Specialist Mathematics 2019 v1.2

General Senior Syllabus

This syllabus is for implementation with Year 11 students in 2019.

Queensland Government
Queensland Curriculum & Assessment Authority

180843

For all Queensland schools

Contents

1 Course overview

1.1 Introduction

1.1.1 Rationale

Mathematics is a unique and powerful intellectual discipline that is used to investigate patterns, order, generality and uncertainty. It is a way of thinking in which problems are explored and solved through observation, reflection and logical reasoning. It uses a concise system of communication, with written, symbolic, spoken and visual components. Mathematics is creative, requires initiative and promotes curiosity in an increasingly complex and data-driven world. It is the foundation of all quantitative disciplines.

To prepare students with the knowledge, skills and confidence to participate effectively in the community and the economy requires the development of skills that reflect the demands of the 21st century. Students undertaking Mathematics will develop their critical and creative thinking, oral and written communication, information & communication technologies (ICT) capability, ability to collaborate, and sense of personal and social responsibility — ultimately becoming lifelong learners who demonstrate initiative when facing a challenge. The use of technology to make connections between mathematical theory, practice and application has a positive effect on the development of conceptual understanding and student disposition towards mathematics.

Mathematics teaching and learning practices range from practising essential mathematical routines to develop procedural fluency, through to investigating scenarios, modelling the real world, solving problems and explaining reasoning. When students achieve procedural fluency, they carry out procedures flexibly, accurately and efficiently. When factual knowledge and concepts come to mind readily, students are able to make more complex use of knowledge to successfully formulate, represent and solve mathematical problems. Problem-solving helps to develop an ability to transfer mathematical skills and ideas between different contexts. This assists students to make connections between related concepts and adapt what they already know to new and unfamiliar situations. With appropriate effort and experience, through discussion, collaboration and reflection of ideas, students should develop confidence and experience success in their use of mathematics.

The major domains of mathematical knowledge in Specialist Mathematics are Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus. Topics are developed systematically, with increasing levels of sophistication, complexity and connection, building on functions, calculus, statistics from Mathematical Methods, while vectors, complex numbers and matrices are introduced. Functions and calculus are essential for creating models of the physical world. Statistics are used to describe and analyse phenomena involving probability, uncertainty and variation. Matrices, complex numbers and vectors are essential tools for explaining abstract or complex relationships that occur in scientific and technological endeavours.

Students who undertake Specialist Mathematics will develop confidence in their mathematical knowledge and ability, and gain a positive view of themselves as mathematics learners. They will gain an appreciation of the true nature of mathematics, its beauty and its power.

Assumed knowledge, prior learning or experience

Specialist Mathematics is designed to be taken in conjunction with, or on completion of, Mathematical Methods. It is assumed that work covered in Mathematical Methods will be known before it is required in Specialist Mathematics.

Assumed knowledge refers to the subject matter that teachers can expect students to know prior to beginning this subject. Emphasis is placed on the mastery of content, ensuring key concepts or procedures are learnt fully so they will not need reteaching.

Developing mastery often involves multiple approaches to teaching and conceptualising the same mathematical concept. When students have a good understanding of a key concept or procedure, they are more easily able to make connections to related new subject matter and apply what they already know to new problems.

Subject matter from previous unit/s is assumed for subsequent unit/s.

The following is a non-exhaustive list of assumed knowledge from the P–10 Australian Curriculum that must be learnt or revised and maintained as required:

- describe the results of two- and three-step chance experiments
- determine probabilities of events
- substitute values into formulas to determine an unknown
- solve right-angled triangle problems
- describe, interpret and sketch hyperbolas and circles
- translate word problems to mathematical form
- factorise, expand and simplify algebraic expressions, including monic quadratic expressions using a variety of strategies
- apply the four operations to simple algebraic fractions with numerical denominators.

Recommended knowledge

Recommended knowledge refers to the subject matter from the Year 10A Australian Curriculum that will enhance students' understanding of this subject's foundational content.

The following in a non-exhaustive list of recommended knowledge from the Year 10A Australian Curriculum:

- rational and irrational numbers
- properties of circles
- trigonometry
- sketching functions
- factor and remainder theorem.

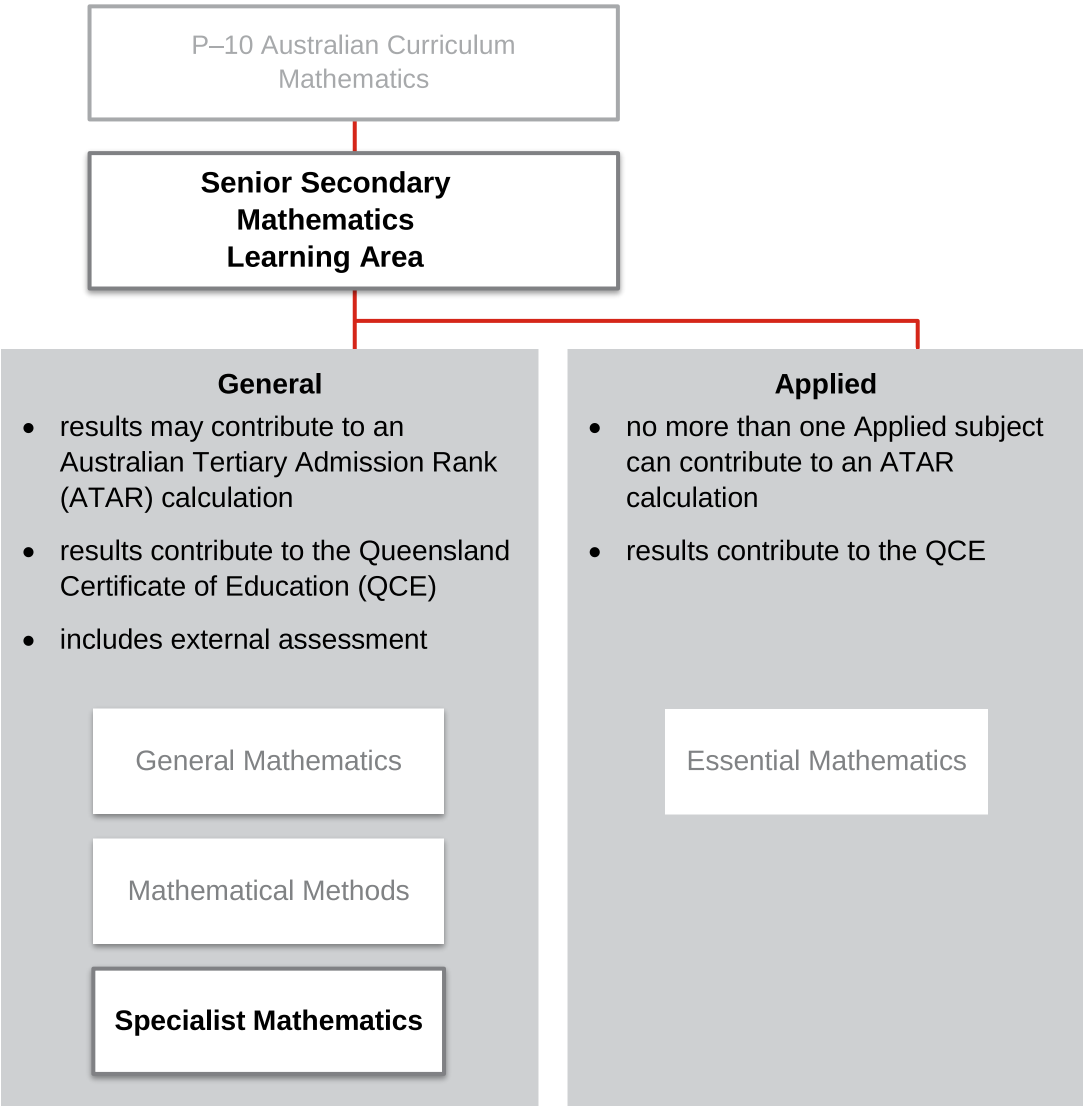
Pathways

Specialist Mathematics is a General subject suited to students who are interested in pathways beyond school that lead to tertiary studies, vocational education or work. A course of study in Specialist Mathematics can establish a basis for further education and employment in the fields of science, all branches of mathematics and statistics, computer science, medicine, engineering, finance and economics.

1.1.2 Learning area structure

All learning areas build on the P–10 Australian Curriculum.

Figure 1: Learning area structure



Specialist Mathematics is to be undertaken in conjunction with, or on completion of, Mathematical Methods.

1.1.3 Course structure

Specialist Mathematics is a course of study consisting of four units. Subject matter, learning experiences and assessment increase in complexity from Units 1 and 2 to Units 3 and 4 as students develop greater independence as learners.

Units 1 and 2 provide foundational learning, which allows students to experience all syllabus objectives and begin engaging with the course subject matter. Students should complete Units 1 and 2 before beginning Unit 3. It is recommended that Unit 3 be completed before Unit 4.

Units 3 and 4 consolidate student learning. Only the results from Units 3 and 4 will contribute to ATAR calculations.

Figure 2 outlines the structure of this course of study.

Each unit has been developed with a notional time of 55 hours of teaching and learning, including assessment.

Figure 2: Course structure

Specialist Mathematics

Unit 1	Unit 2	Unit 3	Unit 4
Unit 1 **Combinatorics, vectors and proof**	**Unit 2** **Complex numbers, trigonometry, functions and matrices**	**Unit 3** **Mathematical induction, and further vectors, matrices and complex numbers**	**Unit 4** **Further calculus and statistical inference**
• Topic 1: Combinatorics • Topic 2: Vectors in the plane • Topic 3: Introduction to proof	• Topic 1: Complex numbers 1 • Topic 2: Trigonometry and functions • Topic 3: Matrices	• Topic 1: Proof by mathematical induction • Topic 2: Vectors and matrices • Topic 3: Complex numbers 2	• Topic 1: Integration and applications of integration • Topic 2: Rates of change and differential equations • Topic 3: Statistical inference
Assessment Formative internal assessment/s	**Assessment** Formative internal assessment/s	**Assessment** Summative internal assessment 1 Problem-solving and modelling task (20%) Summative internal assessment 2: Examination (15%)	**Assessment** Summative internal assessment 3: Examination (15%)

Students should have opportunities in Units 1 and 2 to experience and respond to the types of assessment they will encounter in Units 3 and 4.

For reporting purposes, schools should develop at least *one* assessment per unit, with a maximum of *four* assessments across Units 1 and 2.

Summative external assessment:
Examination (50%)

1.2 Teaching and learning

1.2.1 Syllabus objectives

The syllabus objectives outline what students have the opportunity to learn. Assessment provides evidence of how well students have achieved the objectives.

Syllabus objectives inform unit objectives, which are contextualised for the subject matter and requirements of the unit. Unit objectives, in turn, inform the assessment objectives, which are further contextualised for the requirements of the assessment instruments. The number of each objective remains constant at all levels, i.e. Syllabus objective 1 relates to Unit objective 1 and to Assessment objective 1 in each assessment instrument.

Syllabus objectives are described in terms of actions that operate on the subject matter. Students are required to use a range of cognitive processes in order to demonstrate and meet the syllabus objectives. These cognitive processes are described in the explanatory paragraph following each objective in terms of four levels: retrieval, comprehension, analytical processes (analysis), and knowledge utilisation, with each process building on the previous processes (see Marzano & Kendall 2007, 2008). That is, comprehension requires retrieval, and knowledge utilisation requires retrieval, comprehension and analytical processes (analysis).

By the conclusion of the course of study, students will:

Syllabus objective	Unit 1	Unit 2	Unit 3	Unit 4
1. select, recall and use facts, rules, definitions and procedures drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus	●	●	●	●
2. comprehend mathematical concepts and techniques drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus	●	●	●	●
3. communicate using mathematical, statistical and everyday language and conventions	●	●	●	●
4. evaluate the reasonableness of solutions	●	●	●	●
5. justify procedures and decisions by explaining mathematical reasoning	●	●	●	●
6. solve problems by applying mathematical concepts and techniques drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus	●	●	●	●

1. **select, recall and use facts, rules, definitions and procedures drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus**

   When students select, recall and use facts, rules, definitions and procedures, they recognise particular features of remembered information and consider its accuracy and relevance. They choose relevant facts, rules, definitions and procedures and put them into effect, performing calculations with and without the use of technology.

2. **comprehend mathematical concepts and techniques drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus**

   When students comprehend, they understand the meaning, nature and purpose of the mathematics they are learning. They identify, articulate and symbolise the critical elements of the relevant concepts and techniques, making connections between topics and between the 'why' and the 'how' of mathematics.

3. **communicate using mathematical, statistical and everyday language and conventions**

   When students communicate, they use mathematical and statistical terminology, symbols, conventions and everyday language to organise and present information in graphical and symbolic form, and describe and represent mathematical and statistical models.

4. **evaluate the reasonableness of solutions**

   When students evaluate the reasonableness of solutions, they interpret their mathematical results in the context of the situation. They reflect on whether the problem has been solved by using estimation skills and checking calculations using their knowledge of relevant facts, rules, definitions and procedures. They make an appraisal by assessing strengths, implications and limitations of solutions and/or models with and without technology, and use this to consider if alternative methods or refinements are required.

5. **justify procedures and decisions by explaining mathematical reasoning**

   When students justify procedures and decisions by explaining mathematical reasoning, they describe their mathematical thinking in detail, identifying causes and making relationships evident, constructing mathematical arguments, proving propositions, and providing reasons for choices made and conclusions reached. Students use their conceptual understanding to connect what they already know to new information. Mathematical reasoning is rigorous and requires clarity, precision, completeness and due regard to the order of statements.

6. **solve problems by applying mathematical concepts and techniques drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus**

   When students solve problems by applying mathematical concepts and techniques, they analyse the context of the problem and make decisions about the concepts, techniques and technology that must be used to develop a solution. They analyse, generalise and translate information into a mathematically workable format, synthesise and refine models, and generate and test hypotheses with primary or secondary data and information.

1.2.2 Underpinning factors

There are three skill sets that underpin senior syllabuses and are essential for defining the distinctive nature of subjects:

- literacy — the set of knowledge and skills about language and texts essential for understanding and conveying Specialist Mathematics content
- numeracy — the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations, to recognise and understand the role of

mathematics in the world, and to develop the dispositions and capacities to use mathematical knowledge and skills purposefully

- 21st century skills — the attributes and skills students need to prepare them for higher education, work and engagement in a complex and rapidly changing world.

These skill sets, which overlap and interact, are derived from current education, industry and community expectations. They encompass the knowledge, skills, capabilities, behaviours and dispositions that will help students live and work successfully in the 21st century.

Together these three skill sets shape the development of senior subject syllabuses. Although coverage of each skill set may vary from syllabus to syllabus, students should be provided with opportunities to learn through and about these skills over the course of study. Each skill set contains identifiable knowledge and skills that can be directly assessed.

Literacy in Specialist Mathematics

Literacy skills and strategies enable students to express, interpret and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their abilities to read, write, visualise and talk about complex situations involving a range of mathematical ideas.

Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

To understand and use Specialist Mathematics content, teaching and learning strategies include:

- breaking the language code to make meaning of Specialist Mathematics language and texts
- comprehending language and texts to make literal and inferred meanings about Specialist Mathematics content
- using Specialist Mathematics ideas and information in classroom, real-world and/or lifelike contexts to progress students' learning.

To analyse and evaluate Specialist Mathematics content, teaching and learning strategies include:

- making conclusions about the purpose and audience of Specialist Mathematics language and texts
- analysing the ways language is used to convey ideas and information in Specialist Mathematics texts
- transforming language and texts to convey Specialist Mathematics ideas and information in particular ways to suit audience and purpose.

These aspects of literacy knowledge and skills are embedded in the syllabus objectives, unit objectives and subject matter, and instrument-specific marking guides (ISMGs) for Specialist Mathematics.

Numeracy in Specialist Mathematics

Numeracy relates to the capacity to deal with quantitative aspects of life (Goos, Geiger & Dole 2012). It involves accessing, using, interpreting and communicating mathematical information and ideas when engaging with and managing the mathematical demands of real contexts — everyday and civic life, the world of work, and opportunities for further learning (OECD 2012). Numerate citizens who are constructive, engaged and reflective are able to use mathematics to help make credible judgments and reasoned decisions (OECD 2015).

Unlike mathematics, numeracy must be understood as inseparable from context:

> Mathematics climbs the ladder of abstraction to see, from sufficient height, common patterns in seemingly different things. Abstraction is what gives mathematics its power; it is what enables methods derived in one context to be applied in others. But abstraction is not the focus of numeracy. Instead, numeracy clings to specifics, marshalling all relevant aspects of setting and context to reach conclusions.
>
> To enable students to become numerate, teachers must encourage them to see and use mathematics in everything they do. Numeracy is driven by issues that are important to people in their lives and work, not by future needs of the few who may make professional use of mathematics or statistics (Steen 2001, pp. 17–18).

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in P–10. For example, this subject contains topics that will equip students for the ever-increasing demands of the information age.

These aspects of numeracy knowledge and skills are embedded in the syllabus objectives, unit objectives and subject matter, and ISMGs for Specialist Mathematics.

21st century skills

The 21st century skills identified in the following table reflect a common agreement, both in Australia and internationally, on the skills and attributes students need to prepare them for higher education, work and engagement in a complex and rapidly changing world.

21st century skills	Associated skills	21st century skills	Associated skills
critical thinking	• analytical thinking • problem-solving • decision-making • reasoning • reflecting and evaluating • intellectual flexibility	creative thinking	• innovation • initiative and enterprise • curiosity and imagination • creativity • generating and applying new ideas • identifying alternatives • seeing or making new links
communication	• effective oral and written communication • using language, symbols and texts • communicating ideas effectively with diverse audiences	collaboration and teamwork	• relating to others (interacting with others) • recognising and using diverse perspectives • participating and contributing • community connections
personal and social skills	• adaptability/flexibility • management (self, career, time, planning and organising) • character (resilience, mindfulness, open- and fair-mindedness, self-awareness) • leadership • citizenship • cultural awareness • ethical (and moral) understanding	information & communication technologies (ICT) skills	• operations and concepts • accessing and analysing information • being productive users of technology • digital citizenship (being safe, positive and responsible online)

Specialist Mathematics helps develop the following 21st century skills:

- critical thinking
- creative thinking
- communication
- information & communication technologies (ICT) skills.

These elements of 21st century skills are embedded in the syllabus objectives, unit objectives and subject matter, and ISMGs for Specialist Mathematics.

Use of digital technology

An important aspect of teaching and learning in the 21st century is to embed digital technologies so that they are not seen as optional tools. Digital technologies allow new approaches to explaining and presenting mathematics, and can assist in connecting representations and deepening understanding. They can make previously inaccessible mathematics accessible and increase the opportunities for teachers to make mathematics interesting to a wider range of students. The computational and graphing capabilities of digital technologies enable students to engage in active learning through exploratory work and experiments using realistic data. The ability to visualise solutions can give problems more meaning. Digital technologies can support the development of conceptual understanding that can lead to enhanced procedural fluency.

To meet the requirements of this syllabus, students must make use of a range of digital technologies, such as:

- general-purpose computer software that can be used for mathematics teaching and learning, e.g. spreadsheet software, applications
- computer software designed for mathematics teaching and learning, e.g. dynamic graphing software, dynamic geometry software
- handheld (calculator) technologies designed for mathematics teaching and learning, e.g. scientific, graphics (non-CAS or CAS) calculators, smartphone and tablet apps.

Students must make choices about various forms of technology and develop the ability to work with these flexibly. Technology use must go beyond simple computation or word processing.

Access to a handheld graphics calculator (no CAS functionality) is a requirement for Paper 2 of the external assessment. Scientific calculators may also be used.

1.2.3 Aboriginal perspectives and Torres Strait Islander perspectives

The QCAA is committed to reconciliation in Australia. As part of its commitment, the QCAA affirms that:

- Aboriginal peoples and Torres Strait Islander peoples are the first Australians, and have the oldest living cultures in human history
- Aboriginal peoples and Torres Strait Islander peoples have strong cultural traditions and speak diverse languages and dialects, other than Standard Australian English
- teaching and learning in Queensland schools should provide opportunities for students to deepen their knowledge of Australia by engaging with the perspectives of Aboriginal peoples and Torres Strait Islander peoples
- positive outcomes for Aboriginal students and Torres Strait Islander students are supported by successfully embedding Aboriginal perspectives and Torres Strait Islander perspectives across planning, teaching and assessing student achievement.

Guidelines about Aboriginal perspectives and Torres Strait Islander perspectives and resources for teaching are available at www.qcaa.qld.edu.au/k-12-policies/aboriginal-torres-strait-islander-perspectives.

Where appropriate, Aboriginal perspectives and Torres Strait Islander perspectives have been embedded in the subject matter.

To understand and use mathematics content, teaching and learning strategies may include:

- using pedagogies such as Maths as Storytelling (MAST)

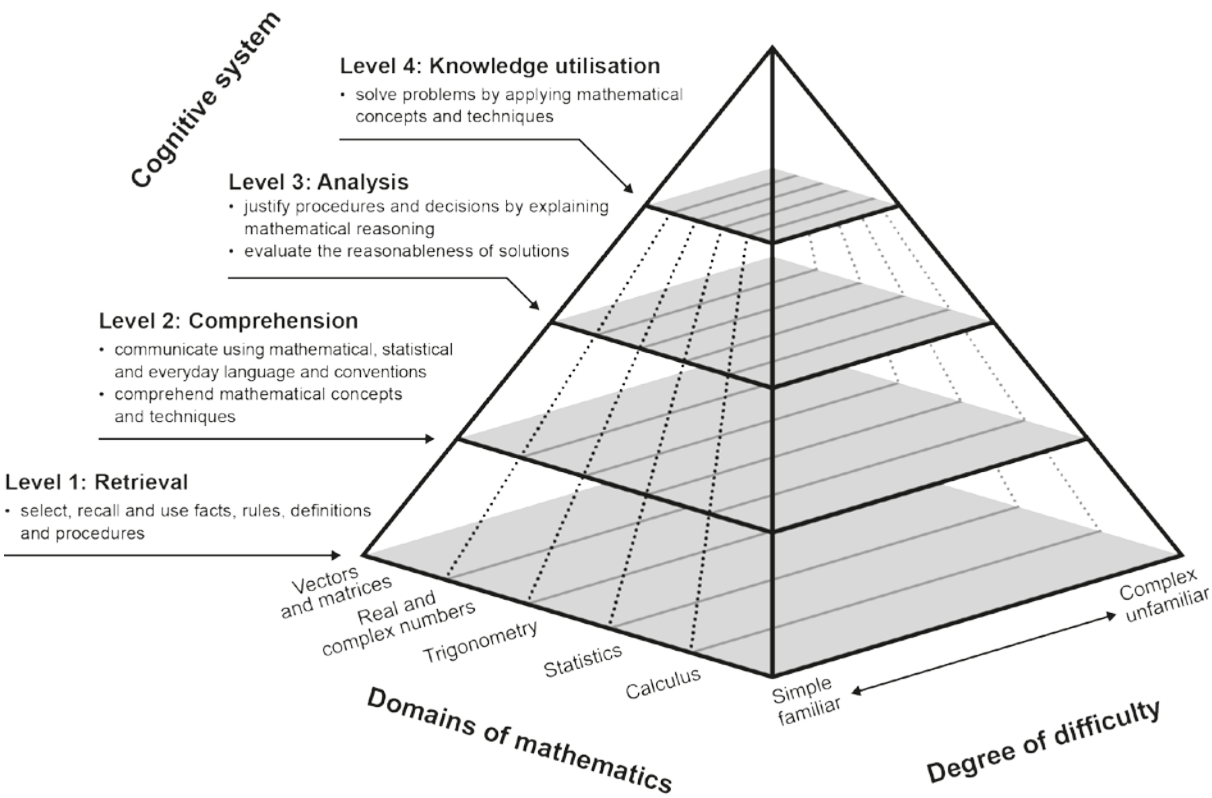
- using mathematics subject matter in real-world Aboriginal contexts and Torres Strait Islander contexts
- identifying the specific issues that may affect Aboriginal peoples and Torres Strait Islander peoples that are relevant to the mathematics topics being covered
- providing learning experiences and opportunities that support the application of students' general mathematical knowledge and problem-solving processes in an Aboriginal context and Torres Strait Islander context.

1.2.4 Pedagogical and conceptual frameworks

The relationship between foundational knowledge and problem-solving

To succeed in mathematics assessment, students must understand the subject matter (organised in domains of mathematics), draw on a range of cognitive skills, and apply these to problems of varying degrees of difficulty, from simple and routine, through to unfamiliar situations, complex contexts, and multi-step solutions (Grønmo et al. 2015). The relationship between the domains of mathematics in Specialist Mathematics, level of cognitive skill required (syllabus objective) and degree of difficulty is represented in three dimensions for mathematics problems in the following diagram.

Figure 3: Assessment pyramid



Adapted from Verhage & de Lange (1997) and Marzano & Kendall (2007).

Principles of developing mathematics problems

This representation, known as the ‘assessment pyramid’, shows the relative distribution of thinking and range of difficulty of mathematics problems.[1] It places an emphasis on building up from the basics. Success in mathematics is built on knowledge of basic facts and proficiency with foundational processes (Norton & O’Connor 2016). With a solid foundation, students can then be asked to apply higher level cognitive processes in more complex and unfamiliar situations that require the application of a wider range of concepts and skills.

The degree of difficulty

The difficulty of a problem is defined by its complexity and a student’s familiarity with it, not the level of cognitive process required to solve it. The complexity of a particular type of problem doesn't change, but familiarity does. With practice, students become more familiar with a process and can execute it more quickly and easily (Marzano & Kendall 2007).

The cognitive system

To solve a full range of mathematics problems, students are required to engage the cognitive system at all four levels of processing knowledge: retrieval, comprehension, analysis, and knowledge utilisation (Marzano & Kendall 2007). The syllabus objectives are represented in the pyramid model through their alignment to these levels.

Using a full range of questions

The pyramid model shows that problems requiring Level 1 processes to solve them can be hard and relatively complex, even though they are based on ‘retrieval’ and therefore might seem easy and straightforward (Shafer & Foster 1997). Problems requiring higher level processes to solve them are not necessarily more difficult than those in Level 1. There are some students who find Level 1 processes more challenging and have more success in solving problems requiring Levels 2, 3 and 4 (Webb 2009).

The distance along the domains of mathematics dimension and the degree of difficulty dimension decreases for higher levels. Problems requiring Level 1 processes can more easily be based on distinct subject matter and the difference between easy and hard can be great. Problems that require students to use more levels of cognition tend to also involve making connections with subject matter within and across the domains of mathematics. They are often placed in contexts that require strategic mathematical decisions and making representations according to situation and purpose. At higher levels the difference between easy and hard is smaller (Shafer & Foster 1997; Webb 2009). Students should master basic facts and processes through practising simple familiar problems, before moving on to those that are more complex and unfamiliar, at any level.[2]

The assessment pyramid helps visualise what is necessary for a complete assessment program. Problems in a complete mathematics program need to assess a student’s growth and achievement in all domains of mathematics and across the full range of objectives. Over time, through a teaching and learning period, students will be exposed to problems that 'fill the pyramid'. Each assessment instrument will reflect this for the relevant subject matter, providing

[1] In an assessment instrument for Mathematics, a ‘problem’ is synonymous with ‘assessment item’ (a question, task or command that forms part of an assessment technique).

[2] Complex unfamiliar questions that require more levels of cognitive skills should not be equated with elaborate problem-solving tasks and modelling questions only. A single-answer, conventional question, such as: ‘Find the equation of the line passing through the points (2,1) and (1,3)’ can be adapted to a more open ended question, such as: ‘Write the equations of at least five lines passing through the point (2,1)’ (Goos 2014). This revised question targets the identical subject matter but provides the possibility of easily identifying a variety of student understanding and skills by moving it towards complex unfamiliar and assessing more cognitive skills. For further examples, see White et al. (2000).

students with the opportunity to demonstrate what they know and can do at all levels of thinking and at varying degrees of difficulty (Shafer & Foster 1997).

Problem-solving and mathematical modelling

A key aspect of learning mathematics is to develop strategic competence; that is, to formulate, represent and solve mathematical problems (Kilpatrick, Swafford & Bradford 2001). As such, problem-solving is a focus of mathematics education research, curriculum and teaching (Sullivan 2011). This focus is not to the exclusion of routine exercises, which are necessary for practising, attaining mastery and being able to respond automatically. But mathematics education in the 21st century goes beyond this to include innovative problems that are complex, unfamiliar and non-routine (Mevarech & Kramarski 2014).

Problem-solving in mathematics can be set in purely mathematical contexts or real-world contexts. When set in the real world, problem-solving in mathematics involves mathematical modelling.

Problem-solving

Problem-solving is required when a task or goal has limiting conditions placed upon it or an obstacle blocking the path to a solution (Marzano & Kendall 2007). It involves:

- knowledge of the relevant details
- using generalisations and principles to identify, define and interpret the problem
- mental computation and estimation
- critical, creative and lateral thinking
- creating or choosing a strategy
- making decisions
- testing, monitoring and evaluating solutions.

Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples without understanding. Through problem-solving, students will make connections between mathematics topics, across the curriculum and with the real world, and see the value and usefulness of mathematics. Problems may be real-world or abstract, and presented to students as issues, statements or questions that may require them to use primary or secondary data.

Mathematical modelling

Mathematical modelling begins from an assumption that mathematics is everywhere in the world around us — a challenge is to identify where it is present, access it and apply it productively. Models are developed in order to better understand real-world phenomena, to make predictions and answer questions. A mathematical model depicts a situation by expressing relationships using mathematical concepts and language. It refers to the set of simplifying assumptions (such as the relevant variables or the shape of something); the set of assumed relationships between variables; and the resulting representation (such as a formula) that can be used to generate an answer (Stacey 2015).

Mathematical modelling involves:

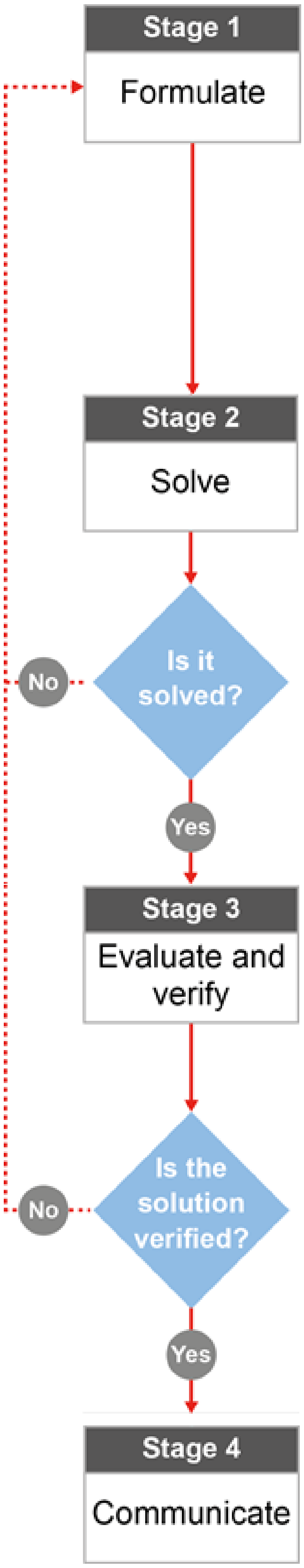
- formulating a mathematical representation of a problem derived from within a real-world context
- using mathematics concepts and techniques to obtain results
- interpreting the results by referring back to the original problem context
- revising the model (where necessary) (Geiger, Faragher & Goos 2010).

Through developing and applying mathematical models, students cumulatively become real-world problem-solvers. Ultimately, this means that they not only can productively address problems set by others, but also that they develop the ability to identify and address problems and answer questions that matter to them.

The following section outlines an approach to problem-solving and mathematical modelling.[3] Problems must be real-world, and can be presented to students as issues, statements or questions that may require them to use primary or secondary data.

[3] A wide variety of frameworks for problem-solving and modelling exist in mathematics education literature. The approach outlined here aligns with and is informed by other approaches, such as Polya (1957) in *How to Solve It: A new aspect of mathematical method* (1957), the Australian Curriculum (ACARA 2015a) *Statistical investigation process*, the OECD/PISA Mathematics framework (OECD 2015, 2003) and *A framework for success in implementing mathematical modelling in the secondary classroom* (Stillman et al. 2007). For further reading see Blum et al. (2007); Kaiser et al. (2011); and Stillman et al. (2013).

Figure 4: An approach to problem-solving and mathematical modelling



Once students understand what the problem is asking, they must design a plan to solve the problem. Students translate the problem into a mathematically purposeful representation by first determining the applicable mathematical and/or statistical principles, concepts, techniques and technology that are required to make progress with the problem. Appropriate assumptions, variables and observations are identified and documented, based on the logic of a proposed solution and/or model.

In mathematical modelling, formulating a model involves the process of mathematisation — moving from the real world to the mathematical world.

Students select and apply mathematical and/or statistical procedures, concepts and techniques previously learnt to solve the mathematical problem to be addressed through their model. Possible approaches are wide-ranging and include synthesising and refining existing models, and generating and testing hypotheses with primary or secondary data and information, as well as using standard mathematical techniques to produce a valid solution.

Solutions can be found using algebraic, graphic, arithmetic and/or numeric methods, with and/or without technology.

Once a possible solution has been achieved, students need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. They evaluate their results and make a judgment about the solution/s to the problem in relation to the original issue, statement or question.

This involves exploring the strengths and limitations of the solution and/or model. Where necessary, this will require going back through the process to further refine the solution and/or model. In mathematical modelling, students must check that the output of their model provides a valid solution to the real-world problem it has been designed to address.

This stage emphasises the importance of methodological rigour and the fact that problem-solving and mathematical modelling is not usually linear and involves an iterative process.

The development of solutions and models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated clearly and fully. Students communicate findings systematically and concisely using mathematical, statistical and everyday language. They draw conclusions, discussing the key results and the strengths and limitations of the solution and/or model. Students could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem.

Approaches to problem-solving and mathematical modelling in the classroom

When teaching problem-solving and mathematical modelling, teachers should consider *teaching for* and *learning through* problem-solving and mathematical modelling. When *teaching for*, students are taught the specific mathematical rules, definitions, procedures, problem-solving strategies and critical elements of the model that are needed to solve a given problem. When *learning through*, students are presented with problems to solve, but must apply the knowledge and skills they have previously been taught to solve it. By solving these problems, students are able to develop new mathematical understanding and skills. This requires an explicit and connected approach to teaching problem-solving and mathematical modelling that necessitates fluency of critical facts and processes at each step.

The following describes three different approaches to teaching problem-solving and mathematical modelling[4] along the continua between *teaching for* and *learning through*:

Approach	Description	Teaching *for* or learning *through*
Dependent	The teacher explicitly demonstrates and teaches the concepts and techniques required to solve the problem, and/or develop a mathematical model. This usually involves students solving (stage 2), and evaluating and verifying (stage 3).	*Teaching for*
Guided	The teacher influences the choice of concepts and techniques, and/or model that students use to solve the problem. Guidance is provided and all stages of the approach are used.	Moving towards *learning through*
Independent	The teacher cedes control and students work independently, choosing their own solution and/or model, and working at their own level of mathematics. The independent approach is the most challenging.	*Learning through*

These approaches are not mutually exclusive. An independent approach (*learning through*) might be undertaken as an extension of a dependent or guided activity that students have previously undertaken (*teaching for*). Students need to have attained the relevant foundational understanding and skills before working independently during the problem-solving and modelling task. This capacity needs to be built over time through the course of study with teachers closely monitoring student progress.

1.2.5 Subject matter

Subject matter is the body of information, mental procedures and psychomotor procedures (see Marzano & Kendall 2007, 2008) that are necessary for students' learning and engagement with Specialist Mathematics. It is particular to each unit in the course of study and provides the basis for student learning experiences.

Subject matter has a direct relationship to the unit objectives, but is of a finer granularity and is more specific. These statements of learning are constructed in a similar way to objectives. Each statement:

- describes an action (or combination of actions) — what the student is expected to do
- describes the element — expressed as information, mental procedures and/or psychomotor procedures
- is contextualised for the topic or circumstance particular to the unit.

Subject matter in Specialist Mathematics is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

[4] Based on Galbraith (1989).

1.3 Assessment — general information

Assessments are formative in Units 1 and 2, and summative in Units 3 and 4.

Assessment	Unit 1	Unit 2	Unit 3	Unit 4
Formative assessments	●	●		
Summative internal assessment 1			●	
Summative internal assessment 2			●	
Summative internal assessment 3				●
Summative external assessment*			●	●

* Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of the supervised examination.

1.3.1 Formative assessments — Units 1 and 2

Formative assessments provide feedback to both students and teachers about each student's progress in the course of study.

Schools develop internal assessments for each senior subject, based on the learning described in Units 1 and 2 of the subject syllabus. Each unit objective must be assessed at least once.

For reporting purposes, schools should devise at least *two* but no more than *four* assessments for Units 1 and 2 of this subject. At least *one* assessment must be completed for *each* unit.

The sequencing, scope and scale of assessments for Units 1 and 2 are matters for each school to decide and should reflect the local context.

Teachers are encouraged to use the A–E descriptors in the reporting standards (Section 1.4) to provide formative feedback to students and to report on progress.

1.3.2 Summative assessments — Units 3 and 4

Students will complete a total of *four* summative assessments — three internal and one external — that count towards their final mark in each subject.

Schools develop *three* internal assessments for each senior subject, based on the learning described in Units 3 and 4 of the syllabus.

The three summative internal assessments will be endorsed and the results confirmed by the QCAA. These results will be combined with a single external assessment developed and marked by the QCAA. The external assessment results for Specialist Mathematics will contribute 50% towards a student's result.

Summative internal assessment — instrument-specific marking guides

This syllabus provides ISMGs for the three summative internal assessments in Units 3 and 4.

The ISMGs describe the characteristics evident in student responses and align with the identified assessment objectives. Assessment objectives are drawn from the unit objectives and are contextualised for the requirements of the assessment instrument.

Criteria

Each ISMG groups assessment objectives into criteria. An assessment objective may appear in multiple criteria, or in a single criterion of an assessment.

Making judgments

Assessment evidence of student performance in each criterion is matched to a performance-level descriptor, which describes the typical characteristics of student work.

Where a student response has characteristics from more than one performance level, a best-fit approach is used. Where a performance level has a two-mark range, it must be decided if the best fit is the higher or lower mark of the range.

Authentication

Schools and teachers must have strategies in place for ensuring that work submitted for internal summative assessment is the student's own. Authentication strategies outlined in QCAA guidelines, which include guidance for drafting, scaffolding and teacher feedback, must be adhered to.

Summative external assessment

The summative external assessment adds valuable evidence of achievement to a student's profile. External assessment is:

- common to all schools
- administered under the same conditions at the same time and on the same day
- developed and marked by the QCAA according to a commonly applied marking scheme.

The external assessment contributes 50% to the student's result in Specialist Mathematics. It is not privileged over the school-based assessment.

1.4 Reporting standards

Reporting standards are summary statements that succinctly describe typical performance at each of the five levels (A–E). They reflect the cognitive taxonomy and objectives of the course of study.

The primary purpose of reporting standards is for twice-yearly reporting on student progress. These descriptors can also be used to help teachers provide formative feedback to students and to align ISMGs.

Reporting standards

A
The student demonstrates a comprehensive knowledge and understanding of the subject matter; recognises, recalls and uses facts, rules, definitions and procedures; and comprehends and applies mathematical concepts and techniques to solve problems drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar, complex familiar and complex unfamiliar situations. The student explains mathematical reasoning to justify procedures and decisions; evaluates the reasonableness of solutions; communicates using mathematical, statistical and everyday language and conventions; and makes decisions about the choice of technology, and uses the technology, to solve problems in simple familiar, complex familiar and complex unfamiliar situations.
B
The student demonstrates a thorough knowledge and understanding of the subject matter; recognises, recalls and uses facts, rules, definitions and procedures; and comprehends and applies mathematical concepts and techniques to solve problems drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar and complex familiar situations. The student explains mathematical reasoning to justify procedures and decisions; evaluates the reasonableness of solutions; communicates using mathematical, statistical and everyday language and conventions; and makes decisions about the choice of technology, and uses the technology, to solve problems in simple familiar and complex familiar situations.
C
The student demonstrates knowledge and understanding of the subject matter; recognises, recalls and uses facts, rules, definitions and procedures; and comprehends and applies mathematical concepts and techniques to solve problems drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar situations. The student explains mathematical reasoning to justify procedures and decisions; evaluates the reasonableness of solutions; communicates using mathematical, statistical and everyday language and conventions; and uses technology to solve problems in simple familiar situations.
D
The student demonstrates partial knowledge and understanding of the subject matter; recognises, recalls and uses some facts, rules, definitions and procedures; and comprehends and applies aspects of mathematical concepts and techniques to solve some problems drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar situations. The student explains some mathematical reasoning to justify procedures and decisions; sometimes evaluates the reasonableness of solutions; communicates using some mathematical, statistical and everyday language and conventions; and uses technology to solve some problems in simple familiar situations.
E
The student demonstrates isolated knowledge and understanding of the subject matter; infrequently recognises, recalls and uses some facts, rules, definitions and procedures; and infrequently comprehends and applies aspects of mathematical concepts and techniques drawn from Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus in simple familiar situations. The student infrequently describes aspects of mathematical reasoning relevant to procedures and decisions; rarely evaluates the reasonableness of solutions; infrequently communicates using some aspects of mathematical, statistical and everyday language and conventions; and uses aspects of technology in simple familiar situations.

2 Unit 1: Combinatorics, vectors and proof

2.1 Unit description

In Unit 1, students will develop the mathematical understandings and skills to solve problems relating to the topics:

- Topic 1: Combinatorics
- Topic 2: Vectors in the plane
- Topic 3: Introduction to proof.

Combinatorics provides techniques that are useful in many areas of mathematics, including probability and algebra. Vectors in the plane provides new perspectives for working with two-dimensional space, and serves as an introduction to techniques that will extend to three-dimensional space in Unit 3. Introduction to proof provides the opportunity to summarise and extend students' studies in deductive Euclidean geometry, and is of great benefit in the study of other topics in the course, including vectors and complex numbers.

These three topics considerably broaden students' mathematical experience and enhance their awareness of the breadth and utility of the subject. They contain procedures and processes that will be required for later topics. All these topics develop students' ability to construct mathematical arguments and enable students to increase their mathematical flexibility and versatility.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 1. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

2.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 1 topics
2. comprehend mathematical concepts and techniques drawn all Unit 1 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 1 topics.

2.3 Topic 1: Combinatorics

Subject matter
The inclusion–exclusion principle for the union of two sets and three sets (4 hours) In this sub-topic, students will: • determine and use the formulas (including the addition principle) for finding the number of elements in the union of two and the union of three sets • use the multiplication principle.
Permutations (ordered arrangements) and combinations (unordered selections) (9 hours) In this sub-topic, students will: • solve problems involving permutations • use factorial notation • use the notation ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ • solve problems involving permutations with restrictions • solve problems involving combinations • use the notation $\binom{n}{r}$ and ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ • derive and use simple identities associated with Pascal's triangle • solve problems involving combinations with restrictions • apply permutations and combinations to probability problems.
The pigeon-hole principle (2 hours) In this sub-topic, students will: • solve problems and prove results using the pigeon-hole principle.

2.4 Topic 2: Vectors in the plane

Subject matter
Representing vectors in the plane by directed line segments (6 hours) In this sub-topic, students will: • examine examples of vectors • understand the difference between a scalar and a vector • define and use the magnitude and direction of a vector • understand and use vector equality • understand and use both the Cartesian form and polar form of a vector • represent a scalar multiple of a vector • use the triangle rule to find the sum and difference of two vectors.
Algebra of vectors in the plane (11 hours) In this sub-topic, students will: • use ordered pair notation and column vector notation to represent a vector • understand and use vector notation: $\overrightarrow{AB}$, $\underset{\sim}{c}$, $\boldsymbol{d}$, unit vector notation $\hat{\boldsymbol{n}}$ • convert between Cartesian form and polar form • determine a vector between two points • define and use unit vectors and the perpendicular unit vectors $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ • express a vector in component form using the unit vectors $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ • examine and use addition and subtraction of vectors in component form • define and use multiplication by a scalar of a vector in component form • define and use a vector representing the midpoint of a line segment • define and use scalar (dot) product

Subject matter
• apply the scalar product to vectors expressed in component form • examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular • define and use projections of vectors • solve problems involving displacement, force, velocity, equilibrium and relative velocity involving the above concepts.

2.5 Topic 3: Introduction to proof

Subject matter
The nature of proof (5 hours) In this sub-topic, students will: • use implication, converse, equivalence, negation, contrapositive • use proof by contradiction • use the symbols for implication ($\Rightarrow$), equivalence ($\Leftrightarrow$), and equality (=) • use the quantifiers 'for all' ($\forall$) and 'there exists' ($\exists$) • use examples and counterexamples.
Rational and irrational numbers (4 hours) In this sub-topic, students will: • prove simple results involving numbers • express rational numbers as terminating or eventually recurring decimals and vice versa • prove irrationality by contradiction.
Circle properties and their proofs (8 hours) In this sub-topic, students will: • prove circle properties, such as - an angle in a semicircle is a right angle - the angle at the centre subtended by an arc of a circle is twice the angle at the circumference subtended by the same arc - angles at the circumference of a circle subtended by the same arc are equal - the opposite angles of a cyclic quadrilateral are supplementary - chords of equal length subtend equal angles at the centre and conversely chords subtending equal angles at the centre of a circle have the same length - a tangent drawn to a circle is perpendicular to the radius at the point of contact - the alternate segment theorem - when two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord and its converse - when a secant (meeting the circle at A and B) and a tangent (meeting the circle at T) are drawn to a circle from an external point M, the square of the length of the tangent equals the product of the lengths to the circle on the secant; ($AM \times BM = TM^2$) and its converse • solve problems finding unknown angles and lengths and prove further results using the circle properties listed above.
Geometric proofs using vectors (6 hours) In this sub-topic, students will: • prove the diagonals of a parallelogram meet at right angles if and only if it is a rhombus • prove midpoints of the sides of a quadrilateral join to form a parallelogram • prove the sum of the squares of the lengths of a parallelogram's diagonals is equal to the sum of the squares of the lengths of the sides • prove an angle in a semicircle is a right angle.

2.6 Assessment guidance

In constructing assessment instruments for Unit 1, schools should ensure that the objectives cover, or are chosen from, the unit objectives. If one assessment instrument is developed for a unit, it must assess all the unit objectives; if more than one assessment instrument is developed, the unit objectives must be covered across those instruments.

It is suggested that schools develop:

- a problem-solving and modelling task that assesses either Unit 1 Topic 1 or Unit 1 Topic 2, and
- an internal examination that representatively samples subject matter from Unit 1 not assessed in the problem-solving and modelling task.

3 Unit 2: Complex numbers, trigonometry, functions and matrices

3.1 Unit description

In Unit 2, students will develop the mathematical understandings and skills to solve problems relating to:

- Topic 1: Complex numbers 1
- Topic 2: Trigonometry and functions
- Topic 3: Matrices.

Complex numbers 1 introduces the complex plane, complex arithmetic and complex algebra. Trigonometry and functions builds on the nature of proof and models periodic phenomena. Matrices introduces basic operations and extends to transformations in the plane.

These topics further develop the thinking techniques and mathematical rigour introduced in Unit 1, and provide opportunities to further nurture curiosity about the nature and utility of mathematics.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 2. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

3.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 2 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 2 topics
3. communicate using mathematical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 2 topics.

3.3 Topic 1: Complex numbers 1

Subject matter
Complex numbers (4 hours) In this sub-topic, students will: • define the imaginary number i as a root of the equation $x^2 = -1$ • use complex numbers in the form $a + bi$ where a and b are the real and imaginary parts • determine and use complex conjugates • perform complex-number arithmetic: addition, subtraction, multiplication and division.
The complex plane (the Argand plane) (5 hours) In this sub-topic, students will: • consider complex numbers as points in a plane with real and imaginary parts as Cartesian coordinates • examine and use addition of complex numbers as vector addition in the complex plane • understand and use location of complex conjugates in the complex plane • examine and use multiplication as a linear transformation in the complex plane.
Complex arithmetic using polar form (3 hours) In this sub-topic, students will: • use the modulus $\|z\|$ of a complex number z and the argument $Arg(z)$ of a non-zero complex number z • convert between Cartesian form and polar form • define and use multiplication, division and powers of complex numbers in polar form and the geometric interpretation of these.
Roots of equations (3 hours) In this sub-topic, students will: • use the general solution of real quadratic equations • determine complex conjugate solutions of real quadratic equations • determine linear factors of real quadratic polynomials.

3.4 Topic 2: Trigonometry and functions

Subject matter
The basic trigonometric functions (2 hours) In this sub-topic, students will: • find all solutions of $f\big(a(x-b)\big) = c$ where $f(\theta)$ is one of $\sin(\theta)$, $\cos(\theta)$ or $\tan(\theta)$ • sketch and graph functions with rules of the form $y = f\big(a(x-b)\big)$ where $f(\theta)$ is one of $\sin(\theta)$, $\cos(\theta)$ or $\tan(\theta)$.
Sketching graphs (6 hours) In this sub-topic, students will: • use and apply the notation $\|x\|$ for the absolute value for the real number x and the graph of $y = \|x\|$ • examine the relationship between the graph of $y = f(x)$ and the graphs of $y = \frac{1}{f(x)}$, $y = \|f(x)\|$ and $y = f(\|x\|)$ • sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree.
The reciprocal trigonometric functions, secant, cosecant and cotangent (3 hours) In this sub-topic, students will: • define the reciprocal trigonometric functions, sketch their graphs, and graph simple transformations of them.

Subject matter
Trigonometric identities (9 hours) In this sub-topic, students will: • prove and apply the Pythagorean identities • prove and apply the angle sum, difference and double-angle identities for sines and cosines • prove and apply the identities for products of sines and cosines expressed as sums and differences • convert sums $a\cos(x) + b\sin(x)$ to $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ and apply these to sketch graphs, solve equations of the form $a\cos(x) + b\sin(x) = c$ and solve real-world problems • use the binomial theorem to prove and apply multi-angle trigonometric identities up to $\sin(4x)$ and $\cos(4x)$.
Applications of trigonometric functions to model periodic phenomena (5 hours) In this sub-topic, students will: • model periodic motion using sine and cosine functions, and understand the relevance of the period and amplitude of these functions in the model.

3.5 Topic 3: Matrices

Subject matter
Matrix arithmetic (6 hours) In this sub-topic, students will: • understand the matrix definition and notation • define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and multiplicative inverse • calculate the determinant and inverse of 2 x 2 matrices algebraically and solve matrix equations of the form $\mathbf{AX} = \mathbf{B}$, where $\mathbf{A}$ is a 2 x 2 matrix and $\mathbf{X}$ and $\mathbf{B}$ are column vectors • calculate the determinant and inverse of higher order matrices and solve matrix equations using technology.
Transformations in the plane (9 hours) In this sub-topic, students will: • understand translations and their representation as column vectors • define and use basic linear transformations: dilations of the form $(x, y) \longrightarrow (ax, by)$, rotations about the origin and reflection in a line that passes through the origin, and the representations of these transformations by 2 x 2 matrices • apply these transformations to points in the plane and geometric objects • define and use composition of linear transformations and the corresponding matrix products • define and use inverses of linear transformations and the relationship with the matrix inverse • examine the relationship between the determinant and the effect of a linear transformation on area • establish geometric results by matrix multiplications.

3.6 Assessment guidance

In constructing assessment instruments for Unit 2, schools should ensure that the objectives cover, or are chosen from, the unit objectives. If one assessment instrument is developed for a unit, it must assess all the unit objectives; if more than one assessment instrument is developed, the unit objectives must be covered across those instruments.

It is suggested that schools develop:

- an internal examination that representatively samples subject matter from all Unit 2 topics, and/or
- an internal examination that representatively samples subject matter from Units 1 and 2.

4 Unit 3: Mathematical induction, and further vectors, matrices and complex numbers

4.1 Unit description

In Unit 3, students will develop the mathematical understandings and skills to solve problems relating to:

- Topic 1: Proof by mathematical induction
- Topic 2: Vectors and matrices
- Topic 3: Complex numbers 2.

Proof by mathematical induction continues the developmental concept of proof from Units 1 and 2. Unit 1 introduced a study of vectors with a focus on vectors in two-dimensional space. Unit 2 introduced complex numbers; Unit 3 extends the study of complex numbers to include complex arithmetic using polar form.

In this unit, students explore applications of matrices, study three-dimensional vectors, and are introduced to vector equations and vector calculus, with the latter extending students' knowledge of calculus from Mathematical Methods. Cartesian equations and vector equations, together with equations of planes, enable students to solve geometric problems and problems involving motion in three-dimensional space.

These topics build on prior knowledge to enable a greater depth of analytical thinking and metacognition.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 3. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

4.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

Unit objective	IA1	IA2	EA
1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics	•	•	•
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics	•	•	•
3. communicate using mathematical and everyday language and conventions	•	•	•
4. evaluate the reasonableness of solutions	•	•	•
5. justify procedures and decisions by explaining mathematical reasoning	•	•	•
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.	•	•	•

4.3 Topic 1: Proof by mathematical induction

Subject matter
Mathematical induction (7 hours) In this sub-topic, students will: • understand the nature of inductive proof including the 'initial statement' and inductive step • prove results for sums for any positive integer n. • prove divisibility results for any positive integer n.

4.4 Topic 2: Vectors and matrices

Subject matter
The algebra of vectors in three dimensions (4 hours) In this sub-topic, students will: • review the concepts of vectors from Unit 1 and extend to three dimensions by introducing the unit vector $\hat{\boldsymbol{k}}$ and the altitude φ • prove geometric results (review from the topic Geometric proofs using vectors) in the plane and construct simple proofs in three dimensions.
Vector and Cartesian equations (10 hours) In this sub-topic, students will: • introduce Cartesian coordinates for three-dimensional space, including plotting points and the equations of spheres • use vector equations of curves in two or three dimensions involving a parameter, and determine a 'corresponding' Cartesian equation in the two-dimensional case • determine a vector, parametric and Cartesian equation of a straight line and straight-line segment given the position of two points, or equivalent information, in both two and three dimensions • examine the position of two particles, each described as a vector function of time, and determine if their

Subject matter
paths cross or if the particles meet • define and use the vector (cross) product to determine a vector normal to a given plane • use vector methods in applications, including areas of shapes and determining vector and Cartesian equations of a plane and of regions in a plane.
Systems of linear equations (6 hours) In this sub-topic, students will: • recognise the general form of a system of linear equations in several variables and use Gaussian techniques of elimination to solve a system of linear equations • solve systems of linear equations using matrix algebra • examine the three cases for solutions of systems of equations — a unique solution, no solution and infinitely many solutions — and the geometric interpretation of a solution of a system of equations with three variables.
Applications of matrices (7 hours) In this sub-topic, students will: • model real-life situations using matrices, including Dominance and Leslie • investigate how matrices have been applied in other real-life situations, e.g. Leontief, Markov, area, cryptology, eigenvectors and eigenvalues. **Note:** The external examination may assess only Dominance and Leslie matrices.
Vector calculus (5 hours) In this sub-topic, students will: • consider position of vectors as a function of time • derive the Cartesian equation of a path given as a vector equation in two dimensions, including circles, ellipses and hyperbolas • differentiate and integrate a vector function with respect to time • determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration • apply vector calculus to motion in a plane, including projectile and circular motion.

4.5 Topic 3: Complex numbers 2

Subject matter
Cartesian forms (4 hours) In this sub-topic, students will: • review real and imaginary parts $Re(z)$ and $Im(z)$ of a complex number z • review Cartesian form • review complex arithmetic using Cartesian form.
Complex arithmetic using polar form (3 hours) In this sub-topic, students will: • prove the identities involving modulus and argument • prove and use De Moivre's theorem for integral powers.
The complex plane (the Argand plane) (2 hours) In this sub-topic, students will: • identify subsets of the complex plane determined by straight lines and circles.

Subject matter
Roots of complex numbers (3 hours) In this sub-topic, students will: • determine and examine the nth roots of unity and their location on the unit circle • determine and examine the nth roots of complex numbers and their location in the complex plane.
Factorisation of polynomials (4 hours) In this sub-topic, students will: • prove and apply the factor theorem and the remainder theorem for polynomials • consider conjugate roots for polynomials with real coefficients • solve polynomial equations to order 4.

4.6 Assessment

4.6.1 Summative internal assessment 1 (IA1): Problem-solving and modelling task (20%)

Description

This assessment focuses on the interpretation, analysis and evaluation of ideas and information. It is an independent task responding to a particular situation or stimuli. While students may undertake some research in the writing of the problem-solving and modelling task, it is not the focus of this technique. This assessment occurs over an extended and defined period of time. Students will use class time and their own time to develop a response.

The problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Vectors and matrices
- Topic 3: Complex numbers 2.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3.

Specifications

Description

A problem-solving and modelling task is an assessment instrument developed in response to a mathematical investigative scenario or context. It requires students to respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Students must provide a response to a specific task or issue that is set in a context that highlights a real-life application of mathematics. The task requires students to use relevant stimulus material involving the selected subject matter and must have sufficient scope to allow students to address all the stages of the problem-solving and modelling approach (see Section 1.2.4). Technology must be used.

The response is written and must be able to be read and interpreted independently of the instrument task sheet.

Conditions

- Write:
  - up to 10 pages (including tables, figures and diagrams)
  - maximum of 2000 words
  - appendixes can include raw data, repeated calculations, evidence of authentication and student notes (appendixes are not to be marked).
- Duration: 4 weeks (including 3 hours of class time).
- Other:
  - opportunity may be provided for group work, but unique responses must be developed by each student
  - use of technology is required; schools must specify the technology used, e.g. scientific calculator, graphics calculator (CAS or non-CAS), spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation or word processing
  - the teacher provides the mathematical investigative scenario or context for the problem-solving and modelling task.

Task examples

Examples of problem-solving and modelling tasks include:

- a report that investigates the applications of matrices in a particular field
- a persuasive report to convince the reader of a preferred model to predict population growth between Leslie matrices and polynomials
- a magazine article to explain the best squares to buy in *Monopoly*
- a report showing how matrices can be used to predict the eventual winner for a competition
- an investigative report on eigenvectors and eigenvalues.

Summary of the instrument-specific marking guide

The following table summarises the criteria, assessment objectives and mark allocation for the problem-solving and modelling task.

Criterion	Objectives	Marks
Formulate	1, 2 and 5	4
Solve	1 and 6	7
Evaluate and verify	4 and 5	5
Communicate	3	4
Total		**20**

Instrument-specific marking guide

Criterion: Formulate

Assessment objectives

1. select, recall and use facts, rules definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques.	3–4
• statement of some assumptions • statement of some observations • translation of simple aspects of the problem by identifying mathematical concepts and techniques.	1–2
• does not satisfy any of the descriptors above.	0

Criterion: Solve

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3

The student work has the following characteristics:	Marks
• accurate use of complex procedures to reach a valid solution • discerning application of mathematical concepts and techniques relevant to the task • accurate and appropriate use of technology.	6–7
• use of complex procedures to reach a reasonable solution • application of mathematical concepts and techniques relevant to the task • use of technology.	4–5
• use of simple procedures to make some progress towards a solution • simplistic application of mathematical concepts and techniques relevant to the task • superficial use of technology.	2–3
• inappropriate use of technology or procedures.	1
• does not satisfy any of the descriptors above.	0

Criterion: Evaluate and verify

Assessment objectives

4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	4–5
• statements about the reasonableness of solutions by considering the context of the task • statements about relevant strengths and limitations of the solution and/or model • statements about decisions made relevant to the context of the task.	2–3
• statement about a decision and/or the reasonableness of a solution.	1
• does not satisfy any of the descriptors above.	0

Criterion: Communicate

Assessment objective

3. communicate using mathematical, statistical and everyday language and conventions

The student work has the following characteristics:	Marks
• correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4
• use of some appropriate language and conventions to develop the response • adequate organisation of the response.	1–2
• does not satisfy any of the descriptors above.	0

4.6.2 Summative internal assessment 2 (IA2): Examination (15%)

Description

The examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 3 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics

Specifications

Description

The examination will representatively sample subject matter from all Unit 3 topics. Where relevant, the focus of this assessment should be on subject matter not assessed in the problem-solving and modelling task.

Subject matter from Units 1 and 2 is considered assumed knowledge.

The examination must ensure that all assessment objectives are assessed. The examination should be designed using the principles of developing mathematics problems in Section 1.2.4. The total number of marks used in an examination marking scheme is a school decision. However, in order to correctly apply the ISMG, the percentage allocation of marks must match the following specifications.

Mark allocations

Percentage of marks	Degree of difficulty
~ 20%	**Complex unfamiliar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is - the required procedure is not clear from the way the problem is posed; and - in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
~ 20%	**Complex familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is - the required procedure is clear from the way the problem is posed, *or* - in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
~ 60%	**Simple familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is - the required procedure is clear from the way the problem is posed, *or* - is in a context that has been a focus of prior learning. Students are *not* required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.

Conditions

- Time: 120 minutes plus 5 minutes perusal.
- Length: the number of short-response items should allow students to complete the response in the set time.
- Short-response format, consisting of a number of items that ask students to respond to the following activities:
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams
  - short items requiring single-word, sentence or short-paragraph responses
  - justifying solutions using appropriate mathematical language where applicable
  - responding to seen or unseen stimulus materials
  - interpreting ideas and information.
- Other:
  - the instrument must be designed in such a way as to ensure that items provide for a balance of both technology-free and technology-active responses
  - seen stimulus — teachers must ensure the purpose of the technique is not compromised
  - unseen stimulus — materials or questions must not be copied from information or texts that students have previously been exposed to or have directly used in class
  - when stimulus materials are used, they will be succinct enough to allow students sufficient time to engage with them; for stimulus materials that are lengthy, complex or large in number, they will be shared with students prior to the administration of the assessment instrument
  - only the QCAA formula sheet must be provided
  - notes are not permitted
  - use of technology is required; schools must specify the technology used, e.g. scientific calculator, graphics calculator (CAS or non-CAS), spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation.

Summary of the instrument-specific marking guide

The following table summarises the mark allocation for the objectives assessed in the examination.

Criterion	Objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5 and 6	15
Total		**15**

Instrument-specific marking guide

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2

The student work has the following characteristics:	Cut-off	Marks
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0

4.6.3 Summative external assessment (EA): Examination (50%)

General information

Summative external assessment is developed and marked by the QCAA. In Specialist Mathematics, it contributes 50% to a student's overall subject result.

Summative external assessment assesses learning from Units 3 and 4. Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of the supervised examination.

The external assessment in Specialist Mathematics is common to all schools and administered under the same conditions, at the same time, on the same day.

See Section 5.6.2.

5 Unit 4: Further calculus and statistical inference

5.1 Unit description

In Unit 4, students will develop the mathematical understandings and skills to solve problems relating to:

- Topic 1: Integration and applications of integration
- Topic 2: Rates of change and differential equations
- Topic 3: Statistical inference.

The study of Integration and applications of integration and Rates of change and differential equations examine the complex processes of integration techniques. In this unit, students' previous experience working with statistics in Mathematical Methods is drawn together in the study of statistical inference for the distribution of sample means and confidence intervals for sample means. The study of differentiation and integration of functions continues, and the calculus techniques developed in this and previous topics are applied to simple differential equations in contexts found in areas such as biology and kinematics.

Learning in this unit reinforces the real-world applications of the mathematics used throughout Specialist Mathematics. These topics build on the critical and creative thinking techniques introduced in the previous units to facilitate the transition to further studies.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 4. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

5.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

Unit objective	IA3	EA
1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics	●	●
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics	●	●
3. communicate using mathematical, statistical and everyday language and conventions	●	●
4. evaluate the reasonableness of solutions	●	●
5. justify procedures, decisions by explaining mathematical reasoning	●	●
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.	●	●

5.3 Topic 1: Integration and applications of integration

Subject matter
Integration techniques (10 hours) In this sub-topic, students will: • integrate using the trigonometric identities $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$ and $1 + \tan^2(x) = \sec^2(x)$ • use substitution $u = g(x)$ to integrate expressions of the form $f(g(x))\, g'(x)$ • establish and use the formula $\int \frac{1}{x}\, dx = \ln\lvert x\rvert + c$, for $x \neq 0$ • find and use the inverse trigonometric functions: arcsine, arccosine and arctangent • find and use the derivative of the inverse trigonometric functions: arcsine, arccosine and arctangent • integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$ • use partial fractions where necessary for integration in simple cases • integrate by parts.
Applications of integral calculus (9 hours) In this sub-topic, students will: • calculate areas between curves determined by functions • determine volumes of solids of revolution about either axis • use the numerical integration method of Simpson's rule, using technology • use and apply the probability density function, $f(t) = \lambda e^{-\lambda t}$ for $t \geq 0$, of the exponential random variable with parameter $\lambda > 0$, and use the exponential random variables and associated probabilities and quantiles to model data and solve practical problems.

5.4 Topic 2: Rates of change and differential equations

Subject matter
Rates of change (10 hours) In this sub-topic, students will: • use implicit differentiation to determine the gradient of curves whose equations are given in implicit form • use related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ • solve simple first-order differential equations of the form $\frac{dy}{dx} = f(x)$, differential equations of the form $\frac{dy}{dx} = g(y)$ and, in general, differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables • examine slope (direction or gradient) fields of a first-order differential equation • formulate and use differential equations, including the logistic equation, e.g. examples in chemistry, biology and economics.
Modelling motion (10 hours) In this sub-topic, students will: • examine momentum, force, resultant force, action and reaction • consider constant and non-constant force • understand motion of a body under concurrent forces • consider and solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and the use of expressions $\frac{dv}{dt}$, $\frac{d^2x}{dt^2}$, $v\,\frac{dv}{dx}$ and $\frac{d(\frac{1}{2}v^2)}{dx}$ for acceleration.

5.5 Topic 3: Statistical inference

Subject matter
Sample means (8 hours) In this sub-topic, students will: • examine the concept of the sample mean $\bar{X}$ as a random variable whose value varies between samples where X is a random variable with mean μ and the standard deviation σ • simulate repeated random sampling from a variety of distributions and a range of sample sizes to illustrate properties of the distribution of $\bar{X}$ across samples of a fixed size n, including its mean μ, its standard deviation $\frac{\sigma}{\sqrt{n}}$ (where μ and σ are the mean and standard deviation of X) and its approximate normality if n is large • simulate repeated random sampling from a variety of distributions and a range of sample sizes to illustrate the approximate standard normality of $\frac{\bar{X}-\mu}{(s/\sqrt{n})}$ for large samples $(n \geq 30)$, where s is the sample standard deviation.
Confidence intervals for means (8 hours) In this sub-topic, students will: • understand the concept of an interval estimate for a parameter associated with a random variable • examine the approximate confidence interval $\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$, as an interval estimate for μ, the population mean, where z is the appropriate quantile for the standard normal distribution • use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ • use $\bar{x}$ and s to estimate μ and σ, to obtain approximate intervals covering desired proportions of values of a normal random variable and compare with an approximate confidence interval for μ • collect data and construct an approximate confidence interval to estimate a mean and to report on survey procedures and data quality.

5.6 Assessment

5.6.1 Summative internal assessment 3 (IA3): Examination (15%)

Description

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 4 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

Specifications

Description

The examination will representatively sample subject matter from all Unit 4 topics.

Subject matter from Units 1, 2 and 3 is considered assumed knowledge.

The examination must ensure that all assessment objectives are assessed. The examination should be designed using the principles of developing mathematics problems in Section 1.2.4. The total number of marks used in the examination marking scheme is a school decision. However, in order to correctly apply the ISMG, the percentage allocation of marks must match the specifications below.

Mark allocations

Percentage of marks	Degree of difficulty
~ 20%	**Complex unfamiliar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is - the required procedure is not clear from the way the problem is posed; and - in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
~ 20%	**Complex familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is - the required procedure is clear from the way the problem is posed, *or* - in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
~ 60%	**Simple familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is - the required procedure is clear from the way the problem is posed, *or* - is in a context that has been a focus of prior learning. Students are *not* required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.

Conditions

- Time: 120 minutes plus 5 minutes perusal
- Length: the number of short-response items should allow students to complete the response in the set time.
- Short-response format, consisting of a number of items that ask students to respond to the following activities
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams
  - short items requiring single-word, sentence or short-paragraph responses
  - justifying solutions using appropriate mathematical language where applicable
  - responding to seen or unseen stimulus materials
  - interpreting ideas and information.
- Other:
  - the instrument must be designed in such a way as to ensure that items provide for a balance of both technology-free and technology-active responses
  - seen stimulus — teachers must ensure the purpose of the technique is not compromised
  - unseen stimulus — materials or questions must not be copied from information or texts that students have previously been exposed to or have directly used in class
  - when stimulus materials are used, they will be succinct enough to allow students sufficient time to engage with them; for stimulus materials that are lengthy, complex or large in number, they will be shared with students prior to the administration of the assessment instrument
  - only the QCAA formula sheet must be provided
  - notes are not permitted
  - use of technology is required; schools must specify the technology used, e.g. scientific calculator, graphics calculator (CAS or non-CAS), spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation.

Summary of the instrument-specific marking guide

The following table summarises the mark allocation for the objectives assessed in the examination.

Criterion	Objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5 and 6	15
Total		**15**

Instrument-specific marking guide

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2

The student work has the following characteristics:	Cut-off	Marks
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0

5.6.2 Summative external assessment (EA): Examination (50%)

General information

Summative external assessment is developed and marked by the QCAA. In Specialist Mathematics it contributes 50% to a student's overall subject result.

Summative external assessment assesses learning from Units 3 and 4. Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of the examination.

The external assessment in Specialist Mathematics is common to all schools and administered under the same conditions, at the same time, on the same day.

Description

This assessment consists of two papers: technology-free (Paper 1) and technology-active (Paper 2). The examination assesses the application of a range of cognitions to a number of items drawn from Units 3 and 4. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Specifications

Description

The external assessment for Specialist Mathematics will representatively sample subject matter from Units 3 and 4.

The percentage allocation of marks for each paper of the external examination will match the specifications below.

Mark allocations

<table>
<tr><th>Percentage of marks</th><th>Degree of difficulty</th></tr>
<tr><td>~ 20%</td><td>Complex unfamiliar
Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where:
• relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and
• all the information to solve the problem is not immediately identifiable; that is
- the required procedure is not clear from the way the problem is posed, and
- in a context in which students have had limited prior experience.
Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.</td></tr>
<tr><td>~ 20%</td><td>Complex familiar
Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where:
• relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and
• all of the information to solve the problem is identifiable; that is
- the required procedure is clear from the way the problem is posed, or
- in a context that has been a focus of prior learning.
Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.</td></tr>
<tr><td>~ 60%</td><td>Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where:
• relationships and interactions are obvious and have few elements; and
• all of the information to solve the problem is identifiable; that is
- the required procedure is clear from the way the problem is posed, or
- in a context that has been a focus of prior learning.
Students are not required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.</td></tr>
</table>

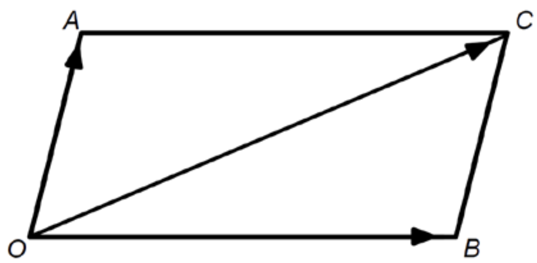
Conditions

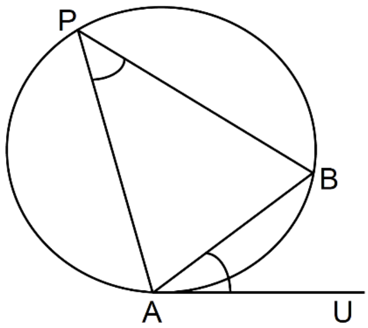
- Time:
  - Paper 1 (technology-free, 25%); 90 minutes plus 5 minutes perusal
  - Paper 2 (technology-active, 25%); 90 minutes plus 5 minutes perusal.
- Length: the number of short-response items should allow students to complete the response in the set time.
- Short-response format, consisting of a number of items that ask students to respond to the following activities
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams
  - short items requiring multiple-choice, single-word, sentence or short-paragraph responses
  - justifying solutions using appropriate mathematical language where applicable
  - responding to seen or unseen stimulus materials
  - interpreting ideas and information.
- Other:
  - the QCAA formula sheet will be provided for both papers
  - no calculator or technology of any type is permitted in Paper 1 (technology-free); access to a QCAA-approved handheld graphics calculator (no CAS functionality) is a requirement for Paper 2 (technology-active) of the external assessment, and scientific calculators may also be used.

Instrument-specific marking guide

No ISMG is provided for the external assessment.

6 Glossary

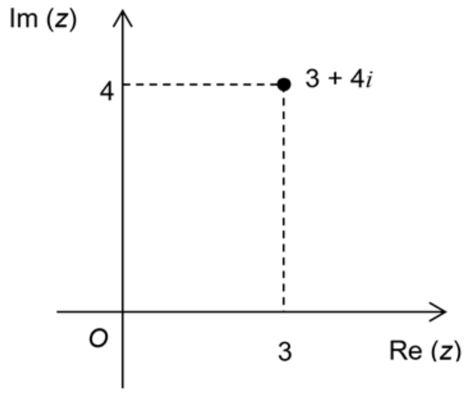
Term	Explanation
A	
accomplished	highly trained or skilled in a particular activity; perfected in knowledge or training; expert
accuracy	the condition or quality of being true, correct or exact; freedom from error or defect; precision or exactness; correctness; in science, the extent to which a measurement result represents the quantity it purports to measure; an accurate measurement result includes an estimate of the true value and an estimate of the uncertainty
accurate	precise and exact; to the point; consistent with or exactly conforming to a truth, standard, rule, model, convention or known facts; free from error or defect; meticulous; correct in all details
adept	very/highly skilled or proficient at something; expert
adequate	satisfactory or acceptable in quality or quantity equal to the requirement or occasion
addition and subtraction of matrices	Addition of matrices if **A** and **B** are matrices with the same dimensions and the entries of **A** are a_{ij} and the entries of **B** are b_{ij} then the entries of **A** + **B** are $a_{ij} + b_{ij}$ Subtraction of matrices if **A** and **B** are matrices with the same dimensions and the entries of **A** are a_{ij} and the entries of **B** are b_{ij} then the entries of **A** – **B** are $a_{ij} - b_{ij}$
addition and subtraction of vectors	given vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be directed line segments that represent $\boldsymbol{a}$ and $\boldsymbol{b}$; they have the same initial point O; the sum of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is the directed line segment $\overrightarrow{OC}$ where C is a point such that $OACB$ is a parallelogram; this is known as the **parallelogram rule**  if $\boldsymbol{a} = \binom{a_1}{a_2}$ and $\boldsymbol{b} = \binom{b_1}{b_2}$ then $\boldsymbol{a} + \boldsymbol{b} = \binom{a_1 + b_1}{a_2 + b_2}$ in component form if $\boldsymbol{a} = a_1\hat{\boldsymbol{\imath}} + a_2\hat{\boldsymbol{\jmath}}$ and $\boldsymbol{b} = b_1\hat{\boldsymbol{\imath}} + b_2\hat{\boldsymbol{\jmath}}$ then $\boldsymbol{a} + \boldsymbol{b} = (a_1 + b_1)\hat{\boldsymbol{\imath}} + (a_2 + b_2)\hat{\boldsymbol{\jmath}}$ Subtraction of vectors $\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$
algorithm	a precisely defined procedure that can be applied and systematically followed through to a conclusion

Term	Explanation
alternate segment	the word *alternate* means *other*; the chord AB divides the circle into two segments and AU is tangent to the circle; angle APB 'lies in' the segment on the other side of chord AB from angle BAU; we say that it is in the **alternate segment**  ∴ angle APB = angle BAU
analyse	dissect to ascertain and examine constituent parts and/or their relationships; break down or examine in order to identify the essential elements, features, components or structure; determine the logic and reasonableness of information; examine or consider something in order to explain and interpret it, for the purpose of finding meaning or relationships and identifying patterns, similarities and differences
angle sum, difference and double-angle identities for sine and cosine ratios	angle sum and difference identities $\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$ $\sin(A-B) = \sin(A)\cos(B) - \sin(B)\cos(A)$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$ double-angle identities $\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 2\cos^2(A) - 1$ $= 1 - 2\sin^2(A)$
applied learning	the acquisition and application of knowledge, understanding and skills in real-world or lifelike contexts that may encompass workplace, industry and community situations; it emphasises learning through doing and includes both theory and the application of theory, connecting subject knowledge and understanding with the development of practical skills
Applied subject	a subject whose primary pathway is work and vocational education; it emphasises applied learning and community connections; a subject for which a syllabus has been developed by the QCAA with the following characteristics: results from courses developed from Applied syllabuses contribute to the QCE; results may contribute to ATAR calculations
apply	use knowledge and understanding in response to a given situation or circumstance; carry out or use a procedure in a given or particular situation
appraise	evaluate the worth, significance or status of something; judge or consider a text or piece of work

Term	Explanation
appreciate	recognise or make a judgment about the value or worth of something; understand fully; grasp the full implications of
appropriate	acceptable; suitable or fitting for a particular purpose, circumstance, context, etc.
apt	suitable to the purpose or occasion; fitting, appropriate
arcosine function	if the domain of the cosine function $y = \cos(x)$ is restricted to the interval $[0,\ \pi]$, a one to one function is formed and so an inverse function exists denoted by $y\ = \cos^{-1}(x)$ or $\arccos(x)$. The arccosine function is defined by: $\cos^{-1}: [-1, 1] \to R, \cos^{-1}(x) = y$ where $\cos(y) = x, y \in [0,\ \pi]$
arcsine function	if the domain of the sine function $y = \sin(x)$ is restricted to the interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, a one to one function is formed and so an inverse function exists denoted by $y\ = \sin^{-1}(x)$ or $\arcsin(x)$. The arcsine function is defined by: $\sin^{-1}: [-1, 1] \to R, \sin^{-1}(x) = y$ where $\sin(y) = x, y \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
arctangent function	if the domain of the tangent function $y = \tan(x)$ is restricted to the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, a one to one function is formed and so an inverse tangent function exists denoted by $y\ = \tan^{-1}(x)$ or $\arctan(x)$. The arctangent function is defined by: $\tan^{-1}: R \to R, \tan^{-1}(x) = y$ where $\tan(y) = x, y \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
area of study	a division of, or a section within a unit
argue	give reasons for or against something; challenge or debate an issue or idea; persuade, prove or try to prove by giving reasons
argument	(of a complex number) if represented by a point P in the complex plane, then the argument of z, denoted arg z, is the angle θ that OP makes with the positive real axis O_X, with the angle measured anticlockwise from O_X; the principal value of the argument is the one in the interval $(-\pi,\ \pi]$
aspect	a particular part of a feature of something; a facet, phase or part of a whole
assess	measure, determine, evaluate, estimate or make a judgment about the value, quality, outcomes, results, size, significance, nature or extent of something
assessment	purposeful and systematic collection of information about students' achievements
assessment instrument	a tool or device used to gather information about student achievement
assessment objectives	drawn from the unit objectives and contextualised for the requirements of the assessment instrument (see also 'syllabus objectives', 'unit objectives')
assessment technique	the method used to gather evidence about student achievement, (e.g. examination, project, investigation)

Term	Explanation
assumptions	conditions that are stated to be true when beginning to solve a problem
astute	showing an ability to accurately assess situations or people; of keen discernment
ATAR	Australian Tertiary Admission Rank
authoritative	able to be trusted as being accurate or true; reliable; commanding and self-confident; likely to be respected and obeyed
B	
balanced	keeping or showing a balance; not biased; fairly judged or presented; taking everything into account in a fair, well-judged way
basic	fundamental
binomial theorem	the expansion $(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + y^n$
C	
calculate	determine or find (e.g. a number, answer) by using mathematical processes; obtain a numerical answer showing the relevant stages in the working; ascertain/determine from given facts, figures or information
Cartesian equation of a straight line	let $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ be the position vector of any point on a straight line in three-dimensional space and $\boldsymbol{d} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ be any vector with direction along the line; the line consists of all points P(x, y, z) whose Cartesian equation is $\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$
Cartesian equation of a plane	when $\boldsymbol{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is a vector normal to a plane in three-dimensional space; the plane consists of all points P(x, y, z) whose Cartesian equation is $ax + by + cz + d = 0$
categorise	place in or assign to a particular class or group; arrange or order by classes or categories; classify, sort out, sort, separate
challenging	difficult but interesting; testing one's abilities; demanding and thought-provoking; usually involving unfamiliar or less familiar elements
characteristic	a typical feature or quality
clarify	make clear or intelligible; explain; make a statement or situation less confused and more comprehensible
clarity	clearness of thought or expression; the quality of being coherent and intelligible; free from obscurity of sense; without ambiguity; explicit; easy to perceive, understand or interpret

Term	Explanation
classify	arrange, distribute or order in classes or categories according to shared qualities or characteristics
clear	free from confusion, uncertainty, or doubt; easily seen, heard or understood
clearly	in a clear manner; plainly and openly, without ambiguity
coherent	having a natural or due agreement of parts; connected; consistent; logical, orderly; well-structured and makes sense; rational, with parts that are harmonious; having an internally consistent relation of parts
cohesive	characterised by being united, bound together or having integrated meaning; forming a united whole
combinations	the number of selections of *n* objects taken *r* at a time (that is, the number of ways of selecting *r* objects out of *n*) is denoted by ${}^{n}C_{r} = \binom{n}{r}$ and is equal to $\frac{n!}{r!(n-r)!}$
comment	express an opinion, observation or reaction in speech or writing; give a judgment based on a given statement or result of a calculation
communicate	convey knowledge and/or understandings to others; make known; transmit
compare	display recognition of similarities and differences and recognise the significance of these similarities and differences
competent	having suitable or sufficient skills, knowledge, experience, etc. for some purpose; adequate but not exceptional; capable; suitable or sufficient for the purpose; having the necessary ability, knowledge or skill to do something successfully; efficient and capable (of a person); acceptable and satisfactory, though not outstanding
competently	in an efficient and capable way; in an acceptable and satisfactory, though not outstanding, way
complex	composed or consisting of many different and interconnected parts or factors; compound; composite; characterised by an involved combination of parts; complicated; intricate; a complex whole or system; a complicated assembly of particulars
complex arithmetic	if $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$ $z_1 + z_2 = x_1 + x_2 + (y_1 + y_2)i$ $z_1 - z_2 = x_1 - x_2 + (y_1 - y_2)i$ $z_1 \times z_2 = x_1 x_2 - y_1 y_2 + (x_1 y_2 + x_2 y_1)i$
complex conjugate	for any complex number $z = x + yi$, its **conjugate** is $\bar{z} = x - yi$

Term	Explanation
complex familiar	problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable; that is - the required procedure is clear from the way the problem is posed, *or* - in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
complex number forms	complex numbers can be expressed in various forms including: $z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\,cis(\theta)$ where $r = \|z\| = \sqrt{x^2 + y^2}$ and $\arg(z) = \theta,\ \tan(\theta) = \frac{y}{x}, -\pi < \theta \leq \pi$.
complex plane	a geometric representation of the complex numbers established by the **real axis** and the orthogonal **imaginary axis**; sometimes called the Argand plane 
complex unfamiliar	problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable; that is - the required procedure is not clear from the way the problem is posed, and - in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
comprehend	understand the meaning or nature of; grasp mentally
comprehensive	inclusive; of large content or scope; including or dealing with all or nearly all elements or aspects of something; wide-ranging; detailed and thorough, including all that is relevant

Term	Explanation
concise	expressing much in few words; giving a lot of information clearly and in a few words; brief, comprehensive and to the point; succinct, clear, without repetition of information
concisely	in a way that is brief but comprehensive; expressing much in few words; clearly and succinctly
conduct	direct in action or course; manage; organise; carry out
confidence interval	provides a range of values that describe the uncertainty surrounding an estimate
consider	think deliberately or carefully about something, typically before making a decision; take something into account when making a judgment; view attentively or scrutinise; reflect on
considerable	fairly large or great; thought about deliberately and with a purpose
considered	formed after careful and deliberate thought
consistent	agreeing or accordant; compatible; not self-opposed or self-contradictory, constantly adhering to the same principles; acting in the same way over time, especially so as to be fair or accurate; unchanging in nature, standard, or effect over time; not containing any logical contradictions (of an argument); constant in achievement or effect over a period of time
construct	create or put together (e.g. an argument) by arranging ideas or items; display information in a diagrammatic or logical form; make; build
context	a group of related situations, phenomena, technical applications and social issues likely to be encountered by students; can provide a meaningful application of concepts in real-world applications; in Specialist Mathematics, a framework for linking concepts and learning experiences that enables students to identify and understand the application of mathematics to their world
contradiction	in Specialist Mathematics, assume the opposite (negation) of what you are trying to prove; then proceed through a logical chain of argument until you reach a demonstrably false conclusion; since all the reasoning is correct and a false conclusion has been reached, the only thing that could be wrong is the initial assumption; therefore, the original statement is true
contrapositive	the contrapositive of the statement 'If P then Q' is 'If not Q then not P'; the contrapositive of a true statement is also true (not Q is the negation of the statement Q)
contrast	display recognition of differences by deliberate juxtaposition of contrary elements; show how things are different or opposite; give an account of the differences between two or more items or situations, referring to both or all of them throughout
controlled	shows the exercise of restraint or direction over; held in check; restrained, managed or kept within certain bounds
convention	the generally agreed upon way in which something is done; in a mathematical context this refers to notation, symbols, abbreviations, usage and setting out

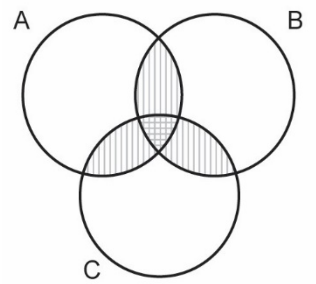
Term	Explanation
converse	in Specialist Mathematics, the converse of the statement 'if p then q' is 'if q then p'; symbolically the converse of p $\Rightarrow$ q is: q $\Rightarrow$ p or p $\Leftarrow$ q; the converse of a true statement need not be true
convert	to change into a different form
convincing	persuaded by argument or proof; leaving no margin of doubt; clear; capable of causing someone to believe that something is true or real; persuading or assuring by argument or evidence; appearing worthy of belief; credible or plausible
counter-example	an example that demonstrates that a statement is not true
course	a defined amount of learning developed from a subject syllabus or alternative sequence
create	bring something into being or existence; produce or evolve from one's own thought or imagination; reorganise or put elements together into a new pattern or structure or to form a coherent or functional whole
creative	resulting from originality of thought or expression; relating to or involving the use of the imagination or original ideas to create something; having good imagination or original ideas
credible	capable or worthy of being believed; believable; convincing
criterion	the property or characteristic by which something is judged or appraised
critical	involving skilful judgment as to truth, merit, etc.; involving the objective analysis and evaluation of an issue in order to form a judgment; expressing or involving an analysis of the merits and faults of a work of literature, music, or art; incorporating a detailed and scholarly analysis and commentary (of a text); rationally appraising for logical consistency and merit
critique	review (e.g. a theory, practice, performance) in a detailed, analytical and critical way
cursory	hasty, and therefore not thorough or detailed; performed with little attention to detail; going rapidly over something, without noticing details; hasty; superficial
cyclic quadrilateral	a quadrilateral whose vertices all lie on a circle
D	
decide	reach a resolution as a result of consideration; make a choice from a number of alternatives
deduce	reach a conclusion that is necessarily true, provided a given set of assumptions is true; arrive at, reach or draw a logical conclusion from reasoning and the information given
defensible	justifiable by argument; capable of being defended in argument
define	give the meaning of a word, phrase, concept or physical quantity; state meaning and identify or describe qualities

Term	Explanation
De Moivre's theorem	for all integers $n, (\cos(\theta) + i\sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$, or alternatively if $z = rcis(\theta)$ then $z^n = r^n\, cis\,(n\theta)$
demonstrate	prove or make clear by argument, reasoning or evidence, illustrating with practical example; show by example; give a practical exhibition
derive	arrive at by reasoning; manipulate a mathematical relationship to give a new equation or relationship; in mathematics, obtain the derivative of a function
describe	give an account (written or spoken) of a situation, event, pattern or process, or of the characteristics or features of something
design	produce a plan, simulation, model or similar; plan, form or conceive in the mind; in English, select, organise and use particular elements in the process of text construction for particular purposes; these elements may be linguistic (words), visual (images), audio (sounds), gestural (body language), spatial (arrangement on the page or screen) and multimodal (a combination of more than one)
detailed	executed with great attention to the fine points; meticulous; including many of the parts or facts
determinant	for a 2×2 matrix, if $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the determinant of $\mathbf{A}$ denoted as det $\mathbf{A} = ad - bc$
determine	establish, conclude or ascertain after consideration, observation, investigation or calculation; decide or come to a resolution
develop	elaborate, expand or enlarge in detail; add detail and fullness to; cause to become more complex or intricate
devise	think out; plan; contrive; invent
differentiate	identify the difference/s in or between two or more things; distinguish, discriminate; recognise or ascertain what makes something distinct from similar things; in mathematics, obtain the derivative of a function
discerning	discriminating; showing intellectual perception; showing good judgment; making thoughtful and astute choices; selected for value or relevance
discriminate	note, observe or recognise a difference; make or constitute a distinction in or between; differentiate; note or distinguish as different
discriminating	differentiating; distinctive; perceiving differences or distinctions with nicety; possessing discrimination; perceptive and judicious; making judgments about quality; having or showing refined taste or good judgment
discuss	examine by argument; sift the considerations for and against; debate; talk or write about a topic, including a range of arguments, factors or hypotheses; consider, taking into account different issues and ideas, points for and/or against, and supporting opinions or conclusions with evidence

Term	Explanation
disjointed	disconnected; incoherent; lacking a coherent order/sequence or connection
distinguish	recognise as distinct or different; note points of difference between; discriminate; discern; make clear a difference/s between two or more concepts or items
diverse	of various kinds or forms; different from each other
document	support (e.g. an assertion, claim, statement) with evidence (e.g. decisive information, written references, citations)
domains of mathematics	a particular taxonomic classification used to group similar mathematics concepts, ideas, knowledge, understandings and skills; the scope and range of mathematics subject matter
draw conclusions	make a judgment based on reasoning and evidence
E	
effective	successful in producing the intended, desired or expected result; meeting the assigned purpose
efficient	working in a well-organised and competent way; maximum productivity with minimal expenditure of effort; acting or producing effectively with a minimum of waste, expense or unnecessary effort
element	a component or constituent part of a complex whole; a fundamental, essential or irreducible part of a composite entity
elementary	simple or uncompounded; relating to or dealing with elements, rudiments or first principles (of a subject); of the most basic kind; straightforward and uncomplicated
equivalent statements	statements P and Q are equivalent if $P \Rightarrow Q$ and $Q \Rightarrow P$; the symbol $\Leftrightarrow$ is used; it is also written as P if and only if Q or P iff Q
erroneous	based on or containing error; mistaken; incorrect
essential	absolutely necessary; indispensable; of critical importance for achieving something
evaluate	make an appraisal by weighing up or assessing strengths, implications and limitations; make judgments about ideas, works, solutions or methods in relation to selected criteria; examine and determine the merit, value or significance of something, based on criteria
examination	a supervised test that assesses the application of a range of cognitions to one or more provided items such as questions, scenarios and/or problems; student responses are completed individually, under supervised conditions, and in a set timeframe
examine	investigate, inspect or scrutinise; inquire or search into; consider or discuss an argument or concept in a way that uncovers the assumptions and interrelationships of the issue
experiment	try out or test new ideas or methods, especially in order to discover or prove something; undertake or perform a scientific procedure to test a hypothesis, make a discovery or demonstrate a

Term	Explanation
	known fact
explain	make an idea or situation plain or clear by describing it in more detail or revealing relevant facts; give an account; provide additional information
explicit	clearly and distinctly expressing all that is meant; unequivocal; clearly developed or formulated; leaving nothing merely implied or suggested
explore	look into both closely and broadly; scrutinise; inquire into or discuss something in detail
express	convey, show or communicate (e.g. a thought, opinion, feeling, emotion, idea or viewpoint); in words, art, music or movement, convey or suggest a representation of; depict
extended response	an open-ended assessment technique that focuses on the interpretation, analysis, examination and/or evaluation of ideas and information in response to a particular situation or stimulus; while students may undertake some research when writing of the extended response, it is not the focus of this technique; an extended response occurs over an extended and defined period of time
Extension subject	a two-unit subject for which a syllabus has been developed by QCAA, that is an extension of one or more General or alternative sequence subject/s, studied concurrently with, the final two units of that subject or after completion of, the final two units of that subject
extensive	of great extent; wide; broad; far-reaching; comprehensive; lengthy; detailed; large in amount or scale
external assessment	summative assessment that occurs towards the end of a course of study and is common to all schools; developed and marked by the QCAA according to a commonly applied marking scheme
external examination	a supervised test, developed and marked by the QCAA, that assesses the application of a range of cognitions to multiple provided items such as questions, scenarios and/or problems; student responses are completed individually, under supervised conditions, and in a set timeframe
extrapolate	infer or estimate by extending or projecting known information; conjecture; infer from what is known; extend the application of something (e.g. a method or conclusion) to an unknown situation by assuming that existing trends will continue or similar methods will be applicable

Term	Explanation
F	
factual	relating to or based on facts; concerned with what is actually the case; actually occurring; having verified existence
familiar	well-acquainted; thoroughly conversant with; well known from long or close association; often encountered or experienced; common; (of materials, texts, skills or circumstances) having been the focus of learning experiences or previously encountered in prior learning activities
feasible	capable of being achieved, accomplished or put into effect; reasonable enough to be believed or accepted; probable; likely
fluent	spoken or written with ease; able to speak or write smoothly, easily or readily; articulate; eloquent; in artistic performance, characteristic of a highly developed and excellently controlled technique; flowing; polished; flowing smoothly, easily and effortlessly
fluently	in a graceful and seemingly effortless manner; in a way that progresses smoothly and readily
formative assessment	assessment whose major purpose is to improve teaching and student achievement
fragmented	disorganised; broken down; disjointed or isolated
frequent	happening or occurring often at short intervals; constant, habitual, or regular
fundamental	forming a necessary base or core; of central importance; affecting or relating to the essential nature of something; part of a foundation or basis
G	
General subject	a subject for which a syllabus has been developed by the QCAA with the following characteristics: results from courses developed from General syllabuses contribute to the QCE; General subjects have an external assessment component; results may contribute to ATAR calculations
generate	produce; create; bring into existence
H	
hypothesise	formulate a supposition to account for known facts or observed occurrences; conjecture, theorise, speculate; especially on uncertain or tentative grounds
I	
identify	distinguish; locate, recognise and name; establish or indicate who or what someone or something is; provide an answer from a number of possibilities; recognise and state a distinguishing factor or feature

Term	Explanation
identities for products of sine and cosine ratios	$\cos(A)\cos(B) = \frac{1}{2}(\cos(A-B) + \cos(A+B))$ $\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))$ $\sin(A)\cos(B) = \frac{1}{2}(\sin(A+B) + \sin(A-B))$ $\cos(A)\sin(B) = \frac{1}{2}(\sin(A+B) - \sin(A-B))$
illogical	lacking sense or sound reasoning; contrary to or disregardful of the rules of logic; unreasonable
imaginary part of a complex number	a complex number z may be written as $x + yi$, where x and y are real, and then y is the imaginary part of z; it is denoted by $Im(z)$
implement	put something into effect, e.g. a plan or proposal
implication	in Specialist Mathematics, if P then Q symbol: P $\Rightarrow$ Q
implicit	implied, rather than expressly stated; not plainly expressed; capable of being inferred from something else
implicit differentiation	In Specialist Mathematics, implicit differentiation consists of differentiating each term of an equation as it stands and making use of the chain rule; this can lead to a formula for $\frac{dy}{dx}$; for example, if $x^2 + xy^3 - 2x + 3y = 0$, then $2x + x(3y^2)\frac{dy}{dx} + y^3 - 2 + 3\frac{dy}{dx} = 0$, and so $\frac{dy}{dx} = \frac{2-2x-y^3}{3xy^2+3}$
improbable	not probable; unlikely to be true or to happen; not easy to believe
inaccurate	not accurate
inappropriate	not suitable or proper in the circumstances
inclusion–exclusion principle	suppose A and B are subsets of a finite set X then $\lvert A \cup B\rvert = \lvert A\rvert + \lvert B\rvert - \lvert A \cap B\rvert$  suppose A, B and C are subsets of a finite set X then $\lvert A \cup B \cup C\rvert = \lvert A\rvert + \lvert B\rvert + \lvert C\rvert - \lvert A \cap B\rvert - \lvert A \cap C\rvert - \lvert B \cap C\rvert + \lvert A \cap B \cap C\rvert$ this result can be generalised to four or more sets

Term	Explanation
inconsistent	lacking agreement, as one thing with another, or two or more things in relation to each other; at variance; not consistent; not in keeping; not in accordance; incompatible, incongruous
independent	thinking or acting for oneself, not influenced by others
in-depth	comprehensive and with thorough coverage; extensive or profound; well-balanced or fully developed
infer	derive or conclude something from evidence and reasoning, rather than from explicit statements; listen or read beyond what has been literally expressed; imply or hint at
informed	knowledgeable; learned; having relevant knowledge; being conversant with the topic; based on an understanding of the facts of the situation (of a decision or judgment)
innovative	new and original; introducing new ideas; original and creative in thinking
insightful	showing understanding of a situation or process; understanding relationships in complex situations; informed by observation and deduction
instrument-specific marking guide	ISMG; a tool for marking that describes the characteristics evident in student responses and aligns with the identified objectives for the assessment (see 'assessment objectives')
integer	the numbers $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
integral	*adjective* necessary for the completeness of the whole; essential or fundamental; *noun* in mathematics, the result of integration; an expression from which a given function, equation, or system of equations is derived by differentiation
intended	designed; meant; done on purpose; intentional
internal assessment	assessments that are developed by schools; summative internal assessments are endorsed by the QCAA before use in schools and results externally confirmed contribute towards a student's final result
interpret	use knowledge and understanding to recognise trends and draw conclusions from given information; make clear or explicit; elucidate or understand in a particular way; bring out the meaning of, e.g. a dramatic or musical work, by performance or execution; bring out the meaning of an artwork by artistic representation or performance; give one's own interpretation of; identify or draw meaning from, or give meaning to, information presented in various forms, such as words, symbols, pictures or graphs
investigation	an assessment technique that requires students to research a specific problem, question, issue, design challenge or hypothesis through the collection, analysis and synthesis of primary and/or

Term	Explanation
	secondary data; it uses research or investigative practices to assess a range of cognitions in a particular context; an investigation occurs over an extended and defined period of time
investigate	carry out an examination or formal inquiry in order to establish or obtain facts and reach new conclusions; search, inquire into, interpret and draw conclusions about data and information
irrational numbers	a real number is irrational if it cannot be expressed as a quotient of two integers
irrelevant	not relevant; not applicable or pertinent; not connected with or relevant to something
ISMG	instrument-specific marking guide; a tool for marking that describes the characteristics evident in student responses and aligns with the identified objectives for the assessment (see 'assessment objectives')
isolated	detached, separate, or unconnected with other things; one-off; something set apart or characterised as different in some way
J	
judge	form an opinion or conclusion about; apply both procedural and deliberative operations to make a determination
justified	sound reasons or evidence are provided to support an argument, statement or conclusion
justify	give reasons or evidence to support an answer, response or conclusion; show or prove how an argument, statement or conclusion is right or reasonable
L	
learning area	a grouping of subjects, with related characteristics, within a broad field of learning, e.g. the Arts, sciences, languages
linear transformations in two dimensions	a linear transformation in the plane is a mapping of the form $T(x, y) = (ax + by, cx + dy)$; a transformation T is linear if and only if $T(\alpha(x_1, y_1) + \beta(x_2, y_2)) = \alpha T(x_1, y_1) + \beta T(x_2, y_2)$; linear transformations include: • dilations modelled by the matrix $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ • rotations of angle θ about the origin modelled by the matrix $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ • reflections in the line $y = x\tan(\theta)$ modelled by the matrix $\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$ translations are not linear transformations
logical	rational and valid; internally consistent; reasonable; reasoning in accordance with the principles/rules of logic or formal argument; characterised by or capable of clear, sound reasoning; (of an action, decision, etc.) expected or sensible under the circumstances

Term	Explanation
logically	according to the rules of logic or formal argument; in a way that shows clear, sound reasoning; in a way that is expected or sensible
logistic equation	the logistic equation has applications in a range of fields, including biology, biomathematics, economics, chemistry, mathematical psychology, probability, and statistics. One form of this differential equation is: $\frac{dy}{dt} = ay - by^2$ (where $a > 0$ and $b > 0$) the general solution of this is: $y = \frac{a}{b+Ce^{-at}}$, where C is an arbitrary constant
M	
make decisions	select from available options; weigh up positives and negatives of each option and consider all the alternatives to arrive at a position
magnitude of a vector	the magnitude of a vector $\boldsymbol{a}$ is the length of any directed line segment that represents $\boldsymbol{a}$; it is denoted by $\lvert\boldsymbol{a}\rvert$; this can be represented in two dimensions by $\lvert\boldsymbol{a}\rvert = \left\lvert\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\right\rvert = \sqrt{a_1^2 + a_2^2}$ and in three dimensions by $\lvert\boldsymbol{a}\rvert = \left\lvert\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\right\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$.
manipulate	adapt or change to suit one's purpose
mathematical model	a depiction of a situation that expresses relationships using mathematical concepts and language, usually as an algebraic, diagrammatic, graphical or tabular representation
mathematical modelling	involves: • formulating a mathematical representation of a problem derived from within a real-world context • using mathematics concepts and techniques to obtain results • interpreting the results by referring back to the original problem context • revising the model (where necessary)
matrix	a **matrix** is a rectangular array of elements or entries displayed in rows and columns; a **square** matrix has the same number of rows and columns; a **column** matrix (or vector) has only one column; a **row** matrix (or vector) has only one row

Term	Explanation
matrix algebra	for 2 by 2 matrix if $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are 2×2 matrices, $\mathbf{I}$ the 2×2 (multiplicative) identity matrix and $\mathbf{O}$ the 2×2 zero matrix then: $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$ (commutative law for addition) $(\mathbf{A}+\mathbf{B})+\mathbf{C}=\mathbf{A}+(\mathbf{B}+\mathbf{C})$ (associative law for addition) $\mathbf{A}+\mathbf{O}=\mathbf{A}$ (additive identity) $\mathbf{A}+(-\mathbf{A})=\mathbf{O}$ (additive inverse) $(\mathbf{AB})\mathbf{C}=\mathbf{A}(\mathbf{BC})$ (associative law for multiplication) $\mathbf{AI}=\mathbf{A}=\mathbf{IA}$ (additive identity) $\mathbf{A}\,\mathbf{A}^{-1}=\mathbf{A}^{-1}\mathbf{A}=\mathbf{I}$ (multiplicative inverse) $\mathbf{A}(\mathbf{B}+\mathbf{C})=\mathbf{AB}+\mathbf{AC}$ (left distributive law) $(\mathbf{B}+\mathbf{C})\mathbf{A}=\mathbf{BA}+\mathbf{CA}$ (right distributive law)
matrix multiplication	matrix multiplication is the process of multiplying a matrix by another matrix; the product $\boldsymbol{AB}$ of two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ with dimensions $m \times n$ and $p \times q$ is defined if $n = p$; if it is defined, the product $\boldsymbol{AB}$ is an $m \times q$ matrix and it is computed as shown in the following example: $\begin{bmatrix}1 & 8 & 0\\2 & 5 & 7\end{bmatrix}\begin{bmatrix}6 & 10\\11 & 3\\12 & 4\end{bmatrix}=\begin{bmatrix}94 & 34\\151 & 63\end{bmatrix}$ the entries are computed as shown $1\times 6+8\times 11+0\times 12=94$ $1\times 10+8\times 3+0\times 4=34$ $2\times 6+5\times 11+7\times 12=151$ $2\times 10+5\times 3+7\times 4=63$ the entry in row i and column j of the product $\mathbf{AB}$ is computed by 'multiplying' row i of $\mathbf{A}$ by column j of $\mathbf{B}$ as shown If $\mathbf{A}=\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\\a_{31} & a_{32}\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}b_{11} & b_{12} & b_{13}\\b_{21} & b_{22} & b_{23}\end{bmatrix}$ then $\mathbf{AB}=\begin{bmatrix}a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} & a_{11}b_{13}+a_{12}b_{23}\\a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} & a_{21}b_{13}+a_{22}b_{23}\\a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} & a_{31}b_{13}+a_{32}b_{23}\end{bmatrix}$
mental procedures	a domain of knowledge in Marzano's taxonomy, and acted upon by the cognitive, metacognitive and self-systems; sometimes referred to as 'procedural knowledge'; there are three distinct phases to the acquisition of mental procedures — the cognitive stage, the associative stage, and the autonomous stage; the two categories of mental procedures are skills (single rules, algorithms and tactics) and processes (macroprocedures)
methodical	performed, disposed or acting in a systematic way; orderly; characterised by method or order; performed or carried out systematically
minimal	least possible; small, the least amount; negligible
modify	change the form or qualities of; make partial or minor changes to something
modulus of a complex number	if z is a complex number and $z = x + iy$ then the modulus of z is the distance of z from the origin in the Argand plane; the modulus of z denoted by $\lvert z\rvert = \sqrt{x^2 + y^2}$

Term	Explanation
momentum	the momentum $\boldsymbol{p}$ of a particle is the vector quantity $\boldsymbol{p} = m\boldsymbol{v}$ where m is the mass and $\boldsymbol{v}$ is the velocity
multimodal	uses a combination of at least two modes (e.g. spoken, written), delivered at the same time, to communicate ideas and information to a live or virtual audience, for a particular purpose; the selected modes are integrated so that each mode contributes significantly to the response
multiplication by a scalar	let $\boldsymbol{a}$ be a non-zero vector and k a positive real number (scalar); then the scalar multiple of $\boldsymbol{a}$ by k is the vector $k\boldsymbol{a}$, which has magnitude $\lvert k\rvert\lvert\boldsymbol{a}\rvert$ and the same direction as $\boldsymbol{a}$; if k is a negative real number, then $k\,\boldsymbol{a}$ has magnitude $\lvert k\rvert\lvert\boldsymbol{a}\rvert$ but is directed in the opposite direction to $\boldsymbol{a}$ (see **negative of a vector**)
multiplication principle	suppose a choice is to be made in two stages; if there are a choices for the first stage and b choices for the second stage, no matter what choice has been made at the first stage, then there are ab choices altogether; if the choice is to be made in n stages and if for each i, there are a_i choices for the i^{th} stage then there are $a_1 a_2 \ldots\ a_n$ choices altogether
multiplicative identity matrix	a multiplicative identity matrix is a square matrix in which all the elements in the leading diagonal are ones and the remaining elements are zeros. Identity matrices are designated by the letter $\boldsymbol{I}$; there is an identity matrix for each order of square matrix; when clarity is needed, the order is written with a subscript: $\boldsymbol{I_n}$
multiplicative inverse	Multiplicative inverse of a square matrix the inverse of a square matrix $\mathbf{A}$ is written as $\mathbf{A^{-1}}$ and has the property that $$\mathbf{A\,A^{-1} = A^{-1}A = I}$$ not all square matrices have an inverse; a matrix that has an inverse is said to be invertible Multiplicative inverse of a 2 × 2 matrix the inverse of the matrix $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, when $\det \mathbf{A} \neq 0$.
N	
narrow	limited in range or scope; lacking breadth of view; limited in amount; barely sufficient or adequate; restricted
negation	if P is a statement then the statement ‘not P’, denoted by ¬P is the negation of P; if P is the statement ‘it is snowing’, then ¬P is the statement ‘it is not snowing’
nuanced	showing a subtle difference or distinction in expression, meaning, response, etc.; finely differentiated; characterised by subtle shades of meaning or expression; a subtle distinction, variation or quality; sensibility to, awareness of, or ability to express delicate shadings, as of meaning, feeling, or value
O	
objectives	see ‘syllabus objectives’, ‘unit objectives’, ‘assessment objectives’

Term	Explanation
observation	data or information required to solve a mathematical problem and/or develop a mathematical model; empirical evidence
obvious	clearly perceptible or evident; easily seen, recognised or understood
optimal	best, most favourable, under a particular set of circumstances
organise	arrange, order; form as or into a whole consisting of interdependent or coordinated parts, especially for harmonious or united action
organised	systematically ordered and arranged; having a formal organisational structure to arrange, coordinate and carry out activities
outstanding	exceptionally good; clearly noticeable; prominent; conspicuous; striking
P	
parametric equation of a straight line	when $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ is the position vector of a point on a straight line in three-dimensional space and $\boldsymbol{d} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ is any vector with direction along the line; the line consists of all points P(x, y, z) whose parametric form is given by $x = a_1 + kd_1$, $y = a_2 + kd_2$, $z = a_3 + kd_3$ for some real number *k.*
partial	not total or general; existing only in part; attempted, but incomplete
particular	distinguished or different from others or from the ordinary; noteworthy
Pascal's triangle	a triangular arrangement of binomial coefficients in which the n^{th} row consists of the binomial coefficients $\binom{n}{r}$; for $0 \le r \le n$, each interior entry is the sum of the two entries above it, and the sum of the entries in the n^{th} row is 2^n identities include the recurrence relation ${}^{n}C_{k} = {}^{n-1}C_{k-1} + {}^{n-1}C_{k}$
perceptive	having or showing insight and the ability to perceive or understand; discerning (see also 'discriminating')
performance	an assessment technique that requires students to demonstrate a range of cognitive, technical, creative and/or expressive skills and to apply theoretical and conceptual understandings, through the psychomotor domain; it involves student application of identified skills when responding to a task that involves solving a problem, providing a solution or conveying meaning or intent; a performance is developed over an extended and defined period of time

Term	Explanation
permutations	a permutation of n objects is an arrangement or rearrangement of n objects (order is important); the number of arrangements of n objects is $n!$ the number of permutations of n objects taken r at a time is denoted ${}^{n}P_{r}$; $${}^{n}P_{r} = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$$
persuasive	capable of changing someone's ideas, opinions or beliefs; appearing worthy of approval or acceptance; (of an argument or statement) communicating reasonably or credibly (see also 'convincing')
perusal time	time allocated in an assessment to reading items and tasks and associated assessment materials; no writing is allowed; students may not make notes and may not commence responding to the assessment in the response space/book
pigeon-hole principle	if there are n pigeon holes and $n+1$ pigeons to go into them, then at least one pigeon hole must get two or more pigeons
planning time	time allocated in an assessment to planning how to respond to items and tasks and associated assessment materials; students may make notes but may not commence responding to the assessment in the response space/book; notes made during planning are not collected, nor are they graded or used as evidence of achievement
polished	flawless or excellent; performed with skilful ease
precise	definite or exact; definitely or strictly stated, defined or fixed; characterised by definite or exact expression or execution
precision	accuracy; exactness; exact observance of forms in conduct or actions
predict	give an expected result of an upcoming action or event; suggest what may happen based on available information
probability density function	the probability density function (pdf) of a continuous random variable is the function that when integrated over an interval gives the probability that the continuous random variable having that pdf lies in that interval; the **probability density function** is therefore the derivative of the (cumulative probability) distribution function
problem-solving strategies	may include estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data
procedural vocabulary	instructional terms used in a mathematical context (e.g. calculate, convert, determine, identify, justify, show, sketch, solve, state).
procedure	a list of sequential steps that are used to solve a problem or perform a task
product	an assessment technique that focusses on the output or result of a process requiring the application of a range of cognitive, physical, technical, creative and/or expressive skills, and theoretical and conceptual understandings; a product is developed over an extended and defined period of time

Term	Explanation
proficient	well advanced or expert in any art, science or subject; competent, skilled or adept in doing or using something
project	an assessment technique that focusses on a problem-solving process requiring the application of a range of cognitive, technical and creative skills and theoretical understandings; the response is a coherent work that documents the iterative process undertaken to develop a solution and includes written paragraphs and annotations, diagrams, sketches, drawings, photographs, video, spoken presentations, physical prototypes and/or models; a project is developed over an extended and defined period of time
propose	put forward (e.g. a point of view, idea, argument, suggestion) for consideration or action
prove	use a sequence of steps to obtain the required result in a formal way
psychomotor procedures	a domain of knowledge in Marzano's taxonomy, and acted upon by the cognitive, metacognitive and self-systems; these are physical procedures used to negotiate daily life and to engage in complex physical activities; the two categories of psychomotor procedures are skills (foundational procedures and simple combination procedures) and processes (complex combination procedures)
purposeful	having an intended or desired result; having a useful purpose; determined; resolute; full of meaning; significant; intentional
Pythagorean identities	$\cos^2(A) + \sin^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \text{cosec}^2(A)$
Q	
QCE	Queensland Certificate of Education
qualitative statements	statements relating to a quality or qualities; of a non-numerical nature
quantifiers	for all (for each) • symbol ∀ • for all real numbers x, $x^2 \geq 0$ (∀ real numbers x, $x^2 \geq 0$) there exists • symbol ∃ • there exists a real number that is not positive (∃ a real number that is not positive)
quantitative analysis	use of mathematical measurements and calculations, including statistics, to analyse the relationships between variables; may include use of the correlation coefficient, coefficient of determination, simple residual analysis or outlier analysis
R	
random sample	a set of data in which the value of each observation is governed by some chance mechanism that depends on the situation; the most common situation in which the term *random sample* is used refers to a set of independent and identically distributed observations

Term	Explanation
rarely	infrequently; in few instances
rational function	a function such that $f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials; usually $g(x)$ and $h(x)$ are chosen so as to have no common factor of degree greater than or equal to one, and the domain of f is usually taken to be $R \backslash \{x: h(x) = 0\}$
rational numbers	a real number is rational if it can be expressed as a quotient of two integers; otherwise, it is called irrational; all rational numbers can be expressed as decimal expansions that are either terminating or eventually recurring
realise	create or make (e.g. a musical, artistic or dramatic work); actualise; make real or concrete; give reality or substance to
real numbers	the numbers generally used in mathematics, in scientific work and in everyday life are the real numbers; they can be pictured as points on a number line, with the integers evenly spaced along the line, and a real number a to the right of a real number b if $a > b$; a real number is either rational or irrational; the set of real numbers consists of the set of all rational and irrational numbers; every real number has a decimal expansion; rational numbers are the ones whose decimal expansions are either terminating or eventually recurring
reasonable	endowed with reason; having sound judgment; fair and sensible; based on good sense; average; appropriate, moderate
reasonableness of solutions	to justify solutions obtained with or without technology using everyday language, mathematical language or a combination of both; may be applied to calculations to check working, or to questions that require a relationship back to the context
reasoned	logical and sound; based on logic or good sense; logically thought out and presented with justification; guided by reason; well-grounded; considered
recall	remember; present remembered ideas, facts or experiences; bring something back into thought, attention or into one's mind
reciprocal trigonometric functions	$\sec(A) = \frac{1}{\cos(A)}, \cos(A) \neq 0$ $\operatorname{cosec}(A) = \frac{1}{\sin(A)}, \sin(A) \neq 0$ $\cot(A) = \frac{\cos(A)}{\sin(A)}, \sin(A) \neq 0$
recognise	identify or recall particular features of information from knowledge; identify that an item, characteristic or quality exists; perceive as existing or true; be aware of or acknowledge
refined	developed or improved so as to be precise, exact or subtle
reflect on	think about deeply and carefully
rehearsed	practised; previously experienced; practised extensively
related	associated with or linked to
relevance	being related to the matter at hand

Term	Explanation
relevant	bearing upon or connected with the matter in hand; to the purpose; applicable and pertinent; having a direct bearing on
repetitive	containing or characterised by repetition, especially when unnecessary or tiresome
reporting	providing information that succinctly describes student performance at different junctures throughout a course of study
representatively sample	in this syllabus, a selection of subject matter that accurately reflects the intended learning of a topic
resolve	in the Arts, consolidate and communicate intent through a synthesis of ideas and application of media to express meaning
routine	often encountered, previously experienced; commonplace; customary and regular; well-practised; performed as part of a regular procedure, rather than for a special reason
rudimentary	relating to rudiments or first principles; elementary; undeveloped; involving or limited to basic principles; relating to an immature, undeveloped or basic form
S	
safe	secure; not risky
sample mean	the arithmetic average of the sample values
scalar multiplication	the process of multiplying a matrix by a scalar (number); in general, for the matrix $\mathbf{A}$ with entries a_{ij} the entries of $k\mathbf{A}$ are ka_{ij}
scalar (dot) product	let $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$; $\boldsymbol{a} \,.\, \boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ is the scalar (dot) product; when expressed in $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}}$ notation, $\boldsymbol{a} = a_1\hat{\boldsymbol{\imath}} + a_2\hat{\boldsymbol{\jmath}} + a_3\hat{\boldsymbol{k}}$ and $\boldsymbol{b} = b_1\hat{\boldsymbol{\imath}} + b_2\hat{\boldsymbol{\jmath}} + b_3\hat{\boldsymbol{k}}$ then $\boldsymbol{a} \,.\, \boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$; the scalar (dot) product has the following geometric interpretation; $\boldsymbol{a} \,.\, \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert cos\,(\theta)$ where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$; note $\lvert\boldsymbol{a}\rvert = \sqrt{\boldsymbol{a} \,.\, \boldsymbol{a}}$
secure	sure; certain; able to be counted on; self-confident; poised; dependable; confident; assured; not liable to fail
select	choose in preference to another or others; pick out
sensitive	capable of perceiving with a sense or senses; aware of the attitudes, feelings or circumstances of others; having acute mental or emotional sensibility; relating to or connected with the senses or sensation
separation of variables	differential equations of the form $\frac{dy}{dx} = g(x)h(y)$ can be rearranged as long as $h(y) \neq 0$ to obtain $\frac{1}{h(y)}\frac{dy}{dx} = g(x)$

Term	Explanation
sequence	place in a continuous or connected series; arrange in a particular order
show	provide the relevant reasoning to support a response
significant	important; of consequence; expressing a meaning; indicative; includes all that is important; sufficiently great or important to be worthy of attention; noteworthy; having a particular meaning; indicative of something
simple	easy to understand, deal with and use; not complex or complicated; plain; not elaborate or artificial; may concern a single or basic aspect; involving few elements, components or steps
simple familiar	problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable; that is - the required procedure is clear from the way the problem is posed, *or* - in a context that has been a focus of prior learning. Students are *not* required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.
simple harmonic motion	used to model oscillations in two dimensions; occurs when the acceleration is proportional to displacement but in opposite directions modelled by $\frac{d^2x}{dt^2} = -\omega^2 x$ where ω represents the angular frequency; the equations for simple harmonic motion with amplitude A, phase α or β, velocity v, period T and frequency F include: displacement $x = A\sin(\omega t + \alpha)$ or $x = A\cos(\omega t + \beta)$ velocity $v^2 = \omega^2(A^2 - x^2)$ period $T = \frac{2\pi}{\omega}$ frequency $f = \frac{1}{T}$
simplistic	characterised by extreme simplification, especially if misleading; oversimplified
Simpson's rule	a formula for approximating the integral of a function; $\int_a^b f(x)\,dx \approx \frac{w}{3}[f(x_0) + 4[f(x_1) + f(x_3) + \cdots] + 2[f(x_2) + f(x_4) + \cdots] + f(x_n)]$ where the interval $[a, b]$ of the function $f(x)$ is divided into an even number n of equal strips of width w.
sketch	execute a drawing or painting in simple form, giving essential features but not necessarily with detail or accuracy; in mathematics, represent by means of a diagram or graph; the sketch should give a general idea of the required shape or relationship and should include features

Term	Explanation
skilful	having technical facility or practical ability; possessing, showing, involving or requiring skill; expert, dexterous; demonstrating the knowledge, ability or training to perform a certain activity or task well; trained, practised or experienced
skilled	having or showing the knowledge, ability or training to perform a certain activity or task well; having skill; trained or experienced; showing, involving or requiring skill
slope field	slope field (direction or gradient field) is a graphical representation of the solutions of a linear first-order differential equation in which the derivative at a given point is represented by a line segment of the corresponding slope
solution	the result of a mathematical process undertaken to answer or resolve a problem
solve	find an answer to, explanation for, or means of dealing with (e.g. a problem); work out the answer or solution to (e.g. a mathematical problem); obtain the answer/s using algebraic, numerical and/or graphical methods
sophisticated	of intellectual complexity; reflecting a high degree of skill, intelligence, etc.; employing advanced or refined methods or concepts; highly developed or complicated
specific	clearly defined or identified; precise and clear in making statements or issuing instructions; having a special application or reference; explicit, or definite; peculiar or proper to something, as qualities, characteristics, effects, etc.
sporadic	happening now and again or at intervals; irregular or occasional; appearing in scattered or isolated instances
statement	a sentence or assertion
straightforward	without difficulty; uncomplicated; direct; easy to do or understand
structure	*verb* give a pattern, organisation or arrangement to; construct or arrange according to a plan; *noun* in languages, arrangement of words into larger units, e.g. phrases, clauses, sentences, paragraphs and whole texts, in line with cultural, intercultural and textual conventions
structured	organised or arranged so as to produce a desired result
subject	a branch or area of knowledge or learning defined by a syllabus; school subjects are usually based in a discipline or field of study (see also ‘course’)
subject matter	the subject-specific body of information, mental procedures and psychomotor procedures that are necessary for students’ learning and engagement within that subject

Term	Explanation
subject	a branch or area of knowledge or learning defined by a syllabus or alternative sequence; school subjects are usually based in a discipline or field of study (see also ‘course’)
substantial	of ample or considerable amount, quantity, size, etc.; of real worth or value; firmly or solidly established; of real significance; reliable; important, worthwhile
substantiated	established by proof or competent evidence
subtended angle	the angle made by a line, arc or object at a given point
subtle	fine or delicate in meaning or intent; making use of indirect methods; not straightforward or obvious
successful	achieving or having achieved success; accomplishing a desired aim or result
succinct	expressed in few words; concise; terse; characterised by conciseness or brevity; brief and clear
sufficient	enough or adequate for the purpose
suitable	appropriate; fitting; conforming or agreeing in nature, condition, or action
summarise	give a brief statement of a general theme or major point/s; present ideas and information in fewer words and in sequence
summative assessment	assessment whose major purpose is to indicate student achievement; summative assessments contribute towards a student’s subject result
superficial	concerned with or comprehending only what is on the surface or obvious; shallow; not profound, thorough, deep or complete; existing or occurring at or on the surface; cursory; lacking depth of character or understanding; apparent and sometimes trivial
supported	corroborated; given greater credibility by providing evidence
sustained	carried on continuously, without interruption, or without any diminishing of intensity or extent
syllabus	a document that prescribes the curriculum for a course of study
syllabus objectives	outline what the school is required to teach and what students have the opportunity to learn; described in terms of actions that operate on the subject matter; the overarching objectives for a course of study (see also ‘unit objectives’, ‘assessment objectives’)
symbolise	represent or identify by a symbol or symbols
synthesise	combine different parts or elements (e.g. information, ideas, components) into a whole, in order to create new understanding

Term	Explanation
systematic	done or acting according to a fixed plan or system; methodical; organised and logical; having, showing, or involving a system, method, or plan; characterised by system or method; methodical; arranged in, or comprising an ordered system
T	
technical vocabulary	terms that have a precise mathematical meaning (e.g. categorical data, chain rule, decimal fraction, imaginary number, log laws, linear regression, sine rule, whole number); may include everyday words used in a mathematical context (e.g. capacity, differentiate, evaluate, integrate, order, property, sample, union)
test	take measures to check the quality, performance or reliability of something
thorough	carried out through, or applied to the whole of something; carried out completely and carefully; including all that is required; complete with attention to every detail; not superficial or partial; performed or written with care and completeness; taking pains to do something carefully and completely
thoughtful	occupied with, or given to thought; contemplative; meditative; reflective; characterised by or manifesting thought
topic	a division of, or sub-section within a unit; all topics/sub-topics within a unit are interrelated
U	
unclear	not clear or distinct; not easy to understand; obscure
understand	perceive what is meant by something; grasp; be familiar with (e.g. an idea); construct meaning from messages, including oral, written and graphic communication
uneven	unequal; not properly corresponding or agreeing; irregular; varying; not uniform; not equally balanced
unfamiliar	not previously encountered; situations or materials that have not been the focus of prior learning experiences or activities
unit	a defined amount of subject matter delivered in a specific context or with a particular focus; it includes unit objectives particular to the unit, subject matter and assessment direction
unit objectives	drawn from the syllabus objectives and contextualised for the subject matter and requirements of a particular unit; they are assessed at least once in the unit (see also 'syllabus objectives', 'assessment objectives')
unit vector	a vector with magnitude 1; given a vector $\boldsymbol{a}$ the unit vector in the same direction as $\boldsymbol{a}$ is $\frac{1}{\lvert\boldsymbol{a}\rvert}\boldsymbol{a}$; this vector is often denoted as $\hat{\boldsymbol{a}}$
unrelated	having no relationship; unconnected
use	operate or put into effect; apply knowledge or rules to put theory into practice

Term	Explanation
V	
vague	not definite in statement or meaning; not explicit or precise; not definitely fixed, determined or known; of uncertain, indefinite or unclear character or meaning; not clear in thought or understanding; couched in general or indefinite terms; not definitely or precisely expressed; deficient in details or particulars; thinking or communicating in an unfocused or imprecise way
valid	sound, just or well-founded; authoritative; having a sound basis in logic or fact (of an argument or point); reasonable or cogent; able to be supported; legitimate and defensible; applicable
variable	*adjective* apt or liable to vary or change; changeable; inconsistent; (readily) susceptible or capable of variation; fluctuating, uncertain; *noun* in mathematics, a symbol, or the quantity it signifies, that may represent any one of a given set of number and other objects
variety	a number or range of things of different kinds, or the same general class, that are distinct in character or quality; (of sources) a number of different modes or references
vector	in Specialist Mathematics, the term *vector* is used to describe a physical quantity like velocity or force that has a magnitude and direction; a vector is an entity $\boldsymbol{a}$ which has a given length (magnitude) and a given direction; if $\overrightarrow{AB}$ is a directed line segment with this length and direction, then we say that $\overrightarrow{AB}$ represents $\boldsymbol{a}$
vector (cross) product	when expressed in $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}}$ notation, $\boldsymbol{a} = a_1\hat{\boldsymbol{\imath}} + a_2\hat{\boldsymbol{\jmath}} + a_3\hat{\boldsymbol{k}}$ and $\boldsymbol{b} = b_1\hat{\boldsymbol{\imath}} + b_2\hat{\boldsymbol{\jmath}} + b_3\hat{\boldsymbol{k}}$ then $\boldsymbol{a} \times \boldsymbol{b} = (a_2b_3 - a_3b_2)\hat{\boldsymbol{\imath}} + (a_3b_1 - a_1b_3)\hat{\boldsymbol{\jmath}} + (a_1b_2 - a_2b_1)\hat{\boldsymbol{k}}$; the vector (cross) product has the following geometric interpretation; let $\boldsymbol{a}$ and $\boldsymbol{b}$ be two non-parallel vectors then $\lvert\boldsymbol{a} \times \boldsymbol{b}\rvert$ is the area of the parallelogram defined by $\boldsymbol{a}$ and $\boldsymbol{b}$ and $\boldsymbol{a} \times \boldsymbol{b}$ is normal to this parallelogram
vector equation of a straight line	let $\boldsymbol{a}$ be the position vector of a point on a straight line and $\boldsymbol{d}$ be any vector with direction along the line; the line consists of all points P whose position vector $\boldsymbol{r}$ is given by $\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$ for some real number *k.*
vector equation of a plane	let $\boldsymbol{a}$ be the position vector of a point on a plane and $\boldsymbol{n}$ be any vector normal to the plane; the plane consists of all points P whose position vector $\boldsymbol{r}$ is given by $\boldsymbol{r}.\boldsymbol{n} = \boldsymbol{a}.\boldsymbol{n}$

Term	Explanation
vector function	in this course, a vector function is one that depends on a single real number parameter t, often representing time, producing a vector $\boldsymbol{r}(t)$ as the result; in terms of the standard unit vectors $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, $\hat{\boldsymbol{k}}$ of three-dimensional space, the vector-valued functions of this specific type are given by expressions such as $\boldsymbol{r}(t) = f(t)\hat{\boldsymbol{\imath}} + g(t)\hat{\boldsymbol{\jmath}} + h(t)\hat{\boldsymbol{k}}$ where $f(t)$, $g(t)$ and $h(t)$ are real valued functions giving coordinates
vector projection	when $\boldsymbol{a}$ and $\boldsymbol{b}$ are two vectors and θ is the angle between them; the projection of a vector $\boldsymbol{a}$ on a vector $\boldsymbol{b}$ is the vector $\lvert\boldsymbol{a}\rvert\cos(\theta)\,\hat{\boldsymbol{b}}$ where $\hat{\boldsymbol{b}}$ is the unit vector in the direction of $\boldsymbol{b}$; the projection of a vector $\boldsymbol{a}$ on a vector $\boldsymbol{b}$ is $(\boldsymbol{a}.\hat{\boldsymbol{b}})\hat{\boldsymbol{b}}$ where $\hat{\boldsymbol{b}}$ is the unit vector in the direction of $\boldsymbol{b}$; this projection is also given by $\frac{\boldsymbol{a}.\boldsymbol{b}}{\boldsymbol{b}.\boldsymbol{b}}\boldsymbol{b}$
verify	to ascertain the truth or correctness of, especially by examination or comparison
W	
wide	of great range or scope; embracing a great number or variety of subjects, cases, etc.; of full extent
with expression	in words, art, music or movement, conveying or indicating feeling, spirit, character, etc.; a way of expressing or representing something; vivid, effective or persuasive communication

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8 Version history

<table>
<tr><th>Version</th><th>Date of change</th><th>Update</th></tr>
<tr><td rowspan="9">1.1</td><td rowspan="9">July 2018</td><td>Minor editorial amendments to assumed knowledge, prior learning and experience</td></tr>
<tr><td>Minor amendments to:
• syllabus objectives 5 and 6, and their associated explanatory paragraphs
• unit objective 5 across all units
• assessment objective 5 across all assessment instruments</td></tr>
<tr><td>Minor amendments to pedagogical and conceptual frameworks</td></tr>
<tr><td>Unit 1 — minor amendments to:
• Topic 2 title
• subject matter
• assessment guidance</td></tr>
<tr><td>Minor amendments to subject matter across Units 2, 3 and 4</td></tr>
<tr><td>Summative internal assessment 1 (IA1) — minor amendments to:
• description
• conditions
• ISMG</td></tr>
<tr><td>Summative internal assessments 2 and 3 (IA2 & 1A3) — minor amendments to:
• description
• degree of difficulty definitions</td></tr>
<tr><td>Summative external assessment (EA) — minor amendments to:
• degree of difficulty definitions
• conditions</td></tr>
<tr><td>Glossary updates</td></tr>
<tr><td>1.2</td><td>August 2018</td><td>Summative internal assessment 1 (IA1) — minor amendments to assessment objectives 1, 2, and 6</td></tr>
</table>

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