2011 Senior External Examination

Mathematics A
Monday 31 October 2011
Paper Two — Question book
1:15 pm to 4:25 pm

Time allowed
- Perusal time: 10 minutes
- Working time: 3 hours

Examination materials provided
- Paper Two — Question book
- Paper Two — Resource book
- Paper Two — Response book

Equipment allowed
- QSA-approved equipment
- ruler (metric, parallel or rolling)
- protractor
- drawing compass
- set squares
- templates (without formulas)
- non-programmable calculator
- graphing calculator

Not allowed: calculators with computer algebra system (CAS) functionality.

Directions
You may write in this book during perusal time.
Paper Two has five extended-response questions.
Attempt all questions.

Assessment
Assessment standards are at the end of this book.

After the examination session
Take this book when you leave.
Planning space
Extended response

Paper Two has five questions. Each question assesses Knowledge and procedures (KP), Modelling and problem solving (MP) or a combination of both. Communication and justification (CJ) will be assessed by an overall judgment of your responses.

Attempt all questions.
Write your responses in the response book.

Question 1

a. Jing deposits $16500 into an investment account earning 4% per annum compounding quarterly over three years.
   i. Calculate the value of her investment after three years.
   ii. Find the amount of interest earned at the end of the three years.

b. Jack’s distribution company purchased equipment worth $350000 on 1 June 2009. The equipment will reach its salvage value on 1 July 2014. On 1 July 2010 the equipment was depreciated by $60000 and by a further $60000 on 1 July 2011.
   i. What was the depreciated value of the equipment on 2 July 2011?
   ii. On 2 July 2011 Jack changed the method of calculating the depreciated value of his equipment to the diminishing value method with an annual rate of depreciation of 30%. What will be the salvage value of the equipment?

c. Susan wants to buy a desktop computer system for $4000.
   i. Susan considers buying the system on a no-deposit leasing arrangement with terms of 14.4% per annum simple interest over three years with equal monthly repayments. How much would Susan’s monthly repayment be under this proposal?
   ii. Susan also makes an enquiry with her local bank for a loan of $4000 over three years. The bank offers a loan at 12.5% per annum compounding monthly over three years on the reducing balance. To compare the two arrangements, Susan uses the following formula to calculate the effective simple interest rate of the bank’s loan offer:

\[ E\% = \frac{(1 + r)^n - 1}{n} \times 100 \]

where \( r \) is the stated interest rate per period expressed as a decimal, \( E\% \) is the effective percentage simple interest rate per period and \( n \) is the number of periods.

Susan thinks that the leasing arrangement would cost her less than the bank loan. Do you agree?
Show full working and justify your response.
Question 2

a. Medical scientists have developed a test to determine whether people are carriers of a certain mosquito-borne virus. Describe a method of randomly selecting 347 people to be tested for the virus that scientists could use to ensure they have reliable data.

Respond to Question 2b on pages 12 and 13 of your response book.

b. Jim has placed 50 coloured pens in a bucket. Of the 50 pens, 28 are red and the rest are green. Jim has picked two pens at random from the bucket and noted their colours.

i. Complete the probability tree diagram.

ii. Calculate the probability of Jim selecting two pens which are the same colour.

Respond to Question 2c on pages 14 and 15 of your response book.

c. Indra is a scientist at the Mt Coot-tha Botanical gardens. She has been studying the growth of a particular tree over the last eight years. The data she has collected is shown in the table below.

**Note**: No data was recorded at the four year mark.

<table>
<thead>
<tr>
<th>Time since planting, $t$ years</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<tbody>
<tr>
<td>Height of the tree, $H$ metres</td>
<td>0.8</td>
<td>1.3</td>
<td>2.5</td>
<td>4.7</td>
<td>6.8</td>
<td>8.0</td>
<td>8.9</td>
<td></td>
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i. Prepare a scatter plot of the data given in the table above and draw a straight line of best fit.

ii. One of Indra’s colleagues examined the scatter plot and the straight line of best fit and then made the following observation concerning the height of the tree, $H$ (in metres), at time, $t$ (in years):

$$H = 1.4t - 0.6$$

Indra’s colleague claimed that the formula can be applied at any stage to predict the height of the tree.

Discuss the limitations of the model proposed by Indra’s colleague. Suggest a reasonable value for the height of the tree after four years.

**Justify your response.**
Question 3

a. The *Prince* is a rescue craft travelling at 15 knots in the direction 306°T towards a stationary
distressed vessel, *Lady Gaga*. The *Lady Gaga* is located 40 nautical miles from the *Prince*.

![Diagram showing the Prince and Lady Gaga](Not drawn to scale)

i. What is the back bearing of the *Prince* from the *Lady Gaga*?

ii. How far south of the *Lady Gaga* is the *Prince* located? Answer in nautical miles correct to
one decimal place.

iii. If the *Prince* maintains its speed, how long will it take to reach the *Lady Gaga*?

(KP)

Respond to Question 3b on pages 16 and 17 of your response book.
Clearly show all chart work and provide all intermediate calculations.

b. A fishing boat departs Pearly Point and sails until it reaches a point, Q, with latitude 18°35´S and
longitude 153°20´E.

i. Clearly mark point Q on the chart.

ii. Find the magnetic bearing of Bald Peak from point Q in 2011. Answer correct to the nearest
degree.

iii. Correct to the nearest nautical mile, how far is Bald Peak from point Q?

(KP)

Respond to Question 3c on pages 18 and 19 of your response book.
Clearly show all chart work and provide all intermediate calculations.

c. At 9 am, the crew on board the charter boat *Intrepid* sights Port Sound bearing 065°T at a distance
of 10 nautical miles. The skipper changes course to 335°T and sets sail at 15 knots.
At 11:40 am, the *Intrepid* stops and anchors on the edge of a reef. At that moment, the navigator
on board the military vessel *Sea Storm* sights the *Intrepid* bearing 040°T and Port Sound bearing
080°T. To avoid both the *Intrepid* and the reef, the *Sea Storm* changes course to 340°T at
27 knots.
In nautical miles, what is the closest the *Sea Storm* gets to the *Intrepid*? At what time does this happen?

Show full working to justify your response.

(MP)
Question 4

Respond to Question 4a on page 20 of your response book.

a. The following network shows the cost, in dollars, of connecting telecommunications cabling to houses in the local area.

What is the minimum cost to connect all houses?

(b) Emily is a project engineer employed by the local community council. She is currently developing a drainage system for the town’s community park. Emily has prepared the following network for the project. Times are given in weeks.

i. Find the minimum time to complete the project.

ii. State the critical path.

iii. How would the critical path change if activity K is delayed by 4 weeks?

(KP)
c. The graph below illustrates the situation at a post office during a typical 20 minute period on a busy day.

i. Calculate the average waiting time.

ii. If all service times were increased by three minutes, how long will customer E be waiting before being served?

(KP)
Question 5

a. At the start of 2008, Mary had $360000 in an investment account. She used it as a deposit on an investment property with a purchase price of $425000. Mary had sufficient extra financial resources to pay for all costs associated with the purchase but needed to borrow the difference between her deposit and the purchase price. Mary’s financial broker arranged a monthly mortgage repayment for 15 years at 6.5% per annum.

Use the following table to calculate Mary’s monthly loan repayment.

(KP)

b. In the three years after purchase, Mary rented out the investment property for $400 per week. Costs in maintaining the property averaged $4200 per annum. Mary paid tax on the rental income (less costs) of 30 cents in the dollar.

During those three years, house prices increased on average by 6% per annum during 2008 and 2009 but then decreased by 5% in 2010.

Mary could have left her money in the investment account. Over the same three year period, it would have earned 6% per annum compounding half-yearly. Tax would have been paid on the interest earned at the rate of 30 cents in the dollar.

All taxes are paid at the end of the three year period.

Compare both investments. Describe the advantages and disadvantages of each.

Show full working and provide a written explanation to justify your conclusion.

(MP)

End of Paper Two
### Assessment standards from the 2006 senior external syllabus for Mathematics A

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<td>Knowledge and procedures (KP)</td>
<td>The overall quality of a candidate’s achievement across the full range within the contexts of application, technology and complexity, and across topics, <strong>consistently demonstrates</strong>: • accurate recall, selection and use of definitions and rules • use of technology • recall and selection of procedures, and their accurate and proficient use.</td>
<td>The overall quality of a candidate’s achievement across a range within the contexts of application, technology and complexity, and across topics, <strong>generally demonstrates</strong>: • accurate recall, selection and use of definitions and rules • use of technology • recall and selection of procedures, and their accurate use.</td>
<td>The overall quality of a candidate’s achievement in the contexts of application, technology and complexity, <strong>generally demonstrates</strong>: • accurate recall and use of basic definitions and rules • use of some technology • accurate use of basic procedures.</td>
<td>The overall quality of a candidate’s achievement in the contexts of application, technology and complexity, <strong>sometimes demonstrates</strong>: • accurate recall and use of some definitions and rules • use of some technology.</td>
<td>The overall quality of a candidate’s achievement rarely <strong>demonstrates</strong> knowledge and use of procedures.</td>
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<td>Modelling and problem solving (MP)</td>
<td>The overall quality of a candidate’s achievement across the full range within each context, and across topics <strong>generally demonstrates</strong> mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations, and identifying variables • selecting and using effective strategies • informed decision making ... and generally demonstrates mathematical thinking which includes: • selecting and using procedures to solve a wide range of problems • initiative in exploring the problem • recognising strengths and limitations of models.</td>
<td>The overall quality of a candidate’s achievement across a range within each context, and across topics, <strong>generally demonstrates</strong> mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations, and identifying variables • selecting and using strategies ... and sometimes demonstrates mathematical thinking which includes: • selecting and using procedures required to solve a range of problems • informed decision making.</td>
<td>The overall quality of a candidate’s achievement <strong>demonstrates</strong> mathematical thinking which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures.</td>
<td>The overall quality of a candidate’s achievement <strong>demonstrates</strong> mathematical thinking which includes following basic procedures and/or using strategies.</td>
<td>The overall quality of a candidate’s achievement rarely <strong>demonstrates</strong> mathematical thinking which includes following basic procedures and/or using strategies.</td>
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<td>Communication and justification (CJ)</td>
<td>The overall quality of a candidate’s achievement across the full range within each context consistently demonstrates: • accurate use of mathematical terms and symbols • accurate use of language • organisation of information into various forms suitable for a given use • use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or decisions • justification of procedures.</td>
<td>The overall quality of a candidate’s achievement across a range within each context generally demonstrates: • accurate use of mathematical terms and symbols • accurate use of language • organisation of information into various forms suitable for a given use • use of mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or decisions.</td>
<td>The overall quality of a candidate’s achievement in some contexts generally demonstrates: • accurate use of basic mathematical terms and symbols • accurate use of basic language • organisation of information into various forms • use of some mathematical reasoning to develop simple logical arguments.</td>
<td>The overall quality of a candidate’s achievement demonstrates evidence of the use of the basic conventions of language and mathematics.</td>
<td>The overall quality of a candidate’s achievement rarely demonstrates use of the basic conventions of language or mathematics.</td>
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