Preparing for the QCS Test

Sample questions that draw on assumed mathematical knowledge

The following is a 'sample bag' of questions that draw on assumed mathematical knowledge of students sitting the Queensland Core Skills (QCS) Test. The answers are given at the end.

Percentage

1. 87.6% of Queensland's people live less than 50 km from the coast. If the population of Queensland is 4.7 million, how many people live 50 or more km from the coast?
2. The present Queensland population represents an increase of 5.2% on its population three years ago. What was Queensland's population three years ago?
3. The population of Cairns and its region is 297 000. What percentage of Queensland's population lives in Cairns and its region?
4. It is predicted that the population of Cairns and its region will increase by 22% over the next ten years. What will the population of Cairns and its region be in ten years' time?

Time

This table shows the 24-hour clock times at which the sun and moon rose and set over Gladstone on 5 June 2013.

	TIME
Sunrise	0632
Sunset	1716
Moonrise	0318
Moonset	1449

1. For how long, in hours and minutes, was the sun in the sky at Gladstone?
2. For how long were both the sun and the moon visible in the sky at Gladstone?
3. For what percentage of 5 June was the sun visible over Gladstone but not the moon?
4. In Cloncurry, the moon rises 41 minutes later than it does in Gladstone but stays in the sky seven minutes longer. On 5 June, at what time did the moon set in Cloncurry?

Algebraic procedures I

1a. A simple number game involves asking a person to think of a number, add five and then double the result. If the number chosen is symbolised by x, give an expression which indicates the value after these two steps are performed.

1b. The game then asks the person to subtract the number they first thought of. What is required in the fourth step of the trick if the idea is to return to the number the person originally chose?

2. Substitute the values $x = -5$ and $y = 4$ into the following expressions and then rank them in order of size beginning with the biggest.

   A. $x - y$ B. $\sqrt{y} - x^2$ C. $2 - xy$ D. $2xy$

3. The probability of a pyramid landing on its base when it is thrown like a die depends on its height (h) and the side-length of its square base (b).

   This probability (P) is given by the formula $P = \dfrac{13(2b - h)}{50b}$

   If the side-length of the square base is 7cm, what must the vertical height be if we want $P = 0.2$?

Rates and ratio

1. A poll of 600 voters finds that 285 of the people surveyed intend to vote for the Arty Party at the next election, 195 intend to vote for the Nasty Party and the rest will vote for the Birthday Party. If 21 000 people vote in the next election, how many can be expected to vote for the Birthday Party?

2. A biologist checks the health of a swamp in terms of biodiversity by counting some of the species of frogs each year. During 2009, a chemical spill near the swamp affected the whole population of frogs quite badly.

Year	Green Frogs	Brown Frogs	Grey Frogs
2008	120	160	70
2009	80	115	50

   Judging by the statistics in this table, which species of frog was the worst affected by the chemical spill?

3. At the Athens Olympics, cyclist Katie Mactier set an Australian record for the 3000 m individual pursuit with a time of 3 minutes 27.65 seconds. What was her average speed for this race in kilometres per hour?

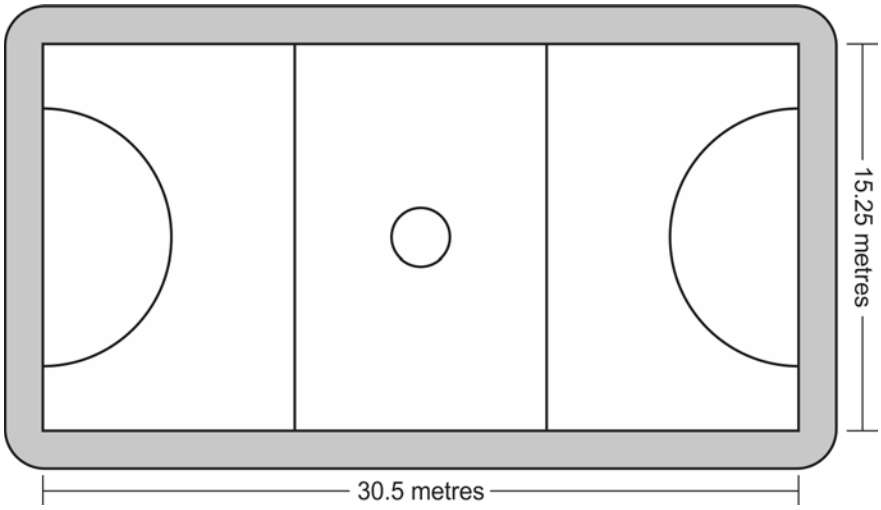
4. A recipe for pancake mixture gives the required ingredients as 150 g of plain flour, one egg and 250 mL of milk. This gives enough mixture for five pancakes. If all you have is a kilogram of flour, a dozen eggs and two litres of milk, how many pancakes can you make following this recipe and how much of each ingredient will be left over?

Algebraic procedures II — measurement questions

Formulas which may be useful in the questions below are:

$A = \frac{1}{2}bh$ $c^2 = a^2 + b^2$ $A = lw$ $V = \pi r^2 h$ $P = 2l + 2w$

1. What is the area of a standard netball court as shown in this diagram?
2. At training, a coach makes her players run sprints from one corner of this court to the opposite corner. How long is each sprint?



3. A painted umpires' track 1.5 m wide surrounds the court and this is shown shaded in the diagram. It helps to keep spectators from coming too close. What is the approximate area of the umpires' track?
4. Each game day, the coach makes up 10 litres of cold sports drink in a cylindrical container 40 cm high. The space between the top of the container and level of the drink is 8 cm. What is the diameter of the container?

Power of ten notation

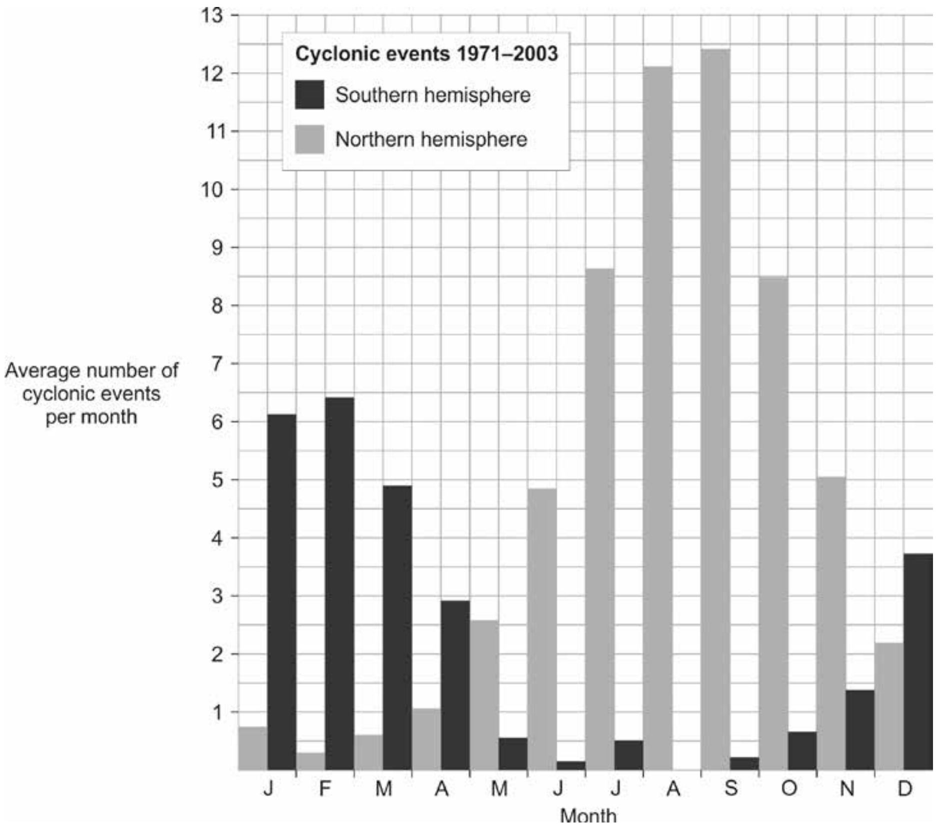
1. The distance from Earth to the sun is about 150 million km. A light year is the distance travelled by light in one year, about 9.46×10^{12} km. How far is it from Earth to the sun in light years? Give your answer as an ordinary number.
2. The distance to the ninth-nearest star, Ross 248, is currently 97 639 000 000 000 km. How many light years away is Ross 248?
3. Over the next 36 000 years, Ross 248 will move towards us to become our closest star, 4.22 light years away. At what average speed, in km/year, will Ross 248 move towards us over this period? Give your answer using power-of-ten notation.
4. The average mass of a centimetre of human hair is 5.4×10^{-5} grams. Blonde people have about 110 000 strands of hair on their scalps. What would be the weight of hair on the head of a blonde person whose average length per strand is 12 cm? Give your answer in kilograms.

Measurement practice

Construct, using a drawing compass, a pair of concentric circles with radii 2 cm and 4 cm. Draw from the centre a radius of the larger circle. At the centre, draw a radius of the smaller circle at an angle of 108° to this line. Join the points on the circle to form a triangle. Measure all the sides of the triangle and all the angles in its corners.

Graphs

In the northern hemisphere, tropical cyclones are called typhoons and hurricanes. Although both the number and severity of these storms vary from year to year, they do follow a pattern in terms of the time of year when they are most likely to arise. The graph below shows the average number of cyclonic events per month in the northern and southern hemispheres for the 33-year study period from 1971 to 2003.



1. In the northern hemisphere, during which month did the most cyclonic events arise? Approximately how many were reported in that month during the whole study period?
2. In which month were there, on average, about five times as many cyclonic events in one hemisphere as there were in the other?
3. In which month/s of the year does the average number of cyclonic events increase in both hemispheres?
4. A meteorologist made the observation: ‘For four consecutive months, the average number of cyclonic events globally stays fairly constant.’ Which four months was she referring to?

Worked answers to sample questions

Percentage

1 87.6% of Queensland's people live less than 50 km from the coast. If the population of Queensland is 4.7 million, how many people live 50 or more km from the coast?

87.6% of 4.7 million = 0.876 x 4 700 000 = 4 117 200 people live near the coast.
So all of the rest live 50 km or further from the coast.
This is 4 700 000 – 4 117 200 = 582 800.

Or

100 – 87.6 = 12.4% of people must live 50 km or further from the coast.
12.4% of 4 700 000 = 0.124 x 4 700 000 = 582 800.

2 The present Queensland population represents an increase of 5.2% on its population three years ago. What was Queensland's population three years ago?

Previous population (call this P) increased by 5.2% = 4 700 000 means that
P x 1.052 = 4 700 000. Divide both sides by 1.052 and $P \approx$ 4 467 681.

[Note that this is slightly, but importantly, different from what you get by decreasing 4.7 million by 5.2% ... 0.948 x 4 700 000 = 4 455 600].

3 The population of Cairns and its region is 297 000. What percentage of Queensland's population lives in Cairns and its region?

The fraction of Queensland's population living in and around Cairns is

$\frac{297\,000}{4\,700\,000} \approx 0.06319\ldots$ Multiply by 100 to convert this to percentage

This gives about 6.32%.

4 It is predicted that the population of Cairns and its region will increase by 22% over the next ten years. What will the population of Cairns and its region be in ten years' time?

Increase in population = 22% of 297 000 = 0.22 x 297 000 = 65 340.
So predicted population in ten years' time is 297 000 + 65 340 = 362 340.

Or

22% more than 1 is 1.22 so predicted population is 1.22 x 297 000 = 362 340.

Time

1 For how long, in hours and minutes, was the sun in the sky at Gladstone?

Sunrise till noon (1200) is 5 h 28 min. Noon till sunset is another 5 h 16 min, making the total time = 10 h 44 min.

2 For how long were both the sun and the moon visible in the sky at Gladstone?

Moonrise at 0318 means that as soon as the sun rose, both were in the sky. The moon set at 1449 (2:49 pm) ending this period. Hence the total time is
5 h 28 min (till noon) + another 2 h 49 min (after noon) = 7 h 77 min = 8 h 17 min.

3 For what percentage of 5 June was the sun visible over Gladstone but not the moon?

There are 11 minutes from 1449 (2:49 pm) to 1500 (3 pm) and another 2 h 16 min to 1716 (5:16 pm).
So 2 h 27 min = 147 min out of the day. 1 day = 24 x 60 = 1440 min so percentage required is (147÷1440) x 100 = 10.21%.

4 In Cloncurry, the moon rises 41 minutes later than it does in Gladstone but stays in the sky seven minutes longer. On 5 June, at what time did the moon set in Cloncurry?

In Cloncurry, moonset would have occurred (41 + 7) = 48 min later than at Gladstone.
i.e. at 1449 + 48 mins = 1497 = 1537 or 3:37 pm.

Algebraic procedures I

1a A simple number game involves asking a person to think of a number, add five and then double the result. If the number chosen is symbolised by *x*, give an expression which indicates the value after these two steps are performed.

$2(x + 5)$ or $2x + 10$.

1b The game then asks the person to subtract the number they first thought of. What is required in the fourth step of the trick if the idea is to return to the number the person originally chose?

Taking away the number first thought of gives $2x + 10 – x = x + 10$. So the fourth step needs to be to take ten away, leaving *x*.

2 Substitute the values $x = -5$ and $y = 4$ into the following expressions and then rank them in order of size beginning with the biggest.

A. -5 – 4 = -9

B. $\sqrt{4} – (-5)^2 = 2 – 25 = -23$

C. 2 – (-5 x 4) = 2 – (-20) = 2 + 20 = 22

D. 2 x -5 x 4 = -40.

So in descending order of size they are C, A, B, D.

3 **The probability of a pyramid landing on its base when it is thrown like a die depends on its height (*h*) and the side-length of its square base (*b*).**

This probability (*P*) is given by the formula $P = \dfrac{13(2b - h)}{50b}$

If the side-length of the square base is 7cm, what must the vertical height be if we want *P* = 0.2?

Substituting $b = 7$ and $P = 0.2$ in the equation gives $0.2 = \dfrac{13\ (2\text{x}7 - h)}{50\text{x}7}$

so $0.2 = \dfrac{13\ (14 - h)}{350}$ and $0.2 \times 350 = 182 - 13h$ which yields $70 = 182 - 13h$

Subtracting 182 from both sides gives $-112 = -13h$ and dividing by -13 gives a vertical height of 8.615… ≈ 8.62 cm.

Rates and ratio

1 **A poll of 600 voters finds that 285 of the people surveyed intend to vote for the Arty Party at the next election, 195 intend to vote for the Nasty Party and the rest will vote for the Birthday Party. If 21 000 people vote in the next election, how many can be expected to vote for the Birthday Party?**

Birthday Party supporters account for 600 – 285 – 195 = 120 voters out of 600.
This is one in every five, so at the election 21 000 ÷ 5 = 4200 can be expected to vote for the Birthday Party.

2. **Judging by the statistics in this table, which species of frog was the worst affected by the chemical spill?**

Every type of frog suffered a population decline so we compare the ratios of survivors.
80 out of 120 or 0.6667 for green frogs, 115 out of 160 or 0.71875 for brown frogs and 50 out of 70 or 0.714 for grey frogs. So the green frogs suffered the most severe decline in population.

3. **At the Athens Olympics, cyclist Katie Mactier set an Australian record for the 3000 m individual pursuit with a time of 3 minutes 27.65 seconds. What was her average speed for this race in kilometres per hour?**

3000 m in 3 min 27.65 sec means she travelled 3000 m in 180 + 27.65 = 207.65 sec. Her speed in metres per second was therefore 3000 ÷ 207.65 ≈ 14.447387…
An hour has 60 x 60 = 3600 seconds so her speed in metres per second is multiplied by 3600 to give 52 010.6 metres per hour. This converts to 52.01 km/h.

4. **A recipe for pancake mixture gives the required ingredients as 150 g of plain flour, one egg and 250 mL of milk. This gives enough mixture for five pancakes. If all you have is a kilogram of flour, a dozen eggs and two litres of milk, how many pancakes can you make following this recipe and how much of each ingredient will be left over?**

1 kg of flour gives 1000 ÷ 150 ≈ 6.6667… serves of flour. A dozen eggs is enough for 12 serves of pancake mix. Two litres of milk gives 2000 ÷ 250 = 8 serves of milk.
So a total of six pancake mixtures can be made which will be enough for 30 pancakes.
Six eggs will be left over as will 2000 – 6 x 250 = 2000 – 1500 = 500 ml of milk. There will also be 1000 – 900 =100 g of plain flour.

[Note that if two thirds of an egg can be measured out, however messily, then $6^2/_3$ mixtures can be made, leaving no flour, $333^1/_3$ ml of milk and $5^1/_3$ eggs. This will be enough for $6^2/_3$ x 5 = $33^1/_3$ pancakes.]

Algebraic procedures II — measurement questions

1. **What is the area of a standard netball court as shown in this diagram?**

   The court is a rectangle 15.25 x 30.5 m. So its area is 30.5 x 15.25 = 465.125 m^2
   [or about 465 m^2].

2. **At training, a coach makes her players run sprints from one corner of this court to the opposite corner. How long is each sprint?**

   The diagonal sprint forms the hypotenuse of a right-angled triangle. So Pythagoras' Rule is used. $c^2 = 30.5^2 + 15.25^2 = 930.25 + 232.5625 = 1162.8125$. Thus $c = \sqrt{1162.8125} = 34.1$ m.

3. **A painted umpires' track 1.5 m wide surrounds the court and this is shown shaded in the diagram. It helps to keep spectators from coming too close. What is the approximate area of the umpires' track?**

   The umpires' track is composed of two rectangles 30.5 x 1.5 m, two rectangles 15.25 x 1.5m and four quarter-circles at the corners which form a full circle of radius 1.5 m. So the track's area is $2 \times 30.5 \times 1.5 + 2 \times 15.25 \times 1.5 + \pi \times 1.5^2 \approx 91.5 + 45.75 + 7.069 \approx 144.3$ m^2.

4. **Each game day, the coach makes up 10 litres of cold sports drink in a cylindrical container 40 cm high. The space between the top of the container and level of the drink is 8 cm. What is the diameter of the container?**

   The depth of drink in the container is 40 – 8 = 32 cm. The drink forms a cylinder of volume 10 x 1000 = 10 000 cm^3 and height 32 cm. The formula for the volume of a cylinder is $V = \pi r^2 h$ and substituting V = 10 000 and h = 32 into this gives
   $10\,000 = \pi \times r^2 \times 32$. Dividing both sides by 32π yields $99.4718\ldots \approx r^2$.
   So $r \approx \sqrt{99.472} \approx 9.9735\ldots$ So the container's diameter is 2 x 9.9735… ≈ 19.95 or approximately 20 cm.

Power of ten notation

1. **The distance from Earth to the sun is about 150 million km. A light year is the distance travelled by light in one year, about 9.46×10^{12} km. How far is it from Earth to the sun in light years? Give your answer as an ordinary number.**

   $(1.5 \times 10^8) \div (9.46 \times 10^{12}) \approx 1.5856\ldots \times 10^{-5} \approx 0.0000159$ light years from Earth to the sun.

2. **The distance to the ninth-nearest star, Ross 248, is currently 97 639 000 000 000 km. How many light years away is Ross 248?**

   $(9.7639 \times 10^{13}) \div (9.46 \times 10^{12}) \approx 10.32$ light years from *Ross 248* to Earth.

3. **Over the next 36 000 years, Ross 248 will move towards us to become our closest star, 4.22 light years away. At what average speed, in km/year, will Ross 248 move towards us over this period? Give your answer using power-of-ten notation.**

   Ross 248 will travel 97 639 000 000 000 – 4.22 x (9.46 x 10^{12}) = 5.77178 x 10^{13} km in 36 000 years. This is equivalent to 1 603 272 222 ≈ 1.603 x 10^{9} km/year.

4. **The average mass of a centimetre of human hair is 5.4 x 10^{-5} grams. Blonde people have about 110 000 strands of hair on their scalps. What would be the weight of hair on the head of a blonde person whose average length per strand is 12 cm? Give your answer in kilograms.**

   Each strand would weigh 12 x (5.4 x 10^{-5}) grams = 6.48 x 10^{-4} grams. 110 000 of these would weigh a total of 71.28 grams = 0.07128 kg.

Measurement practice

The triangle you constructed should have sides 2 cm, 4 cm and 5 cm long. The angles should be 108°, 22° and 50°. The closer your answers are to these figures, the better your measurement skills are. More than 2mm or 2° out either way is not so good.

Graphs

1 **In the northern hemisphere, during which month did the most cyclonic events arise? Approximately how many were reported in that month during the whole study period?**

In September, there were 12.4 cyclonic events on average. Over 33 years, this would mean 33 x 12.4 = 409.2 or about 409 events.

2 **In which month were there, on average, about five times as many cyclonic events in one hemisphere as there were in the other?**

Months where the disparity between the hemispheres is extreme can be discounted, i.e. June to October and February. This leaves January (>6 to 0.75), March (<5 to 0.6), April (1 to <3), May (2.5 to 0.5), November (5 to 1.4) and December (2.7 to 3.7).

So the ratios for January and March are too large, and those for April, November and December are too small. May is about 5:1.

3 **In which month/s of the year does the average number of cyclonic events increase in both hemispheres?**

Both hemispheres' columns for July are taller than for June. Also when September is compared to August. Hence, July and September.

4 **A meteorologist made the observation: 'For four consecutive months, the average number of cyclonic events globally stays fairly constant.' Which four months was she referring to?**

The approximate monthly totals are {7, 7, 5.5, 4, 3, 5, 9, 12, 12.5, 9, 7, 6} so no four months within the calendar year really warrant this comment. But taken cyclically, the months of November, December, January and February {7, 6, 7, 7} exhibit the kind of stability the meteorologist was likely referring to.