mathematical situations, as well as to help the exploration and investigation of real and life-like situations.

Mathematics B aims to provide the opportunity for candidates to be more fully prepared for life-long learning. It provides the opportunity for development of:

- knowledge, procedures and skills in mathematics
- mathematical modelling and problem-solving strategies
- the capacity to justify and communicate in a variety of forms.

Such development should occur in contexts ranging from purely mathematical through life-like to real, from simple through intermediate to complex, from basic to more advanced technology use, and from routine rehearsed through to innovative. Of importance is the development of thinking skills, as well as recognition and use of mathematical patterns.

The intent of Mathematics B is to encourage candidates to develop positive attitudes towards mathematics by approaches involving exploration, investigation, application of knowledge and skills, problem solving and communication. Candidates will be encouraged to mathematically model, to work systematically and logically, to propose conjectures and reflect on them, and to justify and communicate with and about mathematics.

2. Global aims

Candidates of Mathematics B should:

- have significantly broadened their mathematical knowledge and skills
- be able to recognise problems that are suitable for mathematical analysis and solution, and be able to attempt such analysis or solution with confidence
- be aware of the uncertain nature of their world and be able to use mathematics to help make informed decisions in life-related situations
- have experienced diverse applications of mathematics
- have positive attitudes to the learning and practice of mathematics
- comprehend mathematical information that is presented in various forms
- communicate mathematical information in various forms
- be able to use justification in and with mathematics
- be able to benefit from the availability of a wide range of technologies
- be able to choose and use mathematical instruments appropriately
- be able to recognise functional relationships and applications.

3. General objectives

3.1 Introduction

The general objectives of this subject are organised into four categories:

- *Knowledge and procedures*
- *Modelling and problem solving*
- *Communication and justification*
- *Affective*.

These objectives are linked with the criteria described in section 7.2, with the exception of *Affective.*

3.2 Contexts

The objectives of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification* incorporate contexts of Application, Technology, Initiative and Complexity. Each of the contexts has a continuum for the particular aspect of mathematics it represents. Mathematics from this syllabus should be taught and learned using a variety of contexts. It is expected that all candidates experience mathematics along the continuum of each of the contexts outlined below. The examination will cover a number of contexts.

Application

Candidates should recognise the usefulness of mathematics through its application, and the beauty and power of mathematics that comes from the capacity to abstract and generalise. Thus candidates' learning experiences should include mathematical tasks that demonstrate a balance across the range from life-related through to pure abstraction.

Technology

A variety of technological tools should be used in the learning experiences. These range from pen and paper, measuring instruments and tables through to higher technologies such as graphing calculators. The minimum level of higher technology appropriate is a graphing calculator (refer to section 4.4).

Initiative

Candidates should undertake tasks that range from routine and well rehearsed through to those that require demonstration of insight and creativity.

Complexity

Candidates should work on simple, single-step tasks through to tasks that are complex. Complexity may derive from either the nature of the concepts involved or from the number of ideas or techniques that are sequenced to produce a suitable conclusion.

3.3 General objectives

The general objectives of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*, are linked to the exit criteria in section 6.2.

3.3.1 Knowledge and procedures

This objective involves recalling and using results and procedures within the contexts of Application, Technology, Initiative and Complexity (see section 3.2).

Candidates should be able to:

- recall definitions and rules
- access and apply rules and techniques
- demonstrate number and spatial sense
- demonstrate algebraic facility
- demonstrate an ability to select and use technology such as calculators, measuring instruments and tables
- demonstrate an ability to use graphing calculators
- recall, select and use mathematical procedures
- work accurately and manipulate formulae
- break some tasks into smaller components
- transfer and apply mathematical procedures to situations.

3.3.2 Modelling and problem solving

This objective involves the uses of mathematics in which the candidates will model mathematical situations and constructs, solve problems and investigate situations mathematically within the contexts of Application, Technology, Initiative and Complexity (see section 3.2).

Candidates should be able to demonstrate modelling and problem solving through:

Modelling

- understanding that a mathematical model is a mathematical representation of a situation
- identifying the assumptions and variables of a simple mathematical model of a situation
- forming and/or selecting a mathematical model of a life-related situation
- deriving results from consideration of mathematical models
- interpreting results from the mathematical model in terms of the given situation
- exploring the strengths and limitations of a mathematical model.

Problem solving

- identifying a problem
- interpreting, clarifying and analysing a problem
- selecting mathematical procedures required to solve a problem
- using a range of problem-solving strategies such as estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data

- exploring a problem and from emerging patterns creating conjectures or theories
- selecting and using problem-solving strategies to test and validate any conjectures or theories
- reflecting on conjectures or theories, making modifications if needed
- understanding that there may be more than one way to solve a problem
- developing a solution consistent with the problem.

3.3.3 Communication and justification

This objective involves presentation, communication (both mathematical and everyday language), logical arguments, interpretation and justification of mathematics within the contexts of Application, Technology, Initiative and Complexity.

Communication

Candidates should be able to demonstrate communication through:

- organising and presenting information
- communicating ideas, information and results as required
- using mathematical terms and symbols accurately and as required
- using accepted spelling, punctuation and grammar in written communication
- understanding material presented in a variety of forms such as written, symbolic, pictorial and graphical
- translating material from one form to another
- presenting material in a variety of forms (such as written, symbolic, pictorial and graphical)
- recognising necessary distinctions in the meanings of words and phrases according to whether they are used in a mathematical or non-mathematical situation.

Justification

Candidates should be able to demonstrate justification through:

- developing logical arguments expressed in everyday language, mathematical language or a combination of both, as required, to support conclusions, results and/or propositions
- evaluating the validity of arguments designed to convince others of the truth of propositions
- justifying procedures which may include:
  - providing evidence (words, diagrams, symbols, etc.) to support processes used
  - stating a generic formula before specific use
  - providing a reasoned, well-formed, logical sequence within a response
- recognising when and why derived results of a given problem are clearly improbable or unreasonable
- recognising that one counter example is sufficient to disprove a generalisation
- recognising the effect of assumptions on the conclusions that can be reached
- deciding whether it is valid to use a general result in a specific case
- using supporting arguments to justify results obtained by calculator when necessary.

3.3.4 Affective

Affective objectives refer to the attitudes, values and feelings that this subject aims to develop in candidates although they are not assessed in an external examination**.**

Candidates should appreciate the:

- diverse applications of mathematics
- precise language and structure of mathematics
- diverse and evolutionary nature of mathematics and the wide range of mathematics-based vocations
- contribution of mathematics to human culture and progress
- power and beauty of mathematics.

4. Organisation

4.1 Introduction

Candidates need to acquire certain fundamental knowledge and procedures. Some of these are listed under the heading “Basic knowledge and procedures” in Appendix 1. Candidates should revise the fundamental knowledge and procedures within topics, as they are required.

The subject matter has been organised into seven topics, which are discussed in detail in section 5. All topics must be studied. The topics are:

- Introduction to functions
- Rates of change
- Periodic functions and applications
- Exponential and logarithmic functions and applications including financial maths
- Optimisation using derivatives
- Introduction to integration
- Applied statistical analysis.

4.2 Time allocation

For teaching centres preparing candidates for the external examination the recommended number of hours for tuition in the subject developed from this syllabus is 150 to 200 hours. Recommended notional times are given in brackets to indicate the relative emphasis of each topic. The time allocation suggested provides a guide for the effective planning of learning experiences.

Time allocation depends on the method of study. Candidates who elect to study without systematic tuition must organise their time according to syllabus requirements and individual circumstances.

4.3 Sequencing

After considering the subject matter and the relevant learning experiences needed to achieve the general objectives a *spiralling* and *integrated* sequence of learning experiences should be developed that allows candidates to see links between the different topics of mathematics rather than seeing them as discrete.

The order in which the topics are presented in the syllabus is not intended to indicate a sequence of learning, but some topics include subject matter that is developed and extended in the subject matter of other topics. The subject matter should be revisited and spiralled so that candidates internalise their knowledge before developing it further.

The following guidelines for the sequencing of subject matter may be helpful.

- No subject matter should be studied before the relevant prerequisite material has been covered.
- Subject matter across topics should be linked when possible.
- Time will be needed for maintaining basic knowledge and procedures.

4.4 Technology

The advantage of mathematics-enabled technology in mathematics is that it allows for the exploration of the concepts and processes of mathematics. For example, graphing calculators let candidates explore, investigate and more easily understand concepts, especially graphs.

More specifically, the mathematics-enabled technology allows candidates to tackle more diverse, life-related problems. Real-life optimisation problems are more easily solved with this technology. It may be used in statistics to investigate larger datasets and rapidly produce a variety of graphical displays and summary statistics, thus freeing candidates to look for patterns, to detect anomalies in the data and to make informed comments.

The minimum electronic technology requirement is a calculator that has the following operations:

- basic operations
- logs and exponentials
- financial mathematical operations
- calculus capabilities (differential and integral)
- graphing
- statistical functions
- trigonometric functions.

Use of technology in the examination

During the examination the noise of operating a calculator must not disturb other candidates.

Candidates should not rely only on the use of technology to obtain a result. To meet the requirements of the criterion *Communication and justification* candidates should show full working for each question.

5. Topics

5.1 Introduction

Each topic has a focus statement, subject matter and learning experiences which, taken together, clarify the scope, depth and emphasis for the topic.

Focus

This section highlights the intent of the syllabus with respect to the topic and indicates how candidates should develop their understanding of the topic.

Subject matter

This section outlines the subject matter to be studied in the topic. All subject matter listed in the topic must be included, but the order in which it is presented is not intended to imply a sequence of learning.

Learning experiences (LE)

This section provides learning experiences, which may be effective in using the subject matter to achieve the general objectives of the subject. The numbers provided with the subject matter link to learning experiences. Included are experiences that involve life-related applications of mathematics with both real and simulated situations, use of instruments and opportunities for modelling and problem solving.

The learning experiences are suggestions only and are not prescriptive. Candidates should be involved in a variety of activities to extend their understanding.

Note: The learning experiences must provide candidates with the opportunity to experience mathematics along the continuum within each of the contexts.

Language is the means by which meaning is constructed and shared, and communication is effected. It is the central means by which candidates learn. Mathematics B requires candidates to use language in a variety of ways —mathematical, written, graphical, symbolic. Candidates should develop the ability to use effectively the forms of language demanded by this subject. They should develop abilities to:

- select and sequence information
- manage the conventions related to the forms of communication used in Mathematics B as related to an external examination
- use the specialised vocabulary and terminology related to Mathematics B
- use language conventions related to grammar, spelling, punctuation and layout.

The learning of language is a developmental process. When writing, reading, questioning, listening and talking about mathematics, candidates should use the specialised vocabulary of Mathematics B. Candidates should be involved in learning experiences that require them to comprehend and transform data in a variety of forms and, in so doing, use the appropriate language conventions. Attention to language education within Mathematics B should help candidates to meet the language components of the external examination including exit criteria, especially the *Communication and justification* criterion.

5.2 The topics

The order in which topics and items within topics are given should not be seen as implying a teaching or learning sequence.

Introduction to functions (notional time 35 hours)

Focus

Candidates should develop an understanding and appreciation of relationships between variables, be conversant with the three methods of representation (algebraic, graphical, numerical) and interrelate these methods in a variety of modelling situations, ranging from life-related to abstract. Emphasis should be placed on the recognition of functions, sketching, investigating shapes and relationships, and the general forms of functions. The use of technology will help candidates in these processes.

Subject matter

- concepts of function, domain and range (LE 1 – 4)
- mappings, tables and graphs as representations of functions and relations (LE 1–4)
- graphs as a representation of the points whose coordinates satisfy an equation (LE 1, 4, 5, 8 and 9)
- distinction between functions and relations (LE 1, 2, and 11)
- distinctions between continuous functions, discontinuous functions and discrete functions (LE 1 and 3)
- practical applications of linear functions, including:
  - direct variation
  - simple interest
  - linear relationships between variables (LE 8)
- practical applications of quadratic functions, the reciprocal function and inverse variation (LE 8 – 12)
- relationships between the graph of $f(x)$ and the graphs of $f(x)+ a$, $f(x + a)$, $a\,f(x)$, $f(ax)$ for both positive and negative values of the constant a (LE 5 and 7)
- general shapes of graphs of absolute value functions, the reciprocal function and polynomial functions up to and including the fourth degree (LE 4 – 7)
- algebraic and graphical solution of two simultaneous equations in two variables (to be applied only to linear and quadratic functions) (LE 7 and 8)
- concepts of the inverse of a function and composite functions (LE 7 and 13).

Learning experiences

The following learning experiences may be developed.

1. Find the domain and range of functions in both mathematical and life-related contexts, given data in a variety of forms such as graphs, tables of values, mathematical expressions or descriptions of situations.
2. Use the vertical line test to determine if a relation is a function.
3. Draw graphs of step functions such as postal charges for packages against the weight of the packages, telephone charges per unit time against distance.
4. Investigate the general shapes of polynomial functions of the type $y = x^n$, $n = 2, 3, 4$.

5. Using $f(x) = x^n$, for $n = 1$ to 4; investigate the relationships between the graph of $f(x)$ and the graphs of $f(x) + a$, $f(x + a)$, $a f(x)$ and $f(a x)$, by means of a graphing calculator and algebraic methods.
6. Investigate the number of times a straight line intersects the graph of a polynomial of degree n, $n = 1$ to 4 .
7. Use a graphing calculator to investigate the shapes of different functions.
8. Solve simultaneous equations that model life-related situations algebraically and graphically.
9. Locate the position algebraically, numerically and graphically of, for example: a) the highest point on a projectile path defined by a quadratic function, and b) the lowest point of a cost function modelled by a quadratic function.
10. Use quadratic functions in life-related situations such as: find the dose of a chemical needed to obtain a 50% kill of insects, if the percentage of insects killed, p, is related to the dose level, x, by the equation, $p = a + bx + cx^2$ where a, b and c are constants with values such as 2, –1 and 3 .
11. Use a graphing calculator to investigate possible functions for data.
12. Derive the formula for the solution of a general quadratic equation and solve quadratics by completion of the square.
13. Investigate simple inverse and composite functions.

Rates of change (notional time 30 hours)

Focus

Candidates should develop an understanding of average and instantaneous rates of change and of the derivative as a function. This understanding should be developed using both algebraic and graphical approaches. Candidates should apply the rules for differentiation and interpret the results. The use of technology is expected to help in these processes.

Subject matter

- concept of rate of change (LE 1–7)
- calculation of average rates of change in both practical and purely mathematical situations (LE 1 – 3)
- interpretation of the average rate of change as the gradient of the secant (LE 1 and 2)
- intuitive understanding of a limit (LE 1 – 4).

Calculations using limit theorems are not required.

- definition of the derivative of a function at a point (LE 3 and 4)
- derivative of simple algebraic functions from first principles up to and including quadratic functions only (LE 3 and 4)

- rules for differentiation including: (LE 7)

  $\frac{d}{dp}\ p^n$ for rational values of n

  $\frac{d}{dr}[k\ f(r)]$

  $\frac{d}{ds}[f(s)+g\ (s)]$

  $\frac{d}{dt}[f(t)\ g\ (t)]$

  $\frac{d}{dx}\ f\ [g(x)]$

- evaluation of the derivative of a function at a point (LE 3 and 4)
- interpretation of the derivative as an instantaneous rate of change (LE 1, 2 and 5)
- interpretation of the derivative as the gradient function (LE 1, 2, 4 and 6)
- practical applications of instantaneous rates of change (LE 1–5 and 7).

Learning experiences

The following learning experiences may be developed.

1. Determine average and instantaneous speeds from a distance–time graph.
2. Determine average and instantaneous accelerations from a velocity–time graph.
3. Use a numerical technique to estimate a limit or an average rate of change.
4. Graph a function and its gradient function; relate the features of each to the other.
5. Determine the instantaneous rate of change of a variable with respect to another variable in life-related situations given the mathematical model; such as:
   - the rate of population change with respect to time
   - the rate of change of resistance in a wire with respect to temperature
   - the rate of change of the surface area of an object with respect to volume
   - the rate of change of a cost function with respect to the number of items produced.
6. Find the equation of the tangent to a curve under various given conditions.
7. Practical application of rates of change such as:
   - compare the evaporation rate of water in open containers of varying cross-sections
   - calculate the effect of small measurement errors in the calculation of a volume; relate this to a graph.

Periodic functions and applications (notional time 30 hours)

Focus

Candidates should develop an understanding and appreciation of periodic functions, be conversant with the three methods of representation (algebraic, graphical, numerical) and interrelate these methods in a variety of modelling situations, ranging from life-related to abstract. Emphasis should be placed on the recognition of periodic functions, sketching, investigating shapes and relationships, and the general forms of periodic functions.

The use of technology will help candidates in these processes. Trigonometric identities should not be developed beyond the Pythagorean identity.

Subject matter

- trigonometry including the definition and practical applications of the sine, cosine and tangent ratios (LE 1 and 2)
- simple practical applications of the sine and cosine rules (the ambiguous case is not required) (LE 1 and 2)
- definition of a periodic function, the period, frequency and amplitude (LE 3, 4, 7 and 8)
- definition of a radian and its relationship with degrees (LE 5)
- definition of the trigonometric functions sin, cos and tan of any angle in degrees and radians (LE 3, 5 and 7)
- graphs of y = sin x, y = cos x and y = tan x for any angle in degrees ($-360 \leq x \leq 360$) and in radians in the domain of $-2\pi \leq x \leq 2\pi$ (LE 3, 5 and 8)
- significance of the constants A, B, C and D on the graphs of y = A sin(Bx+C)+D, y = A cos(Bx+C) + D (LE 4, 7 and 8)
- applications of periodic functions (LE 3, 4, 7 and 8)
- Pythagorean identity $\sin^2 x + \cos^2 x = 1$ (LE 7)
- solution of simple trigonometric equations within a specified domain (LE 6)
- derivatives of functions involving sin x and cos x (LE 7)
- applications of the derivatives of sin x and cos x in life-related situations (LE7).

Learning experiences

The following learning experiences may be developed.

1. Use sine, cosine and tangent ratios to determine lengths/distances and magnitudes of angles in life-related situations such as:
   - guy ropes for tents or flagpoles
   - distances across rivers and valleys.
2. Use sine and cosine rules to solve triangles in two- and three-dimensional contexts and determine lengths/distances and the magnitude of angles in life-related situations such as:
   - the distance between two ships given the distances and bearings to a fixed point
   - the height of a tower given the direction and angles of inclination from two fixed locations of known distance apart.
3. Find the period, amplitude and frequency of periodic functions involving sine and cosine given their graphs and/or equations.
4. Find the period, amplitude and frequency of trigonometric functions that are used to model phenomena such as biorhythms, tide heights.
5. Using the concept of the unit circle:
   - evaluate the trigonometric functions sin, cos and tan of any angle in degrees and radians
   - explore the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$.
6. Find solutions of trigonometric equations within a specified domain such as $-2\pi \leq \theta \leq 2\pi$ for equations such as $2 \sin \theta = -0.7$, $2 \sin^2 \theta = \cos \theta$.

7. Use practical applications of periodical functions such as:
   - investigating the periodic motion of a mass on the end of a spring; given the mathematical model of the displacement from a fixed position, find mathematical representations of its velocity and acceleration; solve these simple trigonometric equations to find displacement, velocity and acceleration at a given time during the periodic motion.
   - calculating the rate at which the water level is changing on a vertical marker given a sine function as a model of tide height.
   - investigating the periodic motion of a pendulum; mathematically model the motion of a pendulum using trigonometric equations.
   - exploring the period, amplitude and frequency of periodic (oscillatory) phenomena including planetary motion, hormone cycles, ECGs, Halley's comet, reciprocating motion.
   - investigating the path of a point on a moving bicycle wheel.
8. Use the graph of a sinusoidal function to develop the corresponding algebraic form.

Exponential and logarithmic functions and applications including financial mathematics (notional time 35 hours)

Focus

Candidates should develop an understanding and appreciation of exponential and logarithmic functions and the relationships between them. They should be conversant with the three methods of representation (algebraic, graphical, numerical). Emphasis should be placed on the application of these functions to solve problems in a range of life-related situations (e.g. finance and investment, growth and decay). The use of technology will help candidates in these processes.

It is not intended that a great emphasis be placed on simplification of expressions involving indices or logarithms.

Subject matter

- index laws and definitions (LE 1)
- definitions of a^x and $log_a x$, for $a > 1$ (LE 1)
- logarithmic laws and definitions (LE 1)
- change of base rule (LE 7)
- definition of the exponential function e^x (LE 6 and 8)
- graphs of, and the relationships between, $y = a^x$, $y = \log_a x$ for $a = e$ and other values of a (LE 5, 7, 8, 14 and 16)
- graphs of $y = e^{kx}$ for $k \neq 0$ (LE 5)
- solution of equations involving indices (LE 2–7, 12–16)
- use of logarithms to solve equations involving indices (LE 3–6, 8–10, 12 and 15)
- development of algebraic models from appropriate datasets using logarithms and/or exponents (LE 2, 9 and 10)
- derivatives of exponential and logarithmic functions for any base (LE 2–5 and 7)
- applications of exponential and logarithmic functions (LE 2–5, 7 and 9)
- applications of exponential and logarithmic functions that include compound interest; past, present and future values; nominal and effective interest rates (LE 11 – 13).
- geometric progressions in financial contexts (LE 14)
- annuities and amortising a loan (LE 15 and 16).

Learning experiences

The following learning experiences may be developed.

1. Use the identities $x = a^{\log_a x}$ and y = loga(a y) .
2. Determine the function Q given a rate of growth or decay of some quantity Q such as quantity of bacteria, size of epidemic, drug concentration, population size as a simple function of time or Q alone.
3. Investigate life-related situations by practical application of simple exponential functions such as:
   - applications of Newton's Law of Cooling
   - concentration against time in chemistry
   - carbon dating in archaeology
   - decrease of atmospheric pressure with altitude.
4. Investigate change such as radioactive decay, growth of bacteria, or growth of an epidemic, where the rate of change is proportional to the amount of material left or the current population size.
5. Consider that the proportion of a radioactive material remaining after time t has elapsed is e^{-kt}, where k is a positive constant; investigate the relationship between k and the half-life of the material.
6. Use practical applications of logarithmic scales, for example decibels, Richter scale and pH.
7. Develop the derivative of the function of a^x and $log_a x$, to identify the significance of the exponential constant e.
8. Use logarithms to solve equations such as the time taken for an investment to double for a given compound interest rate.
9. Investigate the time at which the quantity of the intermediate substance reaches a maximum in a simple two-step radioactive decay; that is, the original substance decays to an intermediate substance that in turn decays to an inert substance.
10. Plot the logarithm of the population of Australia at censuses (a) from 1891 to 1933 (b) from 1947 to 1971 and (c) from 1971 to 1991; recognise that the linear tendencies of the plot indicate power/exponential relationships in the original.
11. Use a graphing calculator to compare interest accrued and yearly balances for an investment over five years where (a) a flat rate applies, and (b) compound interest applies.
12. Solve a range of financial problems involving the compound interest formula.
13. Calculate the effective interest rate given an annual nominal interest rate and the compounding period.
14. Apply the compound interest formula and the sum of a geometric progression to develop formulae for the future value and present value of an annuity.
15. Solve a range of financial problems involving annuities formulae.
16. Amortise a loan.
17. Investigate various financial models using spreadsheet technology.

Optimisation using derivatives (notional time 25 hours)

Focus

Candidates are encouraged to develop an understanding of the use of differentiation as a tool in a range of situations that involve the optimisation of continuous functions. This understanding should be developed using both algebraic and graphical approaches. The use of technology will help candidates in these processes.

Subject matter

- positive and negative values of the derivative as an indication of the points at which the function is increasing or decreasing (LE 10)
- zero values of the derivative as an indication of stationary points (LE 4)
- concept of relative maxima and minima and greatest and least value of functions (LE 2)
- relationships between the graph of a differentiable function, and the graph of the derivative of that function (LE 10)
- methods of determining the nature of stationary points (LE 1–10)
- greatest and least values of a function in a given interval (LE 1–10)
- recognition of the problem to be optimised (maximised or minimised) (LE 1, 2, 5 – 10)
- identification of variables and construction of the function to be optimised (LE 1–10)
- applications of the derivative to optimisation in life-related situations (LE 1, 2, 5 – 9)
- interpretation of mathematical solutions and their communication in a form appropriate to the given problem (LE 1, 2, 5 – 9).

Learning experiences

The following learning experiences may be developed.

1. Use life-related situations such as enclosing a rectangular area with a fixed length of fencing, to demonstrate the need for calculus to determine optimal values.
2. Interpret a table of data values as to the rate of increase, greatest and least values taken by a smoothly varying function.
3. Interpret a graph as to rates of increase or decrease of a function; relate these observations to the behaviour of the derivative.
4. Use zero values of the derivative to find local optima and points of horizontal inflection in curve sketching of simple functions (the solutions should not be dependent on the factor theorem).
5. Solve shortest distance problems in two-dimensional geometry, such as finding the shortest distance from a given point to a point on a given straight line through the origin (geometry of the straight line and circle is all that will be assumed).
6. Investigate applications of shortest distance problems, such as: a water tap inside the boundary of a property is to be connected to a main that runs along the boundary of the property where the boundary is to be assumed straight; find the shortest length of pipe needed to connect the tap to the main.
7. Maximise areas subject to restrictions on their perimeters; elimination of constraint variables will involve only simple algebra.
8. Minimise perimeters subject to constraints on area; elimination of constraint variables will involve only simple algebra.

9. Investigate situations finding optimal quantities and/or optimal costs such as the optimal use of materials used for the manufacture of various containers of simple shapes.
10. Use graphing calculators in the investigation of optimal points and optimal values in life-related situations.

Introduction to integration (notional time 20–25 hours)

Focus

Candidates should develop an understanding of the concept of integration as a process by which a “whole” can be obtained from the summation of a large number of parts. This understanding should be developed using both numerical and analytical techniques, in life-related situations as well as in purely mathematical situations. The emphasis in the topic should be on the applications of integration rather than on developing a large repertoire of techniques. The use of technology is expected to help candidates in these processes.

Subject matter

- definition of the definite integral and its relation to the area under a curve (LE 1–5)
- the value of the limit of a sum as a definite integral (LE 1–5)
- definition of the indefinite integral (LE 1–5)
- rules for integration including (LE 1–5)

  $\int af(x)\,dx$

  $\int [f(x) \pm g(x)]\,dx$

  $\int f(ax+b)\,dx$
- indefinite integrals of simple polynomial functions, simple exponential functions (base e only), *sin (ax + b)*, *cos (ax + b)* and $\frac{1}{a\;x+b}$ (LE 1–4 and 9)
- use of integration to find area (LE 1 – 4)
- practical applications of the integral (LE 1–9)
- trapezoidal rule for the approximation of a value of a definite integral numerically (LE 3 – 5, 7, and 8).

Learning experiences

The following learning experiences may be developed.

1. Use integration to calculate the areas of regions by finding the area under the curve for suitably chosen functions, including functions that intersect the x axis within the given interval.
2. Use integration to calculate the area enclosed by two intersecting curves.
3. From a velocity time function (or graph), determine a distance or displacement function (or graph); interpret the result.
4. From an acceleration time function (or graph), determine a velocity function (or graph); interpret the result.
5. Apply the trapezoidal rule to integrals of known values and compare the approximate solutions with the exact solutions; investigate the variation in accuracy with the number of strips chosen.

6. Determine the volume of water that could be contained in a trough of given length and parabolic cross-section.
7. Calculate the distance travelled in a car by taking speedometer readings at regular intervals and then using these readings in a numerical formula.
8. Calculate the approximate volume of fill to remove in the construction of a road cutting by approximating the cross-sectional area using a numerical method.
9. Use integration to determine appropriate values in contexts such as learning curves, consumer surplus, Lorenz curves.

Applied statistical analysis (notional time 20–25 hours)

Focus

Candidates should develop a working knowledge of the concepts involved in describing, summarising, comparing and modelling data, and of some elementary concepts in using data to estimate probabilities and parameters, and to answer simple questions. It is expected that calculators will be used routinely for calculations and graphical displays.

Candidates should develop skills in interpreting and commenting on data in context.

Subject matter

- identification of variables and types of variables and data (continuous and discrete) (LE 1, 2, 4 and 5)
- choice and use in context of appropriate graphical and tabular displays for different types of data including pie charts, barcharts, tables, histograms, stem-and-leaf and box plots (candidates are not expected to construct pie charts) (LE 1 – 3)
- use of summary statistics including mean, median, mode, standard deviation, interquartile distance and range as appropriate descriptors of features of data in context (LE 1 – 3, 6, 8 – 10)
- use of graphical displays and summary statistics in describing key features of data, particularly in comparing datasets and exploring possible relationships (LE 1 – 3, 8 – 10)
- use of relative frequencies to estimate probabilities; the notion of probabilities of individual values for discrete variables and intervals for continuous variables (LE 4, 5, 7 and 11)
- probability distribution and expected value for a discrete variable (LE 5 and 11)
- identification of the binomial situation and use of tables or technology for binomial probabilities (LE 4 – 6 and 10)
- concept of a probability distribution for a continuous random variable; notion of expected value and median for a continuous variable (LE 3, 7 and 10)
- the normal model and use of standard normal tables (LE 7)
- normal approximation to the binomial distribution (LE 12).

Learning experiences

The following learning experiences may be developed.

1. Organise a set of data using a variety of approaches such as summary statistics and graphical displays.
2. Given a set of data, produce a concise summary of the main information in the data, referring to graphical displays and summary statistics.
3. Use graphical displays on the same scale to give an effective visual comparison between two or more datasets, and comment on general comparative features, making allowance for variation.
4. Calculate probabilities involving situations with events that could be assumed to be equally likely, such as birthdays, month of birth, male/female births.
5. Use models such as tree diagrams and tables to estimate probabilities; for example, the number of girls in families of two or three children by listing probabilities.
6. Use binomial probabilities in life-related situations such as examining the efficacy of a new drug.
7. Consider a number of life-related situations where a normal distribution may be assumed; standardise variables and use standard normal tables.
8. Examine the use of summary statistics in, for example, newspapers, articles, TV programs such as weather reports and advertisements, government reports.
9. Examine reports by the Real Estate Institute of Queensland (e.g. house prices in different areas) and explain their choice of measure of central tendency.
10. Compare the effects of an outlier on a variety of summary statistics.
11. Investigate probability distributions in life-related contexts such as:
    - use the tabled information given in a newspaper about previous Gold Lotto draws to determine whether the numbers are drawn at random; that is, whether the numbers follow a uniform probability distribution model
    - ask a group of people to try to generate random numbers between, say, 0 and 50, and use graphical displays to investigate how successful they were; for example use a histogram to check rectangular shape, and plot numbers in order of generation to check on trends or patterns.
12. Determine probabilities in binomial situations where n is large, such as the probability of 40 heads when a coin is tossed 100 times using the normal distribution.

6. Assessment

6.1 Summative assessment

Candidates will be assessed according to the criteria set out below. The examination will consist of two papers, each of up to 3 hours working time plus 10 minutes perusal/planning time.

6.1.1 Format of the external examination

The two papers will each cover the three criteria and all four contexts. Both papers will have a range of questions from selected topics that together sample the course. All topics will be covered and some integration of topics will be included.

Each year, the chief examiner will confirm the arrangements for each examination paper. The Chief Examiner will provide advice to candidates about additional conditions or equipment, materials or the like, that will be required for the examination in a particular year.

Use of technology in the examination

During the examination the noise of operating a calculator must not disturb other candidates.

Candidates should not rely only on the use of technology to obtain a result. To meet the requirements of the criterion *Communication and justification* candidates should show full working methods for each question.

6.2 Exit criteria

Within the contexts of Application, Technology and Complexity the following three criteria will be applied when making judgments about levels of achievement.

Criterion 1: Knowledge and procedures

This criterion includes:

- recalling definitions and rules
- accessing and applying rules and techniques
- demonstrating number and spatial sense
- demonstrating algebraic facility
- demonstrating an ability to select and use technologies such as calculators, measuring instruments and tables
- demonstrating an ability to use graphing calculators
- recalling, selecting and using appropriate mathematical procedures
- working accurately and manipulating formulae
- recognising that some tasks may be broken up into smaller components
- transferring and applying mathematical procedures to situations such as those outlined in the learning experiences in this syllabus.

Criterion 2: Modelling and problem solving

This criterion includes:

Modelling

- understanding that a mathematical model is a mathematical representation of a situation
- identifying the assumptions and variables of a simple mathematical model of a situation
- forming and/or selecting a mathematical model of a life-related situation
- deriving results from consideration of the mathematical models
- interpreting results from the mathematical model in terms of the given situation
- exploring the strengths and limitations of mathematical models.

Problem solving

- identifying a problem
- interpreting, clarifying and analysing a problem
- using a range of problem-solving strategies such as estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data
- understanding that there may be more than one way to solve a problem
- selecting appropriate mathematical procedures required to solve problems
- developing a solution consistent with the problem
- developing procedures in problem solving
- exploring a problem and from emerging patterns creating conjectures or theories
- reflecting on conjectures or theories, making modifications if needed
- selecting and using problem-solving strategies to test and validate any conjectures or theories.

Criterion 3: Communication and justification

Communication

This includes:

- organising and presenting information
- communicating ideas, information and results
- using mathematical terms and symbols accurately
- using accepted spelling, punctuation and grammar in written communication
- understanding material presented in a variety of forms such as written, symbolic, pictorial and graphical
- translating material from one form to another
- presenting material for different audiences in a variety of forms (such as written, symbolic, pictorial and graphical)
- recognising necessary distinctions in the meanings of words and phrases according to whether they are used in a mathematical or non-mathematical situation.

Justification

This includes:

- developing logical arguments expressed in everyday language, mathematical language or a combination of both, as required, to support conclusions, results and/or propositions
- evaluating the validity of arguments designed to convince others of the truth of propositions
- justifying procedures which may include:
  - providing evidence (words, diagrams, symbols, etc.) to support processes used
  - stating a generic formula before using specifically
  - providing a reasoned, well-formed, logical sequence within a response
- recognising when and why derived results are clearly improbable or unreasonable
- recognising that one counter example is sufficient to disprove a generalisation
- recognising the effect of assumptions on the conclusions that can be reached
- deciding whether it is valid to use a general result in a specific case
- using supporting arguments to justify results obtained by calculator.

6.3 Special consideration

Under certain circumstances, special arrangements or consideration may be available to candidates for the Senior External Examination. The special consideration provisions are detailed in the annual Handbook for the Senior External Examination, available on the QSA website at
www.qsa.qld.edu.au/testing/extern-exams/handbook.html.

Missing an examination for any reason cannot be the basis for an application for special consideration.

6.4 Awarding levels of achievement

The chief examiner will award each candidate who sits the examination a level of achievement from one of the five categories:

- Very High Achievement (VHA)
- High Achievement (HA)
- Sound Achievement (SA)
- Limited Achievement (LA)
- Very Limited Achievement (VLA).

The process of arriving at a judgment about a candidate's responses to examination questions is essentially a process of matching the candidate's responses against the syllabus standards associated with exit criteria. A level of achievement that best describes the pattern of performance in each criterion across the examination as a whole is then awarded.

Information about how scripts are assessed is provided in the annual Handbook for the Senior External Examination, available on the QSA website www.qsa.qld.edu.au/testing/extern-exams/.

The level of achievement will be based on the exit standard for each of the three criteria *Knowledge and procedures, Modelling and problem solving,* and *Communication and justification.* The criteria are derived from the general objectives and are described in section 3. The standards associated with the three exit criteria are described in Table 2.

When standards have been determined for each of the three criteria, the following table is used to determine the level of achievement, where *A* represents the highest standard and *E* the lowest.

Table 1

VHA	The candidate must achieve a Standard *A* in any two exit criteria and no less than a Standard *B* in the remaining criterion
HA	The candidate must achieve Standard *B* in any two exit criteria and no less than a Standard *C* in the remaining criterion
SA	The candidate must achieve Standard *C* in any two exit criteria, one of which must be the *Knowledge and procedures* criterion and no less than a Standard *D* in the remaining criterion
LA	The candidate must achieve Standard *D* in any two exit criteria, one of which must be the *Knowledge and procedures* criterion
VLA	The candidate must achieve Standard *E* in at least two exit criteria

Table 2: Standards associated with exit criteria

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: *Knowledge and procedures*	The **overall quality** of a candidate's achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, **consistently demonstrates**: • accurate recall, selection and use of definitions and rules • accurate use of technology • recall and selection of procedures and their accurate and proficient use • effective transfer and application of mathematical procedures.	The **overall quality** of a candidate's achievement across a range within the contexts of Application, Technology and Complexity, and across topics, **generally demonstrates**: • accurate recall, selection and use of definitions and rules • accurate use of technology • recall and selection of procedures and their accurate use.	The **overall quality** of a candidate's achievement in the contexts of Application, Technology and Complexity **generally demonstrates**: • accurate recall and use of basic definitions and rules • use of technology • accurate recall, selection and use of basic procedures.	The **overall quality** of a candidate's achievement in the contexts of Application, Technology and Complexity **sometimes demonstrates**: • accurate recall and use of some definitions, and rules • use of technology • use of basic procedures.	The **overall quality** of a candidate's achievement **rarely demonstrates** knowledge and use of procedures.

[This table continues on the next two pages.]

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: *Modelling and problem solving*	The **overall quality** of a candidate's achievement across the full range within each context, and across topics, **generally demonstrates mathematical thinking** which includes: • interpreting, clarifying and analysing a range of situations and identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems … *and* sometimes **demonstrates** mathematical thinking which includes: • suitable synthesis of procedures and strategies to solve problems • initiative and insight in exploring the problem • identifying strengths and limitations of models.	The **overall quality** of a candidate's achievement across a range within each context, and across topics, **generally demonstrates** mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations and identifying assumptions and variables • selecting and using effective strategies • selecting suitable procedures required to solve a range of problems … *and* sometimes **demonstrates** mathematical thinking which includes: • suitable synthesis of procedures and strategies.	The **overall quality** of a candidate's achievement **demonstrates** mathematical thinking which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures required to solve problems.	The **overall quality** of a candidate's achievement **sometimes demonstrates** mathematical thinking which includes: • following basic procedures and/or using strategies.	The **overall quality** of a candidate's achievement **rarely demonstrates** mathematical thinking which includes following basic procedures and/or using strategies.

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: *Communication and justification*	The **overall quality** of a candidate's achievement across the full range within each context **consistently demonstrates**: • accurate use of mathematical terms and symbols • accurate use of language • organisation of information into various forms suitable for a given use • use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or propositions • justification of procedures • recognition of the effects of assumptions • evaluation of the validity of arguments.	The **overall quality** of a candidate's achievement across a range within each context **generally demonstrates**: • accurate use of mathematical terms and symbols • accurate use of language • organisation of information into various forms suitable for a given use • use of mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or propositions • justification of procedures.	The **overall quality** of a candidate's achievement in all contexts **generally demonstrates**: • accurate use of basic mathematical terms and symbols • accurate use of language • organisation of information into various forms • use of some mathematical reasoning to develop simple logical arguments.	The **overall quality** of a candidate's achievement **sometimes demonstrates** evidence of the use of the basic conventions of language and mathematics and occasional use of mathematical reasoning.	The **overall quality** of a candidate's achievement **rarely demonstrates** use of the basic conventions of language or mathematics.

Contexts are explained in section 3.2.

7. Resources

7.1 QSA website

The QSA website provides essential resources for all candidates for the Senior External Examination. The website address is www.qsa.qld.edu.au/testing/extern-exams/index.html or go to www.qsa.qld.edu.au > Testing > Senior External Examination.

The following information (current at time of printing) is available:

Senior External Examination Handbook:

- the handbook gives information about
  - how to nominate to sit the examinations
  - teaching centres that provide tuition for the subjects
  - examination timetable
  - important dates relating to the Senior External Examination.

Subject resources

The syllabus and examination papers for the previous three years are available.

Notices to candidates

Information is provided by chief examiners to help candidates prepare for the examination.

Notices to teaching centres

Information is provided by chief examiners to help tutors and candidates prepare for the examination.

7.2 Textbooks and other resources

A wide variety of textbooks and resource materials can be used to supplement study in Mathematics B. Book suppliers can provide detailed information regarding new publications.

Appendix 1

Basic knowledge and procedures

The following basic knowledge and procedures will be required and must be learned or maintained as required:

- metric measurement including measurement of mass, length, area and volume in practical contexts
- calculation and estimation with and without instruments
- rates, percentages, ratio and proportion
- simple interest
- basic algebraic manipulations
- identities, linear equations and inequalities
- gradient of a straight line
- equation of a straight line
- plotting points using Cartesian coordinates
- solutions of a quadratic equation
- graphs of quadratic functions
- tree diagrams as a tool for defining sample spaces and estimating probabilities
- the summation notation: $\sum_{i=1}^{n} x_i$.
- lengths of arcs and areas of sectors
- tangent to the circle is perpendicular to the radius at the point of contact
- volume and surface areas of regular and compound shapes.

Appendix 2

Explanation of some terms

Amortising a loan

The repayment of a debt with constant repayments at fixed intervals.

Annuities

The accumulation of fixed payments at fixed intervals over a period of time.

Frequency plot

A diagrammatic presentation of the frequency distribution of the observations; for example, bar chart, pie chart, histogram, frequency polygon or an ogive.

Mathematical model

Any representation of a situation that is expressed in mathematical terms. It should be noted that models may be as simple as expressing simple interest as:

$$I = \frac{\mathrm{Pr}t}{100}$$

or showing the relationship between two variables as a scattergram.

Median boxplot (box-and-whisker plot)

A graphical presentation of some main features of a dataset. The simplest version of a boxplot is formed by drawing a box extending from the lower to the upper quartiles, marking the median within that box, and drawing lines (called whiskers) from the box to the smallest and largest data points.

There are slight variations in the possible ways of identifying the median and quartiles of data: these variations make very little difference except for small or sparse datasets. One technique is illustrated in the following example:

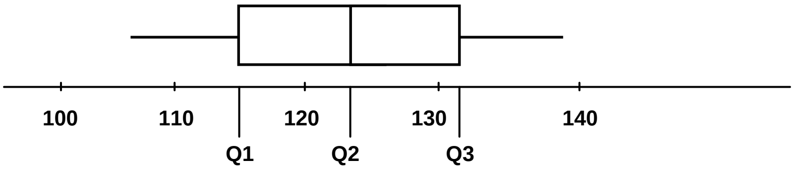
Example: Consider the following 18 systolic blood pressures (bp)

110, 130, 108, 125, 111, 122, 126, 119, 114,
134, 120, 132, 134, 130, 107, 137, 120, 136

These data are ordered and numbered from the smallest to largest below.

x:	107	108	110	111	114	119	120	120	122	125	126	130	130	132	134	134	136	137
order:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

The median, Q2, is taken to be 123.5. The lower quartile, Q1, is the median of the lower 9 observations, which is 114, and the upper quartile, Q3, is the median of the upper 9 observations, which is 132. Thus, there are exactly four observations below Q1, between Q1 and Q2, between Q2 and Q3, and above Q3.



With this technique, datasets with 16, 17, 18 or 19 observations all have exactly four observations below Q1, above Q3 and between Q1, Q2, Q3.

Note 1: A more informative version of the boxplot, particularly with larger datasets, that is also often provided by statistical computer packages, takes the whiskers out to the last data points within a certain distance of the quartiles and then marks individual *data points beyond the whiskers.*

Note 2: A boxplot can be presented vertically or horizontally.

Outlier

An extreme value in the observations, for example an observation that lies beyond the box in the box-and-whisker plot, or a point that is well away from the line of best fit.

Stemplot (stem-and-leaf plot)

An exploratory technique that simultaneously ranks the data and gives an idea of the distribution. Example: The following 16 average daily temperatures have been recorded to the nearest degree Celsius:

31 21 35 30 22 23 9 24

13 41 30 21 29 24 18 28

2 | 1 represents 21

Preliminary stem-and-leaf plot of the temperatures:
Unit = 1

0	9
1	3, 8
2	1, 2, 3, 4, 1, 9, 4, 8
3	0, 0, 1, 5
4	1

Final stem-and-leaf plot of the temperatures:
Unit = 1

0	9
1	3, 8
2	1, 1, 2, 3, 4, 4, 8, 9
3	0, 0, 1, 5
4	1

Summary statistics

Characteristics that describe the sample of observations, for example, the mean, median, or standard deviation.

Trapezoidal rule

The area under the curve above the *x*-axis between the limits $x = a$ and $x = b$, can be approximated by dividing the interval [*a*, *b*] into *n* sub-intervals of equal length, $l = \frac{|b - a|}{n}$, and using the formula:

$$A \approx l\ [\frac{1}{2} f(a) + f(a+l) + (f(a+2l) + \dots + f(a+(n-1)l) + \frac{1}{2} f(b)]$$

Variation

The way in which the observations differ (vary) from each other, often measured by the standard deviation or range.

Appendix 3

Further learning experiences

Introductions to functions

Calculate the amount of simple interest generated over a given period using a graphing calculator; plot these discrete values and generate a function that can be used to represent all values.

Rates of change

Discuss how instantaneous rates of change may be used to measure the sensitivity of the human body to various stimulants or sedatives.

Investigate how the rate of change of air temperature varies during the daylight hours when the relationship is approximated by a quadratic.

Periodic functions and applications

Investigate the repetitive nature of daily temperature and human pulse.

Develop the derivative of *sin x* from the graph of *cos x* and vice versa.

Plot, as a function of the date, the elapsed time between sunrise and sunset for capital cities in Australia.

Graph the slopes of the tangent of a sine curve.

Investigate daily electrical energy consumption over a period of time.

Use a graphing calculator to investigate more complicated periodic functions, for example,

$$\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x \cdots$$

Exponential and logarithmic functions and applications

Solve problems involving sinking funds designed for future purchases.

Discuss responses of newspaper financial columnists to financial questions.

Calculate the term of the loan given the interest rate and the mortgage payment.

Investigate the use of tables by financial institutions in annuity calculations.

Applied statistical analysis

Identify the effect of different sampling situations in pursuit of a random sample, for example a Gallup Poll compared with a phone-in poll.

Discuss different sampling situations and possible difficulties and sources of bias, for example, owing to such things as poor questionnaire design, a lack of random sampling or to practical difficulties such as survey interviewer influence.