

# Mathematics A

## 2017 Senior External Examination: Assessment report

### Statistics

Year	Number of candidates	Level of achievement				
		VHA	HA	SA	LA	VLA
2017	70	11	19	24	12	4
2016	53	12	12	14	11	4
2015	70	10	25	21	9	5
2014	83	7	28	26	13	9
2013	83	6	26	31	13	7

### General comments

The majority of candidates satisfactorily attempted the full range of subject matter in both papers and generally provided detailed responses to the questions they attempted. Candidates who failed to demonstrate a C standard in *Knowledge and procedures* answered insufficient questions across contexts. These candidates also gave no response to *Modelling and problem solving* questions.

### Knowledge and procedures

Overall, candidates provided high-quality responses to all core questions.

Common errors in responses to Paper One resulted from not reading information carefully (particularly Question 1d), and a lack of knowledge of statistical definitions and how to explore and understand data was evident in Question 3.

In the navigation topic, candidates showed an inability to interpret the information.

In Paper Two, candidates were unable to correctly use the dividend formulas provided in Question 1d and work formulas backwards in Questions 1b and 1e. In contrast, candidates showed a much greater understanding of networks in Questions 4a and 4b.

# Modelling and problem solving

Candidate responses mostly demonstrated the C standard across the criteria. However, candidates were unable to correctly interpret a given situation (such as for Paper One Questions 1d (ii) and 4a, and Paper Two Question 4d (ii)), resulting in poor responses to these questions. This displayed a lack of understanding of the processes embedded in topics, including navigation and queueing.

Provision of strengths and limitations of models was quite well attempted across both papers.

# Communication and justification

Many candidates recognised the need to justify their solutions by providing correct formulas and notation within their responses. Generally, working steps were clearly shown and concise explanations given in written responses. The candidates who failed to obtain a satisfactory grade in *Knowledge and procedures* and *Modelling and problem solving* were also unable to provide more than minimal responses to all questions.

The main failures in *Communication and justification* resulted from candidates not using correct notation (e.g. not writing money correctly and not using correct units of measurement) and a lack of organisation of information and mathematical reasoning in extended-response questions such as Paper Two Question 2c.

# Marker responses

The following solutions are not necessarily prescriptive model responses and do not necessarily represent the only way of solving a problem. Other approaches and problem-solving strategies may be just as acceptable.

# Paper One

Paper One has **four** extended-response questions. Attempt **all** questions.

Write your responses in the spaces provided. **Show full working in all responses. Partial credit can be awarded only if working is shown.**

Additional pages for responses are at the back of this book.

## Question 1

- a. Calculate the weekly pay of an employee earning an annual salary of \$75 765. Your answer should be correct to the nearest dollar.

$$\begin{aligned} \text{Weekly pay} &= \frac{\$75765}{52} \\ \text{Weekly pay} &= \$1457 \end{aligned}$$

(KP)

- b. A real-estate agent's charges for selling a property are as follows:
- \$500 advertising fee;
  - 5% of the first \$18 000 of the sale price; 2.5% of the remainder of the sale price.
- How much does the agent charge on a sale worth \$121 000?

..... Advertising Fee:		\$500	.....
..... Commission:	$\frac{5}{100} \times \$18000 =$	\$900	.....
..... Commission:	$\frac{2.5}{100} \times (\$121000 - \$18000) =$	\$29800	.....
..... <b>Total:</b>		<b>\$31200</b>	.....

(KP)

- c. An Antarctic tour is advertised at **US\$5 680** per person. The exchange rate is **AS1 : US\$0.75**. What is the total cost for a **couple** to go on this expedition, in Australian dollars?

$$\begin{aligned} \text{AUS\$} &= \frac{\$5680}{0.75} \\ &= \$7573.33 \\ \text{Cost for couple} &= \$7573.33 \times 2 = \$15146.66 \end{aligned}$$

(KP)

- d. Michelle earns \$810.70 a week as a courier and \$480.50 a week as a masseuse.
- i. If the Medicare Levy is at 2%, use the tax table below to calculate the total weekly PAYG tax liability, including the Medicare Levy, for **each job**.

Taxable income	Tax on this income
\$18201 – \$37000	19c for each \$1 over \$18200
\$37001 – \$87000	\$3572 plus 32.5c for each \$1 over \$37000
\$87001 – \$180000	\$19822 plus 37c for each \$1 over \$87000
\$180001 and over	\$54 232 plus 45c for each \$1 over \$180000

### Courier

$$\begin{aligned}\text{Annual pay} &= \$810.70 \times 52 \\ &= \$42156.40\end{aligned}$$

$$\begin{aligned}\text{Tax Payable} &= \$3572 + (\$42156 - \$37000) \times \$0.325 \\ &= \$5247.70\end{aligned}$$

$$\begin{aligned}\text{Medicare Levy} &= \frac{2}{100} \times \$42156 \\ &= \$843.12\end{aligned}$$

$$\begin{aligned}\text{Weekly PAYG Tax} &= \frac{\$5247.70 + \$843.12}{52} \\ &= \frac{\$6090.82}{52} \\ &= \$117.13\end{aligned}$$

### Masseur

$$\begin{aligned}\text{Annual pay} &= \$480.50 \times 52 \\ &= \$24986\end{aligned}$$

$$\begin{aligned}\text{Tax Payable} &= \$\text{Nil} + (\$24986 - \$18200) \times \$0.19 \\ &= \$1289.34\end{aligned}$$

$$\begin{aligned}\text{Medicare Levy} &= \frac{2}{100} \times \$24986 \\ &= \$499.72\end{aligned}$$

$$\begin{aligned}\text{Weekly PAYG Tax} &= \frac{\$1289.34 + \$499.72}{52} \\ &= \frac{\$1789.06}{52} \\ &= \$34.41\end{aligned}$$

(KP)

One of Michelle's friends told her she would have more money to spend if she just worked one single job.

- ii. Investigate this statement, and determine if it is true that Michelle would earn a higher weekly net income for the same total gross income in a single job.

Fully justify your decision with supporting mathematical evidence.

$$\begin{aligned}\text{Total Annual} &= \$42156 + \$24986 \\ &= \$67142\end{aligned}$$

$$\begin{aligned}\text{Tax Payable} &= \$3573 + (\$67142 - \$37000) \times 0.325 \\ &= \$13368.15\end{aligned}$$

$$\begin{aligned}\text{Medicare Levy} &= \frac{2}{100} \times \$67142 \\ &= \$1342.84\end{aligned}$$

$$\begin{aligned}\text{Total Weekly PAYG Tax} &= \frac{\$13368.15 + \$1342.84}{52} \\ &= \frac{\$14710.99}{52} \\ &= \$282.90\end{aligned}$$

$$\begin{aligned}\text{Weekly Net Income} &= (\$810.70 + \$480.50) - \$282.90 \\ &= \$1291.20 - \$282.90 \\ &= \$1008.30\end{aligned}$$

$$\begin{aligned}\text{In two jobs, Weekly Net} &= \$693.67 + \$446.09 \\ &= \$1139.76\end{aligned}$$

**Decision:** Statement not correct as earn more money as weekly net in two jobs.

- e. Steve is paid \$20.50 per hour for a 35-hour week. For working wet days, he is paid a wet weather allowance of \$5.65 per hour extra.

Calculate Steve's pay for a week in which he works 10 hours in the rain.

$$\text{Normal pay: } \$20.50 \times 35 = \$717.50$$

$$\text{Rain: } \$5.65 \times 10 = \$56.50$$

$$\text{Total pay for week} = \$717.50 + \$56.50$$

$$= \$774$$

(KP)

- f. A shop marks up its T-shirts by 45% from the wholesale price. These T-shirts were then put on sale after a 12.5% discount at a sale price of \$19. Determine the original wholesale price of the T-shirts.

$$\begin{aligned} \text{Shop price} &= \frac{\$19}{0.875} \\ &= \$21.71 \end{aligned}$$

$$\begin{aligned} \text{Wholesale price} &= \frac{\$21.71}{1.45} \\ &= \$14.98 \end{aligned}$$

- g. In 2014, a farmer had a large property with 10 000 head of cattle. Because of drought, his stock was reduced by 25% in 2015, and floods caused a further 10% reduction in 2016. The farmer has estimated that he needs to purchase a 50% increase on his **current** herd of cattle to regain his losses.

Determine if this percentage increase in cattle will get the farmer back to his original stock level of 10 000.

Fully justify your decision with supporting mathematical working.

State **one limitation** of this model.

$$\text{2015: } \frac{25}{100} \times 10000 = 2500 \text{ cows}$$

$$\begin{aligned} \text{Number of cows} &= 10000 - 2500 \\ &= 7500 \text{ cows} \end{aligned}$$

$$\text{2016: } \frac{10}{100} \times 7500 = 750 \text{ cows}$$

$$\begin{aligned} \text{Number of cows} &= 7500 - 750 \\ &= 6750 \text{ cows} \end{aligned}$$

$$\begin{aligned} \text{Loss} &= 10000 - 6750 \\ &= 3250 \text{ cows} \end{aligned}$$

$$\text{Current 2017: } \frac{50}{100} \times 6750 = 3375 \text{ cows}$$

Therefore the **farmer can regain his original herd numbers**

with this percentage increase in cattle.

**Limitation:** No further natural disasters to affect herd numbers

## Question 2

- a. A sheep shearer recorded the following shearing totals for a 10-day period:

47    51    58    59    60    63    66    68    70    71

For this data find the:

- i. mean

$$\bar{x} = 61.3 \quad (KP)$$

- ii. median

$$\begin{aligned} \text{Median} &= \frac{60+63}{2} \\ &= \frac{123}{2} \\ &= 61.5 \quad (KP) \end{aligned}$$

- iii. interquartile range.

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 68 - 58 \\ \text{IQR} &= 10 \quad (KP) \end{aligned}$$

- b. Twenty-four households were surveyed to find the amount of their quarterly electricity account in dollars. The results are listed below.

197    185    183    200    232    179    184    163  
216    198    169    176    187    209    234    172  
167    195    166    211    194    192    176    163

- i. Construct an **ordered** stem-and-leaf plot to represent this data. Use the key 19/7 = \$197.

Stem	Leaves
16	3 3 6 7 9
17	2 6 6 9
18	3 4 5 7
19	2 4 5 7 8
20	0 9
21	1 6
22	
23	2 4

(KP)

ii. Calculate the five-number summary for this data.

$$\text{Median} = \frac{185 + 187}{2}$$

$$= \frac{372}{2}$$

$$= 186$$

$$Q_1 = \frac{172 + 176}{2}$$

$$Q_1 = \frac{348}{2}$$

$$Q_1 = 174$$

$$Q_3 = \frac{198 + 200}{2}$$

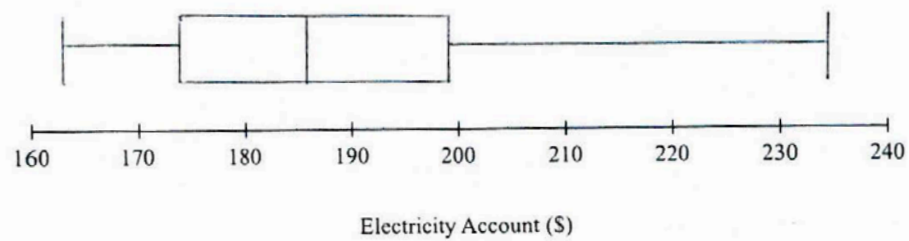
$$Q_3 = \frac{398}{2}$$

$$Q_3 = 199$$

**Five Number Summary = 163, 174, 186, 199, 234**

(KP)

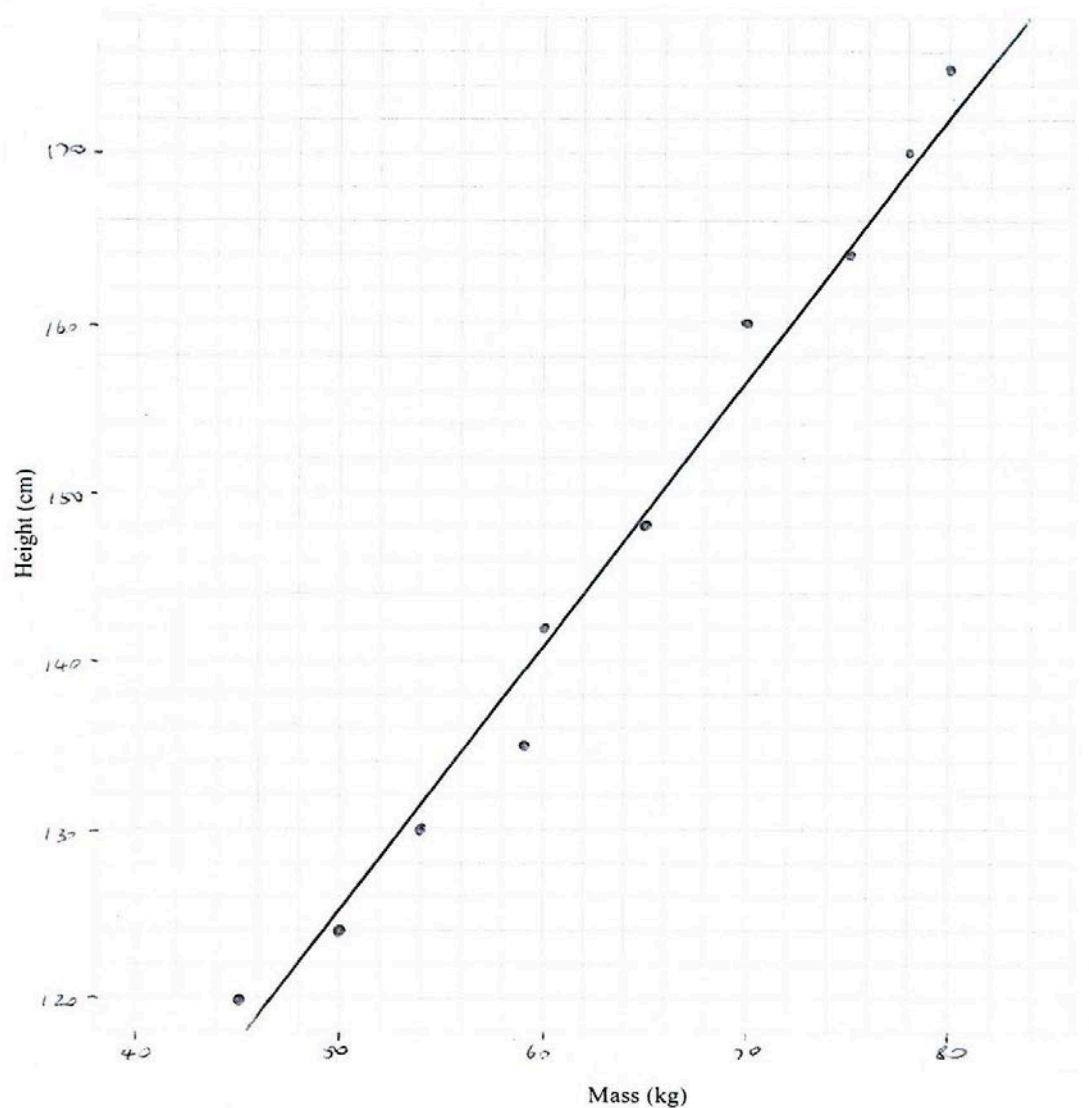
iii. Illustrate this data using a box-and-whisker plot.



- c. The table below shows the height and mass of ten Year 11 students.

Mass (kg)	45	50	54	59	60	65	70	75	78	80
Height (cm)	120	124	130	135	142	148	160	164	170	175

- i. Using the grid below, display the data on a scatterplot.



(KP)

- ii. Draw a line of best fit to represent is data.

(KP)

- iii. What type of correlation is this?

Positive Linear Correlation

(KP)

### Question 3

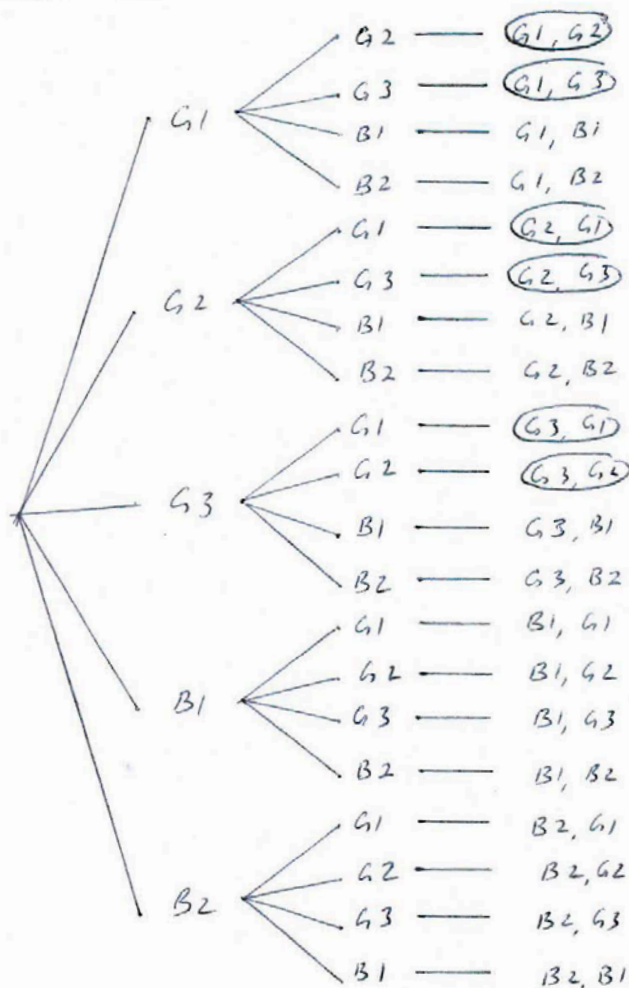
- a. A sample of 20 students must be selected from a school population of 850 students. The school lists students in alphabetical order and all have a three-digit student number. Describe one random sampling method that could be used to choose the students.

..... Random Number generator. ....

..... Systematic sampling. ....

(KP)

- b. Three girls and two boys are members of the Environmental Committee. Two of them are randomly chosen to represent the committee at State Council.
- i. Draw a tree diagram to show all the possible pairs that could be chosen to represent the committee at State Council.



(KP)

- ii. Find the probability that the pair chosen consists of two girls.

.....  $P(2 \text{ girls}) = \frac{6}{20}$  .....

.....

(KP)

$$P(2 \text{ girls}) = \frac{3}{10}$$

- c. The probability that Jane wins a raffle is  $\frac{5}{65}$ .

If Jane bought 12 tickets in the raffle, what is the total number of tickets that must have been sold?

$$\begin{aligned} \frac{5}{65} &= \frac{12}{x} \\ 5x &= 65 \times 12 \\ 5x &= 780 \\ x &= 156 \end{aligned}$$

(KP)

- d. Kate and Kris are playing a card game where one card is randomly chosen from a standard pack of cards.

Determine if Kate is correct when she tells Kris he has more chance of drawing a heart than a picture card (King, Queen or Jack).

Justify your decision with mathematical reasoning.

$$P(\text{Heart}) = \frac{13}{52}$$

$$P(\text{Heart}) = \frac{1}{4}$$

$$P(\text{Picture Card}) = \frac{12}{52}$$

$$P(\text{Picture Card}) = \frac{3}{13}$$

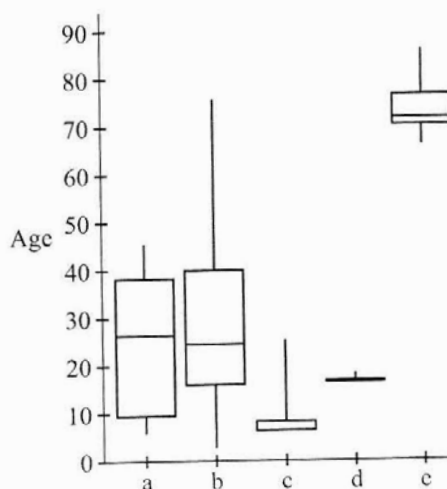
Therefore **Kate is correct** – there is more chance of drawing a heart than a picture card.

- e. Which of the following would be **least** affected by the addition of an extreme score to a dataset that has a large number of entries?  
(Circle the letter of the correct response.)

- A Range  
☒ B Interquartile range  
☒ C Mode  
 D Mean

(KP)

- f. Below are five box-and-whisker plots which illustrate the age data of five groups of 20 people passing a street corner.



Match each group (1, 2, 3, 4, 5) to its appropriate box-and-whisker plot (a, b, c, d, e)

- | Group | Box-and-whisker plot   |
|-------|--|
| 1     | A teacher taking nineteen Year 2 students to the shops                                   |
| 2     | A group of elderly citizens  |
| 3     | A random group of twenty citizens  |
| 4     | A group of twenty Year 11 students on an excursion                                       |
| 5     | Five families of four people, each with two adults and two primary- school-aged children |

Group	Box-and-whisker plot
1	c
2	e
3	b
4	d
5	a

(KP)

## Question 4

Use the chart on the following page to respond to Question 4a. A copy of the chart is reproduced at the back of this book if required.

- a. After being out on a friend's boat you are heading up the coast on a bearing of  $330^\circ$  T and at a speed of 12 knots. At this time Cape Moreton is at a bearing of  $269^\circ$  T. Two hours later it is now at a bearing of  $197^\circ$  T.

Your friend worries that the boat has only about 100 litres of petrol left and uses 40 litres of petrol per hour when travelling at a speed of 12 knots.

You decide to immediately return to Noosa Heads but are concerned that there may be insufficient petrol to make the direct journey. Your friend suggests it may be better to head to Point Cartwright to refuel and then continue from Point Cartwright to Noosa Heads.

Determine which option you should choose.

Justify your decision with mathematical reasoning.

State **one strength** and **one limitation** of the mathematical model.

### Boat to Point Cartwright

Distance to Point Cartwright = 24 nm

$$T = \frac{D}{S}$$

$$T = \frac{24}{12}$$

$$T = 2 \text{ hours}$$

$$\begin{aligned} \text{Petrol} &= 2 \times 40 \\ &= 80 \text{ L} \end{aligned}$$

### Boat to Noosa Heads

Distance to Noosa Heads = 33 nm

$$T = \frac{D}{S}$$

$$T = \frac{33}{12}$$

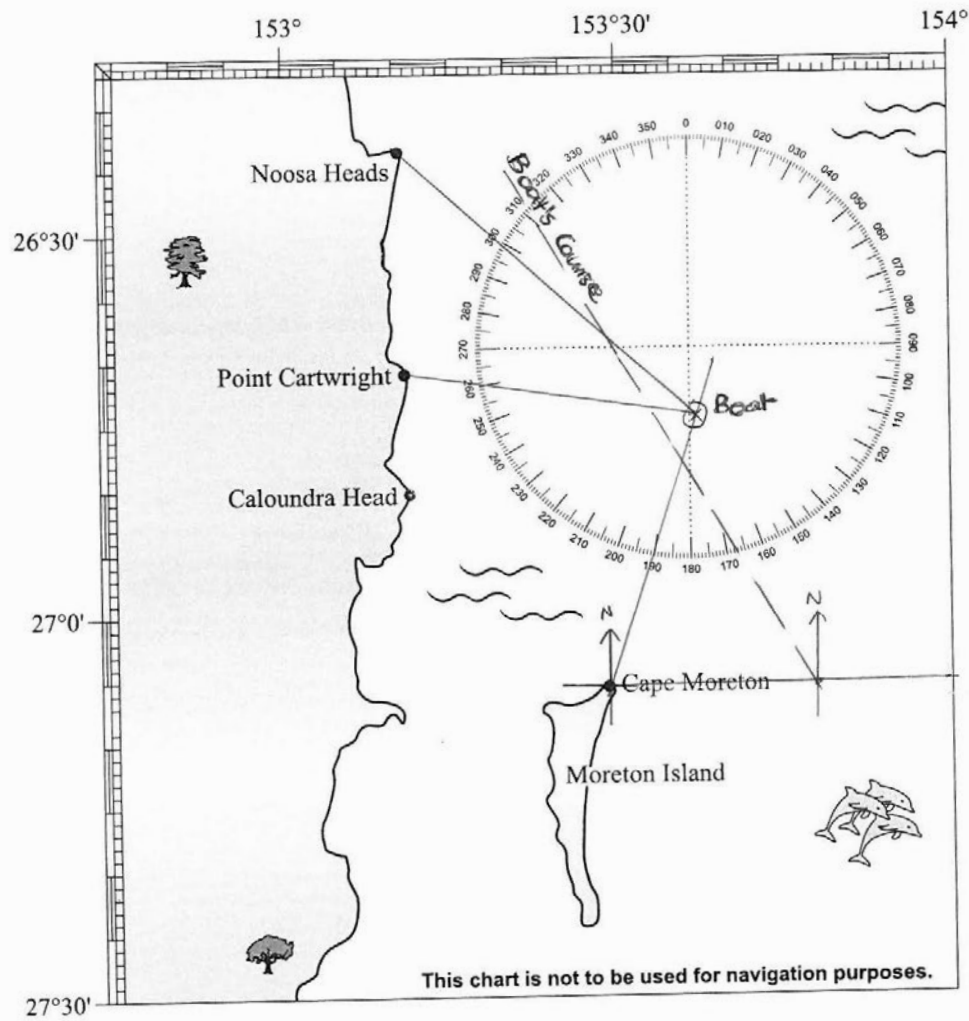
$$T = 2.75 \text{ hours}$$

$$\begin{aligned} \text{Petrol} &= 2.75 \times 40 \\ &= 110 \text{ L} \end{aligned}$$

$\therefore$  Need to head to **Point Cartwright** to refuel first.

**Strength:** Direction of travel to Point Cartwright and Noosa Heads is similar so tides, currents should not effect fuel efficiency.

**Limitation:** Need to travel at constant speed.

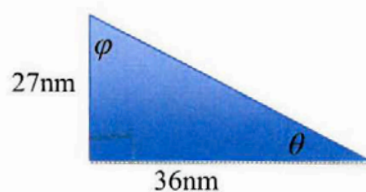


- b. At 9 am a boat, sailing on a course of  $270^\circ$  T, sights a lighthouse at  $000^\circ$  T.  
 At 10.30 am, the bearing of the lighthouse is  $045^\circ$  T.  
 The boat is cruising at 18 knots.  
 If the boat continues on this course at the same constant speed, determine the expected true bearing of the lighthouse from the boat at 11 am.

$$D = S \times T$$

$$D = 18 \times 1.5$$

$$D = 27 \text{ nm}$$



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{27}{36}$$

$$\theta = \tan^{-1} \left( \frac{27}{36} \right)$$

$$\theta = 37^\circ$$

$$\phi = 53^\circ \quad (\text{complementary angles})$$

$\therefore$  **Bearing** of boat to Lighthouse is  **$053^\circ$  T**.

# Paper Two

Paper Two has **four** extended-response questions. Attempt **all** questions.

Write your responses in the spaces provided. **Show full working in all responses. Partial credit can be awarded only if working is shown.**

Additional pages for responses are at the back of this book.

### Question 1

- a. If the inflation rate is 1.5% p.a. find the expected value in two years of a car that has a current price of \$22 950.

$$A = P(1 + r)^n$$

$$A = \$22950(1 + 0.015)^2$$

$$A = \$22950 \times 1.030225$$

$$A = \$23643.66$$

(KP)

- b. Find the principal needed to accumulate a total of \$10000 at 12% p.a. compounded quarterly for 5 years.

$$A = P(1 + r)^n$$

$$\$10000 = P \left( 1 + \frac{0.12}{4} \right)^{5 \times 4}$$

$$\$10000 = P \times 1.80611235$$

$$P = \$5536.76$$

(KP)

- c. The purchase price of a boat is \$85 000.

A 10% deposit is paid and the balance is charged simple interest of 4.3% p.a. over 5 years.

- i. Calculate the amount of the deposit.

$$\text{Deposit} = \frac{10}{100} \times \$85000$$

$$\text{Deposit} = \$8500$$

(KP)

- ii. Calculate the amount of each monthly repayment during the 5 years.

$$\text{Balance} = \text{Purchase price} - \text{Deposit}$$

$$\text{Balance} = \$85000 - \$8500$$

$$\text{Balance} = \$76500$$

$$I = Prn$$

$$I = \$76500 \times 0.043 \times 5$$

$$I = \$16447.50$$

$$A = P + I$$

$$A = \$76500 + \$16447.50$$

$$A = \$92947.50$$

$$\text{Monthly repayment} = \frac{\$92947.50}{5 \times 12}$$

$$\text{Monthly repayment} = \$1549.13$$

- d. A company's share price is \$16.40.  
Calculate the dividend paid if the percentage yield is 6.25%.

$$\text{Percentage yield} = \frac{\text{Dividend per share}}{\text{Market price per share}} \times 100\%$$

$$6.25\% = \frac{\text{Dividend per share}}{16.40} \times 100\%$$

$$\text{Dividend per share} = \frac{6.25 \times 16.40}{100}$$

$$\text{Dividend per share} = \$1.03 \text{ per share}$$

(KP)

- e. A couple want to buy a house. Their combined gross income is \$4540 per fortnight. They can afford 25% of their gross monthly income for loan repayments. The couple are already paying off a car loan at \$260 per month.

The bank offers a loan with monthly repayments of \$8.36 per \$1000 borrowed.

Calculate the largest loan that this couple can afford.

$$\text{Gross monthly income} = \text{fortnightly income} \times 26 \div 12$$

$$\text{Gross monthly income} = \$4540 \times 26 \div 12$$

$$\text{Gross monthly income} = \$9836.67$$

$$\text{Affordable repayment} = 0.25 \times \$9836.67$$

$$\text{Affordable repayment} = \$2459.17 \text{ per month}$$

$$\text{Amount available} = \text{Affordable repayment} - \text{Current repayment}$$

$$\text{Amount available} = \$2459.17 - \$260$$

$$\text{Amount available} = \$2199.17 \text{ per month}$$

$$\text{Potential loan} = \frac{\$2199.17}{\$8.36} \times 100$$

$$\text{Potential loan} = \$263058.61$$

$\therefore$  the largest loan available is \$263058.

f. Michael has the choice of investing his money in two different investment funds:

i. 4.9% p.a. compounding quarterly

ii. 4.75% p.a. compounding daily

Determine which investment fund would provide Michael with the greatest return over two years.

State **one strength** and **one limitation** that may affect this situation. Justify your decisions with mathematical reasoning.

.....

.....

.....

Assume a \$1000 investment in each fund and compare the amount after two years.

Fund A:

$$A = P(1 + r)^n$$

$$A = \$1000 \left( 1 + \frac{0.049}{4} \right)^{2 \times 4}$$

$$A = \$1000 \times 1.102306215$$

$$A = \$1102.31$$

Fund B:

$$A = P(1 + r)^n$$

$$A = \$1000 \left( 1 + \frac{0.0474}{365} \right)^{2 \times 365}$$

$$A = \$1000 \times 1.099652058$$

$$A = \$1099.65$$

$\therefore$  **Fund A** provides the better return over two years.

**Strength:** provides investment information for a fixed interest rate situation.

**Limitation:** interest rates must remain constant and no changes made to money investment.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

## Question 2

- a. A Christmas tree decoration is in the shape of a sphere. It has a diameter of 4.5 cm. Twelve of these decorations are to fit into a rectangular box that is 10 cm wide and 5 cm high. What is the minimum length that the box can be if it is to hold all 12 decorations, allowing an extra centimetre in the length of the box for packing?

1 row height; 2 rows width

Length of box = diameter  $\times$  6 + 1cm for packing

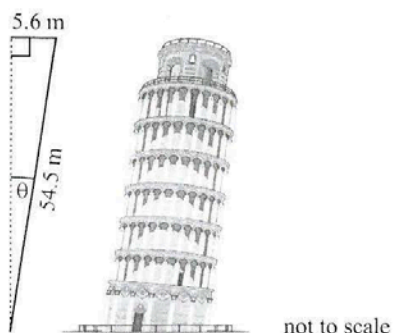
Length of box = 4.5  $\times$  6 + 1

**Length of box = 28cm.**

(KP)

- b. A side of the Leaning Tower of Pisa is 54.5 metres long. The building is 5.6 metres 'off vertical' at the top.

Calculate the size of  $\theta$ , the angle the Tower is 'off vertical'.



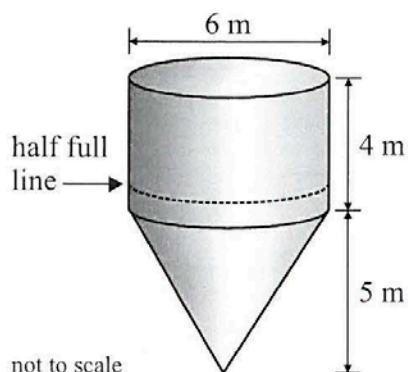
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin \theta = \frac{5.6}{54.5}$$

$$\theta = \sin^{-1} \left( \frac{5.6}{54.5} \right)$$

$$\theta = 5.9^\circ$$

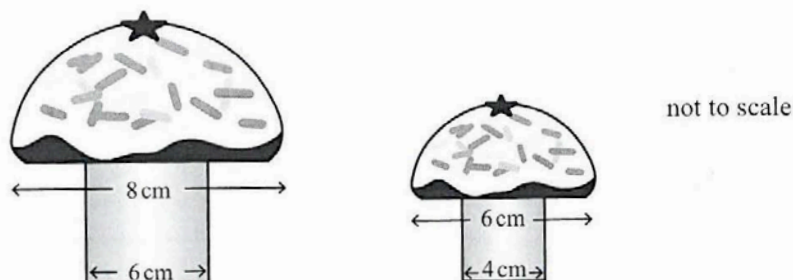
- c. A wheat silo is made from a cylinder and cone as shown. An engineer wishes to determine the position of a line on the outside of the silo to show when the silo is half full.



How far from the bottom of the silo should the line be marked?

$$\begin{aligned}
 \text{Volume of cone: } V &= \frac{1}{3} \pi r^2 h \\
 V &= \frac{1}{3} \pi \times 3^2 \times 5 \\
 V &= 47.1238898 \text{ m}^3 \\
 \\
 \text{Volume of cylinder: } V &= \pi r^2 h \\
 V &= \pi \times 3^2 \times 4 \\
 V &= 113.0973355 \text{ m}^3 \\
 \\
 \text{Total volume of silo} &= \text{volume of cone} + \text{volume of cylinder} \\
 \text{Total volume of silo} &= 47.1238898 + 113.0973355 \\
 \text{Total volume of silo} &= 160.2212253 \text{ m}^3 \\
 \\
 \text{Half full volume} &= \frac{160.2212253}{2} \\
 &= 80.11061265 \text{ m}^3 \\
 \\
 \text{Line on cylinder: } \text{Volume} &= \text{Half full volume} - \text{volume of cone} \\
 \text{Volume} &= 80.11061265 - 47.1238898 \\
 \text{Volume} &= 32.98672285 \text{ m}^3 \\
 \\
 \text{To determine line from bottom of cylinder: } V &= \pi r^2 h \\
 32.98672285 &= \pi \times 3^2 \times h \\
 h &= \frac{32.98672285}{28.27433388} \\
 h &= 1.17 \text{ m} \\
 \\
 \therefore \text{ the line should be marked } 1.17 + 5 &= \mathbf{6.17 \text{ m from the bottom of the cone.}}
 \end{aligned}$$

- d. A café sells two sizes of muffins. The bottom section of the larger size muffin is shaped as a cube with side length 6 cm. When cooked, a hemisphere is formed on the top with a diameter of 8 cm. The bottom section of the smaller size muffin is shaped as a cube with side length 4 cm. When cooked, a hemisphere is formed on the top with a diameter of 6 cm.



The price is based on the volume of cake mixture used in the muffin. The large muffin costs \$5.40.

Determine the cost of the small muffin.

..... Large muffin: Volume = Volume of cube base + volume of hemisphere top .....

$$\begin{aligned}
 \text{Volume} &= L \times W \times H + \frac{4}{3} \times \pi \times r^3 \times \frac{1}{2} \\
 &= 6 \times 6 \times 6 + \frac{4}{3} \times \pi \times 4^3 \times \frac{1}{2} \\
 &= 216 + 134.0412866 \\
 &= 350.0412866 \text{ cm}^3
 \end{aligned}$$

$$\begin{aligned}
 \text{Price of muffin mixture} &= \frac{\text{Price}}{\text{Volume}} \\
 &= \frac{5.40}{350.0412866} \\
 &= 1.54 \text{ c / cm}^3
 \end{aligned}$$

..... Small Muffin: Volume = Volume of cube base + volume of hemisphere top .....

$$\begin{aligned}
 \text{Volume} &= L \times W \times H + \frac{4}{3} \times \pi \times r^3 \times \frac{1}{2} \\
 &= 4 \times 4 \times 4 + \frac{4}{3} \times \pi \times 3^3 \times \frac{1}{2} \\
 &= 64 + 56.54866776 \\
 &= 120.5486678 \text{ cm}^3
 \end{aligned}$$

..... Cost of small muffin = Volume  $\times$  Price of muffin mixture .....

(KP)

$$\text{Cost of small muffin} = 120.5486678 \times 1.54$$

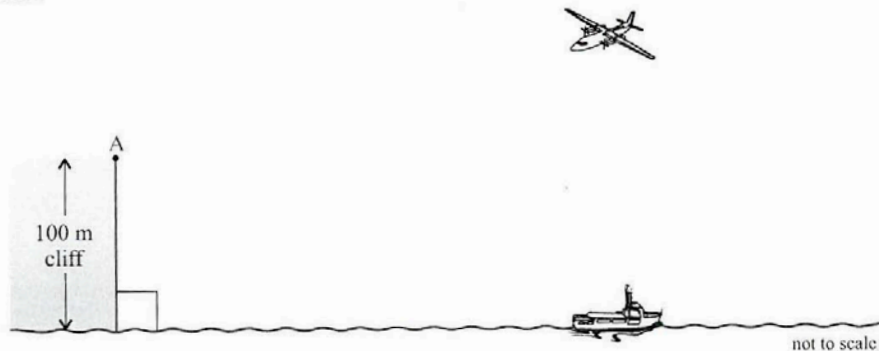
$$\text{Cost of small muffin} = \$1.86$$

$\therefore$  Cost of small muffin is \$1.90.

- e. A person standing at point A on a cliff 100 metres high notices a boat at an angle of depression of  $12^\circ$ .

An approaching plane is about to pass directly over the boat. At the exact time it passes over the boat, the altitude of the plane is 550 metres.

Calculate the angle of elevation from the top of the cliff to the plane at the moment it passes over the boat.



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 12^\circ = \frac{100}{x}$$

$$x = \frac{100}{\tan 12^\circ}$$

$$x = 470.4630109 \text{ m}$$

Angle:  $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$$\tan \theta = \frac{450}{470.4630109}$$

$$\theta = \tan^{-1} \left( \frac{450}{470.4630109} \right)$$

$$\theta = 43.8^\circ$$

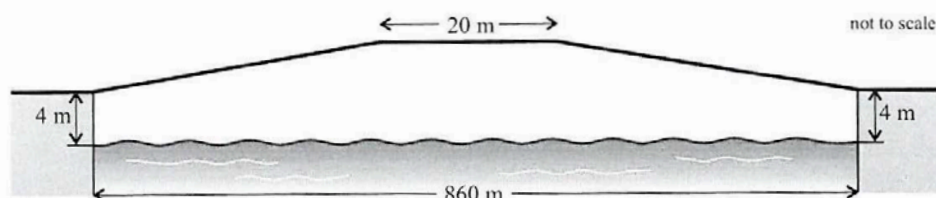
$$\theta = 44^\circ$$

- f. The total length of a bridge is 862 metres. It slopes up from each end at an angle of  $3^\circ$ . It has a flat section 20 metres long in the middle. The ends of the bridge are 4 metres above the water as shown in the diagram below.

A yacht needs 25 metres clearance to safely navigate under this bridge.

Determine if the yacht will be able to sail under this bridge safely.

List **two limitations** that may affect this situation.



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 3^\circ = \frac{x}{420}$$

$$x = 420 \times \tan 3^\circ$$

$$x = 22.0112673$$

$$x = 22 \text{ m}$$

Height of centre of bridge above the water =  $22 + 4$

Height of centre of bridge above the water = 26 m

$\therefore$  **Yacht can pass** under bridge safely as it requires 25m to pass under the bridge.

**Limitation:** boat must pass exactly under the middle 20m section of the bridge

Tides, strong winds, swell of sea, tide differences can all affect height under bridge  
And there is only 1m clearance available.

- g. Danielle lives in Perth (32° S, 116° E) and wishes to watch a live telecast of a cricket test match played in Christchurch (44° S, 173° E). The first session of the match commences at 10:30 am (Christchurch local time) and lasts for 2 hours.

What time will it be in Perth when the first session of the match **finishes**?

$$\begin{aligned} \text{Angle difference} &= 173^\circ - 116^\circ \\ \text{Angle difference} &= 57^\circ \\ \text{Time difference} &= 4 \times 57 \\ \text{Time difference} &= 228 \text{ mins} \\ \text{Time difference} &= 3 \text{ h } 48 \text{ mins} \\ \text{Time in Perth} &= \text{Time in Christchurch} - 3 \text{ h } 48 \text{ mins} \\ \text{Time in Perth} &= 12.30 \text{ p.m.} - 3 \text{ h } 48 \text{ mins} \\ \text{Time in Perth} &= \mathbf{8.42 \text{ a.m.}} \end{aligned} \quad \text{(KP)}$$

- h. A plane flying at 600 km/h travelled directly north for 8 hours and 20 minutes before making an emergency landing. If the plane took off from Hobart (43° S, 147° E), what are the coordinates of where the plane makes the emergency landing?

$$\begin{aligned} D &= S \times T \\ D &= 600 \times 8 \text{ h } 20 \text{ mins} \\ D &= 5000 \text{ km} \\ \text{Distance} &= \text{Angle difference} \times 111.2 \\ 5000 &= \text{Angle difference} \times 111.2 \\ \text{Angle difference} &= \frac{5000}{111.2} \\ \text{Angle difference} &= 45^\circ \\ \text{New Longitude co-ordinate} &= 43^\circ \text{ S} + 45^\circ \text{ N} \\ \text{New Longitude co-ordinate} &= 2^\circ \text{ N} \\ \therefore \text{Emergency landing is made at } & \mathbf{(2^\circ \text{ N}, 147^\circ \text{ E})} \end{aligned} \quad \text{(KP)}$$

### Question 3

a. A house plan is drawn to a scale of 1:100.

i. What would be the dimensions of a bedroom measuring 3 cm square on the plan?

Scale is 1 cm : 100 cm

3 cm :  $3 \times 100$  cm

3 cm : 300 cm

$\therefore$  The bedroom is 3 m  $\times$  3 m

(KP)

ii. The patio is 4500 mm wide. What would be its measurement on the plan?

Patio width =  $4500 \div 1000$

Patio width = 4.5 m

$\therefore$  Width of patio on plan is 4.5 cm.

(KP)

iii. The interior ceiling is 2.4 metres high and all windows are 1 metre square.

The doors of the wardrobes are mirrored glass, cover one complete wall, and reach to the ceiling. The bedroom has one door and one window apart from the wardrobes.

A painter charges \$25 per square metre. This includes all trim work and the door.

Determine the cost of painting the bedroom if the walls are 3 cm square.

Painted area = Area of 3 walls - Area of window

Painted area =  $9 \times 2.4$  -  $1 \times 1$

Painted area =  $20.6 \text{ m}^2$

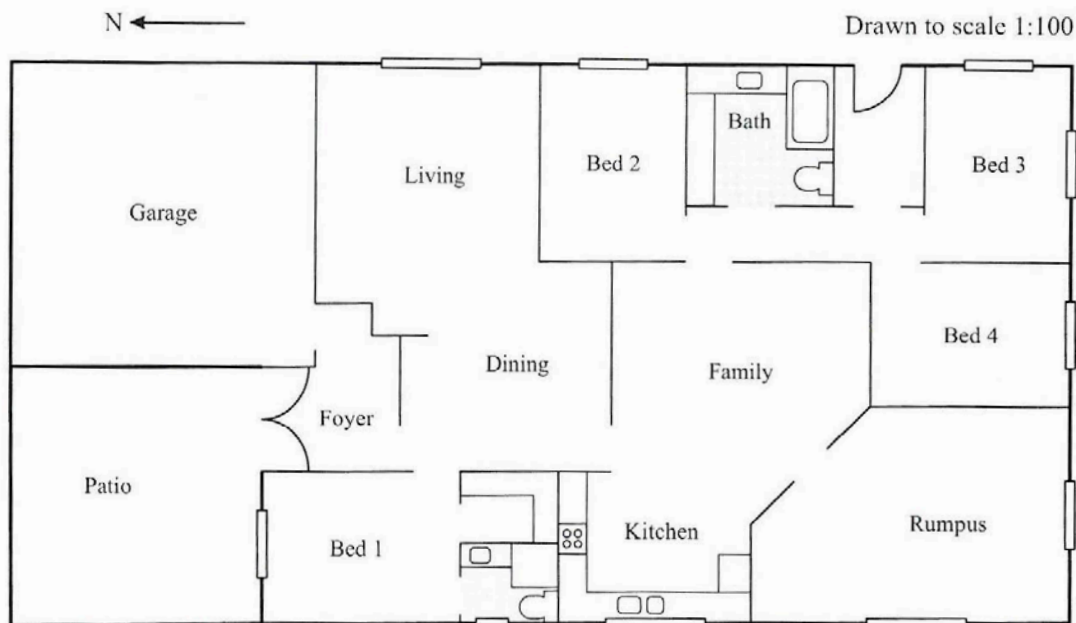
Cost = Number of  $\text{m}^2$   $\times$  cost per  $\text{m}^2$

Cost =  $20.6 \times \$25$

Cost = \$515

$\therefore$  The cost of painting the bedroom walls is \$515.

b.



The above house is on a rectangular block of land. The front of the block faces north.

The front of the block is 20 metres wide and the property is 25 metres long.

The house is positioned 4 metres from the front boundary and 5 metres from the fence on the western side of the property.

- i. Using the graph paper opposite, draw a site plan of the block with the house correctly positioned on it using a scale of 1:125.  
(It is sufficient to represent the house using a rectangle only.)

(KP)

- ii. Calculate if there is sufficient space to build a square 4 metre shed in the south-east corner of the property.

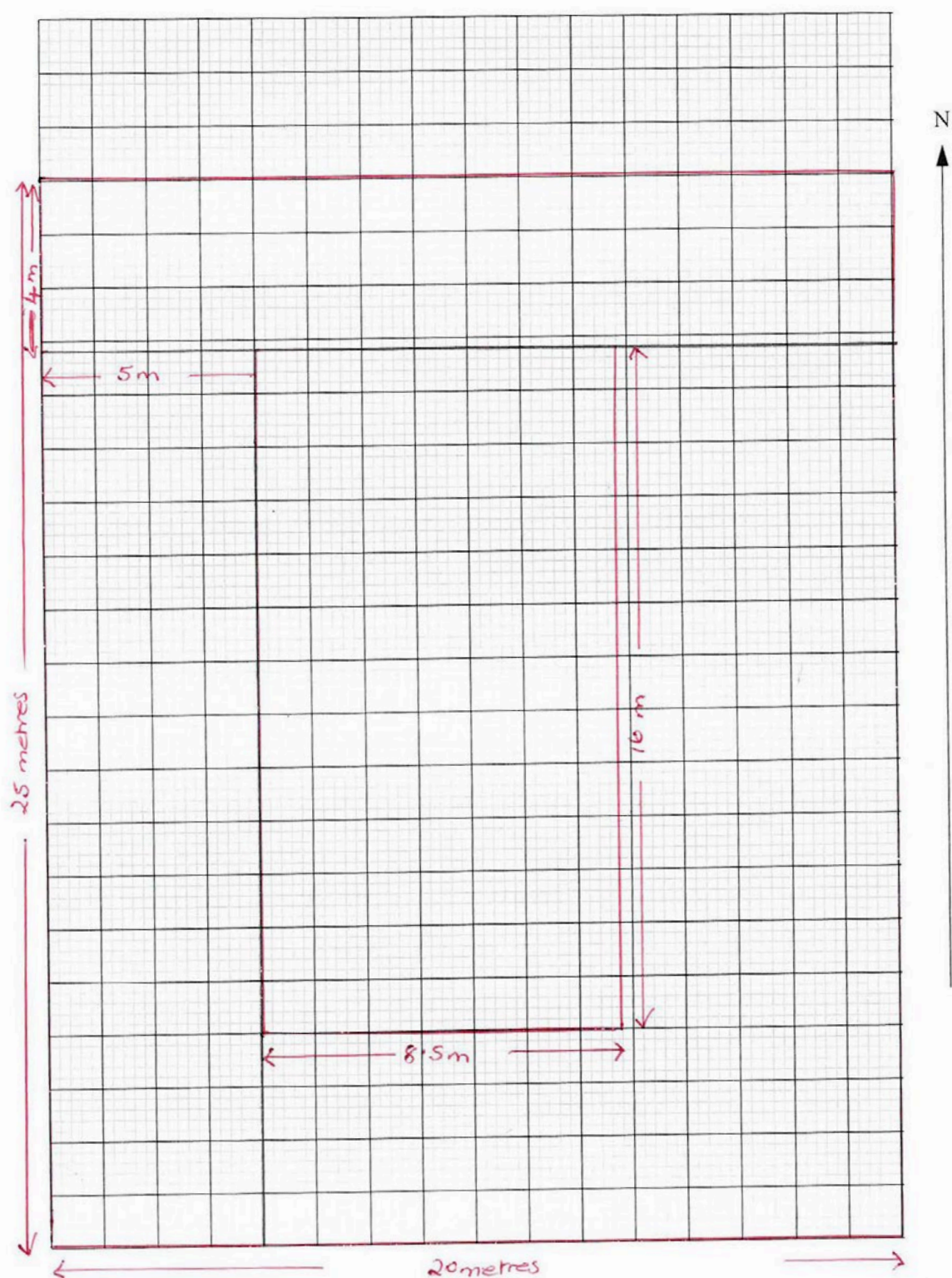
Remaining area on block:

$$\begin{aligned} \text{Using scale of } 1 : 125 \quad & 4\text{cm} \times 125 \\ & = 500\text{cm} \\ & = 5\text{m} \end{aligned}$$

$$\begin{aligned} & 5.2\text{cm} \times 125 \\ & = 650\text{cm} \\ & = 6.5\text{m} \end{aligned}$$

**$\therefore$  There is sufficient room to build a square 4m shed.**

Question 3 b. — response area (Spare graph paper is at the end of this paper.)



- c. Two pieces of timber 480 mm and 640 mm in length are to be used to make a builder's square. How long should the other piece of timber be cut for the length of the hypotenuse?

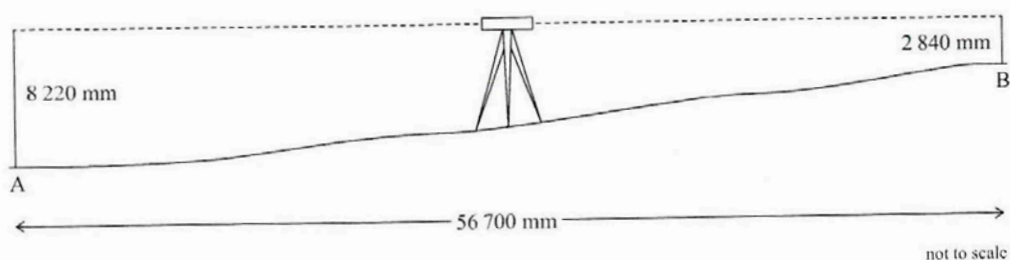
**Note:** A builder's square is a large right angle triangle used to produce square corners on buildings.

$$\begin{aligned}\text{Using Pythagoras, } c^2 &= a^2 + b^2 \\ c^2 &= 480^2 + 640^2 \\ c^2 &= 640000 \\ c &= \sqrt{640000} \\ c &= 800 \text{ mm}\end{aligned}$$

**∴ The other piece of timber should be cut 800 mm long.**

(KP)

- d. Below is a cross-sectional view of a building site where levels were taken.



Calculate the angle at which the ground rises.

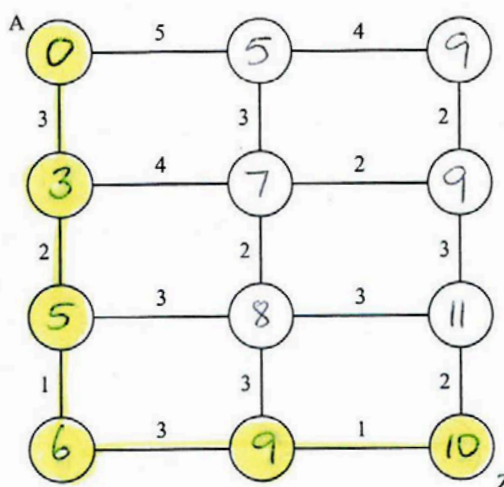
$$\begin{aligned}\sin \theta &= \frac{\text{opp}}{\text{hyp}} \\ \sin \theta &= \frac{5380}{56700} \\ \theta &= \sin^{-1}\left(\frac{5380}{56700}\right) \\ \theta &= 5.4^\circ\end{aligned}$$

**∴ The angle at which the ground rises is 5.4°.**

(KP)

#### Question 4

- a. The network below shows the different routes and distances (in km) to get from A to Z. Find the **shortest** distance from A to Z. Highlight this on the diagram and state this distance.

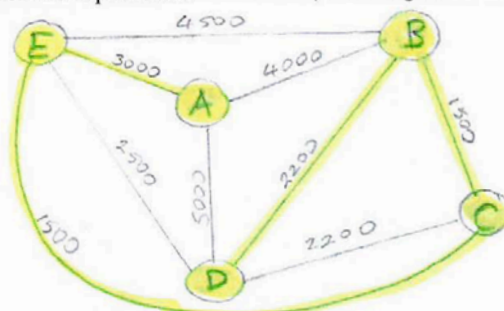


Distance:.... **Shortest distance is 10km.** ..... (KP)

- b. The costs (in \$) of connecting various locations on a school campus with computer cable are given in the table below. A blank space indicates no direct connection.

	A	B	C	D	E
A	—	4000		5000	3000
B	—	—	1500	2200	4500
C	—	—	—	2200	1500
D	—	—	—	—	2500

- i. Draw a network to represent this situation, showing the cost of connection along each arc.



(KP)

- ii. Using a minimum spanning tree, find the least cost of connecting the cable.

.... Least cost = \$3000 + \$1500 + \$1500 + \$2200 ..... (KP)

Least cost = \$8200

∴ Least cost of connecting the cable = \$8200

15

- c. Customers arrive with an inter-arrival time of 1 minute. There are two staff serving customers.

Time	Arrivals	Customer served (server 1)	Customer served (server 2)	Customer in queue	Queue length
10.00	C	A	B	C	1
10.01	D	C	B	D	1
10.02	E, F	C	D	E, F	2
10.03	—	E	D	F	1
10.04	G	E	F	G	1
10.05	H	E	F	G, H	2
10.06	—	G	H	—	0
10.07	I	G	H	I	1
10.08	J	I	H	J	1
10.09	K, L	I	J	K, L	2
10.10	M	K	J	L, M	2
10.11	N	K	L	M, N	2
10.12	O, P	M	N	O, P	2
10.13	Q, R	O	N	P, Q, R	3
10.14	S	O	P	Q, R, S	3
10.15	T	Q	P	R, S, T	3

Complete the 'Customer in queue' and 'Queue length' columns in the table above.

(KP)

- d. A local petrol station has 4 petrol pumps available for customer use. This is a one-way station, where cars drive in for petrol, go to the vacant pump or form a queue. As soon as a pump becomes available another car drives up to the pump.

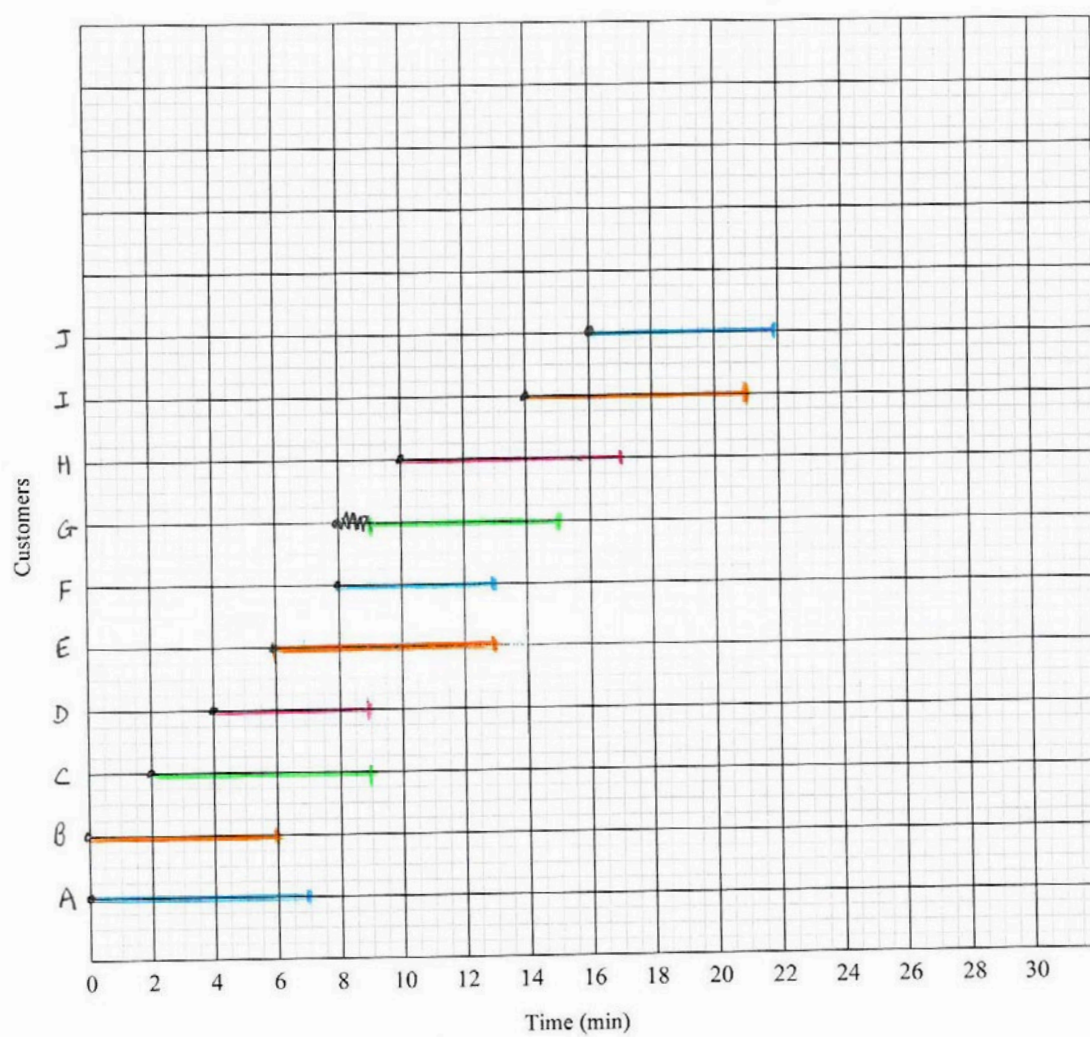
In addition to pumping fuel, it takes approximately 3 minutes for each car to make payment and depart.

The following table represents the time of arrival and the time taken to pump fuel for each customer.

All pumps are vacant when the first car arrives at 8.30 a.m.

Customer	A	B	C	D	E	F	G	H	I	J
Arrival time (am)	8.30	8.30	8.32	8.34	8.36	8.38	8.38	8.40	8.44	8.46
Time taken to pump fuel (mins)	4	3	4	2	4	2	3	4	4	3

- i. Use the graph paper below to graph this scenario.



- ii. One of the petrol pumps stops working at 8.30 am.  
Use the graph paper opposite to graph this new scenario.  
State **one strength** and **one limitation** of this new scenario.

**Strengths:** Longest queuing time with 4 pumps was 1 min (Car G)

Maximum queue length 1 car with 4 pumps.

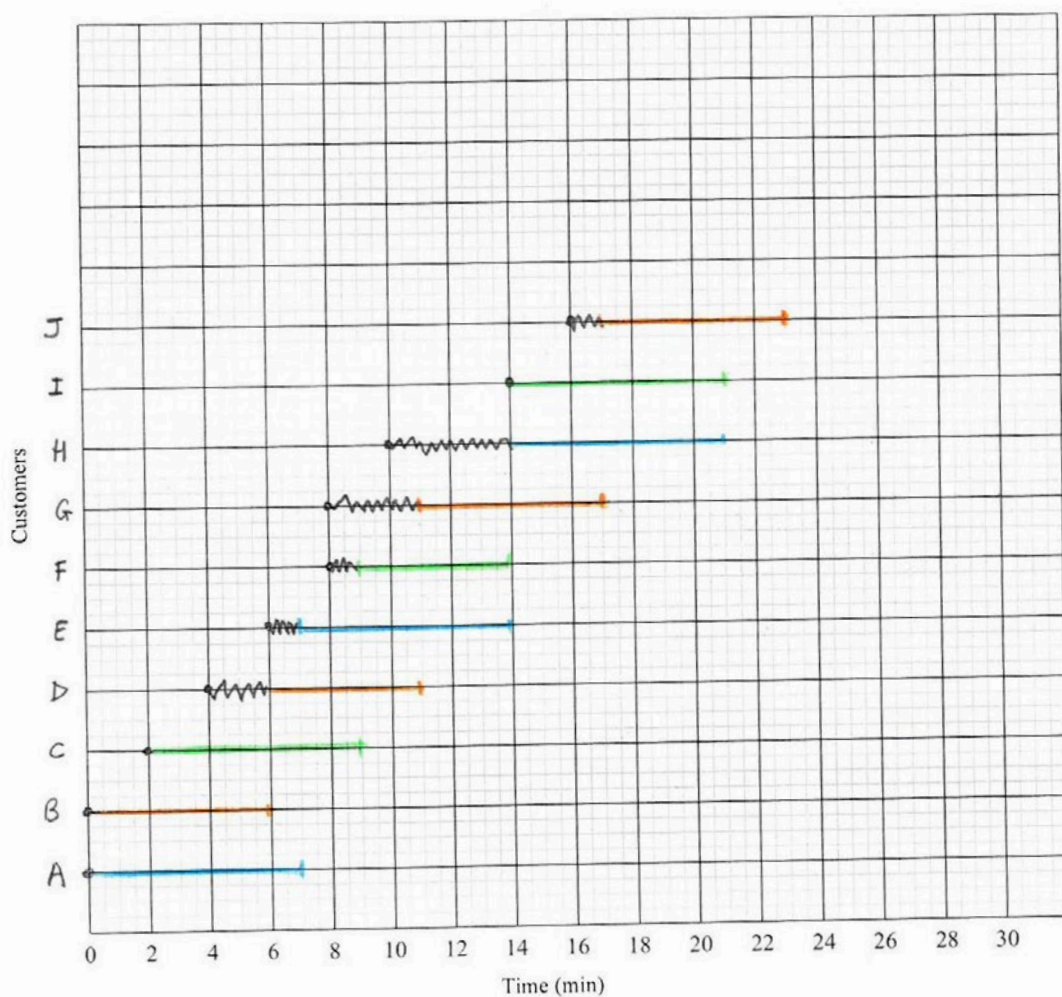
No Idle time with 3 pumps.

**Limitations:** Closing one pump would cause queuing time for most customers to be longer  
(Car D: 2 mins; Car E 1 min; Car F: 1 min; Car G: 3 mins; Car H: 4 mins; Car J: 1 min;)

Maximum queue length 2 cars with 3 pumps.

6 mins Idle Time with 4 pumps (Pump 1: 1+3=4mins; Pump 2: 1 min; Pump 3: Nil;  
Pump 4: 1 min;)

Question 4 d ii. — response area (Spare graph paper is at the end of this paper.)



(KP)

**End of Paper Two**