Specialist Mathematics 2025 v1.0

General senior syllabus

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Queensland Government
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For all Queensland schools

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Queensland syllabuses for senior subjects

In Queensland, a syllabus for a senior subject is an official ‘map’ of a senior school subject. A syllabus’s function is to support schools in delivering the Queensland Certificate of Education (QCE) system through high-quality and high-equity curriculum and assessment.

Syllabuses are based on design principles developed from independent international research about how excellence and equity are promoted in the documents teachers use to develop and enliven the curriculum.

Syllabuses for senior subjects build on student learning in the Prep to Year 10 Australian Curriculum and include General, General (Extension), Senior External Examination (SEE), Applied, Applied (Essential) and Short Course syllabuses.

More information about syllabuses for senior subjects is available at www.qcaa.qld.edu.au/senior/senior-subjects and in the ‘Queensland curriculum’ section of the *QCE and QCIA policy and procedures handbook*.

Teaching, learning and assessment resources will support the implementation of a syllabus for a senior subject. More information about professional resources for senior syllabuses is available on the QCAA website and via the QCAA Portal.

Course overview

Rationale

Mathematics is a unique and powerful intellectual discipline that is used to investigate patterns, order, generality and uncertainty. It is a way of thinking in which problems are explored and solved through observation, reflection and logical reasoning. It uses a concise system of communication, with written, symbolic, spoken and visual components. Mathematics is creative, requires initiative and promotes curiosity in an increasingly complex and data-driven world. It is the foundation of all quantitative disciplines.

To prepare students with the knowledge, skills and confidence to participate effectively in the community and the economy requires the development of skills that reflect the demands of the 21st century. Students undertaking Mathematics will develop their critical and creative thinking, oral and written communication, information & communication technologies (ICT) capability, ability to collaborate, and sense of personal and social responsibility — ultimately becoming lifelong learners who demonstrate initiative when facing a challenge. The use of technology to make connections between mathematical theory, practice and application has a positive effect on the development of conceptual understanding and student disposition towards mathematics.

Mathematics teaching and learning practices range from practising essential mathematical routines to develop procedural fluency, through to investigating scenarios, modelling the real world, solving problems and explaining reasoning. When students achieve procedural fluency, they carry out procedures flexibly, accurately and efficiently. When factual knowledge and concepts come to mind readily, students are able to make more complex use of knowledge to successfully formulate, represent and solve mathematical problems. Problem-solving helps to develop an ability to transfer mathematical skills and ideas between different contexts. This assists students to make connections between related concepts and adapt what they already know to new and unfamiliar situations. With appropriate effort and experience, through discussion, collaboration and reflection of ideas, students should develop confidence and experience success in their use of mathematics.

The major domains of mathematical knowledge in Specialist Mathematics are Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus. Topics are developed systematically, with increasing levels of sophistication, complexity and connection, building on functions, calculus, statistics from Mathematical Methods, while vectors, complex numbers and matrices are introduced. Functions and calculus are essential for creating models of the physical world. Statistics are used to describe and analyse phenomena involving probability, uncertainty and variation. Matrices, complex numbers and vectors are essential tools for explaining abstract or complex relationships that occur in scientific and technological endeavours.

Students who undertake Specialist Mathematics will develop confidence in their mathematical knowledge and ability, and gain a positive view of themselves as mathematics learners. They will gain an appreciation of the true nature of mathematics, its beauty and its power.

Syllabus objectives

The syllabus objectives outline what students have the opportunity to learn.

1. **Recall mathematical knowledge.**

   When students recall mathematical knowledge, they recognise features of remembered information. They recognise relevant concepts, rules, definitions, techniques and algorithms.

2. **Use mathematical knowledge.**

   When students use mathematical knowledge, they put into effect relevant concepts, rules, definitions, techniques and algorithms. They perform calculations with and without technology.

3. **Communicate mathematical knowledge.**

   When students communicate mathematical knowledge, they use mathematical language (terminology, symbols, conventions and representations) and everyday language. They organise and present information in graphical and symbolic form, and describe and represent mathematical models.

4. **Evaluate the reasonableness of solutions.**

   When students evaluate the reasonableness of solutions, they interpret their mathematical results in the context of the situation and reflect on whether the problem has been solved. They verify results by using estimation skills and checking calculations, with and without technology. They make an appraisal by assessing implications, strengths and limitations of solutions and/or models, and use this to consider if alternative methods or refinements are required.

5. **Justify procedures and decisions.**

   When students justify procedures and decisions, they explain their mathematical reasoning in detail. They make relationships evident, logically organise mathematical arguments, and provide reasons for choices made and conclusions reached.

6. **Solve mathematical problems.**

   When students solve mathematical problems, they analyse the context of the problem to translate information into mathematical forms. They make decisions about the concepts, techniques and technology to be used and apply these to develop a solution. They develop, refine and use mathematical models, where applicable.

Designing a course of study in Specialist Mathematics

Syllabuses are designed for teachers to make professional decisions to tailor curriculum and assessment design and delivery to suit their school context and the goals, aspirations and abilities of their students within the parameters of Queensland's senior phase of learning.

The syllabus is used by teachers to develop curriculum for their school context. The term *course of study* describes the unique curriculum and assessment that students engage with in each school context. A course of study is the product of a series of decisions made by a school to select, organise and contextualise subject matter, integrate complementary and important learning, and create assessment tasks in accordance with syllabus specifications.

It is encouraged that, where possible, a course of study is designed such that teaching, learning and assessment activities are integrated and enlivened in an authentic setting.

Course structure

Specialist Mathematics is a General senior syllabus. It contains four QCAA-developed units from which schools develop their course of study.

Each unit has been developed with a notional time of 55 hours of teaching and learning, including assessment.

Students should complete Unit 1 and Unit 2 before beginning Units 3 and 4. Units 3 and 4 are studied as a pair.

More information about the requirements for administering senior syllabuses is available in the 'Queensland curriculum' section of the *QCE and QCIA policy and procedures handbook*.

Curriculum

Senior syllabuses set out only what is essential while being flexible so teachers can make curriculum decisions to suit their students, school context, resources and expertise.

Within the requirements set out in this syllabus and the *QCE and QCIA policy and procedures handbook*, schools have autonomy to decide:

- how and when subject matter is delivered
- how, when and why learning experiences are developed, and the context in which learning occurs
- how opportunities are provided in the course of study for explicit and integrated teaching and learning of complementary skills.

These decisions allow teachers to develop a course of study that is rich, engaging and relevant for their students.

Assessment

Senior syllabuses set out only what is essential while being flexible so teachers can make assessment decisions to suit their students, school context, resources and expertise.

General senior syllabuses contain assessment specifications and conditions for the assessment instruments that must be implemented with Units 3 and 4. These specifications and conditions ensure comparability, equity and validity in assessment.

Within the requirements set out in this syllabus and the *QCE and QCIA policy and procedures handbook*, schools have autonomy to decide:

- specific assessment task details
- assessment contexts to suit available resources
- how the assessment task will be integrated with teaching and learning activities
- how authentic the task will be.

In Unit 1 and Unit 2, schools:

- develop at least two but no more than four assessments
- complete at least one assessment for each unit
- ensure that each unit objective is assessed at least once.

In Units 3 and 4, schools develop three assessments using the assessment specifications and conditions provided in the syllabus.

More information about assessment in senior syllabuses is available in 'The assessment system' section of the *QCE and QCIA policy and procedures handbook*.

Subject matter

Each unit contains a unit description, unit objectives and subject matter. Subject matter is the body of information, mental procedures and psychomotor procedures (see Marzano & Kendall 2007, 2008) that are necessary for students' learning and engagement with the subject. Subject matter itself is not the specification of learning experiences but provides the basis for the design of student learning experiences.

Subject matter has a direct relationship with the unit objectives and provides statements of learning that have been constructed in a similar way to objectives.

Aboriginal perspectives and Torres Strait Islander perspectives

The QCAA is committed to reconciliation. As part of its commitment, the QCAA affirms that:

- Aboriginal peoples and Torres Strait Islander peoples are the first Australians, and have the oldest living cultures in human history
- Aboriginal peoples and Torres Strait Islander peoples have strong cultural traditions and speak diverse languages and dialects, other than Standard Australian English
- teaching and learning in Queensland schools should provide opportunities for students to deepen their knowledge of Australia by engaging with the perspectives of Aboriginal peoples and Torres Strait Islander peoples
- positive outcomes for Aboriginal students and Torres Strait Islander students are supported by successfully embedding Aboriginal perspectives and Torres Strait Islander perspectives across planning, teaching and assessing student achievement.

Guidelines about Aboriginal perspectives and Torres Strait Islander perspectives and resources for teaching are available at www.qcaa.qld.edu.au/k-12-policies/aboriginal-torres-strait-islander-perspectives.

Where appropriate, Aboriginal perspectives and Torres Strait Islander perspectives have been embedded in the subject matter.

Complementary skills

Opportunities for the development of complementary skills have been embedded throughout subject matter. These skills, which overlap and interact with syllabus subject matter, are derived from current education, industry and community expectations and encompass the knowledge, skills, capabilities, behaviours and dispositions that will help students live and work successfully in the 21st century.

These complementary skills are:

- literacy — the knowledge, skills, behaviours and dispositions about language and texts essential for understanding and conveying English language content
- numeracy — the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations, to recognise and understand the role of mathematics in the world, and to develop the dispositions and capacities to use mathematical knowledge and skills purposefully
- 21st century skills — the attributes and skills students need to prepare them for higher education, work, and engagement in a complex and rapidly changing world. These skills include critical thinking, creative thinking, communication, collaboration and teamwork, personal and social skills, and digital literacy. The explanations of associated skills are available at www.qcaa.qld.edu.au/senior/senior-subjects/general-subjects/21st-century-skills.

It is expected that aspects of literacy, numeracy and 21st century skills will be developed by engaging in the learning outlined in this syllabus. Teachers may choose to create additional explicit and intentional opportunities for the development of these skills as they design the course of study.

Additional subject-specific information

Additional subject-specific information has been included to support and inform the development of a course of study.

Assumed knowledge, prior learning or experience

Specialist Mathematics is to be undertaken in conjunction with, or on completion of, Mathematical Methods. It is assumed that work covered in Mathematical Methods will be known before it is required in Specialist Mathematics.

Assumed knowledge refers to the subject matter that teachers can expect students to know prior to beginning this subject. Emphasis is placed on the mastery of content, ensuring key concepts or procedures are learnt fully so they will not need reteaching.

Developing mastery often involves multiple approaches to teaching and conceptualising the same mathematical concept. When students have a good understanding of a key concept or procedure, they are more easily able to make connections to related new subject matter and apply what they already know to new problems.

Subject matter from previous unit/s is assumed for subsequent unit/s.

The following is a non-exhaustive list of assumed knowledge based on the subject matter in the P–10 Australian Curriculum version 9.

- Describe the results of two- and three-step chance experiments.
- List all outcomes for compound events, both with and without replacement.
- Determine probabilities of events.
- Substitute values into formulas to determine an unknown.
- Recognise the effect of using approximations of real numbers in repeated calculations.
- Calculate and interpret percentage errors including investigating error as a percentage of the exact value.
- Rational and irrational numbers.
- Solve problems involving the surface area of right prisms and cylinders using appropriate units.
- Apply angle properties and scale.
- Solve right-angled triangle problems.
- Identify conditions and apply properties of congruence and similarity of triangles.
- Apply Pythagoras' theorem and trigonometry in right-angled triangles including problems involving direction and angles of elevation and depression.
- Apply deductive reasoning to proofs involving shapes in the plane.
- Translate word problems to mathematical form.
- Solve linear inequalities and simultaneous linear equations in 2 variables.
- Expand, factorise and simplify algebraic expressions, including monic quadratic expressions.
- Solve equations algebraically, applying exponent laws involving products, quotients and powers of variables, and apply the commutative and the distributive laws.
- Apply the four operations to simple algebraic fractions with numerical denominators.

Recommended knowledge

Recommended knowledge refers to the subject matter from the optional content for post–Year 10 mathematics pathways that will enhance students' understanding of this subject's foundational content.

The following is a non-exhaustive list of recommended knowledge based on the subject matter in the optional content for post–Year 10 mathematics pathways from the P–10 Australian Curriculum version 9.

- Operations on numbers involving fractional exponents and surds including
  - rationalising the denominator of a quotient, e.g. $\dfrac{3+\sqrt{7}}{3-\sqrt{7}}=8+3\sqrt{7}$
- The solution of a variety of algebraic representations of quadratic functions including
  - use of the quadratic formula
  - use of the discriminant to identify the number and nature of the roots
  - recognising conjugate pairs of irrational roots of a quadratic equation.
- Properties of circles including
  - identifying relationships, angles between tangents and chords,
  - identifying angles subtended by a chord with respect to the centre of a circle and with respect to a point on the circumference
  - exploring how deductive reasoning and diagrams are used in proving geometric theorems related to circles.
- Counting principles that provide efficient counting in multiplicative contexts including
  - the application of the multiplication principle
  - use of the definition of $n!$
  - calculation of probabilities related to sampling, with and without replacement.
- Sketching functions including quadratic, sine and cosine functions.

Problem-solving and mathematical modelling

A key aspect of learning mathematics is to develop strategic competence; that is, to formulate, represent and solve mathematical problems (Kilpatrick, Swafford & Bradford 2001). As such, problem-solving is a focus of mathematics education research, curriculum and teaching (Sullivan 2011). This focus is not to the exclusion of routine exercises, which are necessary for practising, attaining mastery and being able to respond automatically. But mathematics education in the 21st century goes beyond this to include innovative problems that are complex, unfamiliar and non-routine (Mevarech & Kramarski 2014).

Problem-solving in mathematics can be set in purely mathematical contexts or real-world contexts. When set in the real world, problem-solving in mathematics involves mathematical modelling.

Problem-solving

Problem-solving is required when a task or goal has limiting conditions placed upon it or an obstacle blocking the path to a solution (Marzano & Kendall 2007). It involves:

- knowledge of the relevant details
- using generalisations and principles to identify, define and interpret the problem
- mental computation and estimation
- critical, creative and lateral thinking
- creating or choosing a strategy
- making decisions
- testing, monitoring and evaluating solutions.

Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples without understanding. Through problem-solving, students will make connections between mathematics topics, across the curriculum and with the real world, and see the value and usefulness of mathematics. Problems may be real-world or abstract, and presented to students as issues, statements or questions that may require them to use primary or secondary data.

Mathematical modelling

Mathematical modelling begins from an assumption that mathematics is everywhere in the world around us — a challenge is to identify where it is present, access it and apply it productively. Models are developed in order to better understand real-world phenomena, to make predictions and answer questions. A mathematical model depicts a situation by expressing relationships using mathematical concepts and language. It refers to the set of simplifying assumptions (such as the relevant variables or the shape of something); the set of assumed relationships between variables; and the resulting representation (such as a formula) that can be used to generate an answer (Stacey 2015).

Mathematical modelling involves:

- formulating a mathematical representation of a problem derived from within a real-world context
- using mathematics concepts and techniques to obtain results
- interpreting the results by referring back to the original problem context
- revising the model (where necessary) (Geiger, Faragher & Goos 2010).

Through developing and applying mathematical models, students cumulatively become real-world problem-solvers. Ultimately, this means that not only can they productively address problems set by others, but also that they develop the ability to identify and address problems and answer questions that matter to them.

The following section outlines an approach to problem-solving and mathematical modelling.[1] Problems must be real-world, and can be presented to students as issues, statements or questions that may require them to use primary or secondary data.

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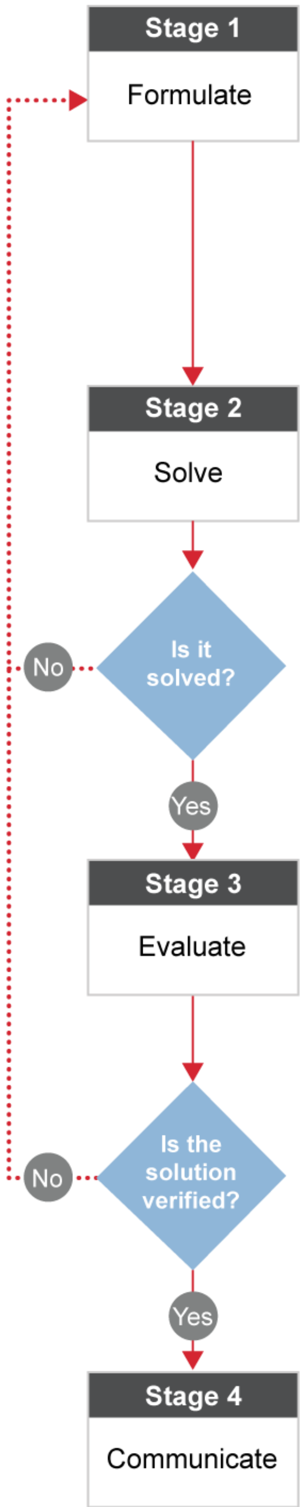
[1] A wide variety of frameworks for problem-solving and modelling exist in mathematics education literature. The approach outlined here aligns with and is informed by other approaches, such as Polya (1957) in *How to Solve It: A new aspect of mathematical method* (1957), the Australian Curriculum (ACARA 2015a) *Statistical investigation process*, the OECD/PISA Mathematics framework (OECD 2015, 2003) and *A framework for success in implementing mathematical modelling in the secondary classroom* (Stillman et al. 2007). For further reading see Blum et al. (2007); Kaiser et al. (2011); and Stillman et al. (2013).

Figure 1: An approach to problem-solving and mathematical modelling



Once students understand what the problem is asking, they must design a plan to solve the problem. Students translate the problem into a mathematically purposeful representation by first determining the applicable mathematical knowledge that is required to make progress with the problem. Important assumptions, variables and observations are identified and justified, based on the logic of a proposed solution and/or model.

In mathematical modelling, formulating a model involves the process of mathematisation — moving from the real world to the mathematical world.

Students select and apply mathematical knowledge previously learnt to solve the problem. Possible approaches are wide-ranging and include synthesising and refining existing models, and generating and testing hypotheses with primary or secondary data and information, to produce a complete solution.

Solutions can be found using algebraic, graphic, arithmetic and/or numeric methods, with and/or without technology.

Once a possible solution has been achieved, students need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. They verify their results and evaluate the reasonableness of the solution to the problem in relation to the original issue, statement or question.

This involves exploring the strengths and limitations of the solution and/or model. Where necessary, this will require going back through the process to further refine the solution and/or model. In mathematical modelling, students must check that the output of their model provides a complete solution to the real-world problem it has been designed to address.

This stage emphasises the importance of methodological rigour and the fact that problem-solving and mathematical modelling is not usually linear and involves an iterative process.

The development of solutions and/or models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated and justified clearly and fully. Students communicate findings logically and concisely using mathematical and everyday language. They draw conclusions, discussing the results, strengths and limitations of the solution and/or model. Students could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem.

Approaches to problem-solving and mathematical modelling in the classroom

When teaching problem-solving and mathematical modelling, teachers should consider tea*ching for* and *learning through* problem-solving and mathematical modelling. When *teaching for*, students are taught the specific mathematical rules, definitions, procedures, problem-solving strategies and critical elements of the model that are needed to solve a given problem. When *learning through*, students are presented with problems to solve, but must apply the knowledge and skills they have previously been taught to solve it. By solving these problems, students are able to develop new mathematical understanding and skills. This requires an explicit and connected approach to teaching problem-solving and mathematical modelling that necessitates fluency of critical facts and processes at each step.

The following describes three different approaches to teaching problem-solving and mathematical modelling[2] along the continua between *teaching for* and *learning through*:

Approach	Description	*Teaching for* or *learning through*
Dependent	The teacher explicitly demonstrates and teaches the concepts and techniques required to solve the problem, and/or develop a mathematical model. This usually involves students solving (stage 2) and evaluating and verifying (stage 3).	*Teaching for*
Guided	The teacher influences the choice of concepts and techniques, and/or model that students use to solve the problem. Guidance is provided and all stages of the approach are used.	Moving towards *learning through*
Independent	The teacher cedes control and students work independently, choosing their own solution and/or model, and working at their own level of mathematics. The independent approach is the most challenging.	*Learning through*

These approaches are not mutually exclusive. An independent approach (*learning through*) might be undertaken as an extension of a dependent or guided activity that students have previously undertaken (*teaching for*). Students need to have attained the relevant foundational understanding and skills before working independently during a problem-solving and modelling task. This capacity needs to be built over time through the course of study with teachers closely monitoring student progress.

[2] Based on Galbraith (1989).

Strategies for retaining and recalling information for assessment

The following practices[3] can support preparation for senior assessment in Specialist Mathematics.

The spacing effect

The spacing effect draws on research about forgetting and learning curves. By recalling and revisiting information at intervals, rather than at the end of a study cycle, students remember a greater percentage of the information with a higher level of accuracy. Exposing students to information and materials numerous times over multiple spaced intervals solidifies long-term memory, positively affecting retention and recall.

Teachers should plan teaching and learning sequences that allow time to revisit previously taught information and skills at several intervals. These repeated learning opportunities also provide opportunities for teachers to provide formative feedback to students.

The retrieval effect

The retrieval effect helps students to practise remembering through quick, regular, low-stakes questioning or quizzes that exercise their memories and develop their ability to engage in the deliberate act of recalling information. This has been shown to be more effective at developing long-term memories than activities that require students to search through notes or other resources.

Students may see an inability to remember as an obstacle, but they should be encouraged to understand that this is an opportunity for learning to take place. By trying to recall information, students exercise or strengthen their memory and may also identify gaps in their learning. The more difficult the retrieval practice, the better it can be for long-term learning.

Interleaving

Interleaving involves interspersing the concepts, categories, skills or types of questions that students focus on in class or revision. This is in contrast to blocking, in which these elements are grouped together in a block of time. For example, for concepts A, B and C:

- Blocking A A A A A B B B B B C C C C C
- Interleaving A B C B C A B A C A C B C A B

Studies have found that interleaving in instruction or revision produces better long-term recall of subject matter. Interleaving also ensures that spacing occurs, as instances of practice are spread out over time.

Additionally, because exposure to one concept is interleaved with exposure to another, students have more opportunities to distinguish between related concepts. This highlighting of differences may explain why studies have found that interleaving enhances inductive learning, where participants use exemplars to develop an understanding of broader concepts or categories. Spacing without interleaving does not appear to benefit this type of learning.

Interleaving can seem counterintuitive — even in studies where interleaving enhanced learning, participants often felt that they had learnt more with blocked study. Despite this, their performance in testing indicated greater learning through the interleaving approach.

[3] Based on Agarwal, Roediger, McDaniel & McDermott (2020); Birnbaum, Kornell, Ligon Bjork & Bjork (2013); Carpenter & Agarwal (2020); Chen, Paas & Sweller (2021); Ebbinghaus (1885); Rohrer (2012); Taylor & Rohrer (2010).

Reporting

General information about determining and reporting results for senior syllabuses is provided in the ‘Determining and reporting results’ section of the *QCE and QCIA policy and procedures handbook*.

Reporting standards

Reporting standards are summary statements that describe typical performance at each of the five levels (A–E).

A
The student recalls, uses and communicates comprehensive mathematical knowledge drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar, complex familiar and complex unfamiliar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar, complex familiar and complex unfamiliar situations.
B
The student recalls, uses and communicates thorough mathematical knowledge drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar and complex familiar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar and complex familiar situations.
C
The student recalls, uses and communicates mathematical knowledge drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar situations.
D
The student recalls, uses and communicates partial mathematical knowledge drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar situations. The student sometimes evaluates the reasonableness of solutions, sometimes justifies procedures and decisions, and solves some mathematical problems in simple familiar situations.
E
The student recalls, uses and communicates isolated mathematical knowledge drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar situations. The student rarely evaluates the reasonableness of solutions, and infrequently justifies procedures and decisions in simple familiar situations.

Determining and reporting results

Unit 1 and Unit 2

Schools make judgments on individual assessment instruments using a method determined by the school. They may use the reporting standards or develop an instrument-specific marking guide (ISMG). Marks are not required for determining a unit result for reporting to the QCAA.

The unit assessment program comprises the assessment instrument/s designed by the school to allow the students to demonstrate the unit objectives. The unit judgment of A–E is made using reporting standards.

Schools report student results for Unit 1 and Unit 2 to the QCAA as satisfactory (S) or unsatisfactory (U). Where appropriate, schools may also report a not rated (NR).

Units 3 and 4

Schools mark each of the three internal assessment instruments implemented in Units 3 and 4 using ISMGs.

Schools report a provisional mark by criterion to the QCAA for each internal assessment.

Once confirmed by the QCAA, these results will be combined with the result of the external assessment developed and marked by the QCAA.

The QCAA uses these results to determine each student's subject result as a mark out of 100 and as an A–E.

Units

Unit 1: Combinatorics, proof, vectors and matrices

In Unit 1, students will develop the mathematical understandings and skills to solve problems relating to:

- Topic 1: Combinatorics
- Topic 2: Introduction to proof
- Topic 3: Vectors in the plane
- Topic 4: Algebra of vectors in two dimensions
- Topic 5: Matrices.

Combinatorics provides techniques that are useful in many areas of mathematics, including probability and algebra. Introduction to proof provides the opportunity to establish students' understanding of the nature of proof, and is of great benefit in the study of other topics in the course. Vectors in the plane and the Algebra of vectors in two dimensions provide new perspectives for working with two-dimensional space, and serve as an introduction to techniques that will extend to three-dimensional space in Unit 3. Matrices introduces basic operations and matrix algebra properties.

These five topics considerably broaden students' mathematical experience and enhance their awareness of the breadth and utility of the subject. They contain procedures and processes that will be required for later topics. All these topics develop students' ability to construct mathematical arguments and enable students to increase their mathematical flexibility and versatility.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Combinatorics

Sub-topic: Introduction to counting techniques (4 hours)

- Use the inclusion-exclusion principle formulas to determine the number of elements in the union of two and the union of three sets.
  - $|A \cup B| = |A| + |B| - |A \cap B|$
  - $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
- Use the multiplication principle.
- Use the addition principle.

Sub-topic: Permutations (ordered arrangements) and combinations (unordered selections) (8 hours)

- Define and use permutations.
- Use factorial notation.
- Use the notation ${}^nP_r$ to represent the number of ways of selecting r objects from n distinct objects where order is important.
  - ${}^nP_r = \dfrac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ \ldots \ \times (n-r+1)$
- Solve problems that involve permutations.
- Solve problems that involve permutations with restrictions including repeated objects, specific objects grouped together and selection from multiple groups.
- Define and use combinations.
- Use the notation $\begin{pmatrix} n \\ r \end{pmatrix}$ and ${}^nC_r$ to represent the number of ways of selecting r objects from n distinct objects where order is not important.
  - ${}^nC_r = \dfrac{n!}{r!(n-r)!}$
- Solve problems that involve combinations.
- Solve problems that involve combinations with restrictions including specific objects grouped together and selection from multiple groups.
- Model and solve problems that involve permutations and combinations including probability problems, with and without technology.

Topic 2: Introduction to proof

Sub-topic: The nature of proof (5 hours)

- Use implication, converse, equivalence, negation, contrapositive.
- Use proof by contradiction.
- Use the symbols for implication ($\Rightarrow$), equivalence ($\Leftrightarrow$), and equality (=).
- Use the quantifiers 'for all' ($\forall$) and 'there exists' ($\exists$) .
- Define and use set notation of number systems, including integers ($\mathbb{Z}$), positive integers ($\mathbb{Z}^+$), negative integers ($\mathbb{Z}^-$), rational numbers ($\mathbb{Q}$), irrational numbers ($\mathbb{Q}'$), and real numbers ($\mathbb{R}$).
- Use the set notation symbol 'is an element of' ($\in$) .
- Use examples and counterexamples.

Sub-topic: Rational and irrational numbers (5 hours)

- Prove results involving integers, e.g. proving that the product of two consecutive odd numbers is an odd number and $5n^2 + 3n + 6 \ \forall n \in \mathbb{Z}$ is an even number.
- Express rational numbers as terminating or eventually recurring decimals and vice versa.
- Prove irrationality by contradiction.

Topic 3: Vectors in the plane

Sub-topic: Representing vectors in the plane by directed line segments (5 hours)

- Examine examples of vectors including displacement, velocity and force.
- Understand the difference between a scalar and a vector including distance and displacement, speed and velocity, and magnitude of force and force.
- Define and use the magnitude and direction of a vector.
- Understand and use vector notation: $\overrightarrow{AB}$, $\underset{\sim}{c}$, $\boldsymbol{d}$ and unit vector notation $\widehat{\boldsymbol{n}}$.
- Understand and use vector equality.
- Represent and use a scalar multiple of a vector.
- Use the triangle rule to represent the resultant vector from the sum and difference of two vectors.
- Represent a vector in the plane using a combination of the sum, difference and scalar multiple of other vectors.

Sub-topic: Vectors in two dimensions (5 hours)

- Use ordered pair notation $(x,\ y)$ and column vector notation $\begin{pmatrix} x \\ y \end{pmatrix}$ to represent a position vector in two dimensions.
- Calculate the magnitude and direction of a vector.
  - $|\boldsymbol{a}| = \left|\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\right| = \sqrt{a_1^{\,2} + a_2^{\,2}}$
  - $\tan(\theta) = \dfrac{y}{x},\ x \neq 0$
- Calculate and use a unit vector, $\widehat{\boldsymbol{n}}$, in the plane.
  - $\widehat{\boldsymbol{n}} = \dfrac{\boldsymbol{n}}{|\boldsymbol{n}|}$
- Define and use unit vectors and the perpendicular unit vectors $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$.
- Express a vector in Cartesian (component) form using the unit vectors $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$.
- Understand and express a vector in the plane in polar form using the notation $(r,\ \theta)$.
- Convert between Cartesian form and polar form, with and without technology.
- Understand and use the Cartesian form and polar form of a vector.

Topic 4: Algebra of vectors in two dimensions

Sub-topic: Algebra of vectors in two dimensions (12 hours)

- Examine and use addition and subtraction of vectors in Cartesian form.
- Define and use multiplication by a scalar of a vector in Cartesian form.
- Determine a vector between two points.
- Define and use a vector representing a section of a line segment, including the midpoint of a line segment.
- Define and use the scalar (dot) product.
  - $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos(\theta)$
  - $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = a_1 b_1 + a_2 b_2$
- Examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular.
- Define and use scalar and vector projections of vectors.
  - scalar projection of $\boldsymbol{a}$ on $\boldsymbol{b}$: $|\boldsymbol{a}|\cos(\theta) = \boldsymbol{a} \cdot \widehat{\boldsymbol{b}}$
  - vector projection of $\boldsymbol{a}$ on $\boldsymbol{b}$: $|\boldsymbol{a}|\cos(\theta)\,\widehat{\boldsymbol{b}} = (\boldsymbol{a} \cdot \widehat{\boldsymbol{b}})\,\widehat{\boldsymbol{b}} = \left(\dfrac{\boldsymbol{a}\cdot\boldsymbol{b}}{\boldsymbol{b}\cdot\boldsymbol{b}}\right)\boldsymbol{b}$
- Apply the scalar product to vectors expressed in Cartesian form.
- Resolve vectors into $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$ components.

- Model and solve problems that involve displacement, force, velocity and relative velocity using the above concepts.
- Model and solve problems that involve motion of a body in equilibrium situations, including vector applications related to smooth inclined planes (excluding situations with pulleys and connected bodies).

Topic 5: Matrices

Sub-topic: Matrix arithmetic and algebra (11 hours)

- Understand the matrix definition and notation.
- Define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and multiplicative inverse.
- Use matrix algebra properties, including
  - $\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{B}+\boldsymbol{A}$ (commutative law for addition)
  - $\boldsymbol{A}+\boldsymbol{0}=\boldsymbol{A}$ (additive identity)
  - $\boldsymbol{A}+(-\boldsymbol{A})=\boldsymbol{0}$ (additive inverse)
  - $\boldsymbol{AI}=\boldsymbol{A}=\boldsymbol{IA}$ (multiplicative identity)
  - $\boldsymbol{AA}^{-1}=\boldsymbol{I}=\boldsymbol{A}^{-1}\boldsymbol{A}$ (multiplicative inverse)
  - $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})=\boldsymbol{AB}+\boldsymbol{AC}$ (left distributive law)
  - $(\boldsymbol{B}+\boldsymbol{C})\boldsymbol{A}=\boldsymbol{BA}+\boldsymbol{CA}$ (right distributive law)
- Recognise that matrix multiplication in general is not commutative.
- Calculate the determinant and multiplicative inverse of 2×2 matrices, with and without technology.
  - If $\boldsymbol{A}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\boldsymbol{A})=ad-bc$
  - $\boldsymbol{A}^{-1}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}=\dfrac{1}{\det(\boldsymbol{A})}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix},\ \det(\boldsymbol{A})\neq 0$
- Use matrix algebra to solve matrix equations that involve matrices of up to dimension 2×2, including those of the form $\boldsymbol{AX}=\boldsymbol{B}$, $\boldsymbol{XA}=\boldsymbol{B}$ and $\boldsymbol{AX}+\boldsymbol{BX}=\boldsymbol{C}$, with and without technology.
- Model and solve problems that involve matrices of up to dimension 2×2, including the solution of systems of linear equations, with and without technology.

Unit 2: Complex numbers, further proof, trigonometry, functions and transformations

In Unit 2, students will develop the mathematical understandings and skills to solve problems relating to:

- Topic 1: Complex numbers
- Topic 2: Complex arithmetic and algebra
- Topic 3: Circle and geometric proofs.
- Topic 4: Trigonometry and functions
- Topic 5: Matrices and transformations.

Complex numbers and Complex arithmetic and algebra introduce complex arithmetic, the complex (Argand) plane, and complex algebra, including conjugate roots. Circle proof provides the opportunity to summarise and extend students' studies in circles and deductive Euclidean geometry. Geometric proof serves as an introduction to vector proofs in two-dimensional space that will extend to three-dimensional space in Unit 3. Trigonometry and functions introduces the absolute value function and reciprocal trigonometric functions, the sketching of graphs and builds on the nature of proof using trigonometric identities. Matrices and transformations extends the application of matrices to linear transformations in the plane.

These topics further develop the thinking techniques and mathematical rigour introduced in Unit 1, and provide opportunities to further nurture curiosity about the nature and utility of mathematics.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Complex numbers

Sub-topic: Introduction to complex numbers (4 hours)

- Define the imaginary number i as a root (solution) of the equation $x^2 = -1$.
- Define and use set notation of the number system for complex numbers ($\mathbb{C}$).
- Use complex numbers in the form $a + bi$ where a and b are the real and imaginary parts (components) $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ of a complex number z.
- Determine and use complex conjugates.
- Perform complex-number arithmetic: addition, subtraction, multiplication and division, with and without technology.

Sub-topic: The complex plane (the Argand plane) (6 hours)

- Sketch and use complex numbers as points in the complex plane with real and imaginary parts as Cartesian coordinates.
- Understand and use addition of complex numbers as vector addition in the complex plane.
- Understand and use location of complex conjugates in the complex plane.
- Understand and use multiplication by a complex number as a linear transformation in the complex plane.

Topic 2: Complex arithmetic and algebra

Sub-topic: Complex arithmetic using polar form (4 hours)

- Use the modulus $|z|$ of a complex number z and the principal argument $\mathrm{Arg}(z)$ of a non-zero complex number z.
  - $|z| = \sqrt{a^2 + b^2}$
  - $\mathrm{Arg}(z) = \theta,\ \tan(\theta) = \frac{b}{a},\ -\pi < \theta \le \pi,\ a \ne 0$
- Understand the difference between the argument, $\arg(z)$, and the principal argument, $\mathrm{Arg}(z)$ of a non-zero complex number z.
  - $\arg(z) = Arg(z) + 2\pi n,\ n \in \mathbb{Z}$
- Express a complex number in Cartesian form $z = a + bi$ and polar form.
  - $z = r(\cos(\theta) + i\sin(\theta))$ or $z = r\,\mathrm{cis}(\theta)$
- Convert between Cartesian form and polar form.
- Understand and use multiplication, division of complex numbers in polar form and the geometric interpretation of these.
  - $z_1 z_2 = r_1 r_2\,\mathrm{cis}\left(\theta_1 + \theta_2\right)$
  - $\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\mathrm{cis}\left(\theta_1 - \theta_2\right)$
- Sketch and use complex numbers in polar form as polar coordinates.

Sub-topic: Subsets of the complex plane (the Argand plane) (3 hours)

- Identify and sketch subsets of the complex plane determined by straight lines and circles, e.g. $|z-3i|<4$, $\frac{\pi}{4} \le \text{Arg}(z) \le \frac{3\pi}{4}$, $\text{Re}(z) > \text{Im}(z)$ and $|z-1| = 2|z-i|$.

Sub-topic: Roots of real quadratic equations (4 hours)

- Determine complex conjugate solutions of real quadratic equations with real coefficients using factorisation, completing the square and the quadratic formula, with and without technology.
- Determine and use linear factors of quadratic polynomials with real coefficients that involve the complex conjugate root theorem, e.g. determine the coefficients of a real quadratic equation given one complex root.

Topic 3: Circle and geometric proofs

Sub-topic: Circle properties and their proofs (6 hours)

- Prove the circle properties
  - the angle at the centre subtended by an arc of a circle is twice the angle at the circumference subtended by the same arc
  - an angle in a semicircle is a right angle
  - angles at the circumference of a circle subtended by the same arc are equal
  - the alternate segment theorem
  - the opposite angles of a cyclic quadrilateral are supplementary and its converse
  - a tangent drawn to a circle is perpendicular to the radius at the point of contact and its converse.
- Solve problems finding unknown angles and lengths and prove further results using the circle properties listed above.

Sub-topic: Geometric proofs using vectors (6 hours)

- Prove the diagonals of a parallelogram meet at right angles if and only if it is a rhombus.
- Prove midpoints of the sides of a quadrilateral join to form a parallelogram.
- Prove the sum of the squares of the lengths of a parallelogram's diagonals is equal to the sum of the squares of the lengths of the sides.
- Prove an angle in a semicircle is a right angle.

Topic 4: Trigonometry and functions

Sub-topic: Sketching graphs (3 hours)

- Use and apply the notation $|x|$ for the absolute value for the real number x and the graph of $y=|x|$.
- Understand and use the relationship between the graph of $y=f(x)$ and the graphs of $y=\frac{1}{f(x)}$, $y=|f(x)|$ and $y=f(|x|)$.

Sub-topic: The reciprocal trigonometric functions, secant, cosecant and cotangent (3 hours)

- Define and use the reciprocal trigonometric functions to determine their simplified exact values and sketch their graphs.

Sub-topic: Trigonometric identities (6 hours)

- Prove and apply the Pythagorean identities.
  - $\sin^2(A)+\cos^2(A)=1$
  - $\tan^2(A)+1=\sec^2(A)$
  - $\cot^2(A)+1=\operatorname{cosec}^2(A)$
- Prove and apply the angle sum, difference and double-angle identities for sines and cosines.
  - $\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$
  - $\sin(A-B)=\sin(A)\cos(B)-\cos(A)\sin(B)$
  - $\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$
  - $\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B)$
  - $\sin(2A)=2\sin(A)\cos(A)$
  - $\begin{aligned}\cos(2A)&=\cos^2(A)-\sin^2(A)\\&=1-2\sin^2(A)\\&=2\cos^2(A)-1\end{aligned}$
- Prove and apply the identities for products of sines and cosines expressed as sums and differences.
  - $\sin(A)\sin(B)=\frac{1}{2}(\cos(A-B)-\cos(A+B))$
  - $\cos(A)\cos(B)=\frac{1}{2}(\cos(A-B)+\cos(A+B))$
  - $\sin(A)\cos(B)=\frac{1}{2}(\sin(A+B)+\sin(A-B))$
  - $\cos(A)\sin(B)=\frac{1}{2}(\sin(A+B)-\sin(A-B))$
- Convert sums $a\cos(x)+b\sin(x)$ to $R\cos(x\pm\alpha)$ or $R\sin(x\pm\alpha)$ and apply these to sketch graphs.
- Model and solve problems that involve equations of the form $a\cos(x)+b\sin(x)=c$.
- Prove and apply multi-angle trigonometric identities up to angles of $4x$ using the identities listed above, e.g. $\cos(4x)=8\cos^4(x)-8\cos^2(x)+1$ and $\operatorname{cosec}(2x)-\cot(2x)=\tan(x)$.

Topic 5: Matrices and transformations

Sub-topic: Transformations in the plane (10 hours)

- Understand translations and their representation as column vectors.
- Use basic linear transformations: dilations of the form $(x, y) \rightarrow (ax, by)$, rotations about the origin and reflection in a line that passes through the origin, and the representations of these transformations by 2×2 matrices.
  - dilation of factor a parallel to the x-axis and factor b parallel to the y-axis: $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
  - rotation of angle θ anticlockwise about the origin: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
  - reflection in the line $y = x\tan(\theta)$: $\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$
- Apply these transformations to points in the plane and polygons.
- Understand and use composition of linear transformations and the corresponding matrix products.
- Understand and use inverses of linear transformations and the relationship with the matrix inverse.
- Understand and use the relationship between the determinant and the effect of a linear transformation on area.
- Determine geometric results by matrix multiplications, e.g. showing that the combined effect of two reflections in lines through the origin is a rotation.

Unit 3: Further complex numbers, proof, vectors and matrices

In Unit 3, students will develop the mathematical understandings and skills to solve problems relating to:

- Topic 1: Further complex numbers
- Topic 2: Mathematical induction and trigonometric proofs
- Topic 3: Vectors in two and three dimensions
- Topic 4: Vector calculus
- Topic 5: Further matrices.

Unit 1 introduced vectors with a focus on vectors in two-dimensional space, matrices with a focus on basic operations and matrix algebra properties that involve matrices of up to dimension 2 x 2. In Unit 3, students study three-dimensional vectors, are introduced to Cartesian equations, vector equations and equations of planes, enabling students to solve geometric problems, and explore applications of matrices beyond dimension 2 x 2. Vector calculus extends students' knowledge of calculus from Mathematical Methods and allows them to apply calculus to solve problems involving motion in two- and three-dimensional space.

Unit 2 introduced complex numbers; Unit 3 extends the study of complex numbers to include complex arithmetic using polar form and roots of complex numbers. Mathematical induction and trigonometric proofs continue the developmental concept of proof from Units 1 and 2.

These topics build on prior knowledge to enable a greater depth of analytical thinking and metacognition.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Further complex numbers

Sub-topic: Complex arithmetic using polar form (3 hours)

- Prove complex number identities involving modulus and argument, e.g. $z\bar{z}=|z|^2$, $|z_1||z_2|=|z_1 z_2|$ and $\arg(z_1 z_2)=\arg(z_1)+\arg(z_2)$.
- Use De Moivre's theorem for integral powers.
  - $z^n = r^n \operatorname{cis}(n\theta)$

Sub-topic: Roots of complex numbers (3 hours)

- Determine and examine the nth roots of unity and their location on the unit circle.
- Determine and examine the nth roots of complex numbers and their location in the complex plane.

Sub-topic: Factorisation of polynomials (5 hours)

- Apply the factor theorem and the remainder theorem for polynomials.
- Understand and use the complex conjugate root theorem for polynomials with real coefficients, e.g. factorise a cubic polynomial with real coefficients given one factor.
- Solve polynomial equations over $\mathbb{C}$ to order 4 including those with real and imaginary coefficients, e.g. solve $z^4+z^3-z^2+z-2=0$ and $z^3-2iz^2+z-2i=0$.

Topic 2: Mathematical induction and trigonometric proofs

Sub-topic: Mathematical induction (8 hours)

- Understand the nature of inductive proof including the use of initial statement, assumption statement, inductive step and conclusion.
- Use sigma notation $\left(\sum\right)$ to represent a sum, e.g. $\sum_{i=1}^{3} x_i = x_1+x_2+x_3$ and $\sum_{j=0}^{n} 2^{2j+1} = 2^1+2^3+2^5+\ldots+2^{2n+1}$.
- Prove results for sums for any positive integer n.
- Prove divisibility results for any positive integer n.
- Prove De Moivre's theorem for powers of positive integers.

Sub-topic: Trigonometric proofs using De Moivre's theorem (2 hours)

- Prove multi-angle trigonometric identities up to angles of $4x$ by equating parts using the binomial expansion and De Moivre's theorem, e.g. $\cos(3x)=4\cos^3(x)-3\cos(x)$ and $\sin(3x)=3\sin(x)-4\sin^3(x)$.

Topic 3: Vectors in two and three dimensions

Sub-topic: Vectors in three dimensions (3 hours)

- Use Cartesian coordinates for three-dimensional space, including plotting points.
- Use ordered pair notation (x, y, z) and column vector notation $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ to represent a position vector in three dimensions.
- Calculate the magnitude of a vector
  - $|\boldsymbol{a}| = \left|\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\right| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$
- Calculate and use a unit vector, $\widehat{\boldsymbol{n}}$, in three-dimensional space.
  - $\widehat{\boldsymbol{n}} = \dfrac{\boldsymbol{n}}{|\boldsymbol{n}|}$
- Define and use unit vectors and the perpendicular unit vectors $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$ and $\hat{\boldsymbol{k}}$.
- Express a vector in Cartesian (component) form using the unit vectors $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$ and $\hat{\boldsymbol{k}}$.
- Define and use the altitude φ.

Sub-topic: Algebra of vectors in three dimensions (3 hours)

- Examine and use addition and subtraction of vectors in Cartesian form.
- Use multiplication by a scalar of a vector in Cartesian form.
- Determine a vector between two points.
- Use a vector representing a section of a line segment, including the midpoint of a line segment.
- Use the scalar (dot) product.
  - $a \cdot b = |a||b|\cos(\theta)$
  - $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$
- Examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular.
- Use scalar and vector projections of vectors.
  - scalar projection of $\boldsymbol{a}$ on $\boldsymbol{b}$: $|\boldsymbol{a}|\cos(\theta) = \boldsymbol{a} \cdot \widehat{\boldsymbol{b}}$
  - vector projection of $\boldsymbol{a}$ on $\boldsymbol{b}$: $|\boldsymbol{a}|\cos(\theta)\,\widehat{\boldsymbol{b}} = \left(\boldsymbol{a} \cdot \widehat{\boldsymbol{b}}\right)\widehat{\boldsymbol{b}} = \left(\dfrac{\boldsymbol{a} \cdot \boldsymbol{b}}{\boldsymbol{b} \cdot \boldsymbol{b}}\right)\boldsymbol{b}$
- Apply the scalar product to vectors expressed in Cartesian form.
- Model and solve problems that involve displacement, force, velocity and relative velocity using the above concepts.
- Use vectors to prove geometric results in two dimensions (other than those listed in Unit 2 Topic 3) and in three dimensions.

Sub-topic: Vector and Cartesian equations (7 hours)

- Understand and use equations of spheres.
  - equation of sphere: $(x-h)^2+(y-k)^2+(z-l)^2=r^2$
- Use vector equations of curves in two or three dimensions involving a parameter, and determine a 'corresponding' Cartesian equation in the two-dimensional case.
- Determine vector, parametric and Cartesian equations of straight lines and straight-line segments given the position of two points, or equivalent information, in both two and three dimensions.
  - vector equation of line: $\boldsymbol{r}=\boldsymbol{a}+t\,\boldsymbol{d}$
  - parametric equations of line: $$x=a_1+t\,d_1$$ $$y=a_2+t\,d_2$$ $$z=a_3+t\,d_3$$
  - Cartesian equation of line: $\dfrac{x-a_1}{d_1}=\dfrac{y-a_2}{d_2}=\dfrac{z-a_3}{d_3}$
- Define and use the vector (cross) product to determine a vector normal to a given plane, with and without technology.
  - $\boldsymbol{a}\times\boldsymbol{b}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$
- Use vector methods in applications, including areas of shapes and determining vector and Cartesian equations of a plane and of regions in a plane.
  - vector equation of plane: $\boldsymbol{r}\cdot\boldsymbol{n}=\boldsymbol{a}\cdot\boldsymbol{n}$
  - Cartesian equation of plane: $ax+by+cz+d=0$.

Topic 4: Vector calculus

Sub-topic: Vector calculus (9 hours)

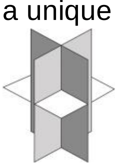
- Understand and use position of vectors as a function of time.
- Understand and use the Cartesian equation of a path given as a vector equation in two dimensions, including circles, ellipses and hyperbolas.
  - equation of circle: $(x-h)^2+(y-k)^2=r^2$
  - equation of ellipse: $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$
  - equation of hyperbola: $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$ or $\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1$
- Understand and use the position of two particles, each described as a vector function of time, and determine if their paths cross or if the particles meet.
- Differentiate and integrate a vector function with respect to time.

- Use vector calculus to determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration.
- Apply vector calculus to model and solve problems that involve motion in a plane, including projectile and circular motion, with and without technology.

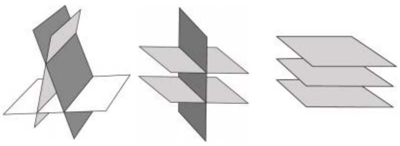
Topic 5: Further matrices

Sub-topic: Matrix algebra and systems of equations (5 hours)

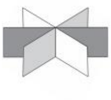
- Calculate the determinant and inverse of square matrices of any order, with technology.
- Use the determinant to determine whether a square matrix of any order is singular or non-singular.
- Use matrix algebra to solve matrix equations that involve matrices of beyond dimension 2×2, including those of the form $\boldsymbol{AX}=\boldsymbol{B}$, $\boldsymbol{XA}=\boldsymbol{B}$ and $\boldsymbol{AX}+\boldsymbol{BX}=\boldsymbol{C}$, with technology.
- Model and solve problems that involve matrices of beyond dimension 2×2, including the solution of systems of linear equations, with technology.
- Recognise the general form of a system of linear equations in several variables and use Gaussian techniques of elimination on an augmented matrix to solve a system of linear equations, with and without technology.
- Examine the three cases for solutions of systems of equations — a unique solution, no solution and infinitely many solutions — and the geometric interpretation of a solution of a system of equations with three variables including
  - a unique solution



  - no solution



  - infinitely many solutions



Sub-topic: Applications of matrices (7 hours)

- Model and solve problems that involve real-life situations using matrices, including Dominance and Leslie matrices.
- Investigate how matrices have been applied in other real-life situations, e.g. Leontief, Markov, area, cryptology, eigenvectors and eigenvalues.

Note: The external examination may assess only Dominance and Leslie matrices.

Unit 4: Further calculus and statistical inference

In Unit 4, students will develop the mathematical understandings and skills to solve problems relating to:

- Topic 1: Integration techniques
- Topic 2: Applications of integral calculus
- Topic 3: Rates of change and differential equations
- Topic 4: Modelling motion
- Topic 5: Statistical inference.

Integration techniques and Applications of integral calculus extend students' knowledge of calculus and examine the complex processes of integration techniques and their uses. Rates of change and differential equations and Modelling motion further extend students' knowledge of calculus and examine the complex processes of calculus techniques. This knowledge is applied to simple differential equations in contexts found in areas such as biology and kinematics. In this unit, the students' previous experience working with probability and statistics is drawn together in the study of Statistical inference for the distribution of sample means and confidence intervals for sample means. Learning in this unit reinforces the real-world applications of the mathematics used throughout Specialist Mathematics. These topics build on the critical and creative thinking techniques introduced in the previous units to facilitate the transition to further studies.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Integration techniques

Sub-topic: Integration techniques (10 hours)

- Integrate using the trigonometric identities $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$, $1 + \tan^2(x) = \sec^2(x)$ and $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$.
- Use substitution $u = g(x)$ to integrate expressions of the form $f(g(x))\,g'(x)$.
- Establish and use the formula $\int \frac{1}{x}\,dx = \ln|x| + c$ for $x \neq 0$ and $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$ for $f(x) \neq 0$.
- Understand and use the inverse trigonometric functions: arcsine, arccosine and arctangent.
- Use the derivative of the inverse trigonometric functions: arcsine, arccosine and arctangent.
  - $\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$
  - $\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$
  - $\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$
- Integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$.
  - $\int \frac{1}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$
  - $\int \frac{-1}{\sqrt{a^2 - x^2}} = \cos^{-1}\left(\frac{x}{a}\right) + c$
  - $\int \frac{a}{a^2 + x^2} = \tan^{-1}\left(\frac{x}{a}\right) + c$
- Use partial fractions for integration involving two distinct linear factors in the denominator, e.g. $\frac{2x - 1}{(x + 1)(x - 2)}$.
- Integrate by parts.
  - $\int u \frac{dv}{dx}\,dx = uv - \int v \frac{du}{dx}\,dx$

Topic 2: Applications of integral calculus

Sub-topic: Applications of integral calculus (10 hours)

- Apply techniques from Unit 4 Topic 1 Sub-topic: Integration techniques to calculate areas between curves determined by functions, with and without technology.
- Determine volumes of solids of revolution about either axis, with and without technology.
  - about the x-axis: $V = \pi \int_a^b [f(x)]^2 \, dx$
  - about the y-axis: $V = \pi \int_a^b [f(y)]^2 \, dy$
- Use Simpson's rule to approximate an area and the value of a definite integral, with and without technology.
  - $\int_a^b f(x) \, dx \approx \frac{w}{3}\left[f(x_0) + 4[f(x_1) + f(x_3) + \ldots] + 2[f(x_2) + f(x_4) + \ldots] + f(x_n)\right]$
    where $w = \frac{b-a}{n}$
- Understand and use the probability density function, $f(t) = \lambda e^{-\lambda t}$ for $t \geq 0$, of the exponential random variable with parameter $\lambda > 0$.
  - mean: $\frac{1}{\lambda}$
  - standard deviation: $\frac{1}{\lambda}$
- Model and solve problems that involve exponential random variables and associated probabilities and quantiles, with and without technology.

Topic 3: Rates of change and differential equations

Sub-topic: Rates of change (3 hours)

- Use implicit differentiation to determine the gradient of curves whose equations are given in implicit form.
- Model and solve related rates problems as instances of the chain rule including situations that involve surface area and volume of cones, pyramids and spheres, with and without technology.

Sub-topic: Differential equations (8 hours)

- Determine general and particular solutions of first-order differential equations of the form $\frac{dy}{dx}=f(x)$, differential equations of the form $\frac{dy}{dx}=g(y)$ and differential equations of the form $\frac{dy}{dx}=f(x)g(y)$ using separation of variables.
- Understand and use slope (direction or gradient) fields of a first-order differential equation.
- Model and solve problems using provided differential equations, including the logistic equation, Newton’s law of cooling and radioactive decay, with and without technology.

Topic 4: Modelling motion

Sub-topic: Modelling motion (11 hours)

- Understand and use momentum, constant force, non-constant force, resultant force, action and reaction.
- Understand and use motion of a body in non-equilibrium situations under concurrent forces.
- Understand and use the expressions $\frac{dv}{dt}$, $\frac{d^2x}{dt^2}$, $v\frac{dv}{dx}$ and $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ to represent the acceleration of an object moving in a straight line.
- Model and solve problems that involve motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion, vertical motion under gravity with and without air resistance, and motion of a body in non-equilibrium situations on a smooth inclined plane (excluding situations with pulleys and connected bodies).
  - If $\frac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin(\omega t+\alpha)$ or $x=A\cos(\omega t+\beta)$
  - $v^2=\omega^2\left(A^2-x^2\right)$
  - $T=\frac{2\pi}{\omega}$
  - $f=\frac{1}{T}$

Topic 5: Statistical inference

Sub-topic: Sample means (7 hours)

- Understand the concept of the sample mean $\overline{X}$ as a random variable whose value varies between samples where X is a random variable with mean μ and the standard deviation σ.
- Use repeated random sampling data from a variety of distributions and a range of sample sizes to examine properties of the distribution of $\overline{X}$ across samples of a fixed size n, including its mean μ, its standard deviation $\frac{\sigma}{\sqrt{n}}$ (where μ and σ are the mean and standard deviation of X) and its approximate normality if n is large.
- Recognise and use the link between the normal distribution of the sample mean and the statistical notation $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.
- Use repeated random sampling data from a variety of distributions and a range of sample sizes to examine the approximate standard normality of $\frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$ for large samples ($n \geq 30$), where s is the sample standard deviation (Central limit theorem).
- Model and solve problems that involve sample means, with and without technology.

Sub-topic: Confidence intervals for means (6 hours)

- Understand the concept of an interval estimate for a parameter associated with a random variable.
- Understand and use the approximate confidence interval $\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$, as an interval estimate for μ, the population mean, where z is the appropriate quantile for the standard normal distribution.
- Understand and use the approximate margin of error.
  - $E = z\frac{s}{\sqrt{n}}$
- Understand and use the relationship between margin of error, level of confidence and sample size.
- Understand and use the concept that there are variations in confidence intervals between samples and that most but not all confidence intervals contain μ.
- Use $\bar{x}$ and s to estimate μ and σ, to obtain approximate intervals covering desired proportions of values of a normal random variable and compare with an approximate confidence interval for μ.
- Model and solve problems that involve interval estimates for sample means, with and without technology.

Assessment

Internal assessment 1: Problem-solving and modelling task (20%)

Students provide a written response to a specific mathematical investigative scenario or context using subject matter from at least one of the topics in Unit 3 or Unit 4. While students may undertake some research, it is not the focus of this task.

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

This task requires students to:

- independently respond to a specific mathematical investigative scenario or context that highlights a real-life application of mathematics
- use relevant stimulus material involving the selected subject matter
- address all the stages of the problem-solving and mathematical modelling approach
- respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Conditions

- Students will use 3 hours of class time and their own time out of class to develop their response.
- This is an individual task.
- Data may be provided or collected individually or collected in groups.
- Appendixes can include raw data, repeated calculations, evidence of authentication and student notes (appendixes are not marked).
- Students must use technology, e.g. scientific calculator, graphics calculator, spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation or word processing.

Response requirements

Written: up to 10 A4 pages, up to 2000 words

Mark allocation

Criterion	Assessment objectives	Marks
Formulate	1, 5	4
Solve	1, 2, 6	7
Evaluate	4, 5	5
Communicate	3, 5	4
	Total marks:	20

Instrument-specific marking guide

Formulate	Marks
The student response has the following characteristics:	
• justified statements of important assumptions • justified statements of important observations • justified mathematical translation of important aspects of the task	3–4
• statement of a relevant assumption • statement of a relevant observation • mathematical translation of an aspect of the task.	1–2
The student response does not match any of the descriptors above.	0

Solve	Marks
The student response has the following characteristics:	
• accurate use of mathematical knowledge for important aspects of the task • efficient use of technology • a complete solution	6–7
• use of mathematical knowledge for an important aspect of the task • use of technology • substantial progress towards a solution	4–5
• simplistic use of mathematical knowledge relevant to the task • simplistic use of technology • progress towards a solution	2–3
• inappropriate use of mathematical knowledge or technology.	1
The student response does not match any of the descriptors above.	0

Evaluate	Marks
The student response has the following characteristics:	
• verified results • justified statements about the reasonableness of the solution by considering the assumptions • justified statements about the reasonableness of the solution by considering the observations • justified statements of relevant strengths of the solution • justified statements of relevant limitations of the solution	4–5
• a verified result • statement about the reasonableness of the solution by considering an assumption or observation • statement of a relevant strength or relevant limitation of the solution	2–3
• statement about the reasonableness of a result or the solution • statement of a strength or limitation.	1
The student response does not match any of the descriptors above.	0

Communicate	Marks
The student response has the following characteristics:	
• correct use of appropriate mathematical language • logical organisation of the response, which can be read independently of the task sheet • justification of decisions using mathematical reasoning	3–4
• use of some appropriate mathematical language • adequate organisation of the response • statement of a decision using mathematical reasoning.	1–2
The student response does not match any of the descriptors above.	0

Internal assessment 2: Examination — short response (15%)

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

The teacher provides an examination that:

- asks students to respond to a number of unseen short response questions
- representatively samples subject matter from any three of the five topics in Unit 3
- provides opportunities for both technology-free and technology-active responses
- may ask students to respond using single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams
  - use assumed knowledge from Units 1 and 2
  - use assumed knowledge from Mathematical Methods Units 1 and 2.

Question specifications

The examination must be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	60%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	20%	These questions can focus on any of the objectives.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	20%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience.

Conditions

- This is an individual supervised task.
- The task may be delivered in two consecutive sessions only if
  - questions in each session are unseen
  - teaching or feedback is not provided between the sessions.
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The teacher must provide the QCAA Specialist Mathematics formula book.
- Students
  - are required to use technology
  - must not bring notes into the examination.

Mark allocation

Criterion	Assessment objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5, 6	15
	Total marks:	15

Instrument-specific marking guide

Foundational knowledge and problem-solving	Cut-off	Marks
The student response has the following characteristics:		
• consistently correct recall and use of mathematical knowledge; authoritative and accurate communication of mathematical knowledge; astute evaluation of the reasonableness of solutions; use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical knowledge to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations	> 93%	15
	> 87%	14
• correct recall and use of mathematical knowledge; clear communication of mathematical knowledge; considered evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical knowledge to solve problems in simple familiar, complex familiar and complex unfamiliar situations	> 80%	13
	> 73%	12
• thorough recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and application of mathematical knowledge to solve problems in simple familiar and complex familiar situations	> 67%	11
	> 60%	10
• recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of some solutions; some use of mathematical reasoning; and some application of mathematical knowledge to make progress towards solving problems in simple familiar situations	> 53%	9
	> 47%	8
• some recall and use of mathematical knowledge; and basic communication of mathematical knowledge	> 40%	7
	> 33%	6
• infrequent recall and use of mathematical knowledge; and basic communication of some mathematical knowledge	> 27%	5
	> 20%	4
• isolated recall and use of mathematical knowledge; and partial communication of rudimentary mathematical knowledge	> 13%	3
	> 7%	2
• isolated and inaccurate recall and use of mathematical knowledge; and disjointed and unclear communication of mathematical knowledge.	> 0%	1
The student response does not match any of the descriptors above.		0

Internal assessment 3: Examination — short response (15%)

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

The teacher provides an examination that:

- asks students to respond to a number of unseen short response questions
- representatively samples subject matter from any three of the five topics in Unit 4
- provides opportunities for both technology-free and technology-active responses
- may ask students to respond using single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams
  - use assumed knowledge from Units 1, 2 and 3
  - use assumed knowledge from Mathematical Methods Units 1, 2 and 3.

Question specifications

The examination must be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	60%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	20%	These questions can focus on any of the objectives.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	20%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience.

Conditions

- This is an individual supervised task.
- The task may be delivered in two consecutive sessions only if
  - questions in each session are unseen
  - teaching or feedback is not provided between the sessions.
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The teacher must provide the QCAA Specialist Mathematics formula book.
- Students
  - are required to use technology
  - must not bring notes into the examination.

Mark allocation

Criterion	Assessment objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5, 6	15
	Total marks:	15

Instrument-specific marking guide

Foundational knowledge and problem-solving	Cut-off	Marks
The student response has the following characteristics:		
• consistently correct recall and use of mathematical knowledge; authoritative and accurate communication of mathematical knowledge; astute evaluation of the reasonableness of solutions; use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical knowledge to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations	> 93%	15
	> 87%	14
• correct recall and use of mathematical knowledge; clear communication of mathematical knowledge; considered evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical knowledge to solve problems in simple familiar, complex familiar and complex unfamiliar situations	> 80%	13
	> 73%	12
• thorough recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and application of mathematical knowledge to solve problems in simple familiar and complex familiar situations	> 67%	11
	> 60%	10
• recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of some solutions; some use of mathematical reasoning; and some application of mathematical knowledge to make progress towards solving problems in simple familiar situations	> 53%	9
	> 47%	8
• some recall and use of mathematical knowledge; and basic communication of mathematical knowledge	> 40%	7
	> 33%	6
• infrequent recall and use of mathematical knowledge; and basic communication of some mathematical knowledge	> 27%	5
	> 20%	4
• isolated recall and use of mathematical knowledge; and partial communication of rudimentary mathematical knowledge	> 13%	3
	> 7%	2
• isolated and inaccurate recall and use of mathematical knowledge; and disjointed and unclear communication of mathematical knowledge.	> 0%	1
The student response does not match any of the descriptors above.		0

External assessment: Examination — combination response (50%)

External assessment is developed and marked by the QCAA. The external assessment in Specialist Mathematics is common to all schools and administered under the same conditions, at the same time, on the same day.

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

This examination:

- consists of two papers: Paper 1 — technology-free, Paper 2 — technology-active
- asks students to respond to a number of unseen short response questions relating to Units 3 and 4
- may ask students to respond using
  - multiple choice
  - single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams
  - use assumed knowledge from Units 1 and 2
  - use assumed knowledge from Mathematical Methods Units 1, 2, 3 and 4.

Paper 1

- Weighted at 25%
- Contains short response questions, including multiple choice
- Technology-free

Paper 2

- Weighted at 25%
- Contains short response questions, including multiple choice
- Technology-active

Question specifications

The examination will be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	60%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	20%	These questions can focus on any of the objectives.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	20%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience.

Conditions

Paper 1

- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The QCAA provides the QCAA Specialist Mathematics formula book.
- Students must not bring notes, calculators, technology or other resources into the examination.

Paper 2

- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The QCAA provides the QCAA Specialist Mathematics formula book.
- Students
  - may use a handheld QCAA-approved graphics calculator (no CAS functionality) and/or a handheld QCAA-approved scientific calculator
  - must not bring notes or other resources into the examination.

Glossary

The syllabus glossary is available at www.qcaa.qld.edu.au/downloads/senior-qce/common/snr_glossary_cognitive_verbs.pdf.

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Version history

Version	Date of change	Information
1.0	January 2024	Released for familiarisation and planning (with implementation starting in 2025)