# Assessment highlights 2021 

## General Mathematics

## Internal assessment 1

Problem-solving and modelling task

## Ronin Starmer

## Centenary State High School

## Assessment overview

## Context

The context for this problem-solving and modelling task was based on the premise that speeding is one of the major contributing factors to injuries and fatalities on Queensland roads and that driving a vehicle within the speed limit reduces stopping distance.

Students were provided with access to statistical information about driving speed, reaction distance and braking distance. They were also given links to online simulators to collect extra test results.

The syllabus conditions require a written response of up to 10 pages (including tables, figures and diagrams) and a maximum of 2000 words. The appendixes can include raw data, repeated calculations, evidence of authentication and student notes. However, any information provided in the appendixes is not included in the word count and is not marked.

## Task

Students were asked to investigate the relationship between driving speeds and stopping distances in two different locations.

In a written report, students were required to:

- verify and evaluate a model of the relationship between driving speeds and stopping distances in two locations
- make recommendations for safe following distances and speeds in the two identified locations
- demonstrate understanding of subject matter from Unit 3, Topic 1: Bivariate data analysis
- demonstrate the use of technology, analytic procedures and the syllabus problem-solving and mathematical modelling approach to develop their response.


## Student response

Note: The following sample is an unedited authentic student response produced with permission. Any images or sources that do not have copyright approval have been redacted from the response. The response may contain errors and/or omissions that do not affect its overall match to the characteristics indicated in the top performance levels of the instrument-specific marking guide (ISMG).

## Table of Contents

1. Introduction ..... 2
1.1 Observations ..... 2
1.2 Assumptions .....  .2
2. Mathematical Concepts and Results .....  3
2.1 Raw data .....  3
2.2 Development of scatterplots and linear regression model .....  3
2.2.1 Built-up area - Dry conditions ..... 3
2.2.2 Motorway - Dry conditions .....
2.2.3 Built-up area - Wet conditions .....  7
2.2.4 Motorway - Wet conditions. ..... 8
3. Discussion ..... 9
4. Evaluation ..... 10
5. Conclusion ..... 11
6. Appendix ..... 11
7. Bibliography ..... 12

## 1. Introduction

Due to an increasing number of speeding and road incidents in Australia, the urgent need for quantitative recommendations for safe driving speeds and following distances has become significantly prevalent (Queensland Government, 2018). Therefore, utilising the Least Squares regression method and residual plot analysis, this PSMT will investigate the relationship between driving speed and stopping distance at various speeds in various conditions and driving areas. By developing a linear regression model for the data, a relevant recommendation for safe driving speeds and following distances in wet and dry driving conditions in built-up areas and motorways will be deduced and evaluated in this report.

### 1.1 Observations

- Insufficient sample space reduces the reliability of research and increases the margin of error (Deziel, 2018).
- An increase in stopping distance is directly related to an increase in Driving Speed (Road Safety Commission, 2020).
- External factors such as driver impairment, vehicle type, tires and the road type can additionally affect the stopping distance.
- Driving Speeds within $40-60 \mathrm{~km} / \mathrm{h}$ is considered to be the speed of a built-up area (Queensland Government, 2020).
- Speeds within 60-100 km/h are considered to be the speeds of motorway areas (Queensland Government, 2020).
- Due to a decreased friction, stopping distance in wet weather is increased (Queensland Government, 2020)
- The safest stopping distance in dry conditions is 2 seconds (Queensland Government, 2020).
- The safest stopping distance in wet weather is 4 seconds to allow for appropriate reaction (Queensland Government, 2020).
- 


### 1.2 Assumptions

- Excel will be utilised as the main graphing tool.
- Driving speed will be the Explanatory variable ( x ) while stopping distance is the response variable (y).
- 20 data points, as a representative sample of $1 / 5$ of 100 for driving speed, reaction distance and braking distance for each roadway type and condition will be collected via the website 'It's a drag' to provide continuity in the statistics determined (Education Services Australia, 2016).
- Stopping distances for built-up areas will be calculated within the driving speed of $40-60 \mathrm{~km} / \mathrm{h}$ to represent driving in this area.
- Stopping distances for motorways will be calculated within the driving speed of $80-100 \mathrm{~km} / \mathrm{h}$ to represent driving in this area.
- Wet and dry road conditions will be used to constitute the bivariate relationship between driving speeds and stopping distances in a 'range of road or driving conditions'.
- Tires, road type and car type/weight will not be considered in calculations.
- The following assumptions of a linear regression model are relevant:
- The relationship between the explanatory variable $(\mathrm{X})$ and the response variable $(\mathrm{Y})$ is linear.
- No outliers present.
- Homoscedasticity is present within the data.
- Values are numerical.


## 2. Mathematical Concepts and Results

### 2.1 Raw data

The stopping distance will be determined via the following formula using the raw data:

$$
\text { Stopping distance }(m)=\text { Reaction distance }(m)+\text { Braking distance }(m)
$$

This calculation was conducted for each data set, view in the appendix.

### 2.2 Development of scatterplots and linear regression model

To determine if there is a linear relationship between driving speed and stopping distance, the data will be graphed as a scatterplot in Excel with a linear trendline. The formula of the linear regression, $y=a+b x$, will be calculated by determining the slope of the line (b) and the intercept point (a). These calculations, whilst being validified by Excel, will then be applied to the linear formula, whereby driving speed as the explanatory variable will be substituted into $x$, and then used calculate the predicted values. To evaluate the reliability of the model, the Correlation Coefficient (r), Coefficient of Determination $\left(R^{2}\right)$ and residuals will be calculated, with the residuals displayed in a residual plot.

### 2.2.1 Built-up area - Dry conditions

Driving Speed (km/h)

| Mean $(\bar{x}):$ | $\bar{x}=\frac{\sum x}{n}$ <br> $=\frac{1035}{20}$ |
| :--- | :---: |
|  | $=51.75 \mathrm{~km} / \mathrm{h}$ |

## Stopping Distance (m)

| $(\bar{y})$ | $\bar{y}=\frac{\sum x}{n}$ |
| :--- | :---: |
|  | $=\frac{616}{20}$ |
|  | $=30.80 \mathrm{~m}$ |
| $\left(S_{y}\right)$ | STDEV of $\bar{y}=7.2227710093 \mathrm{~km} / \mathrm{h}$ |

Least Squares Regression

| Correlation Coefficient $(\mathrm{r})$ | Calculated using the CORREL function in excel: <br> $r=0.6105195033$ |
| :--- | :---: |
| Coefficient of Determination $\left(\mathrm{R}^{2}\right)$ | $R^{2}=r \times r$ |
|  | $=0.3727340639$ |

$y=a+b x \quad y=$ Response variable $x=$ Explanatory variable

| Slope of the line $(b)$ | $b=\frac{r s_{y}}{s_{x}}$ |
| :--- | :---: |
|  | $b=\frac{0.6105195033 x 7.2227710093}{6.6481101391}$ |
| $=0.6633$ |  |

Therefore, the equation of the line is:

$$
y=-3.5254+0.6633 x
$$

This is then graphed with a linear trendline, as a scatterplot in Excel.


Figure 1 DS v. SD - Dry Built-up area
To determine the estimated values, and to evaluate the models' effectiveness, this formula will be reverse-engineered by substituting the driving speed values into $x$.

| Enter | B | C | D |
| :---: | :---: | :---: | :---: |
| Driving Speed (in km/h) |  | Stopping distance (in metres) | Estimated values |
|  | 53 | 28 | 31.63 |

These values will then be utilised to determine the residuals via the formula:
residual $=$ observed - estimated


Figure 2 Residual plot - Dry Built-up area

| C | D | E | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Stopping distance (in metres) | Estimated values | Residuals | Stopping distance (in metres) | Estimated values | Residuals |
| 28 | 31.63 | -3.63 | 28 | $=-3.5254+(0.6633 * 53)$ | =C2-D2 |
| 29 | 30.97 | -1.97 | 29 | =\$B\$43+\$B\$44*B3 | =C3-D3 |
| 38 | 36.27 | 1.73 | 38 | =\$B\$43+\$B\$44*B4 | =C4-D4 |
| 24 | 35.61 | -11.61 | 24 | =\$B\$43+\$B\$44*B5 | =C5-D5 |
| 21 | 26.32 | -5.32 | 21 | =\$B\$43+\$B\$44*B6 | =C6-D6 |
| 44 | 36.27 | 7.73 | 44 | $=\$ \mathrm{~B}$ \$ $43+\$ \mathrm{~B}$ \$44*B7 | =C7-D7 |
| 28 | 25.00 | 3.00 | 28 | =\$B\$43+\$B\$44*B8 | =C8-D8 |
| 30 | 30.97 | -0.97 | 30 | =\$B\$43+\$B\$44*B9 | =C9-D9 |
| 22 | 25.66 | -3.66 | 22 | $=\$ \mathrm{~B}$ \$43+\$B\$44*B10 | =C10-D10 |
| 21 | 26.99 | -5.99 | 21 | $=\$ \mathrm{~B}$ \$43+\$B\$44*B11 | =C11-D11 |
| 33 | 36.27 | -3.27 | 33 | $=\$ \mathrm{~B}$ \$43+\$B\$44*B12 | =C12-D12 |
| 43 | 28.98 | 14.02 | 43 | $=\$ \mathrm{~B}$ \$43+\$B\$44*B13 | =C13-D13 |
| 28 | 30.30 | -2.30 | 28 | $=\$ \mathrm{~B}$ \$43+\$B\$44*B14 | =C14-D14 |
| 35 | 28.31 | 6.69 | 35 | $=\$$ B\$43+\$B\$44*B15 | =C15-D15 |
| 33 | 36.27 | -3.27 | 33 | $=\$ \mathrm{~B}$ \$43+\$B\$44*B16 | =C16-D16 |
| 32 | 32.29 | -0.29 | 32 | $=\$$ B $43+\$$ B $444^{*}$ B17 | =C17-D17 |
| 24 | 25.00 | -1.00 | 24 | =\$B\$43+\$B\$44*B18 | =C18-D18 |
| 24 | 23.67 | 0.33 | 24 | =\$B\$43+\$B\$44*B19 | =C19-D19 |
| 39 | 32.96 | 6.04 | 39 | =\$B\$43+\$B\$44*B20 | =C20-D20 |
| 40 | 36.27 | 3.73 | 40 | =\$B\$43+\$B\$44*B21 | =C21-D21 |

Table 1 Residuals - Dry Built-up area

## Following distance:

To provide a recommendable following distance, the average driving speed will first be converted into $\mathrm{m} / \mathrm{s}$, and then multiplied by two (representing the 2 -second recommendation between vehicles) (Smart Drive Test Inc., 2020).

$$
\begin{gathered}
\bar{x}=51.75 \mathrm{~km} / \mathrm{h} \\
=51.75 \div 3.6 \\
=14.375 \mathrm{~m} / \mathrm{s} \\
=14.375 \times 2 \text { seconds }
\end{gathered}
$$

$$
\text { Safe following distance }=28.75 \mathrm{~m}
$$

Therefore, a safe following distance for vehicles in dry conditions in this area is 28.75 m . This calculation is repeated to determine safe following distances on dry motorways. All calculations above will be conducted for each data set via excel.

### 2.2.2 Motorway - Dry conditions

|  | Driving Speed (in $\mathrm{km} / \mathrm{h}$ ) | Stopping distance (in metres | Estimated values | Residuals |
| :---: | :---: | :---: | :---: | :---: |
|  | 82 | 73 | 54.81 | 18.19 |
|  | 81 | 47 | 53.12 | -6.12 |
|  | 100 | 69 | 85.10 | -16.10 |
|  | 82 | 47 | 54.81 | -7.81 |
|  | 100 | 64 | 85.10 | -21.10 |
|  | 82 | 43 | 54.81 | -11.81 |
|  | 100 | 97 | 85.10 | 11.90 |
|  | 94 | 78 | 75.00 | 3.00 |
|  | 81 | 35 | 53.12 | -18.12 |
|  | 94 | 87 | 75.00 | 12.00 |
|  | 90 | 54 | 68.27 | -14.27 |
|  | 86 | 63 | 61.54 | 1.46 |
|  | 92 | 83 | 71.64 | 11.36 |
|  | 88 | 76 | 64.90 | 11.10 |
|  | 82 | 66 | 54.81 | 11.19 |
|  | 100 | 94 | 85.10 | 8.90 |
|  | 84 | 58 | 58.17 | -0.17 |
|  | 86 | 78 | 61.54 | 16.46 |
|  | 100 | 90 | 85.10 | 4.90 |
|  | 91 | 55 | 69.95 | -14.95 |
|  |  |  |  |  |
|  |  |  |  |  |
|  | $\times$ | $y$ |  |  |
| Mean: | 89.75 | 67.85 |  |  |
| STDEV: | 7.3475738925 | 17.7446062969 |  |  |
| r | 0.6968507315 |  |  |  |
| a | -83.1916158908 |  |  |  |
| b | 1.6829149403 |  |  |  |
|  |  |  |  |  |
| $\mid r^{\wedge}$ | 0.4856009420 |  |  |  |



Table 2 Processed data - Dry Motorway

$$
y=-83.192+1.6829 x
$$



Figure 3 DS v. SD - Dry Motorway


Figure 4 Residual plot - Dry Motorway

Following distance:

| x |  |  | X |
| :---: | :---: | :---: | :---: |
| Mean: | 89.75 | Mean: | =AVERAGE(B2:B21) |
| In m/s: | 24.9305555556 | In m/s: | =B43/3.6 |
| Safe following distance (m): | 49.8611111111 | Safe following distance (m): | $=B 44 * 2$ |

2.2.3 Built-up area - Wet conditions

| A | в | c | D | E |
| :---: | :---: | :---: | :---: | :---: |
|  | Driving Speed (in $\mathrm{km} / \mathrm{h}$ ) | Stopping distance (in metros) | Estimated values | Residuals |
|  | 42 | 25 | 22.31 | 2.69 |
|  | 51 | 32 | 30.40 | 1.60 |
|  | 50 | 15 | 29.50 | -14.50 |
|  | 41 | 17 | 21.42 | -4.42 |
|  | 54 | 32 | 33.10 | -1.10 |
|  | 60 | 37 | 38.49 | -1.49 |
|  | 49 | 41 | 28.60 | 12.40 |
|  | 56 | 49 | 34.89 | 14.11 |
|  | 60 | 38 | 38.49 | -0.49 |
|  | 50 | 26 | 29.50 | -3.50 |
|  | 47 | 25 | 26.81 | -1.81 |
|  | 60 | 16 | 38.49 | -22.49 |
|  | 52 | 28 | 31.30 | -3.30 |
|  | 43 | 19 | 23.21 | -4.21 |
|  | 50 | 32 | 29.50 | 2.50 |
|  | 44 | 30 | 24.11 | 5.89 |
|  | 44 | 28 | 24.11 | 3.89 |
|  | 59 | 49 | 37.59 | 11.41 |
|  | 60 | 46 | 38.49 | 7.51 |
|  | 48 | 23 | 27.70 | -4.70 |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  | 51 | 30.40 |  |  |
| STDEV: | 6.4807406984 | 10.3993926948 |  |  |
| ' | 0.5599289617 |  |  |  |
|  |  |  |  |  |
| a | -15.4233082707 |  |  |  |
| b | 0.8984962406 |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| ${ }^{\wedge}{ }^{2}$ | 0.3135204421 |  |  |  |



Table 3 Processed data - Wet Built-up area

$$
y=-15.4213+0.8985 x
$$

Driving Speed (km/h) v. Stopping Diustance in a Wet - Built up area


Figure 5 DS v. SD - Wet Built-up area


Figure 6 Residual plot - Wet Built-up area

## Following Distance:

The calculations for following distances in wet conditions will be appropriately adjusted to coincide with government recommendations (Queensland Government, 2020). The average driving speed in $\mathrm{m} / \mathrm{s}$ will now be multiplied by 4, representing the increased following distance in seconds recommended.

$$
\begin{gathered}
\bar{x}=51 \mathrm{~m} / \mathrm{h} \\
=51 \div 3.6 \\
=14.1667 \mathrm{~m} / \mathrm{s} \\
=14.1667 \times 4 \text { seconds }
\end{gathered}
$$

$$
\text { Safe following distance }=56.6668 \mathrm{~m}
$$

This process is repeated for wet motorway.

### 2.2.4 Motorway - Wet conditions

|  | Driving Speed (in $\mathrm{km} / \mathrm{h}$ ) | Stopping distance (in metret | Estimated values | Residuals |  | Driving Speed (in km/h) | Stopping distance (in metres) | Estimated values | Residuals |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 82 | 66 | 6 66.50 | -0.50 |  | 82 | 66 | =\$B\$30+\$B\$31*B2 | =C2-D2 |
|  | 93 | 77 | 85.47 | -8.47 |  | 93 | 77 | =\$B\$30+\$B\$31*B3 | =C3-D3 |
|  | 80 | 54 | 63.05 | -9.05 |  | 80 | 54 | =\$B\$30+\$B\$31*B4 | =C4-D4 |
|  | 100 | 99 | 97.55 | 1.45 |  | 100 | 99 | =\$B\$30+\$B\$31*B5 | =C5-D5 |
|  | 94 | 87 | 87.20 | -0.20 |  | 94 | 87 | =\$B\$30+\$B\$31*B6 | =C6-D6 |
|  | 82 | 79 | 66.50 | 12.50 |  | 82 | 79 | =\$B\$30+\$B\$31*B7 | =C7-D7 |
|  | 84 | 83 | 69.95 | 13.05 |  | 84 | 83 | =\$B\$30+\$B\$31*B8 | =C8-D8 |
|  | 100 | 99 | 97.55 | 1.45 |  | 100 | 99 | =\$B\$30+\$B\$31*B9 | =C9-D9 |
|  | 81 | 59 | 64.78 | -5.78 |  | 81 | 59 | =\$8\$30+\$B\$31*B10 | =C10-D10 |
|  | 96 | 93 | 90.65 | 2.35 |  | 96 | 93 | =\$8\$30+\$B\$31*B11 | =C11-D11 |
|  | 85 | 68 | 71.68 | -3.68 |  | 85 | 68 | =\$B\$30+\$B\$31*B12 | =C12-D12 |
|  | 100 | 96 | 97.55 | -1.55 |  | 100 | 96 | = $\$$ B $\$ 30+\$ 8 \$ 31 *$ B13 | =C13-D13 |
|  | 87 | 64 | 75.13 | -11.13 |  | 87 | 64 | =\$8\$30+\$B\$31*B14 | =C14-D14 |
|  | 100 | 98 | 97.55 | 0.45 |  | 100 | 98 | =\$8\$30+\$B\$31*B15 | =C15-D15 |
|  | 80 | 56 | 63.05 | -7.05 |  | 80 | 56 | =\$8\$30+\$B\$31*B16 | =C16-D16 |
|  | 84 | 72 | 69.95 | 2.05 |  | 84 | 72 | =\$8\$30+\$B\$31*B17 | =C17-D17 |
|  | 93 | 82 | 85.47 | -3.47 |  | 93 | 82 | =\$8\$30+\$B\$31*B18 | =C18-D18 |
|  | 90 | 85 | 80.30 | 4.70 |  | 90 | 85 | =\$8\$30+\$B\$31*B19 | =C19-D19 |
|  | 100 | 92 | 97.55 | -5.55 |  | 100 | 92 | =\$B\$30+\$B\$31*B20 | =C20-D20 |
|  | 89 | 97 | 78.58 | 18.42 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | $\times$ | $y$ |  |  |
| Mean: | 90.0000000000 | 80.30 |  |  | Mean: | =AVERAGE(B2:B24) | =AVERAGE(C2:C24) |  |  |
| STDEV: | 7.5602840381 | 15.1348326021 |  |  | STDEV: | $=\operatorname{STDEV}(B 2: B 24)$ | $=S T D E V(C 2: C 24)$ |  |  |
|  |  |  |  |  |  |  |  |  |  |
| r | 0.8615261069 |  |  |  | r | =CORREL(B2:B24,C2:C2 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| a | -74.9209944760 |  |  |  | a | =C25-831*B25 |  |  |  |
| b | 1.7246777164 |  |  |  | b | $=\left(\mathrm{B28}{ }^{*} \mathrm{C} 26\right) / \mathrm{B26}$ |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| r^2 | 0.7422272329 |  |  |  | $\mathrm{r}^{\wedge}$ | =828*B28 |  |  |  |

Table 4 Processed data - Wet Motorway

$$
y=-74.921+1.7247 x
$$



Figure 7 DS v. SD - Wet Motorway


Following distance:

|  | $x$ |  | x |
| :--- | ---: | :--- | :--- |
| Mean: | 90.0526315789 | Mean: | =AVERAGE(B2:B20) |
| In m/s: | 25.0146198830 | In m/s: | $=$ =B40/3.6 |
| Safe following distance $(\mathrm{m})$ | 100.0584795322 | Safe following distance $(\mathrm{m})$ | $=$ =B41*4 |

## 3. Discussion

For both road areas in dry conditions, a linear relationship was present between the variables. The builtup area scatterplot demonstrated a moderate, positive, linear association, deduced by the correlation coefficient 0.6105 . The Coefficient of determination however identified only $37 \%$ of the variation in stopping distance is explained by the variation in driving speeds, indicating driving speed is not very explanatory of the variation in the stopping distance. The linear equation, developed as
$y=-3.524+0.6633 x$, indicated that, if the Driving speed of the vehicle was $0 \mathrm{~km} / \mathrm{h}$, the stopping distance would be a predicted -3.524 m . As the speed of the vehicle increased, however, the stopping distance will increase at a corresponding 0.6633 m . This model allowed the calculation of a suitable following distance for a built-up area in dry conditions is an estimated 28.75 m from the car in front. Similarly, the scatterplot for a motorway in dry conditions demonstrated another moderate, positive linear association between the two variables provided by the correlation coefficient of 0.6969 . The coefficient of determination identified that only $48 \%$ of the variation in stopping distance is explained by the variation in driving speeds in a dry motorway. This relatively low $R^{2}$ for both roadways in dry conditions is indicative of the relationship of low explanation of driving speed in the variation of stopping distance in dry conditions (Frost, 2020). However, the linear relationship between the variables in dry conditions is reinforced by the residual plots, which both demonstrate a random pattern and distribution. The regression equation of $y=-83.192+1.6829 x$ identified that, if the driving speed on a motorway during dry conditions was $0 \mathrm{~km} / \mathrm{h}$, the stopping distance would be a predicted -83.192 m and would increase at a rate of 1.6829 m as driving speed increased. Finally, the motorway model identified that the safe following distance on a dry motorway is an estimated 49 m .

For wet road conditions, similar linear relationships were identified. The scatterplot for a built-up area demonstrated a moderate, positive, linear association with a correlation coefficient of 0.5599 . The coefficient of determination identified that only $35 \%$ of the variation in stopping distance is explained by the variation in driving speeds in this area. The residual plot, however, demonstrated a randomly
distributed pattern of residual values, indicating a strong linear relationship. The regression equation for the data, $\mathrm{y}=-15.4213+0.8985 \mathrm{x}$, identified that, if the speed of the vehicle was $0 \mathrm{~km} / \mathrm{h}$, the stopping distance would be -15.4213 m , and would increase at a rate of 0.8985 m as the speed increased. This predicted value is 0.2217 m higher than the estimated slope of the line for the dry built- up area, coinciding with the observations that reduced friction contributes to a longer stopping distance (Queensland Government, 2020). The safe following distance was determined to be 56.69 m . For a wet motorway, the scatterplot and relationship demonstrated an unprecedented strong, positive linear association with an extremely high correlation coefficient value of 0.8615 . This contributes to the coefficient of determination of $74 \%$, demonstrating that most of the variation in stopping distance is explained by the variation in driving speeds. This $R^{2}$ value is $39 \%$ higher than the variability in the wet built-up area, representing the strongest linear model in this investigation. Furthermore, the regression equation of $y=-74.921+1.7247 x$ identified that, at a speed of $0 \mathrm{~km} / \mathrm{h}$, the stopping distance would be a predicted 74.921 m and would increase at a rate of 1.7247 m as the driving speed increased. This relationship between the variables additionally indicated that stopping distance increases a rate of 0.058 m higher than dry motorways. The linear relationship of both models is reinforced by the residual plots, which equally demonstrated a random residual pattern. Finally, the model identified that the safe following distance on a wet motorway is an estimated 49 m .

While this mathematical model allows the identification of a suitable following distance for each roadway and driving condition, it does not provide a recommendation for safe driving speed. The most appropriate recommendation is to follow the signs provided by the state government. In Queensland, for built-up areas, the legal driving speed is $50 \mathrm{~km} / \mathrm{h}$, and for motorways, 80-100km/h (Queensland Government, 2020). For wet weather conditions, the government recommends that the driver lower their speeds from these limits by at least 10km/h (Queensland Government, 2020).

## 4. Evaluation

## Strengths:

A major strength of this model is the conformity of the data to the assumptions of linear regression. Due to the absence of outliers, a linear relationship between the variables, homoscedasticity of the data, and random residual plots, the linear regression model was the appropriate method to investigate the bivariate relationship between driving speed and stopping distance (Stat Trek, 2020).. Instead of sole reliance on $\mathrm{R}^{2}$ to determine the statistical variability, these mathematical models employed the use of residual plots to interpolate the reliability and accuracy of the models determined. Additionally, by maintaining a consistent speed bracket for both road areas that represents the speeds of the state, the models' recommendations apply effectively (Queensland Government, 2020). Furthermore, as the recommendations of this model are founded upon reliable road safe sources, the model recommendations coincide with the governments' guidelines, validifying their suitability in a modern context. Additionally, the provision of reliable data from the website 'It's a Drag' allowed the investigation stopping distance at various speeds in various driving conditions (Education Services Australia, 2016).

Limitations:
The linear regression model has several limitations that impacts its use in the modern context. Because there is no statistical basis to assume that the linear regression model will apply to future driving speeds and subsequent stopping distances, the limitation to only current sample data limits the model's suitability for proving future relationships (Aivaz Kamer-Ainur, 2020). If speeds increased and organisations needed to apply this model to recommend safe driving speed and following distance, the model will be statistically unreliable (Aivaz Kamer-Ainur, 2020). The relationships identified by the two variables are also heavily hindered by this investigations' inadequate sample space. As the sample space for each roadway and condition is limited to 20 data points obtained from the one website and the same person, the margin of error and power of the study is reduced in determining the relationship between the variables (Deziel, 2018). This limitation is exacerbated by the potential
unreliability of the simulator, which could potentially invalidate the mathematical relationships identified in this report (Deziel, 2018). Additionally, by limiting the investigation to only two external conditions, set speeds, and not assessing other conditions such as road type or vehicle, this solutions' practicality in a wide range of road conditions that can impact driving speed and stopping distance is very limited.

## 5. Conclusion

This PSMT aimed to investigate the bivariate relationship between driving speed and stopping distance in a range of driving conditions on built-up areas and motorways. By analysing this relationship, the solution aimed to provide a suitable recommendation for safe driving speed and following distances within the areas under the driving conditions. The calculations identified and developed by this model provided a mathematical solution to the relationship identified and appropriately addressed this relationship. However, as highlighted above, due to the limitations present within the model, the significance and future practicality of the model limits its use in future situations.

## 6. Appendix

Raw data:
Dry conditions

| Driving Speed (in $\mathrm{km} / \mathrm{h}$ ) | Reaction distance (in metres) | ) Braking Distance (in metres) | Stopping distance (in metres) | Driving Speed (in km/h) | Reaction distance (in metres) | Braking Distance (in metres) | Stopping distance (in metres) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 53 | 8 | $8 \quad 20$ | 28 | 82 | 39 | 34 | 73 |
| 52 | 16 | 18 | 29 | 81 | 21 | 26 | 47 |
| 60 | 25 | 25 | 38 | 100 | 16 | 53 | 69 |
| 59 | 11 | $1{ }^{13}$ | 24 | 82 | 13 | 34 | 47 |
| 45 | 7 | $7 \quad 14$ | 21 | 100 | 11 | 53 | 64 |
| 60 | 25 | 519 | 44 | 82 | 17 | 26 | 43 |
| 43 | 20 | ( 8 | 28 | 100 | 44 | 53 | 97 |
| 52 | 26 |  | 30 | 94 | 52 | 26 | 78 |
| 44 | 14 | 14 8 | 22 | 81 | 22 | 13 | 35 |
| 46 | 13 | 3 | 21 | 94 | 44 | 43 | 87 |
| 60 | 14 | $14 \sim 19$ | 33 | 90 | 35 | 19 | 54 |
| 49 | 35 | 58 | 43 | 86 | 20 | 43 | 63 |
| 51 | 15 | 15 13 | 28 | 92 | 40 | 43 | 83 |
| 48 | 31 | 14 | 35 | 88 | 34 | 42 | 76 |
| 60 | 14 | 4 - 19 | 33 | 82 | 40 | 26 | 66 |
| 54 | 10 | - 22 | 32 | 100 | 41 | 53 | 94 |
| 43 | 16 | 洔 | 24 | 84 | 24 | 34 | 58 |
| 41 | 11 | 1 13 | 24 | 86 | 44 | 34 | 78 |
| 55 | 26 | 6 13 | 39 | 100 | 50 | 40 | 90 |
| 60 | 28 | 812 | 40 | 91 | 35 | 20 | 55 |
|  |  |  |  |  |  |  |  |
|  | BUILT UP AREA |  |  |  | MOTORWAY |  |  |

Wet conditions

| Driving Speed (in km/7) | Reaction distance (in metres) | s) Braking Distance (in metres) | s) Stopping distance (in metres) | Driving Speed (in km Reaction distance (in metres) | s) Braking Distance (in metres) | ) Stopping distance (in metres) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 42 |  | 5020 | 20 25 | 82 | 5 51 | 1 66 |
| 51 | 12 | 220 | 2032 | $93 \quad 12$ | 2 65 | 577 |
| 50 |  | 8 7 | $7{ }^{7}$ | 80 - 15 | 5 39 | $9{ }^{\text {a }}$ [ |
| 41 |  | 5 - 12 | 12 - 17 | 100 - 19 | 9 80 | 0 99 |
| 54 | 12 | 220 | $20 \quad 32$ | $94 \quad 22$ | 2 65 | 5 87 |
| 60 |  | 9 28 | 28 37 | $82 \quad 19$ | 9 60 | 0 79 |
| 49 | 29 | 99 | 12 41 | 84 20 | 0 63 | 3 83 |
| 56 | 29 | 29 | $20 \quad 49$ | 100 19 | 98 | 0 99 |
| 60 | 10 | $0 \quad 28$ | 28 38 | 81 8 | 8 51 | 1 59 |
| 50 |  | 620 | $20 \quad 26$ | 96 - 13 | 380 | 0 93 |
| 47 | 13 | $3 \quad 12$ | 12 25 | 85 17 | 7 51 | 1 68 |
| 60 |  | 4 - 12 | 12 16 |  | 6 80 | 0 96 |
| 52 |  | 8 20 | $20 \quad 28$ | 87 - 13 | 3 洔 | 1 64 |
| 43 |  | $7 \quad 12$ | 12 19 | 100 - 16 | 6 82 | 2 98 |
| 50 | 17 | $7{ }^{7}$ | 15 32 | 80 - 17 | $7{ }^{\text {7 }}$ |  |
| 44 | 18 | 8 12 | 1230 | 84 - 10 | 0 62 | 272 |
| 44 |  | $9 \quad 19$ | 19 28 | 93 17 | 7 65 | 582 |
| 59 | 29 | 29 20 | $20 \quad 49$ | 90 20 | ( 65 | 5 85 |
| 60 | 21 | 1 25 | 25 46 | 100 - 18 | 8 74 | $4{ }^{\text {4 }}$ |
| 48 | 11 | 1 12 | 12 23 | $89 \quad 19$ | 978 | 88 |
|  | BUILT UP AREA |  |  | MOTORWAY |  |  |

## 7. Bibliography

Aivaz Kamer-Ainur, M. M., 2020. ERRORS AND LIMITATIONS ASSOCIATED WITH REGRESSION AND CORRELATION ANALYSIS. [Online]
Available at:
https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.580.790\&rep=rep1\&type=pdf [Accessed 1911 2020].
Deziel, C., 2018. The Effects of a Small Sample Size Limitation. [Online]
Available at: https://sciencing.com/calculate-significance-level-
7610714.html [Accessed 511 2020].

Education Services Australia, 2016. It's a drag. [Online]
Available at: http://www.scootle.edu.au/ec/viewing/L51/L51/index.html\# [Accessed 2910 2020].
Frost, J., 2020. How To Interpret $R$-squared in Regression Analysis. [Online]
Available at: https://statisticsbyjim.com/regression/interpret-r-squaredregression/ [Accessed 2311 2020].
Queensland Government, 2018. 2018 Summary Road Crash Report. [Online]
Available at: https://www.tmr.gld.gov.au/-/media/Safety/Transport-and-
road- statistics/Road-safety/Summary Road Crash Report 2018.pdf?la=en
[Accessed 511 2020].
Queensland Government, 2020. Road safety during wet weather. [Online]
Available at: https://www.qld.gov.au/transport/safety/road-safety/wet-weather
[Accessed 1611 2020].
Queensland Government, 2020. Safe following distances. [Online]
Available at: https://www.qld.gov.au/transport/safety/rules/road/distances
[Accessed 1511 2020].
Queensland Government, 2020. Stopping distances: speed and braking. [Online]
Available at: https://www.qld.gov.au/transport/safety/road-safety/driving-safely/stopping-
distances
[Accessed 1311 2020].
Queensland Government, 2020. Suburban Speed Limits. [Online]
Available at: https://www.qld.gov.au/transport/safety/rules/speed-
limits [Accessed 1011 2020].
Road Safety Commission, 2020. Speeding. [Online]
Available at: https://www.rsc.wa.gov.au/Your-Safety/Behaviours/Speeding
[Accessed 2211 2020].
Smart Drive Test Inc., 2020. Why following distance is measured in time?. [Online]
Available at: https://www.smartdrivetest.com/teaching/why-vehicle-following-distance-is- measured-in-time
[Accessed 1611 2020].
Stat Trek, 2020. What is Linear Regression?. [Online]
Available at: $\underline{\text { https://stattrek.com/regression/linear-regression.aspx }}$
[Accessed 1811 2020].

Unless otherwise indicated the student response is © Ronin Starmer 2022. This resource has been made available under the Creative Commons Attribution 4.0 licence (CC BY 4.0)

Attribution Assessment Highlights 2021 General Mathematics IA1 © Ronin Starmer 2022. CC BY 4.0
Published by Queensland Curriculum and Assessment Authority (QCAA) 2022.

