Mathematics A
2012 Senior External Examination — Assessment report

Statistics

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of candidates</th>
<th>VHA</th>
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<th>SA</th>
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Multiple-choice questions

<table>
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<tr>
<th>Question</th>
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<tbody>
<tr>
<td>Correct option</td>
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<td>D</td>
<td>D</td>
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<td>B</td>
<td>C</td>
<td>D</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

The multiple-choice questions sampled the full range of subject matter and contexts from the syllabus. Approximately 58% of candidates responded correctly to at least half of the multiple-choice questions. Common errors included overlooking the scale in Question 7, not knowing whether to add or subtract the magnetic variation in Question 8, and omitting the waiting time of zero minutes for customer A in Question 14.

Multiple-choice questions will not be included in future Mathematics A Senior External Examinations.

Characteristics of good responses

Knowledge and procedures

Most candidates made a satisfactory attempt at the full range of subject matter in both papers. In Paper One Part B, candidates provided good-quality responses to Question 2 and Question 4. In Paper Two, good-quality responses were provided to Question 2 and Question 5. Good responses provided:

- clear justification and working
- an effective use of diagrams
- the significant intermediate calculation steps.
Modelling and problem solving

The questions most successfully attempted involved straight-forward computation. These included Question 1 from Paper One Part B and Question 2 and Question 5 from Paper Two. Candidates are strongly urged to attempt all Modelling and problem solving questions.

Communication and justification

Many candidates appreciated the need to justify and validate their solutions. This was clearly evident in the responses which exhibited a developed argument and some examination of the strengths and/or limitations of models.

Common weaknesses

Knowledge and procedures

Common errors in responses to Paper One included the incorrect calculation of interest in Question 1, the incorrect calculation of the number of posts, bearers and joists in Question 3 and poorly executed drawings involving the box-and-whisker plot and scatter plot in Question 5.

In Paper Two, many candidates could not distinguish between the depreciation methods in Question 1. Question 3 proved difficult; most candidates could not correctly calculate the magnetic variation and accurately complete the chart work. In Question 4, many candidates failed to correctly identify the critical path. The forwards and backwards scans were often not completed on the network diagram provided in the response book.

Modelling and problem solving

To successfully respond to Modelling and problem solving questions, candidates must:

• carefully read and interpret information. (In Paper One Part B Question 2, many candidates did not correctly interpret the table of charges)
• correctly substitute into a given formula. (In Paper One Part B Question 4, a number of candidates could not perform the numerical calculation correctly)
• make use of clearly labelled, neatly drawn diagrams to support the arguments and conclusions reached. These diagrams are a significant part of a candidate’s justification of an argument. In Paper Two — Question 4, many candidates ignored the need to provide an accurate and carefully labelled graph involving the use of two check-outs.
• explore the strengths and limitations of models.

Communication and justification

Candidates generally showed improvement in this criterion compared to previous years. This was evident in the quality of some of the responses to Modelling and problem solving questions, particularly Paper One Part B Questions 1 and 5 and Paper Two Questions 2, 3 and 5.

To achieve success in this criterion, candidates must:

• effectively build supporting argument to show clarity and depth of thinking (across a range of subject matter)
• give more attention to the requirements of the criterion described in the syllabus when developing solutions to problems. This is particularly the case in responses to Modelling and
Problem solving questions but it is also relevant in responses to Knowledge and procedures questions.

Sample solutions

The following solutions are not necessarily prescriptive model responses and are not necessarily the only way of solving a problem. Other approaches and problem-solving strategies may be just as acceptable.
Paper One

Part B – Extended response

Question 1

a. Dress price = $280
   a. Markdown = 0.2 × $280
      = $56
      Markdown price is $224
   b. New reduced price = $224 − $25
      = $199
      Savings = $81
      % Saving = $81 × 100
      $280
      = 28.9%

b. Statement date is 29/10/2012
   Purchase October 7th = 23 days
   Purchase October 18th = 12 days
   Interest charged = $2100 × 0.052
                     100 × 23 + $175.45 × 0.052
                     100 × 12
   = $25.12 + 1.09
   = $26.21
c. MP

Let \( x \) be the number of boucher bundles.

\[
\text{Earnings} = 2.40 \times 4 + 0.6 \times x
\]
\[
\$165 = 9.60 + 0.6x
\]
\[
\therefore x = 259
\]

Question 2

a. Conversions

i. \[
320000 \text{ cm} = \frac{320000}{100000} \text{ m}
\]
\[
\therefore 320000 \text{ cm} = 3.2 \text{ km}
\]

ii. \[
15 \text{ m}^2 = 15 \times 100 \times 100 \text{ cm}^2
\]
\[
\therefore 15 \text{ m}^2 = 150000 \text{ cm}^2
\]

b. i. \[
\tan \theta = \frac{8}{18}
\]
\[
\rightarrow \theta = \tan^{-1}\left(\frac{4}{9}\right)
\]
\[
\therefore \theta = 24^\circ
\]

ii. \[
\sin \theta = \frac{BD}{18}
\]
\[
\rightarrow BD = 18 \sin 24^\circ
\]
\[
ED = 7321 \text{ m m}
\]
c. \( AF = 8.5 \text{ m} \)

i. Area of shaded face
\[
2 \times 8.5 - \frac{0.8 \times (2.5 + 5)}{2}
\]
\[
= 17 - 3
\]
\[
\therefore \text{ Area } = 14 \text{ m}^2
\]

ii. Each tile has an area
\[
0.15^2 \text{ square metres.}
\]

The number of tiles needed is \( \frac{14}{0.0225} \) or 622.2

The number of boxes of thirty needed is 20.74 or 21 boxes.

Ignoring broken tiles and wastage, the cost of the boxes will be $672.

This question assessed mathematical concepts relating to area using the context of tiling a swimming pool wall. As a first approximation to the number of tiles required, a calculation involving the division of the total area by the area of a tile could be undertaken. Candidates with knowledge of calculations involved in tiling, incorporating the actual dimensions of the tile, could have provided a different response. All responses that demonstrated knowledge and understanding of the mathematical concepts being assessed were awarded credit.

iii. MP

Volume of water required is \( 14 \times FG \) or \( 14 \times 15 = 210 \text{ m}^3 \)

The capacity of the pool is then \( 210000 \ell \).

The fill rate is \( 250 \ell/\text{min} \) and this means that it will take \( \frac{210000}{250} \) minutes

Therefore the fill time is 840 minutes or 14 hours.

The cost of filling the pool is \( $200 + 1.85 \times 210 \) or $588.50

Question 3

a. Dimensions are approximately \( 8.9 \times 5.0 \text{ cm} \) and using a scale of \( 1:180 \), these dimensions are \( 16 \times 9 \text{ m} \) approximately.

b. The number of three metre sections to span the wall is \( \frac{9}{3} \rightarrow 3 \).

Bearers
\[
2 \times 3 \rightarrow 6 \times 3 \text{m} = 18 \text{ m} \text{ of timber for the bearers}
\]
Joists

13 → 13 × 1.8 m = 23.4 m of timber for the joists

Posts

Number of posts needed = 4 → 4 × 2.4 m = 9.6 m of timber for the posts.

Cost of the timber excluding wastage is $12.45 \times 18 + 10.45 \times 23.4 + 15.20 \times 9.6$

This amounts to $614.55

c. MP

Cost of surfacing the carpark is $12.50 per square metre. In terms of the budget, the largest area that can be surfaced is \( \frac{120000}{12.50} \) m\(^2\) or 9600 m\(^2\).

There are 40 spaces each three metres wide, so the width of the lot is 120 m.

\[ \text{Depth of lot} = \frac{9600}{120} = 80 \text{ m}. \]

For the given scale, 1:1000, 120m converts to a width of \( \frac{120000}{1000} = 120 \text{ mm} \) and the depth of 80m converts to 80 mm.

The plan will appear approximately as follows.
Question 4

a. Gross income from second job is $2900 \times 12$

i. Total taxable income = $65250 + 2900 \times 12 - 6500$

   = $93550$

ii. Total tax payable = $18612 + 0.47 \times (93550 - 70000)$

   = $29680.50

b. MP

Given \( E = \frac{2 \times R \times n}{n+1} \) and with \( R = 8\% \) p.a., \( n = 5 \) years → 60 months

\[ E = 15.74\% \]

The effective rate is 15.7% correct to one decimal place.

Question 5

a. From the graph,

i. Median = 150

ii. \( Q_1 \approx 145 \) and \( Q_3 \approx 170 \)

   IQR is \( Q_3 - Q_1 \rightarrow 25 \)

iii. Five-number summary

<table>
<thead>
<tr>
<th>Five-number summary</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>140</td>
</tr>
<tr>
<td>Q1</td>
<td>145</td>
</tr>
<tr>
<td>Median</td>
<td>150</td>
</tr>
<tr>
<td>Q2</td>
<td>170</td>
</tr>
<tr>
<td>Max</td>
<td>25</td>
</tr>
</tbody>
</table>
b. From the data in the table,

i. Mean time on television → 34 minutes
   Mean approval rating → 55 %

ii. Politician's approval ratings

iii. MP

Few points lie on or near the line of best fit. It can be argued that the line of best fit falls through the point representing the mean position just as easily as it appears to be rising through it. Clearly, more data is needed to overcome the limitation of the suggested line of best fit. There is insufficient data to support Stephen's thesis that a Federal politician "should appear on television as often as possible to gain a high approval rating."
Paper Two

Question 1

a.  Kevin has a $45000 truck.
   i.  12% of $45000 = 0.12 \times 45000
       = $5400
       After four years, the truck will have depreciated to
       $45000 - 4 \times 5400 \text{ or } $23400.
   ii. Using the diminishing value formula, Kevin’s truck after 10 years will have
       a value given by
       \[ S = V_0 (1 - r)^n \]
       \[ = 45000(1 - 0.18)^{10} \]
       \[ = $6185.16 \]

b.  i.  Tammy buys 7000 Anzas shares.
       Value of the shares = $7000 \times 0.32
       = $2240
       Brokerage = $5 + 0.025 \times 5000 \text{ or } $61

   ii.  Li sold 4000 Hilton Pty Ltd shares at $3.75 each.
       The value of Li’s shares = $3.75 \times 4000 \text{ or } $15000
       Stamp duty on this purchase is given by
       \[ $0.60 \times \frac{15000}{100} \text{ or } $90 \]
       Brokerage = $5 + 0.025 \times 5000 + 0.02 \times 10000
       = $330
       The amount Li receives is given by
       $15000 - (90 + 330) \text{ or } $14580
c. Tahlia has invested $100000.

i. \( A = P(1+i)^n \)

\[ = $100000 \left(1 + \frac{0.06}{12}\right)^{36} \]

\[ \therefore \text{the investment grows to$119668.05} \]

The interest that Tahlia earns is $1119668.05–100000 or $19668.05

ii. MP

Assume that the annual interest rate is \( i \)

\[ 100000 \left(1 + \frac{i}{2}\right)^6 = $119668.05 \]

\[ \left(1 + \frac{i}{2}\right)^6 = 1.1966805 \]

\[ 1 + \frac{i}{2} = \sqrt[6]{1.1966805} \]

\[ \frac{i}{2} = 0.030377509 \]

\[ i = 0.0607550188 \]

\[ \therefore i \approx 0.061 \]

The annual rate of interest required is 6.1%
Question 2

a. 21 45 29 27 19 35 23 58 34 27

For this data set,

i.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
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<tbody>
<tr>
<td>1</td>
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<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

ii. Median is 28, mean is 31.8 and the standard deviation is 11.3

b. From the table provided,

i. \[ A = 155 \] and \[ B = 170 \]

ii. The percentage of accurate results is \( \frac{180}{200} \times 100 = 90\% \)

iii. \( \frac{15}{170} \times 100 = 8.8\% \)

c. MP

Mary had the most to complain about.

Before the highest and lowest scores are removed her average is 7.46 which is higher than the average of William (7.31) and Joe (7.28).

With the removal of the highest and lowest scores, the averages show a shift. This shift shows that William has an average of 7.42 and Joe is still 7.3 with Mary on 7.37. As a consequence William is awarded first place in spite of Mary’s better overall average score.
Question 3

a.

i. \(270^\circ + 32^\circ = 302^\circ\)
\[\therefore 302^\circ \text{ T}\]

ii. Let \(x = BC\)
\[
\tan 58^\circ = \frac{x}{10}
\]
\[x = 10 \tan 58^\circ\]
\[\therefore x = 16 \text{ nm}\]

iii. Total distance travelled is \(10 + 16 \text{ or } 26 \text{ nm}\)

\[
\text{Time taken} = \frac{\text{Distance}}{\text{Speed}}
\]

\[
\text{Time taken} = \frac{26}{13} \text{ or } 2 \text{ hours}
\]

b.

i. Three hours at 8 knots = 24 nm

On the chart, 1 nm = 1 minute of latitude, so 24 nm corresponds to 24 nm which is about 64 mm on the chart.

In the same way, the second section involves a distance of 10 nm which is about 27 mm on the chart.
ii. From P, the direction to Broken Bay is approximately 234° T. The distance of travel is approximately 58mm on the chart and this corresponds to 22 minutes of latitude or approximately 22 nm.
Calculation steps

Magnetic variation since 1989 → 23 years

\[10^\circ 51' + 03' \times 23 = 12^\circ\]
At 10:25 am, Port Phillip bears 088° M or 100° T from the vessel. This is equivalent to a back-bearing of 280° T from Port Phillip. At 2:05 pm Kingfisher Point bears 118° M and Port Phillip bears 068° M from the vessel. These magnetic bearings are equivalent to bearings from the vessel of 130° T and 080° T respectively or back-bearing of 310° T and 260° T.

The distance travelled during the time interval is 44 nm (≈ 44'). The elapsed time is 3 hours and 40 minutes. The average speed of the vessel through the water is then given by

\[ \text{speed} = \frac{\text{distance}}{\text{time}} \]

\[ = \frac{44}{3 \frac{2}{3}} \text{ or } 12 \text{ knots} \]

**Question 4**

a.

The minimum spanning tree is $A \rightarrow G \rightarrow I \rightarrow J \rightarrow K \rightarrow F$; $K \rightarrow L \rightarrow E \rightarrow D \rightarrow C$; $A \rightarrow B$; and $A \rightarrow H$ totalling 28 km.
The critical path is \( A \rightarrow F \rightarrow G \rightarrow J \rightarrow K \) and the minimum completion time is 61 weeks.

c. 

i.  

<table>
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<tr>
<th>Time</th>
<th>Customer served</th>
<th>Arrivals</th>
<th>Queue length</th>
<th>People in queue</th>
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<tr>
<td>0</td>
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<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>A</td>
<td>B, C</td>
<td>2</td>
<td>B, C</td>
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<tr>
<td>2</td>
<td>A</td>
<td>D</td>
<td>3</td>
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<td>3</td>
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<td>E</td>
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<td>E</td>
<td>H</td>
<td>3</td>
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<td>E</td>
<td>I, J</td>
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<td>2</td>
<td>I, J</td>
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### ii. MP

<table>
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<th>Time</th>
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<th>Arrivals</th>
<th>Queue length</th>
<th>People in queue</th>
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<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
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<td>C</td>
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<td>G, H</td>
<td>I, J</td>
<td>2</td>
<td>I, J</td>
</tr>
</tbody>
</table>

*Assuming that H has a service time of at least one minute.*

Two checkouts significantly cut waiting times and this is a strength; but a limitation of this model is that there is also now a fair amount of slack time for one or both checkouts.
Question 5

<table>
<thead>
<tr>
<th>Income</th>
<th>Expenses</th>
</tr>
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<tbody>
<tr>
<td>Wages</td>
<td>Transport and lunches C</td>
</tr>
<tr>
<td>Interest on his investment</td>
<td>Health and entertainment $6060</td>
</tr>
<tr>
<td>Dividends on his shares</td>
<td>Accommodation D</td>
</tr>
</tbody>
</table>

a. Interest on Jim’s investment

$3000 \times 0.06 = $180 ….. A

Dividends on his shares

$0.03 \times 4.50 \times 2500 = $337.50 ….. B

Transport and lunch expenses

$(80 + 60) \times 52 = $7280 ….. C

Accommodation

$375 \times 52 = $19500 ….. D

b. MP

Jim’s income is as follows

<table>
<thead>
<tr>
<th>Income</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages</td>
<td>$36920.00</td>
</tr>
<tr>
<td>Investment</td>
<td>180.00</td>
</tr>
<tr>
<td>Shares</td>
<td>337.50</td>
</tr>
</tbody>
</table>

Total income $37437.50

Jim’s expenses are

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transport &amp; lunches</td>
<td>$7280.00</td>
</tr>
<tr>
<td>Health &amp; entertainment</td>
<td>6060.00</td>
</tr>
<tr>
<td>Accommodation</td>
<td>19500.00</td>
</tr>
</tbody>
</table>

Total expenses $32840.00

Disposable income $37437.50 - $32840.00 or $4597.50

Income shortfall $8000 - 4597.50

$3402.50 per annum or $65.43 weekly.
Jim needs to find average weekly savings of at least $65.43 to enable the purchase. There are several assumptions that need to be addressed by Jim: sustainability of his current employment, availability of cheaper accommodation nearer to his work, and Jim can afford to maintain his car.

Jim could reach his savings goal in the following ways -

• Reduce the entertainment component to $100 per fortnight would give Jim a saving of $1300 per annum.
• Reduce the expenditure on lunches. For example, take lunch to work rather than buy it and if he could do this to save an extra $20 each week this would amount to an annual saving of $1040.
• Find cheaper accommodation nearer to his work. By reducing his accommodation and transport costs by a total of between $25 to $40 each week will allow Jim to reach his target savings.