Mathematics A
2011 Senior External Examination — assessment report

Statistics

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of candidates</th>
<th>VHA</th>
<th>HA</th>
<th>SA</th>
<th>LA</th>
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<tbody>
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Multiple-choice questions

<table>
<thead>
<tr>
<th>Question</th>
<th>Correct response</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>B</td>
<td>C</td>
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<table>
<thead>
<tr>
<th>Question</th>
<th>Correct response</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
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<td>B</td>
<td>A</td>
<td>D</td>
<td>C</td>
<td>A</td>
</tr>
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</table>

The multiple choice questions sampled the full range of subject matter and contexts from the syllabus. Approximately 70% of candidates responded correctly to at least half of the multiple-choice questions. Common errors included incorrectly calculating the GST by subtracting 10% of the given cost (Question 3), not knowing whether to add or subtract the magnetic variation (Question 10) and incorrectly working with the compound interest formula (Question 15).

Characteristics of good responses

Knowledge and procedures (KP)

Generally Knowledge and procedures was well attempted in Paper One: Part B but many candidates found Paper Two very challenging. Candidates provided good-quality responses to Question 2 (Elements of applied geometry), Question 3 (Linking two and three dimensions) and Question 4 (Data collection and presentation). In good responses, candidates characteristically provided:

- clear justification and working to support their solutions
- substantial use of diagrams and attempts to show significant intermediate calculation steps.
Modelling and problem solving (MP)

The questions most successfully attempted involved straightforward computation. These included from Paper One: Part B — Question 1 (Managing money 1) and Paper Two — Question 1 (Managing money 2). Many candidates attempted to identify the strengths and limitations of models; this was best demonstrated in responses to Paper Two — Question 2c. Some elegant solutions were provided in responses to Paper One: Part B — Question 2c. Candidates are urged to attempt all Modelling and problem solving questions.

Communication and justification (CJ)

Candidates appreciated the need to justify and validate their solutions. This was clearly evident in those successful responses which included a developed argument and some examination of the strengths and limitations of models. Markers noted a significant improvement in the quality of the responses in the Communication and justification criterion.

Common weaknesses

Knowledge and procedures (KP)

Common errors in responses to Paper One included the incorrect calculation of the taxable income and Medicare levy (Question 5) and the incorrect interpretation of the timesheet (Question 1).

In Paper Two responses, many candidates did not manage the compound interest calculation (Question 1). The tree diagram calculations were generally poorly done in Question 2 (Exploring and understanding data 2). The Maps and compasses — Navigation topic assessed in Question 3 continues to provide difficulty for many candidates. The associated chart work shows improvement, but too many candidates are unable to correctly calculate the magnetic variation. Full working must always be shown. Candidates continue to struggle with the subject matter of Operations research — networks and queuing (Question 4). Too often, candidates could not correctly identify the critical path. Not only should the critical path be clearly identified, but any necessary intermediate calculations should appear in the response.

Modelling and problem solving (MP)

To achieve success in this criterion, candidates must:

- carefully read and interpret information (in Paper Two — Question 5, candidates did not appear to understand the investment choices)
- make use of clearly labelled, neatly drawn diagrams to support the arguments and conclusions reached. These diagrams are a significant part of a candidate’s justification of an argument, even if only of an exploratory nature
- explore the strengths and limitations of models.
Communication and justification (CJ)

Candidates showed improvement in this criterion compared to previous years. To achieve success in this criterion, candidates must:

- effectively build supporting arguments to show clarity and depth of thinking (ideally across a range of subject matter)
- give more attention to the Communication and justification requirements of the syllabus when developing solutions to problems. This is particularly the case in response to Modelling and problem-solving questions but it is also relevant when responding to Knowledge and procedures questions.

Sample solutions

The following solutions are not necessarily prescriptive model responses and are not necessarily the only way of solving a problem. Other approaches and problem-solving strategies may be just as acceptable.
Paper 1: Part B

Extended response

Question 1

a. Ticket price = $635 with a mark-up of $80.
   i. Original price = $555
   ii. % mark-up = \( \frac{80}{555} \times 100 \)
       = 14.4%

b. As per the rates table,
   i. Hours worked by Susan = 8 + 5\times 1.5 + 7\frac{3}{4}\times 2
       = 31

   ∴ Gross pay = $21 \times 31
                = $651
   ii. PAYG tax = 0.24 \times $651
           = $156.24

c. Exchange rate between currencies: AUD$1 = USD$1.15
   i. 8000 AUD = 8000 \times 1.15 USD
       = 9200 USD
   ii. MP

Suppose Stephen has 1000 USD after six months.

At the current rate of AUD$1 = USD$1.15, Stephen would have 1000 \times \frac{1}{1.15} AUD or 870AUD. Now if the rate changes to AUD$1 = USD$1.10, Stephen would have 1000 \times \frac{1}{1.10} AUD or 909AUD and this represents an increase; so the change would be in Stephen’s favour.
Question 2

a. i. 5.24 metres = 5240 millimetres

ii. $42500 \text{ cm}^2 = 42500 \times \frac{1}{10000} \text{ m}^2$

   $= 4.25 \text{ m}^2$

b. i. The area of the face ABCD is given by $\frac{1}{2} \times 8 \times (1.5 + 2.25) \text{ m}^2$

   Therefore the area of the shaded face is 15 m$^2$.

ii. The volume of the inground swimming pool is then $AE \times 15 \text{ m}^3$.

   $\therefore Volume = 20 \times 15$ or 300 m$^3$.

c. MP

Let the height of the mountain be represented by $CD = h$ metres and let the distance $CB = x$ metres. We need to find $h$.

\begin{align*}
\text{Given that } AB &= 2000 \text{ m}, \quad \tan(5^\circ) = \frac{h}{2000 + x} \\
\text{And since } \tan(10^\circ) &= \frac{h}{x} \Rightarrow h = x \tan(10^\circ).
\end{align*}

So $\tan(5^\circ) = \frac{x \tan(10^\circ)}{2000 + x}$

$x \tan(10^\circ) - x \tan(5^\circ) = 2000 \tan(5^\circ)$

$x = \frac{2000 \tan(5^\circ)}{\tan(10^\circ) - \tan(5^\circ)}$

Therefore $x \approx 1969.616$ metres

Hence, with $h = x \tan(10^\circ)$, the height of the mountain is approximately 347 metres correct to the nearest metre.
Question 3

a. The Johnson family’s house is drawn to a scale of 1:125.

i. Bedroom 1 on the plan is approximately 53 mm $\times$ 49 mm

Applying the scale, the dimensions of the bedroom are $\frac{53 \times 125}{1000}$ by $\frac{49 \times 125}{1000}$ and these are respectively 6.625 m by 6.125 m.

ii. $\text{Area} = 6.625 \times 6.125$

$= 40.578 \text{ m}^2$

Using two coats of paint, the area to be painted is then 81.156 m$^2$

Time required to paint the ceiling twice is $\frac{81.156}{12}$ hours or approximately 6 ¾ hours.

The labour cost will be \$365 to the nearest dollar.

b. The Johnson residence is built on a block 20 m by 2.7 m

Using the scale of 1:125, this converts to a plan size of $\left(20 \times \frac{1000}{125}\right)$ mm by $\left(27 \times \frac{1000}{125}\right)$ mm or 160 mm by 216 mm.

The house has dimensions 87 mm by 155 mm on the plan.

The point A on the plan is 4 m from the front end and 5 m from the fence on the western side. On the plan, $4 = 4 \times \frac{1000}{125}$ mm or 32 mm and $5 = 5 \times \frac{1000}{125}$ mm or 40 mm.
c. MP

The end-view of the roof of the veranda (AB) forms a quadrant of a circle of radius 1.8 metres. The length of AB is given by \( \frac{90}{360} \times 2\pi \times 1.8 \) metres and this is approximately 2.827 m.

The area of the roof is given by \( 2.827 \times \text{length of the southern wall} \).

From the plan the southern wall has a length of 87 mm and, using the scale of 1:125, this wall is \( \left( \frac{87 \times 125}{1000} \right) \) metres or 10.875 m long.

\[ \therefore \text{the area of the roof is } 2.827 \times 10.875 \text{ m}^2 \text{ or } 30.744 \text{ m}^2. \]

At $110 per m^2$, the roof will cost $3381.80 to build.

As four supporting posts, at $65 each, are also required, the total cost of making the veranda will be $(3381.80 + 4 \times 65)$ or $3641.80$
Question 4

a. By using the calculator,

i. \( \mu_x = \$31571.43 \) and \( \sigma_x = \$14331.36 \) (allow \$14434.83)

ii. The new mean is \$32435.90\) and this represents an increase of \$864.47\) on the old mean; and, the new standard deviation is \$15967.59\) (allow \$16070.94\) and this represents an increase of \$1636.23\) (allow \$1636.11\) on the old standard deviation.

b. i.

![Boxplots](image)

ii. MP

The medians are close in value and therefore it is reasonable for the student to assert that there appears to be no difference in the volume results obtained from both classes. However, the results of Class B indicate a larger interquartile range and this difference in the IQRs suggests that there is less consistency in the results obtained by Class B. This becomes more apparent when the ranges of scores from both classes are compared.

<table>
<thead>
<tr>
<th></th>
<th>Class A</th>
<th>Class B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>26.5</td>
<td>26.6</td>
</tr>
<tr>
<td>IQR</td>
<td>2.2</td>
<td>4.1</td>
</tr>
<tr>
<td>Range</td>
<td>3.4</td>
<td>5.9</td>
</tr>
</tbody>
</table>

When the five number summary is explored, it appears that the volume results of Class A are more reliable because they are more consistent.
Question 5

a. Michael has a gross income of $95 540 and he has already paid $21 790 in tax on it.

i. Michael has $16 500 in allowable deductions

\[
\text{Taxable income} = \text{Gross income} - \text{allowable deductions} \\
= $95 540 - 16 500 \\
\therefore \text{Taxable income} = $79 040
\]

ii. Tax payable on Michael’s taxable income = $18 612 + 0.47 \times (79040 - 70000)

\[
\therefore \text{Tax payable} = $22 860.80
\]

iii. Medicare levy based on Michael’s taxable income = 0.015 \times 79040

\[
= $1185.60
\]

Total tax to be paid = Tax payable on income + Medicare levy

\[
= $22 860.80 + 1185.60 \\
= $24 046.40
\]

Michael must pay $(24046.40 - 21790) or $2256.40

b. MP

Compare the two options.

Option 1:

Retainer = $650 and commission of 5.5% on weekly sales.

The more sales Tina makes, the more she earns. Assume that Tina earns $4000. In this case, her income will be $650 + 0.055 \times 4000 or $870. If Tina earns $8000, her weekly income will be $650 + 0.055 \times 8000 or $1090.

Tina will earn between $650 and $1090 weekly.

Option 2:

No retainer but 20% commission on all sales.

Realistically Tina can earn anywhere between $0 and 20% of $8000. This could mean a maximum income of $0.2 \times 8000 or $1600 weekly.
Clearly the incentive to work harder is there in Option 2 providing Tina with an opportunity to earn an extra $(1600-1090) or $510 weekly. This is an obvious strength and a terrific motivator for Tina to work hard; but there is a limitation. How does Tina guarantee this income in harsh economic times or manage her recurring financial obligations in general?

Option 1 does guarantee a regular income through the retainer only. At best she may earn $1090 weekly but typically much less. Using the midpoint sales figure, she would earn $(650+0.055 \times 6000) or $980 weekly.

Without knowing Tina’s full circumstances it is difficult to advise which of the two options is better.
Paper Two: Solutions

Extended response

Question 1

a. Jing deposits $16500 earning 4% per annum compounding quarterly over three years.
   i. \[ V = 16500 \left( 1 + \frac{0.04}{4} \right)^{12} \]
      \[ \therefore V = $18592.61 \]
   ii. Interest \( = V - 16500 \)
      \[ \therefore \text{Interest} = $2092.61 \]

b. Jack’s company purchased equipment worth $35 0000 1 June 2009.
   i. The depreciated value was $ \( (350000 - 2 \times 60000) \) or $23 0000.
   ii. Salvage value is reached in three years and is calculated as follows.
      \[ S = V_0 (1 - r)^n \]
      \[ S = 230000 (1 - 0.3)^3 \]
      \[ \therefore S = $78890 \]

c. Susan’s leasing arrangement requires the calculation of simple interest.
   i. Interest \( = 4000 \times 0.144 \times 3 \)
      \[ \therefore \text{Interest} = $1728 \]
      Total amount to be repaid \( = (4000 + 1728) \)
      \[ = $5728 \]
      Over 3 years this represents a monthly payment of \( \frac{5728}{36} \) or $159.11

   ii. MP
      \[ E = \frac{(1 + i)^n - 1}{n} \]
      For the given data, \( n = 36 \quad i = 12.5\% \) per annum compounding monthly.
\[ E = \left( 1 + \frac{0.125}{12} \right)^{36} - 1 \]

\[ \therefore E = 0.01256 \]

This is equivalent to 1.256% per month or 15.07% per annum.

The calculated effective rate of interest is greater than that offered through the leasing arrangement. Therefore, Susan is advised to purchase her desktop computer using the leasing arrangement.

Question 2

a. To select the sample, statisticians will need to examine the characteristics of the surveyed population in terms of its overall size, gender and age groupings within it. Once this make-up is determined the number of people required to form the sample within each of the groupings can be made. This would represent a stratified sample. The way in which the members are selected within each strata, could include the random selection of people from databases held by council, electoral data, Medicare data, etc. Other ways of gathering the data samples could include door knocking, randomly selecting names from the phone book, etc.

b. Jim has 50 coloured pens of which 28 are red and 22 are green.
   i.
ii. \( \text{Prob(RR or GG)} = \text{Prob(RR)} + \text{Prob(GG)} \)

\[
= \frac{28}{50} \times \frac{27}{49} + \frac{22}{50} \times \frac{21}{49}
\]

\[\therefore \text{Prob(same colour)} = \frac{87}{175} \]

c. The data collected over eight years is as follows.

<table>
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<tr>
<th>Time since planting, ( t ) years</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of the tree, ( H ) metres</td>
<td>0.8</td>
<td>1.3</td>
<td>2.5</td>
<td>4.7</td>
<td>6.8</td>
<td>8.0</td>
<td>8.9</td>
<td></td>
</tr>
</tbody>
</table>

i. The above data represented as a scatter plot follows.
ii. MP

The model proposed has equation $H = 1.4t - 0.6$

A data set has been generated for the model.

<table>
<thead>
<tr>
<th>Time since planting, $t$ years</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree height model where $H$ is given in metres</td>
<td>0.8</td>
<td>2.2</td>
<td>3.6</td>
<td>6.4</td>
<td>7.8</td>
<td>9.2</td>
<td>10.6</td>
<td></td>
</tr>
</tbody>
</table>

A graph of the model appears below together with the graph from Q2ci.
The proposed model, represented by the broken line, is positioned well above all data points except when \( t = 1 \) and this is a limitation of the model. A better fit is the one proposed by the straight-line of best fit and its strength is that it is closer to the plots of the data points and may provide a more realistic value for the height of the tree. From this straight-line of best fit, the height of the tree after four years would be approximately 4.2 metres.

**Question 3**

a. The Prince is travelling at 15 knots towards the Lady Gaga on a bearing 306° T.

i. The bearing of the Prince from the Lady Gaga is \((306° + 180° - 360°) \text{T}\) or 126° T.

ii. 

![Diagram](image)

\[
\cos 54° = \frac{x}{40}
\]

\[x = 40 \cos 54°\]

\[\therefore \ x \approx 23.5 \text{ nautical miles}\]

iii. Provided the Prince maintains the constant rate of 15 knots through the water, then the time to rendezvous with the Lady Gaga will be

\[
\frac{40}{15} \text{ hours or } 2 \frac{2}{3} \text{ hours.}\]
b. The point Q is located at $18^\circ 35' S \times 153^\circ 20' E$. 

i.
ii. The mean variation is increasing since it is moving westerly here. Across the time frame of 22 years, at $0^\circ 03'$ per annum equates to $1^\circ 06'$ over 22 years.

\[ \text{Magnetic bearing} = 210^\circ - (1^\circ 06' + 10^\circ 51') \]

\[ = 198^\circ 03' \text{M} \]

or approximately $198^\circ \text{M}$ correct to the nearest degree.

iii. The line segment from Bald Peak to Q is about 93 nautical miles.
c. MP

At 9 am the crew on board the *Intrepid* sight Port Sound bearing 065° T at a distance of 10 nautical miles. The skipper changes course to 335° T and travels for 2 \( \frac{3}{2} \) hours at a constant speed through the water of 15 knots. This equates to a distance of 40 nautical miles and is indicated by point P on the chart. The *Intrepid* drops anchor.

Locate the *Sea Storm*. The *Sea Storm* is located on a bearing of 220° T from the *Intrepid* and 260° T from Port Sound. At this moment the *Sea Storm* is represented by the point R on the chart. Immediately the *Sea Storm* heads 340° T at 27 knots to avoid the reef.

The shortest distance is the perpendicular distance so a perpendicular to meet the direction of sail of the *Sea Storm* needs to be constructed and to meet it at the point designated S on the chart. The shortest distance is indicated by SP or x on the chart. This distance is approximately 48 nautical miles. To calculate when the *Sea Storm* is closest, find SR and assuming the speed through the water is constant at 27 knots, then the time it reaches S is given by

\[
\frac{SR}{27} \text{ hours}
\]

From the chart, SR is approximately 27 nautical miles and so it will take about 1 hour for this ship to reach S and the time will be 12.40 pm.
Question 4

a. MP

Start at A.

\[ A \to B , \; B \to C , \; C \to J \]

Then

\[ A \to H , \; H \to G , \; G \to M , \; M \to F , \; F \to E \]

Finish

\[ A \to D . \]

Total circuit covers $1720 to cover all nodes.

b. Emily has developed the following project network for the proposed community park.

i.

The minimum time to complete the task is 29 weeks.
ii. The critical path is the circuit $\text{Start} \rightarrow A \rightarrow B \rightarrow J \rightarrow G \rightarrow \text{Finish}$

iii. If activity K is delayed by four weeks, the critical path changes to the circuit $\text{Start} \rightarrow H \rightarrow K \rightarrow J \rightarrow G \rightarrow \text{Finish}$ with a minimum completion time of 32 weeks.

c. Average waiting time must include the very first customer who has waited 0 minutes.

i. This average $= \frac{33}{11}$ or 3 minutes per customer.

ii. The following amendment to the original chart is needed to analyse what happens to customer E.

Customer E has been waiting 16 minutes.
Question 5

a. Purchase price = $425 000  
Deposit = $360 000  
Balance = $65 000  

Mary must borrow the balance at 6.5% p.a over 15 years.  
From the table, this loan requires a monthly payment of $8.71 per $1000 borrowed.  
Mary must repay each month $\left(8.71 \times \frac{65000}{1000}\right)$ or $566.15  

b. Consider the rental investment.  
Income = $(400 \times 52 - 4200) \times 3  
= $49800  
Tax on this income at the end of three years = $(0.3 \times 49800)  
\therefore \text{Tax} = $14940  
\therefore \text{Net gain through the investment} = $(0.7 \times 49800) \text{ or } $34860  

Mary’s investment property would now have a market value of  
$(425000 \times 1.06^2 \times 0.95)$ or $453653.50  
and this represents a market value improvement of $28653.50 across the same period. This represents a potential gain only and may have some disadvantages associated with it like increased maintenance and insurance costs, and increased rates to pay. 

Now consider Mary’s position by just leaving her money in the bank.  
Accrued value = $360000 \times \left(1 + \frac{0.06}{2}\right)^6$ or $429858.83  
Interest earned = $(429858.83 - 360000)$ or $69858.83  
Tax on this income at the end of three years = $(0.3 \times 69858.83)  
\therefore \text{Tax} = $20957.65  
\therefore \text{Net gain through the investment} = $(0.7 \times 69858.83) \text{ or } $48901.18
Mary will receive the benefit of her investment account sooner and it looks a lot better than $34 860 through the rental property. This has to be a significant advantage.

If Mary uses all of her investment income to live on she will only ever have $360 000 in the bank. This could be a disadvantage since the value of her rental property investment will, very likely, continue to grow beyond $425 000 even if she spends all of her available rental income.

Mary will only realise the capital gains in her rental investment if she ever sells the house and only then might she be in a position to see large gains on her original investment. This might be considered a disadvantage as she may not ever sell the property.