Senior External Syllabus

Mathematics A

Syllabus for the Senior External Examination

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Mathematics A Syllabus for the Senior External Examination

This syllabus should be used for the first time in the 2007
Senior External Examination in Mathematics A.

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1. Rationale

Mathematics is an integral part of a general education. It can enhance understanding of our world and the quality of our participation in a rapidly changing society. Mathematics pervades so many aspects of daily life that a sound knowledge is essential for informed citizenship. Through a deep understanding of mathematics, individuals can learn about economics, society and politics in a mathematically oriented society.

Mathematics continues to develop in response to changes in society and, in turn, influences further societal development. Technology, including calculators, computers and electronic means of communicating information, has had a significant impact on this development. This will be increasingly reflected in the way mathematics is taught and learned.

Mathematics A aims to provide the opportunity for candidates to be more fully prepared for life-long learning. It involves the study of Financial Mathematics, Applied Geometry, Statistics and Operations Research. These are used to develop:

- knowledge and skills of computation, estimation and measurement
- a capacity to interpret and analyse information presented in a variety of forms
- the ability to make judgments based on evidence and reasoning
- a capacity to justify and communicate results in a variety of forms.

This subject emphasises the development of positive attitudes towards mathematics. The subject encourages this development by using relevant and life-related learning experiences. It also encourages candidates to develop mathematical knowledge and understanding via investigative and explorative approaches to learning.

2. Global aims

Candidates of Mathematics A should:

- be aware of the uncertain nature of their world and be able to use mathematics to help them make informed decisions in life-related situations
- be able to manage their financial affairs in an informed way
- be able to visualise and represent spatial relationships in both two and three dimensions
- be able to recognise when problems in their everyday life are suitable for mathematical analysis and solution, and be able to attempt such analysis or solution with confidence
- have positive attitudes to the learning and practice of mathematics
- comprehend mathematical information that is presented in a variety of forms
- justify mathematical results and/or procedures
- communicate mathematical information in a variety of forms
- be able to benefit from the availability of various technologies.

3. General objectives

3.1 Introduction

The general objectives of this subject are organised into four categories:

- *Knowledge and procedures*
- *Modelling and problem solving*
- *Communication and justification*
- *Affective*.

These objectives are linked with the criteria described in section 6.2, with the exception of *Affective*.

3.2 Contexts

The objectives of *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification* incorporate contexts of Application, Technology, Initiative and Complexity. Each of the contexts has a continuum for the particular aspect of mathematics it represents. Mathematics from this syllabus should be taught and learned using a variety of contexts. It is expected that all candidates experience mathematics along the continuum of each of the contexts outlined below. The examination will cover a number of contexts.

Application

Candidates should recognise the usefulness of mathematics through its application. Learning experiences should be used that include a variety of mathematical tasks — from real-world through to contrived or simplified life-related tasks. While some purely mathematical situations may be needed, the emphasis should be on life-related tasks and activities.

Technology

A variety of technological tools should be used in the learning experiences. These can range from pen and paper, compasses, measuring instruments and tables, through to technologies such as scientific calculators.

Initiative

Candidates should undertake tasks that range from routine and well rehearsed through to those that require demonstration of insight and creativity.

Complexity

Candidates should work on **a variety of tasks —** simple, single-step tasks, through to complex tasks. Complexity may derive from either the nature of the concepts involved or from the number of ideas or techniques that must be sequenced in order to produce a meaningful and valid conclusion.

3.3 General objectives

The general objectives, *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*, are linked to the exit criteria in section 6.2.

3.3.1 Knowledge and procedures

This objective involves recalling and using results and procedures within the contexts of Application, Technology, Initiative and Complexity (see section 3.2).

Candidates should be able to:

- recall definitions and rules
- access and apply rules
- demonstrate number and spatial sense
- demonstrate an ability to use technologies such as calculators, measuring instruments, geometrical drawing instruments and tables
- recall, select and use mathematical procedures
- apply mathematical procedures to situations that are similar to situations already encountered
- work accurately and manipulate simple formulae
- recognise that some tasks may be broken up into smaller components.

3.3.2 Modelling and problem solving

This objective involves the uses of mathematics in which candidates will model mathematical situations and constructs, solve problems and investigate situations mathematically within the contexts of Application, Technology, Initiative and Complexity (see section 3.2).

The emphasis should be on using observations, data, diagrams, formulae, graphical and other representations to investigate and model situations, and hence make informed decisions.

Candidates should be able to demonstrate modelling and problem solving through:

- understanding that a mathematical model is a mathematical representation of a situation
- interpreting, clarifying and analysing problems
- identifying the variables of a simple mathematical model of a situation
- forming and/or selecting a mathematical model of a life-related situation
- deriving results by considering mathematical models
- interpreting results from a mathematical model in terms of the given situation
- recognising the strengths and limitations of mathematical models
- developing strategies in modelling
- using a range of problem-solving strategies, such as estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data
- understanding that there may be more than one way to solve a problem
- selecting mathematical procedures required to explore, investigate and solve problems
- developing a solution consistent with the problem

- investigating open-ended situations
- reflecting on conjectures and making modifications if needed
- making decisions from a range of choices.

3.3.3 Communication and justification

This objective involves presentation, communication (by means of mathematical and everyday language), logical arguments, interpretation and justification of mathematics within the contexts of Application, Technology, Initiative and Complexity.

Communication

Candidates should be able to demonstrate communication through:

- organising and presenting information
- communicating ideas, information and results as required
- using mathematical terms and symbols accurately and as required
- using correct spelling, punctuation and grammar in written communication
- understanding material presented in a variety of forms, such as written, symbolic, pictorial and graphical
- translating material from one form to another
- presenting material for different audiences, in a variety of forms (such as written, symbolic, pictorial and graphical)
- recognising necessary distinctions in the meanings of words and phrases according to whether they are used in a mathematical or non-mathematical situation.

Justification

Candidates should be able to demonstrate justification through:

- developing logical arguments expressed in everyday language, mathematical language or a combination of both to support conclusions, results and/or propositions
- evaluating the validity of arguments designed to convince others of the truth of propositions
- justifying procedures that may include:
  - providing evidence (words, diagrams, symbols etc.) to support processes used
  - stating a generic formula before using it
  - providing a reasoned, well-formed, logical sequence within a response
- recognising when and why results of a given problem are clearly improbable or unreasonable
- using supporting arguments to justify results obtained by calculator when necessary.

3.3.4 Affective

Affective objectives refer to the attitudes, values and feelings that this subject aims to develop in candidates. Affective objectives are not assessed in an external examination.

Candidates should appreciate the:

- diverse applications of mathematics

- precise language and structure of mathematics
- contribution of mathematics to human culture and progress
- value of mathematics.

4. Organisation

4.1 Introduction

Candidates need to acquire certain fundamental knowledge and procedures. Some of these are listed under the heading "Basic knowledge and procedures required" in Appendix 1. Candidates should revise the fundamental knowledge and procedures within topics, as they are required.

The topics to be studied are arranged into strands and are listed below. Although the first topic listed in each strand contains material that is required in later topics, the order in which the topics are presented does not imply a sequence of learning. Some relevant learning experiences are also listed in Appendix 3.

Topics

The topics within each strand are:

Financial mathematics strand

- Managing money 1
- Managing money 2

Applied geometry strand

- Elements of applied geometry
- Linking two and three dimensions
- Maps and compasses — Navigation

Statistics and probability strand

- Introduction to data and its presentation
- Exploring and understanding data

Operations research

- Networks and queuing.

The topics are discussed in detail in section 5.

4.2 Time allocation

For teaching centres preparing candidates for the external examination the recommended number of hours for tuition in the subject developed from this syllabus is 150 to 200 hours. Recommended notional times are given in brackets to indicate the relative emphasis of each topic. The time allocation suggested provides a guide for the effective planning of learning experiences.

Time allocation depends on the method of study. Candidates who elect to study without systematic tuition must organise their time according to syllabus requirements and individual circumstances.

4.3 Sequencing

After considering the subject matter and the relevant learning experiences needed to achieve the general objectives, a *spiralling* and *integrated* sequence of learning experiences should be developed that allows candidates to see links between the different strands of mathematics rather than seeing them as discrete. For example, costing, budgeting and financing a loan for house renovations provide an obvious link between the Applied geometry and Financial mathematics strands. As a means of sensibly ordering the subject matter to be studied, the syllabus presents the subject matter grouped into topics but it is not necessary, and often not desirable, to complete one topic before starting another. Obviously candidates must cover any prerequisite material in earlier topics before proceeding with material in later topics. As mentioned earlier, time should be allocated for the maintenance of basic mathematics and mathematical techniques.

The following guidelines for the sequencing of subject matter may be helpful.

- No subject matter should be studied before the relevant prerequisite material has been covered.
- To allow development in each area over time, no topic should be studied immediately after another topic in the same strand.
- Material should be organised to encourage revisiting and spiralling of subject matter.
- Subject matter across topics should be linked when possible.
- Time will be needed for maintaining basic mathematics and mathematical techniques.

4.4 Technology

The advantage of mathematics-enabled technology in mathematics is that it allows for the exploration of the concepts and processes of mathematics. For example, calculators let candidates explore and investigate; they help candidates understand concepts, and they complement traditional approaches to study of the subject. More specifically, mathematics-enabled technology allows candidates to tackle more diverse, life-related problems.

The external examination requires the use of a scientific calculator, as indicated in the general objectives.

Using technology in the examination

During the examination the noise of operating a calculator must not disturb other candidates.

Candidates should not rely only on the use of technology to obtain a result. To meet the requirements of the criterion *Communication and justification* candidates should show full working for each question.

5. Topics

5.1 Introduction

Each topic has a focus statement, subject matter and learning experiences which, taken together, clarify the scope, depth and emphasis for the topic.

Focus

This section highlights the intent of the syllabus with respect to the topic and indicates how candidates should be encouraged to develop their understanding of the topic.

Subject matter

This section outlines the subject matter to be studied in the topic. All subject matter listed in the topic must be included, but the order in which it is presented is not intended to imply a sequence of learning.

Learning experiences (LE)

This section provides some learning experiences that may be effective in using the subject matter to achieve the general objectives of the course. The numbers provided with the subject matter link to learning experiences. Included are experiences that involve life-related applications of mathematics with both real and simulated situations, use of instruments and opportunities for modelling and problem solving.

The learning experiences are suggestions only. Teaching centres and self-guided candidates are encouraged to develop further learning experiences, especially those which relate to the candidate's location, environment and resources.

Note: The learning experiences must provide candidates with the opportunity to experience mathematics along the continuum of each of the contexts.

Language is the means by which meaning is expressed and shared, and by which communication is effected. It is the central means by which candidates learn. Mathematics A requires candidates to use language in a variety of ways — mathematically, graphically and symbolically. Candidates should develop abilities to:

- select and sequence information
- manage the conventions of communication used in Mathematics A as related to an external examination
- use the specialised vocabulary and terminology of Mathematics A
- use language conventions related to grammar, spelling, punctuation and layout.

The learning of language is a developmental process. When writing, reading, questioning, listening and talking about mathematics, teachers and candidates should use the specialised vocabulary of Mathematics A. Candidates should be involved in learning experiences that require them to comprehend and transform data in a variety of forms and, in so doing, use the appropriate language conventions. Attention to language education within Mathematics A should help candidates to meet the

language components of the external examination including exit criteria, especially the *Communication and justification* criterion.

5.2 The topics

The order in which topics and items within topics are given does not imply a learning sequence.

Managing money 1 (notional time 25–30 hours)

Focus

Candidates should develop a working knowledge of the types of financial transactions and decisions they are most likely to encounter.

Subject matter

- earnings, including salary, wages, overtime, commission, piece rate (an industrial award should be used where suitable) (LE 1)
- taxation, including gross, net and taxable income, goods and services tax, deductions, rebates/tax offsets, levies (LE 3 and 4)
- budgeting, including the preparation of a personal budget plan (LE 5 – 7)
- spending, including discount, profit, loss, mark-up and foreign exchange (LE 2).

Learning experiences

The following learning experiences may be developed.

1. Use an industrial award that covers workers in the hospitality industry such as the café, restaurant and catering award, to determine the correct rates of pay and the conditions of work; identify the obligations of employers and employees.
2. Convert Australian dollars into another currency (and the reverse) using buy and sell tables.
3. Using a taxation guide, calculate taxation due.
4. Follow the price of an article from its manufacturer to the retail purchaser, giving consideration to taxes applied.
5. Investigate the costs of owning a car, including insurance, registration, running costs, depreciation and maintenance; compare the estimated costs with those published by organisations such as RACQ.
6. Formulate a simple mathematical model for the cost of hiring a car; consider initial fixed costs and per kilometre charges.
7. Adjust a given budget to allow for various purposes.

Managing money 2 (notional time 35–45 hours)

Focus

Candidates should be encouraged to develop a working knowledge of the mathematics involved in financial transactions and to be aware of their underlying conditions to enable them to make informed decisions on credit and investments.

Subject matter

- simple- and compound-interest calculations for various compounding periods, effective and nominal rates (LE 1, 2, 4 and 5)
- inflation, appreciation and depreciation (LE 4 – 6)
- present and future values (LE 4 and 5)
- consumer finance, including personal loans, credit cards, debit cards, housing loans (including fees and charges) (LE 1 – 5, 8 and 9)
- investments, including savings accounts, term deposits, real estate and the stock market (LE 9).

Learning experiences

The following learning experiences may be developed.

1. Use a repayments table to investigate the effects of different interest rates and varying terms of a loan.
2. Use a calculator or computer to determine a repayment schedule for a loan, with interest calculated on a reducing balance, and demonstrate the effects of changing interest rates during the period of the loan.
3. Investigate and compare the relative cost of finance for a vehicle from a bank, credit union and finance company.
4. Use the simple-interest formula to solve problems involving interest, amount, principal, interest rate and time.
5. Use the compound-interest formula to:
   - find the amount, principal and interest
   - calculate effective interest rates and compare them with nominal and quoted interest rates
   - convert a daily interest rate to a yearly one.
6. Compare the straight-line and diminishing value methods of calculating depreciation.
7. Investigate the effect of inflation on the value of money, income and goods.
8. Compare the effects of various conditions quoted on credit cards from different financial institutions.
9. Use other practical applications of investments, such as:
   - investigating the benefits of making repayments weekly instead of monthly
   - calculating the percentage return from a stock-market transaction, taking into account brokerage fees, dividend and yield
   - calculating the percentage return from real-estate transactions, taking into account commission, legal costs and stamp duty
   - calculating the percentage return from an investment with a financial institution, taking into account establishment costs, commission and interest rates
   - comparing the percentage return from a fixed investment in a savings bank account, a building society account, debentures in an insurance company and a unit trust.
10. Investigate various financial models using spreadsheet technology.

Elements of applied geometry (notional time 15–20 hours)

Focus

Candidates should be encouraged to develop a working knowledge of some geometrical concepts and relationships in two and three dimensions. This is to be accomplished in life-related contexts.

Subject matter

- applications of trigonometry using sine, cosine and tangent ratios and Pythagoras' Theorem (LE 1 and 9)
- perimeter, area and volume in life-related situations (LE 2, 4 and 5)
- latitude, longitude and measurement of time (LE 6–9).

Learning experiences

The following learning experiences may be developed.

1. Use trigonometric ratios and Pythagoras' Theorem in life-related situations, such as the calculation of heights of trees, widths of valleys and rivers, and angles of elevation and depression.
2. Calculate areas of compound figures involving rectangles, trapeziums, triangles, circles, semicircles and quadrants in life-related situations.
3. Calculate perimeter of compound figures.
4. Calculate surface areas of prisms and cylinders in life-related situations.
5. Calculate volumes of compound figures involving prisms, pyramids, cylinders, cones and spheres in life-related situations given necessary formulae, for example:
   - calculate the volume of water in swimming pools of different shapes
   - find the volume of earth which must be removed to construct a canal of given dimensions
   - calculate the number of cubic metres of soil to be ordered to top-dress a lawn.
6. Locate positions on the earth's surface, given latitude and longitude.
7. Find the time differences for a range of cities, as given in the telephone directory, and compare them with those predicted from the longitude.
8. Calculate the arrival time in a destination city in a different time zone, given the city of origin, flight duration and departure time; conversely, calculate the flight time given departure and arrival times (flights that cross the International Date Line should be included).
9. Find the distance in kilometres between points on the earth's surface (same longitude).

Linking two and three dimensions (notional time 20–25 hours)

Focus

Candidates should develop a working knowledge of the practical considerations inherent in a variety of construction areas. Particular emphasis should be given to the representation of three-dimensional constructions in two dimensions.

Subject matter

- interpretation of scale drawings and plans (LE 1 and 6–8)
- drawing simple scale drawings and plans (LE 2, 7 and 8)
- the geometry of bracing for rigidity (LE 3 and 6)
- practical tests for squareness, plumbness and levels (LE 4 and 5)
- estimation of quantities and costs in a variety of construction areas (LE 6–8).

Learning experiences

The following learning experiences may be developed.

1. Use a scale plan to obtain actual dimensions.
2. Draw a scale plan and use it to determine layouts, for example, placement of furniture in a room.
3. Study the method of crossbracing for rigidity considering the range of angles involved.
4. Use other methods of application, such as:
   - study the use of clear plastic tubing filled with water to obtain a level height between posts on uneven ground
   - study the use of a builder's plumb-bob to test whether a wall is vertical
   - study the use of a spirit level to test for squareness on a building corner and to test levels.
5. Study the use of Pythagoras' Theorem to test for squareness.
6. Estimate quantities and costs in construction areas, for example:
   - calculate the amount of concrete required and the costs for the footings and slab of a structure (include reinforcing)
   - calculate the cost of covering an open gable roof in metal and in tiles
   - estimate the cost of brickwork for a wall
   - determine the number of litres of paint and the minimum cost for painting a room given the scale plan
   - calculate the minimum number of rolls of wallpaper needed to paper a room
   - given a scale plan and carpet width, determine the number of metres of carpet required to carpet a house.
7. Design an office layout, given the staff and their functions, the furniture and the legal space requirements for each employee.
8. Investigate the design of air-conditioning ducting and the placement of outlets within buildings.

Maps and compasses — Navigation (notional time 25–30 hours)

Focus

Candidates should be encouraged to develop a working knowledge of techniques needed for coastal navigation.

Subject matter

- compass bearings and reverse bearings (LE 1 – 3 and 6)
- magnetic variation (LE1 – 3)
- nautical miles and knots (LE 3)

- use of maps and charts, compasses, dividers and parallel rulers or their equivalent (LE 1, 3 and 7)
- methods of fixing position, including bearing fix, dead reckoning, running fix (LE 3, 6 and 7)
- applications in coastal navigation (LE 3 and 5 – 7).

Learning experiences

The following learning experiences may be developed.

1. Explore the relationships between true north, magnetic north and grid north.
2. Explore the effects on bearings of magnetic variation; calculate true bearing given compass bearing and magnetic variation; calculate compass bearing given true bearing and magnetic variation.
3. Use coastal navigational techniques on charts to fix a position to plot a given course and calculate distances.
4. Determine the final position given a pre-subset of bearings and distances.
5. Investigate the shortest distance on curved surfaces between two points.
6. Use coastal navigational techniques on charts, including transit fixing, cross-bearings, doubling the angle at the bow.

Introduction to data and its presentation (notional time 15–20 hours)

Focus

Candidates should develop knowledge of the practicalities and concepts involved in collecting, handling, preparing, describing, presenting and summarising data, and of some elementary concepts in data quality and exploring data to describe key features. Candidates should develop skills in recognising data quality and practical problems, and in commenting on data in context. It is expected that calculators will be used routinely. The emphasis should be on the practicalities, concepts and interpretation of data through a range of life-related scenarios.

Subject matter

- identification of types of data — continuous and discrete — and the types of discrete data (categorical, ordinal and count) (LE 1 and 3)
- identification of variables and their type as above (LE 1 and 3)
- practical aspects of collecting data in observation, experimentation, and survey (LE 1–3 and 6)
- practical aspects of data preparation and entry, including possible data problems (LE 1–3, 5 and 6)
- choice and use in context of appropriate graphical and tabular displays, including pie charts, barcharts, tables, histograms, stemplots and boxplots (LE 5, 7 – 11 and 12)
- use and interpretation of plots of data and of scatterplots of one variable against another (LE 4, 5, 7, 8, 10 and 11)

- choice and use, in context, of appropriate graphical displays using the same scale for comparing continuous data in two or more groups, such as back-to-back stemplots and adjacent boxplots
  (LE 5, 7, 8 and 11)
- use of sample mean, mode and median as descriptors of central tendency (LE 6, 7, 9 and 11)
- use of sample standard deviation, range and interquartile distance as descriptors of spread
  (LE 6, 7, 9 and 11)
- description of key features of data with reference to suitable selections of graphical and tabular displays (LE 4, 5, 7–9 and 11).

Learning experiences

The following learning experiences may be developed.

1. Critique a survey to collect data relevant to social questions of interest to candidates, such as study patterns, TV and film preferences and habits; consider variables included in the survey, and how they have been recorded (for example, should age be recorded as a continuous variable or grouped to give age categories?); rephrase and rewrite survey questions.
2. Look for leading or misleading questions in a survey.
3. Analyse an observational study, such as:
   - a traffic study
   - prices of different brands and/or at different outlets
   - use of public phones.

Consider variables and methods included to increase data quality.

4. Analyse graphical methods, for example, those used by weather forecasters to predict daily temperatures and rainfall patterns.
5. Analyse material presented in publications, for example, yearbooks, and reports from the Australian Bureau of Statistics.
6. Identify the effect of different sampling situations in pursuit of a random sample, for example, a Gallup poll compared with a phone-in poll.
7. Organise a set of real data into an understandable form using a variety of approaches, such as summary statistics and graphical displays.
8. Use graphical displays on the same scale to give an effective visual comparison between two or more datasets, and comment on general comparative features, making allowance for variation.
9. Examine the use of summary statistics in, for example, newspapers, articles, TV programs such as weather reports and advertisements, and government reports.
10. Construct and/or interpret a scatterplot, understanding the difference between plots of a number of variables, such as prices over time, and plots of one variable against another, such as daily prices of a commodity (for example, petrol) at two outlets.
11. Given a set of data, produce a concise summary of the main information in the data, referring to graphical displays and summary statistics.
12. Identify examples of misleading and/or ambiguous graphical displays.

Exploring and understanding data (notional time 20–25 hours)

Focus

Candidates should develop a working knowledge of some elementary concepts in exploring and interpreting data, and they are encouraged to develop skills in recognising different types of data situations. Accordingly, candidates will become aware of some fundamental concepts used in models for data, including the roles of probability and estimating probabilities. It is expected that calculators will be used routinely. The emphasis of this topic is on exploration, not inference, and should be studied from the point of view of the recipient rather than the presenter, through a range of life-related scenarios.

Subject matter

- suitability of a sample as representative of a population (LE 4)
- interpretation and use of sample statistics as estimates of population parameters such as sample mean and standard deviation (LE 1 – 3)
- interpretation and use of relative frequencies to estimate probabilities of individual values for discrete variables (including categories) and of intervals for continuous variables (LE 5, 6, 8 and 9)
- interpretation and use of probability as a measure of chance in a range of life-related situations
  (LE 5, 6, 8 and 9)
- interpretation in context of row and column percentages for a contingency table (two-way table of frequencies) (LE 7)
- misuse of probabilities, including misinterpretation of row and column percentages in contingency tables (LE 6–8)
- use and misuse of sample mean, mode and median as descriptors of central tendency
- use and misuse of sample standard deviation, range and interquartile distance as descriptors of spread
- interpolation, extrapolation and interpretation relationships in modelling data, using a line of best fit drawn by eye (LE 10).

Learning experiences

The following learning experiences may be developed.

1. Compare two sets of data using various statistics including the range, mean, median, standard deviation and interquartile distance.
2. Examine reports — for example, the Real Estate Institute of Queensland's (R.E.I.Q.) report of house prices in different areas — and explain their choice of measure of central tendency.
3. Compare the effects of an outlier on various summary statistics.
4. Explore different sampling situations and possible difficulties and sources of bias, e.g. poor questionnaire design, a lack of random sampling, or practical difficulties such as influence from the survey interviewer.
5. Estimate probabilities using relative frequency in simple life-related situations.

6. Identify words used in English as expressions of probability (for example, *unlikely*, *certain*, *impossible*, *good chance*) and place these on a scale from zero to one; identify everyday events which fit into these descriptions (for example, it is *unlikely* that it will snow in Brisbane in December; there is a *good chance* that there will be at least one new movie released during the school holidays).
7. Analyse responses to pairs of questions on a survey, with numbers of types of responses summarised in a two-way table (a contingency table). For example, analyse the responses to a question on approving daylight saving, organised according to gender or longitude; interpret row and column percentages.
8. Identify and use discrete variables to estimate probabilities from data; for example, the number of girls in families of two or three children, the number of pets in a family.
9. Identify and use continuous variables to estimate probabilities of intervals of values from data, for example, maximum temperatures, travelling times, pulse rates.
10. Discuss the suitability and use of, as well as influencing factors on, straight-line relationships between variables such as weight and height, running times and distance, income and education level, daily temperature and degrees from the equator.

Operations research — networks and queuing

(notional time 30 hours)

Focus

Candidates should develop a working knowledge of the procedures involved in two types of operations research — networks and queuing. Candidates should become familiar with the shortest-path and minimum-spanning-tree algorithms and, in the applications of these techniques, with modelling and solving life-related problems that involve networks. They should also develop an intuitive understanding of simple critical-path analysis. Candidates should become familiar with the graphical or tabular methods involved in investigating queuing problems that occur in a variety of life-related situations.

Subject matter

- network terminology including *node*, *branch*, *path* and *tree* (LE 1)
- shortest-path algorithm (LE 1, 2 and 6)
- minimum-spanning-tree algorithm (LE 1and 3)
- critical-path analysis involving simple scenarios (LE 1 and 4)
- slack time in a critical-path analysis (LE 1 and 4)
- single and multiple server queues with constant arrival and service times (LE 5)
- the effects on a queuing system of random arrival and service times (LE 5).

Learning experiences

The following learning experiences may be developed.

1. Consider a life-related problem expressed in everyday language, formulate it into a network problem, and solve and interpret the solutions in terms of the original problem.
2. Consider a road map showing two places connected by a network of roads and

find the shortest path between them.

3. A complicated rail system connecting a number of towns is to be rationalised; investigate which lines should be removed so that a minimum amount of line will be left to service all the towns.

4. Consider the times taken for different activities involved in the following problems; determine the critical steps and consider the effects of changing the time taken for certain activities on the critical path:
   - constructing a house
   - using a recipe
   - planning a 21st birthday party.

5. Using tabular or graphical methods, investigate the percentage of time for which a supermarket server is idle. Consider the average waiting time for a supermarket queue with a single server or multiple servers and constant/random arrival and service times. Determine the first time the queue becomes empty.

6. A sales representative is to visit a series of stores; determine the shortest paths the sales representative can take from home to each of the stores.

6. Assessment

6.1 Summative assessment

Candidates will be assessed according to the criteria set out below. The examination will consist of two papers, each of up to three hours working time plus 10 minutes perusal/planning time.

6.1.1 Format of the external examination

The two papers will each assess the three criteria and include all four contexts. Both papers will have a range of questions from selected topics that together sample the course. All topics will be covered and some integration of topics will be included.

Each year, the chief examiner will confirm the arrangements for each examination paper. The chief examiner will provide advice to candidates about additional conditions or equipment, materials, required readings, or the like, that will be required for the examination in a particular year.

Use of technology in the examination

During the examination the noise of operating a calculator must not disturb other candidates. Candidates should not rely only on the use of technology to obtain a result. To meet the requirements of the criterion *Communication and justification* candidates should show full working methods for each question.

6.2 Exit criteria

Within the contexts of Application, Technology and Complexity the following three criteria will be applied when making judgments about levels of achievement.

Criterion 1: Knowledge and procedures

This criterion includes:

- recalling definitions and rules
- accessing and applying rules
- demonstrating number and spatial sense
- demonstrating an ability to use technologies such as calculators, measuring instruments, geometrical drawing instruments and tables
- recalling, selecting and using appropriate mathematical procedures
- applying mathematical procedures to situations that are similar to learning experiences outlined in the syllabus
- working accurately and manipulating simple formulae
- recognising that some tasks may be broken up into smaller components.

Criterion 2: Modelling and problem solving

This criterion includes:

- understanding that a mathematical model is a mathematical representation of a situation
- interpreting, clarifying and analysing problems
- identifying the variables of a simple mathematical model of a situation
- forming and/or selecting a mathematical model of a life-related situation
- deriving results from consideration of mathematical models
- interpreting results from a mathematical model in terms of the given situation
- recognising the strengths and limitations of mathematical models
- developing strategies in modelling
- using a range of problem-solving strategies such as estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data
- understanding that there may be more than one way to solve a problem
- selecting the appropriate mathematical procedures required to explore, investigate and solve problems
- developing a solution consistent with the problem
- investigating open-ended situations
- reflecting on conjectures and making modifications if needed
- making decisions from a range of choices.

Criterion 3: Communication and justification

Communication

This includes:

- organising and presenting information
- communicating ideas, information and results
- using mathematical terms and symbols accurately
- using correct spelling, punctuation and grammar in written communication
- understanding material in a variety of forms such as, written, symbolic, pictorial and graphical
- translating material from one form to another
- presenting material for different audiences, in a variety of forms (such as, written, symbolic, pictorial and graphical)
- recognising necessary distinctions in the meanings of words and phrases according to whether they are used in a mathematical or non-mathematical situation.

Justification

This includes:

- developing logical arguments expressed in everyday language, mathematical language or a combination of both, as required, to support conclusions, results and/or propositions

- evaluating the validity of arguments designed to convince others of the truth of propositions
- justifying procedures by
  - providing evidence (words, diagrams, symbols etc.) to support processes used
  - stating a generic formula before using it
  - providing a reasoned, well-formed, logical sequence within a response
- recognising when and why results of a given problem are clearly improbable or unreasonable
- using supporting arguments to justify results obtained by calculator.

6.3 Special consideration

Under certain circumstances, special arrangements or consideration may be available to candidates for the Senior External Examination. The special consideration provisions are detailed in the annual Handbook for the Senior External Examination, available on the QSA website at www.qsa.qld.edu.au/testing/extern-exams/handbook.html.

Missing an examination for any reason cannot be the basis for an application for special consideration.

6.4 Awarding levels of achievement

The chief examiner will award each candidate who sits the examination a level of achievement from one of the five categories:

- Very High Achievement (VHA)
- High Achievement (HA)
- Sound Achievement (SA)
- Limited Achievement (LA)
- Very Limited Achievement (VLA).

The process of arriving at a judgment about a candidate's responses to examination questions is essentially a process of matching the candidate's responses against the syllabus standards associated with exit criteria. A level of achievement that best describes the pattern of performance in each criterion across the examination as a whole is then awarded.

Information about how scripts are assessed is provided in the annual Handbook for the Senior External Examination, available on the QSA website www.qsa.qld.edu.au/testing/extern-exams/.

The level of achievement will be based on the exit standard for each of the three criteria *Knowledge and procedures*, *Modelling and problem solving*, and *Communication and justification*. The criteria are derived from the general objectives and are described in section 3. The standards associated with the three exit criteria are described in Table 2.

When standards have been determined for each of the three criteria, the following table is used to determine the level of achievement, where *A* represents the highest standard and *E* the lowest.

Table 1

VHA	The candidate must achieve a Standard *A* in any two exit criteria and no less than a Standard *B* in the remaining criterion
HA	The candidate must achieve Standard *B* in any two exit criteria and no less than a Standard *C* in the remaining criterion
SA	The candidate must achieve Standard *C* in any two exit criteria, one of which must be the *Knowledge and procedures* criterion and no less than a Standard *D* in the remaining criterion
LA	The candidate must achieve Standard *D* in any two exit criteria, one of which must be the *Knowledge and procedures* criterion
VLA	The candidate must achieve Standard *E* in at least two exit criteria

Table 2: Standards associated with exit criteria

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: *Knowledge and procedures*	The **overall quality** of a candidate's achievement across the full range within the contexts of Application, Technology and Complexity, and across topics, **consistently demonstrates**: • accurate recall, selection and use of definitions and rules • use of technology • recall and selection of procedures, and their accurate and proficient use.	The **overall quality** of a candidate's achievement across a range within the contexts of Application, Technology and Complexity, and across topics, **generally demonstrates**: • accurate recall, selection and use of definitions and rules • use of technology • recall and selection of procedures, and their accurate use.	The **overall quality** of a candidate's achievement in the contexts of Application, Technology and Complexity, **generally demonstrates**: • accurate recall and use of basic definitions and rules • use of some technology • accurate use of basic procedures.	The **overall quality** of a candidate's achievement in the contexts of Application, Technology and Complexity, sometimes **demonstrates**: • accurate recall and use of some definitions and rules • use of some technology.	The **overall quality** of a candidate's achievement rarely **demonstrates** knowledge and use of procedures.

[This table continues on the next two pages.]

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: *Modeling and problem solving*	The **overall quality** of a candidate's achievement across the full range within each context, and across topics **generally demonstrates** mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations, and identifying variables • selecting and using effective strategies • informed decision making …*and* sometimes **demonstrates** mathematical thinking which includes: • selecting and using procedures to solve a wide range of problems • initiative in exploring the problem • recognising strengths and limitations of models.	The **overall quality** of a candidate's achievement across a range within each context, and across topics, **generally demonstrates** mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations, and identifying variables • selecting and using strategies …*and* sometimes **demonstrates** mathematical thinking which includes: • selecting and using procedures required to solve a range of problems • informed decision making.	The **overall quality** of a candidate's achievement **demonstrates** mathematical thinking which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures.	The **overall quality** of a candidate's achievement sometimes **demonstrates** mathematical thinking which includes following basic procedures and/or using strategies.	The **overall quality** of a candidate's achievement rarely **demonstrates** mathematical thinking which includes following basic procedures and/or using strategies.

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: *Communication and justification*	The **overall quality** of a candidate's achievement across the full range within each context **consistently demonstrates**: • accurate use of mathematical terms and symbols • accurate use of language • organisation of information into various forms suitable for a given use • use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or decisions • justification of procedures.	The **overall quality** of a candidate's achievement across a range within each context **generally demonstrates**: • accurate use of mathematical terms and symbols • accurate use of language • organisation of information into various forms suitable for a given use • use of mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or decisions.	The **overall quality** of a candidate's achievement in some contexts **generally demonstrates**: • accurate use of basic mathematical terms and symbols • accurate use of basic language • organisation of information into various forms • use of some mathematical reasoning to develop simple logical arguments.	The **overall quality** of a candidate's achievement sometimes **demonstrates** evidence of the use of the **basic** conventions of language and mathematics.	The **overall quality** of a candidate's achievement rarely demonstrates use of the **basic** conventions of language or mathematics.

Contexts are explained in section 3.2.

7. Resources

7.1 QSA website

The QSA website provides essential resources for all candidates for the Senior External Examination. The website address is www.qsa.qld.edu.au/testing/extern-exams/index.html or go to www.qsa.qld.edu.au > Testing > Senior External Examination.

The following information (current at time of printing) is available:

Senior External Examination Handbook:

- the handbook gives information about
  - how to nominate to sit the examinations
  - teaching centres that provide tuition for the subjects
  - examination timetable
  - important dates relating to the Senior External Examination.

Subject resources

The syllabus and examination papers for the previous three years are available.

Notices to candidates

Information is provided by chief examiners to help candidates prepare for the examination.

Notices to teaching centres

Information is provided by chief examiners to help tutors and candidates prepare for the examination.

7.2 Textbooks and other resources

Various textbooks and resource materials can be used to supplement study in Mathematics A. Book suppliers can provide detailed information about new publications. Some suggestions are:

Elms, L. Simpson, N and Macpherson, T 2002 *MathsQuest Year 12 Maths A*, Jacaranda Press (John Wiley & Australia, Ltd): ISBN 0 7016 36254.

Elms, L & Simpson, N 2001 *MathsQuest Year 11 Maths A*, Jacaranda Press (John Wiley & Australia, Ltd): ISBN 0 7016 362 46.

Goodman, H & N 1995 *Future Maths 11A*, McGraw Hill Book Company, Roseville, NSW.

Goodman, H & N 1995 *Future Maths 12A*, McGraw Hill Book Company, Roseville, NSW.

Appendix 1

Basic knowledge and procedures

Basic mathematics

The following skills will be required:

- calculation and estimation, including:
  - basic operations
  - powers
  - roots.

This could involve numbers in various notations, including:

- integer
- fraction
- decimal
- standard (scientific)
- metric measurement of
  - length
  - area
  - volume
  - mass
- rates, percentages, ratio and proportion
- simple algebraic manipulations
- graphs and tables.

Maintaining mathematical procedures

The following learning experiences are included as suggestions.

Suggested learning experiences

- Solve a new problem requiring previously learned skills.
- Work without a calculator for a time.
- Design some questions to help others revise a topic.
- Each member of a group writes a summary of a topic, then all share the summaries.

Appendix 2

Explanation of some terms

Cadastral

Pertaining to maps or surveys which primarily describe artificial boundaries, for example, property boundaries, as well as prominent natural features.

Categorical data

Data which are qualitative rather than quantitative; the observations represent categories rather than measurements.

Contingency table

A table that contains the frequencies arising from the cross-tabulation of data consisting of two or more qualitative variables.

Data (and variables)

Can be of two broad types: discrete and continuous:

- **Discrete variables** take individual or separated values.

  Discrete variables are further divided into categorical (or nominal), ordinal, and count.
  - *Categorical (or nominal) variables* have categories. Each observation can be assigned to one of the categories. The categories can be described in words or by arbitrarily assigned codes — for example, male, female — which could have numerical codes assigned to them. Numerical codes assigned to the categories or classes of a categorical variable have no meaning in themselves; they are nothing other than labels.
  - *Ordinal variables* are categorical variables with coded categories for which the order of the codes matters but not their actual values, for example 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, 5 = strongly agree.
  - *Count variables* count a number of items or people. If a count variable can take a wide range of very large numbers, the methods of continuous variables are often used for it. For example, traffic flow at a city location would be in terms of number of vehicles per fixed time interval, but if it involves counts in the thousands or even hundreds, it would tend to be treated as continuous, hence the term "flow".
- **Continuous variables** take values from a "continuum" or in intervals, and can take any real value, for example, time, distance, height, weight. Their measurement almost always involves at least some form of rounding, and observations are often quoted as correct to a certain number of decimal places, or "to the nearest … ". Amounts of money are also usually treated as continuous variables because there is a sense in which they involve rounding, but also because the scale of measurement is extensive and very fine — it is possible to use a very small unit of measurement.

Dead reckoning

The process of determining position using calculated information.

Five-number summary

The combination of the *median* (Q_2), the *lower* and *upper quartiles* (Q_1 and Q_3), and the two extreme values (the *minimum* and the *maximum*).

Frequency plot

A diagrammatic presentation of the frequency distribution of the observations, for example a bar chart, a pie chart, a histogram, a frequency polygon or an ogive.

Inference

Conclusions made about a population from the observations in a sample taken from that population.

Mathematical model

Any representation of a situation that is expressed in mathematical terms; it should be noted that models may be as simple as expressing simple interest as $I = \frac{Prt}{100}$ or showing the relationship between two variables as a scattergram.

Median boxplot (box-and-whisker plot)

A graphical presentation of some main features of a dataset.

The simplest version of a box-and-whisker plot is formed by drawing a box extending from the lower to the upper quartiles, marking the median within that box, and drawing lines (called whiskers) from the box to the smallest and largest data points.

There are slight variations in the possible ways of identifying the median and quartiles of data: these variations make very little difference except for small or sparse datasets. A very sound and popular method enables the data to be divided into four groups with exactly the same number of observations in each group. The technique is illustrated in the following example:

Example: Consider the following 18 systolic blood pressures (bp)

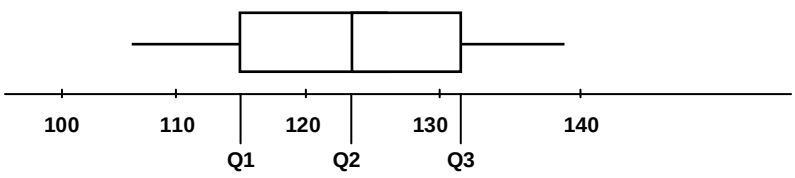
110, 130, 108, 125, 111, 122, 126, 119, 114,

134, 120, 132, 134, 130, 107, 137, 120, 136

These data are ordered and numbered from the smallest to largest below.

x:	107	108	110	111	114	119	120	120	122	125	126	130	130	132	134	134	136	137
order:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

The median (Q2) is taken to be 123.5. The lower quartile (Q1) is the median of the lower 9 observations, viz. 114, and the upper quartile (Q3) is the median of the upper 9 observations, viz. 132. Thus there are exactly four observations below Q1, between Q1 and Q2, between Q2 and Q3, and above Q3.



With this technique, datasets with 16, 17, 18 or 19 observations all have exactly four observations below Q1, above Q3 and between Q1, Q2, Q3.

Note 1: A more informative version of the boxplot, particularly with larger datasets, that is also often provided by statistical computer packages, takes the whiskers out to the last data points within a certain distance of the quartiles and then marks individual data points beyond the whiskers.

Note 2: A boxplot can be presented vertically or horizontally.

Minimum spanning tree

The minimum spanning tree of a network is a collection of arcs which:

- is connected
- has no loops
- covers all nodes
- is of total minimum length.

Operations research

A systematic application of quantitative methods, techniques and tools to solve problems.

Outlier

An extreme value in the observations, for example, an observation which lies beyond the box in the box-and-whisker plot, or a point which is well away from the line of best fit.

Reverse bearing

The bearing of a line is the angle it makes, measured in a clockwise direction, with the meridian, or north–south line. Every line has two bearings according to the direction in which it is regarded. For example, if the bearing of a line from A to B is 45 degrees, its *reverse bearing*, from B to A, will be 225 degrees.

Running fix

A measure of position, taken while in motion, with reference to a fixed point.

Stemplot (stem-and-leaf plot)

An exploratory technique that simultaneously ranks the data and gives an idea of the distribution.

Example: The following 16 average daily temperatures have been recorded to the nearest degree Celsius:

31	21	35	30	22	23	9	24
13	41	30	21	29	24	18	28

Preliminary stem-and-leaf plot of the temperatures:

Example: 2 | 1 represents 21

Unit = 1

0	9
1	3, 8
2	1, 2, 3, 4, 1, 9, 4, 8
3	0, 0, 1, 5
4	1

Final stem-and-leaf plot of the temperatures:

Unit = 1

0	9
1	3, 8
2	1, 1, 2, 3, 4, 4, 8, 9
3	0, 0, 1, 5
4	1

Summary statistics

Characteristics that describe the sample of observations, for example, the mean, median or standard deviation.

Variation

The way in which the observations differ (vary) from each other, often measured by the standard deviation or range.

Appendix 3

Further learning experiences

Topic: Managing money 1

Prepare a budget for a holiday in Australia or overseas.

Simulate the financial experiences of candidates immediately after they complete their studies.

Survey ways in which incomes are derived.

Reconcile a personal financial account, for example, a credit card account or a cheque account.

Investigate the effect of income and family size on family allowance payments.

Investigate the effects of candidates' part-time employment and parents' incomes on youth allowances.

Investigate the costs — rent, phone, insurance, electricity and groceries — involved in living in a flat.

Topic: Managing money 2

Compute present values for superannuation and life insurance lump-sum payments.

Investigate the benefits of making repayments weekly instead of monthly.

Convert the bankcard daily interest rate to a yearly interest rate and compare it with the quoted annual rate.

Investigate the efficient use of credit cards.

Topic: Navigation

Research various navigational techniques (for example, dead reckoning, use of sextant and chronometer, horizontal sextant, doubling the angle on the bow, and satellite navigation systems).

Topic: Linking two and three dimensions

Prepare a quote for the construction of a simple shed, including slab, footings, walls and roof.

Investigate house plans and design packages available from real estate firms or building contractors.

Topic: Navigation

Consider the different problems that arise according to whether one is navigating on land, in the air, or at sea (within sight of land or with no land visibility).