Mathematical Methods 2025 v1.2

General senior syllabus

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Queensland syllabuses for senior subjects

In Queensland, a syllabus for a senior subject is an official ‘map’ of a senior school subject. A syllabus’s function is to support schools in delivering the Queensland Certificate of Education (QCE) system through high-quality and high-equity curriculum and assessment.

Syllabuses are based on design principles developed from independent international research about how excellence and equity are promoted in the documents teachers use to develop and enliven the curriculum.

Syllabuses for senior subjects build on student learning in the Prep to Year 10 Australian Curriculum and include General, General (Extension), Senior External Examination (SEE), Applied, Applied (Essential) and Short Course syllabuses.

More information about syllabuses for senior subjects is available at www.qcaa.qld.edu.au/senior/senior-subjects and in the ‘Queensland curriculum’ section of the *QCE and QCIA policy and procedures handbook*.

Teaching, learning and assessment resources will support the implementation of a syllabus for a senior subject. More information about professional resources for senior syllabuses is available on the QCAA website and via the QCAA Portal.

Course overview

Rationale

Mathematics is a unique and powerful intellectual discipline that is used to investigate patterns, order, generality and uncertainty. It is a way of thinking in which problems are explored and solved through observation, reflection and logical reasoning. It uses a concise system of communication, with written, symbolic, spoken and visual components. Mathematics is creative, requires initiative and promotes curiosity in an increasingly complex and data-driven world. It is the foundation of all quantitative disciplines.

To prepare students with the knowledge, skills and confidence to participate effectively in the community and the economy requires the development of skills that reflect the demands of the 21st century. Students undertaking Mathematics will develop their critical and creative thinking, oral and written communication, information & communication technologies (ICT) capability, ability to collaborate, and sense of personal and social responsibility — ultimately becoming lifelong learners who demonstrate initiative when facing a challenge. The use of technology to make connections between mathematical theory, practice and application has a positive effect on the development of conceptual understanding and student disposition towards mathematics.

Mathematics teaching and learning practices range from practising essential mathematical routines to develop procedural fluency, through to investigating scenarios, modelling the real world, solving problems and explaining reasoning. When students achieve procedural fluency, they carry out procedures flexibly, accurately and efficiently. When factual knowledge and concepts come to mind readily, students are able to make more complex use of knowledge to successfully formulate, represent and solve mathematical problems. Problem-solving helps to develop an ability to transfer mathematical skills and ideas between different contexts. This assists students to make connections between related concepts and adapt what they already know to new and unfamiliar situations. With appropriate effort and experience, through discussion, collaboration and reflection of ideas, students should develop confidence and experience success in their use of mathematics.

The major domains of mathematics in Mathematical Methods are Algebra, Functions, relations and their graphs, Calculus and Statistics. Topics are developed systematically, with increasing levels of sophistication, complexity and connection, and build on algebra, functions and their graphs, and probability from the P–10 Australian Curriculum. Calculus is essential for developing an understanding of the physical world. The domain Statistics is used to describe and analyse phenomena involving uncertainty and variation. Both are the basis for developing effective models of the world and solving complex and abstract mathematical problems. The ability to translate written, numerical, algebraic, symbolic and graphical information from one representation to another is a vital part of learning in Mathematical Methods.

Students who undertake Mathematical Methods will see the connections between mathematics and other areas of the curriculum and apply their mathematical skills to real-world problems, becoming critical thinkers, innovators and problem-solvers. Through solving problems and developing models, they will appreciate that mathematics and statistics are dynamic tools that are critically important in the 21st century.

Syllabus objectives

The syllabus objectives outline what students have the opportunity to learn.

1. **Recall mathematical knowledge.**

   When students recall mathematical knowledge, they recognise features of remembered information. They recognise relevant concepts, rules, definitions, techniques and algorithms.

2. **Use mathematical knowledge.**

   When students use mathematical knowledge, they put into effect relevant concepts, rules, definitions, techniques and algorithms. They perform calculations with and without technology.

3. **Communicate mathematical knowledge.**

   When students communicate mathematical knowledge, they use mathematical language (terminology, symbols, conventions and representations) and everyday language. They organise and present information in graphical and symbolic form, and describe and represent mathematical models.

4. **Evaluate the reasonableness of solutions.**

   When students evaluate the reasonableness of solutions, they interpret their mathematical results in the context of the situation and reflect on whether the problem has been solved. They verify results by using estimation skills and checking calculations, with and without technology. They make an appraisal by assessing implications, strengths and limitations of solutions and/or models, and use this to consider if alternative methods or refinements are required.

5. **Justify procedures and decisions.**

   When students justify procedures and decisions, they explain their mathematical reasoning in detail. They make relationships evident, logically organise mathematical arguments, and provide reasons for choices made and conclusions reached.

6. **Solve mathematical problems.**

   When students solve mathematical problems, they analyse the context of the problem to translate information into mathematical forms. They make decisions about the concepts, techniques and technology to be used and apply these to develop a solution. They develop, refine and use mathematical models, where applicable.

Designing a course of study in Mathematical Methods

Syllabuses are designed for teachers to make professional decisions to tailor curriculum and assessment design and delivery to suit their school context and the goals, aspirations and abilities of their students within the parameters of Queensland's senior phase of learning.

The syllabus is used by teachers to develop curriculum for their school context. The term *course of study* describes the unique curriculum and assessment that students engage with in each school context. A course of study is the product of a series of decisions made by a school to select, organise and contextualise subject matter, integrate complementary and important learning, and create assessment tasks in accordance with syllabus specifications.

It is encouraged that, where possible, a course of study is designed such that teaching, learning and assessment activities are integrated and enlivened in an authentic setting.

Course structure

Mathematical Methods is a General senior syllabus. It contains four QCAA-developed units from which schools develop their course of study.

Each unit has been developed with a notional time of 55 hours of teaching and learning, including assessment.

Students should complete Unit 1 and Unit 2 before beginning Units 3 and 4. Units 3 and 4 are studied as a pair.

More information about the requirements for administering senior syllabuses is available in the 'Queensland curriculum' section of the *QCE and QCIA policy and procedures handbook*.

Curriculum

Senior syllabuses set out only what is essential while being flexible so teachers can make curriculum decisions to suit their students, school context, resources and expertise.

Within the requirements set out in this syllabus and the *QCE and QCIA policy and procedures handbook*, schools have autonomy to decide:

- how and when subject matter is delivered
- how, when and why learning experiences are developed, and the context in which learning occurs
- how opportunities are provided in the course of study for explicit and integrated teaching and learning of complementary skills.

These decisions allow teachers to develop a course of study that is rich, engaging and relevant for their students.

Assessment

Senior syllabuses set out only what is essential while being flexible so teachers can make assessment decisions to suit their students, school context, resources and expertise.

General senior syllabuses contain assessment specifications and conditions for the assessment instruments that must be implemented with Units 3 and 4. These specifications and conditions ensure comparability, equity and validity in assessment.

Within the requirements set out in this syllabus and the *QCE and QCIA policy and procedures handbook*, schools have autonomy to decide:

- specific assessment task details
- assessment contexts to suit available resources
- how the assessment task will be integrated with teaching and learning activities
- how authentic the task will be.

In Unit 1 and Unit 2, schools:

- develop at least two but no more than four assessments
- complete at least one assessment for each unit
- ensure that each unit objective is assessed at least once.

In Units 3 and 4, schools develop three assessments using the assessment specifications and conditions provided in the syllabus.

More information about assessment in senior syllabuses is available in 'The assessment system' section of the *QCE and QCIA policy and procedures handbook*.

Subject matter

Each unit contains a unit description, unit objectives and subject matter. Subject matter is the body of information, mental procedures and psychomotor procedures (see Marzano & Kendall 2007, 2008) that are necessary for students' learning and engagement with the subject. Subject matter itself is not the specification of learning experiences but provides the basis for the design of student learning experiences.

Subject matter has a direct relationship with the unit objectives and provides statements of learning that have been constructed in a similar way to objectives.

Aboriginal perspectives and Torres Strait Islander perspectives

The QCAA is committed to reconciliation. As part of its commitment, the QCAA affirms that:

- Aboriginal peoples and Torres Strait Islander peoples are the first Australians, and have the oldest living cultures in human history
- Aboriginal peoples and Torres Strait Islander peoples have strong cultural traditions and speak diverse languages and dialects, other than Standard Australian English
- teaching and learning in Queensland schools should provide opportunities for students to deepen their knowledge of Australia by engaging with the perspectives of Aboriginal peoples and Torres Strait Islander peoples
- positive outcomes for Aboriginal students and Torres Strait Islander students are supported by successfully embedding Aboriginal perspectives and Torres Strait Islander perspectives across planning, teaching and assessing student achievement.

Guidelines about Aboriginal perspectives and Torres Strait Islander perspectives and resources for teaching are available at www.qcaa.qld.edu.au/k-12-policies/aboriginal-torres-strait-islander-perspectives.

Where appropriate, Aboriginal perspectives and Torres Strait Islander perspectives have been embedded in the subject matter.

Complementary skills

Opportunities for the development of complementary skills have been embedded throughout subject matter. These skills, which overlap and interact with syllabus subject matter, are derived from current education, industry and community expectations and encompass the knowledge, skills, capabilities, behaviours and dispositions that will help students live and work successfully in the 21st century.

These complementary skills are:

- literacy — the knowledge, skills, behaviours and dispositions about language and texts essential for understanding and conveying English language content
- numeracy — the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations, to recognise and understand the role of mathematics in the world, and to develop the dispositions and capacities to use mathematical knowledge and skills purposefully
- 21st century skills — the attributes and skills students need to prepare them for higher education, work, and engagement in a complex and rapidly changing world. These skills include critical thinking, creative thinking, communication, collaboration and teamwork, personal and social skills, and digital literacy. The explanations of associated skills are available at www.qcaa.qld.edu.au/senior/senior-subjects/general-subjects/21st-century-skills.

It is expected that aspects of literacy, numeracy and 21st century skills will be developed by engaging in the learning outlined in this syllabus. Teachers may choose to create additional explicit and intentional opportunities for the development of these skills as they design the course of study.

Additional subject-specific information

Additional subject-specific information has been included to support and inform the development of a course of study.

Assumed knowledge, prior learning or experience

Assumed knowledge refers to the subject matter that teachers can expect students to know prior to beginning this subject. Emphasis is placed on the mastery of content, ensuring key concepts or procedures are learnt fully so they will not need reteaching.

Developing mastery often involves multiple approaches to teaching and conceptualising the same mathematical concept. When students have a good understanding of a key concept or procedure, they are more easily able to make connections to related new subject matter and apply what they already know to new problems.

Subject matter from previous unit/s is assumed for subsequent unit/s.

The following is a non-exhaustive list of assumed knowledge based on the subject matter in the P–10 Australian Curriculum version 9.

- Factorise, expand and simplify expressions including monic quadratic expressions using a variety of strategies.
- Apply the four operations to simple algebraic fractions with numerical denominators.
- Substitute values into formulas to determine an unknown.
- Solve problems involving linear equations, including those derived from formulas and those that involve simple algebraic fractions.
- Recall the equation of a line in the form $y = mx + c$
- Determine if lines are parallel or perpendicular lines, including $m_1 = m_2$ and $m_1 m_2 = -1$
- Explore the connection between algebraic and graphical representations of relations, e.g. simple quadratics, circles and exponentials using digital technology as appropriate.
- Solve simple quadratic equations using a range of strategies.
- Solve linear simultaneous equations, using algebraic and graphical techniques, including using digital technology.
- Solve linear inequalities and graph their solutions on a number line.
- Solve right-angled triangle problems using trigonometric skills.
- Describe the results of two- and three-step chance experiments to determine probabilities of events and investigating the concept of independence and conditional probability.
- Obtain simple statistics from discrete and continuous data, including mean, median, mode, quartiles, range and interquartile range.
- Use scatterplots to investigate and comment on relationships between two numerical variables.
- Investigate and describe bivariate numerical data where the independent variable is time.
- Translate word problems to mathematical form.
- Understand that the real number system includes rational and irrational numbers.
- Use approximations of real numbers by truncating or rounding.
- Solve problems involving the surface area and volume of right prisms, including cylinders.
- Solve problems involving Pythagoras' theorem.

Problem-solving and mathematical modelling

A key aspect of learning mathematics is to develop strategic competence; that is, to formulate, represent and solve mathematical problems (Kilpatrick, Swafford & Bradford 2001). As such, problem-solving is a focus of mathematics education research, curriculum and teaching (Sullivan 2011). This focus is not to the exclusion of routine exercises, which are necessary for practising, attaining mastery and being able to respond automatically. But mathematics education in the 21st century goes beyond this to include innovative problems that are complex, unfamiliar and non-routine (Mevarech & Kramarski 2014).

Problem-solving in mathematics can be set in purely mathematical contexts or real-world contexts. When set in the real world, problem-solving in mathematics involves mathematical modelling.

Problem-solving

Problem-solving is required when a task or goal has limiting conditions placed upon it or an obstacle blocking the path to a solution (Marzano & Kendall 2007). It involves:

- knowledge of the relevant details
- using generalisations and principles to identify, define and interpret the problem
- mental computation and estimation
- critical, creative and lateral thinking
- creating or choosing a strategy
- making decisions
- testing, monitoring and evaluating solutions.

Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples without understanding. Through problem-solving, students will make connections between mathematics topics, across the curriculum and with the real world, and see the value and usefulness of mathematics. Problems may be real-world or abstract, and presented to students as issues, statements or questions that may require them to use primary or secondary data.

Mathematical modelling

Mathematical modelling begins from an assumption that mathematics is everywhere in the world around us — a challenge is to identify where it is present, access it and apply it productively. Models are developed in order to better understand real-world phenomena, to make predictions and answer questions. A mathematical model depicts a situation by expressing relationships using mathematical concepts and language. It refers to the set of simplifying assumptions (such as the relevant variables or the shape of something); the set of assumed relationships between variables; and the resulting representation (such as a formula) that can be used to generate an answer (Stacey 2015).

Mathematical modelling involves:

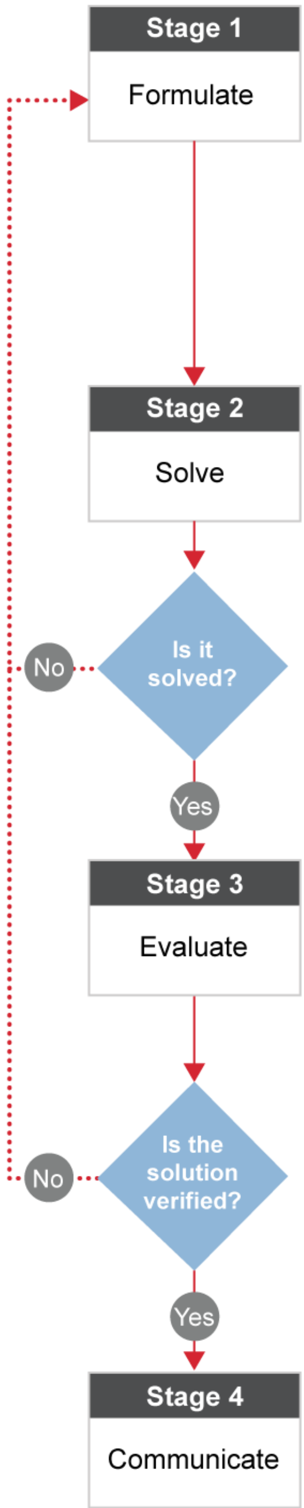
- formulating a mathematical representation of a problem derived from within a real-world context
- using mathematics concepts and techniques to obtain results
- interpreting the results by referring back to the original problem context
- revising the model (where necessary) (Geiger, Faragher & Goos 2010).

Through developing and applying mathematical models, students cumulatively become real-world problem-solvers. Ultimately, this means that not only can they productively address problems set by others, but also that they develop the ability to identify and address problems and answer questions that matter to them.

The following section outlines an approach to problem-solving and mathematical modelling.[1] Problems must be real-world, and can be presented to students as issues, statements or questions that may require them to use primary or secondary data.

[1] A wide variety of frameworks for problem-solving and modelling exist in mathematics education literature. The approach outlined here aligns with and is informed by other approaches, such as Polya (1957) in *How to Solve It: A new aspect of mathematical method* (1957), the Australian Curriculum (ACARA 2015a) *Statistical investigation process*, the OECD/PISA Mathematics framework (OECD 2015, 2003) and *A framework for success in implementing mathematical modelling in the secondary classroom* (Stillman et al. 2007). For further reading see Blum et al. (2007); Kaiser et al. (2011); and Stillman et al. (2013).

Figure 1: An approach to problem-solving and mathematical modelling



Once students understand what the problem is asking, they must design a plan to solve the problem. Students translate the problem into a mathematically purposeful representation by first determining the applicable mathematical knowledge that is required to make progress with the problem. Important assumptions, variables and observations are identified and justified, based on the logic of a proposed solution and/or model.

In mathematical modelling, formulating a model involves the process of mathematisation — moving from the real world to the mathematical world.

Students select and apply mathematical knowledge previously learnt to solve the problem. Possible approaches are wide-ranging and include synthesising and refining existing models, and generating and testing hypotheses with primary or secondary data and information, to produce a complete solution.

Solutions can be found using algebraic, graphic, arithmetic and/or numeric methods, with and/or without technology.

Once a possible solution has been achieved, students need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. They verify their results and evaluate the reasonableness of the solution to the problem in relation to the original issue, statement or question.

This involves exploring the strengths and limitations of the solution and/or model. Where necessary, this will require going back through the process to further refine the solution and/or model. In mathematical modelling, students must check that the output of their model provides a complete solution to the real-world problem it has been designed to address.

This stage emphasises the importance of methodological rigour and the fact that problem-solving and mathematical modelling is not usually linear and involves an iterative process.

The development of solutions and/or models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated and justified clearly and fully. Students communicate findings logically and concisely using mathematical and everyday language. They draw conclusions, discussing the results, strengths and limitations of the solution and/or model. Students could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem.

Approaches to problem-solving and mathematical modelling in the classroom

When teaching problem-solving and mathematical modelling, teachers should consider tea*ching for* and *learning through* problem-solving and mathematical modelling. When *teaching for*, students are taught the specific mathematical rules, definitions, procedures, problem-solving strategies and critical elements of the model that are needed to solve a given problem. When *learning through*, students are presented with problems to solve, but must apply the knowledge and skills they have previously been taught to solve it. By solving these problems, students are able to develop new mathematical understanding and skills. This requires an explicit and connected approach to teaching problem-solving and mathematical modelling that necessitates fluency of critical facts and processes at each step.

The following describes three different approaches to teaching problem-solving and mathematical modelling[2] along the continua between *teaching for* and *learning through*:

Approach	Description	*Teaching for* or *learning through*
Dependent	The teacher explicitly demonstrates and teaches the concepts and techniques required to solve the problem, and/or develop a mathematical model. This usually involves students solving (stage 2) and evaluating and verifying (stage 3).	*Teaching for*
Guided	The teacher influences the choice of concepts and techniques, and/or model that students use to solve the problem. Guidance is provided and all stages of the approach are used.	Moving towards *learning through*
Independent	The teacher cedes control and students work independently, choosing their own solution and/or model, and working at their own level of mathematics. The independent approach is the most challenging.	*Learning through*

These approaches are not mutually exclusive. An independent approach (*learning through*) might be undertaken as an extension of a dependent or guided activity that students have previously undertaken (*teaching for*). Students need to have attained the relevant foundational understanding and skills before working independently during a problem-solving and modelling task. This capacity needs to be built over time through the course of study with teachers closely monitoring student progress.

[2] Based on Galbraith (1989).

Strategies for retaining and recalling information for assessment

The following practices[3] can support preparation for senior assessment in Mathematical Methods.

The spacing effect

The spacing effect draws on research about forgetting and learning curves. By recalling and revisiting information at intervals, rather than at the end of a study cycle, students remember a greater percentage of the information with a higher level of accuracy. Exposing students to information and materials numerous times over multiple spaced intervals solidifies long-term memory, positively affecting retention and recall.

Teachers should plan teaching and learning sequences that allow time to revisit previously taught information and skills at several intervals. These repeated learning opportunities also provide opportunities for teachers to provide formative feedback to students.

The retrieval effect

The retrieval effect helps students to practise remembering through quick, regular, low-stakes questioning or quizzes that exercise their memories and develop their ability to engage in the deliberate act of recalling information. This has been shown to be more effective at developing long-term memories than activities that require students to search through notes or other resources.

Students may see an inability to remember as an obstacle, but they should be encouraged to understand that this is an opportunity for learning to take place. By trying to recall information, students exercise or strengthen their memory and may also identify gaps in their learning. The more difficult the retrieval practice, the better it can be for long-term learning.

Interleaving

Interleaving involves interspersing the concepts, categories, skills or types of questions that students focus on in class or revision. This is in contrast to blocking, in which these elements are grouped together in a block of time. For example, for concepts A, B and C:

- Blocking A A A A A B B B B B C C C C C
- Interleaving A B C B C A B A C A C B C A B

Studies have found that interleaving in instruction or revision produces better long-term recall of subject matter. Interleaving also ensures that spacing occurs, as instances of practice are spread out over time.

Additionally, because exposure to one concept is interleaved with exposure to another, students have more opportunities to distinguish between related concepts. This highlighting of differences may explain why studies have found that interleaving enhances inductive learning, where participants use exemplars to develop an understanding of broader concepts or categories. Spacing without interleaving does not appear to benefit this type of learning.

Interleaving can seem counterintuitive — even in studies where interleaving enhanced learning, participants often felt that they had learnt more with blocked study. Despite this, their performance in testing indicated greater learning through the interleaving approach.

[3] Based on Agarwal, Roediger, McDaniel & McDermott (2020); Birnbaum, Kornell, Ligon Bjork & Bjork (2013); Carpenter & Agarwal (2020); Chen, Paas & Sweller (2021); Ebbinghaus (1885); Rohrer (2012); Taylor & Rohrer (2010).

Reporting

General information about determining and reporting results for senior syllabuses is provided in the ‘Determining and reporting results’ section of the *QCE and QCIA policy and procedures handbook*.

Reporting standards

Reporting standards are summary statements that describe typical performance at each of the five levels (A–E).

A
The student recalls, uses and communicates comprehensive mathematical knowledge drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar, complex familiar and complex unfamiliar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar, complex familiar and complex unfamiliar situations.
B
The student recalls, uses and communicates thorough mathematical knowledge drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar and complex familiar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar and complex familiar situations.
C
The student recalls, uses and communicates mathematical knowledge drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar situations.
D
The student recalls, uses and communicates partial mathematical knowledge drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar situations. The student sometimes evaluates the reasonableness of solutions, sometimes justifies procedures and decisions, and solves some mathematical problems in simple familiar situations.
E
The student recalls, uses and communicates isolated mathematical knowledge drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar situations. The student rarely evaluates the reasonableness of solutions, and infrequently justifies procedures and decisions in simple familiar situations.

Determining and reporting results

Unit 1 and Unit 2

Schools make judgments on individual assessment instruments using a method determined by the school. They may use the reporting standards or develop an instrument-specific marking guide (ISMG). Marks are not required for determining a unit result for reporting to the QCAA.

The unit assessment program comprises the assessment instrument/s designed by the school to allow the students to demonstrate the unit objectives. The unit judgment of A–E is made using reporting standards.

Schools report student results for Unit 1 and Unit 2 to the QCAA as satisfactory (S) or unsatisfactory (U). Where appropriate, schools may also report a not rated (NR).

Units 3 and 4

Schools mark each of the three internal assessment instruments implemented in Units 3 and 4 using ISMGs.

Schools report a provisional mark by criterion to the QCAA for each internal assessment.

Once confirmed by the QCAA, these results will be combined with the result of the external assessment developed and marked by the QCAA.

The QCAA uses these results to determine each student's subject result as a mark out of 100 and as an A–E.

Units

Unit 1: Surds, algebra, functions and probability

In Unit 1, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Surds and quadratic functions
- Topic 2: Binomial expansion and cubic functions
- Topic 3: Functions and relations
- Topic 4: Trigonometric functions
- Topic 5: Probability.

Working with surds provides techniques that are useful in several areas of mathematics. Relationships between variable quantities are reviewed and these are used to introduce the key concepts of a function and its graph. The algebraic expansion of powers of a binomial are found using the binomial theorem. Quadratic, cubic and reciprocal functions are studied. Graphs of relations are introduced. Trigonometric functions, graphs and equations are studied. The study of inferential statistics begins in this unit with a review of the fundamentals of probability and the introduction of the concepts of conditional probability and independence.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Surds and quadratic functions

Sub-topic: Surds (4 hours)

- Understand the concept of a surd as an irrational number represented using a square root or a radical sign.
- Simplify square roots of natural numbers which contain perfect square factors, e.g. $\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$
- Rationalise the denominator of fractional expressions involving square roots, e.g. $\frac{\sqrt{7}}{\sqrt{3}} = \frac{\sqrt{7}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{7}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{\sqrt{21}}{3}$
- Use the four operations to simplify surds, e.g. $\sqrt{5} - 2\sqrt{5} + 4\sqrt{5} = 3\sqrt{5}$ and $2\sqrt{3} \times 5\sqrt{11} = 10\sqrt{33}$

Sub-topic: Quadratic functions (7 hours)

- Recognise and determine features of the graphs of $y = x^2$, $y = ax^2 + bx + c$, $y = a(x-h)^2 + k$ and $y = a(x - x_1)(x - x_2)$, including their parabolic nature, turning points, axes of symmetry and intercepts.
- Solve quadratic equations algebraically using factorisation, the quadratic formula (both exact and approximate solutions), completing the square and using technology.
- Sketch the graphs of quadratic functions, with or without technology.
- Use the discriminant to determine the number of solutions to a quadratic equation.
- Determine turning points and zeros of quadratic functions, with and without technology.
- Model and solve problems that involve quadratic functions, with and without technology.

Topic 2: Binomial expansion and cubic functions

Sub-topic: Binomial expansion (3 hours)

- Understand the notion of a combination as an unordered set of r objects taken from a set of n distinct objects.
- Recognise and use the link between Pascal's triangle and the notation $\binom{n}{r}$
- Use the binomial theorem $(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$ to expand expressions, e.g.$(2x-1)^3$

Sub-topic: Cubic functions (9 hours)

- Identify the coefficients and the degree of a polynomial.
- Expand quadratic and cubic polynomials from factors.
- Recognise and determine features of the graphs of $y = x^3$, $y = a(x-h)^3 + k$ and $y = a(x-x_1)(x-x_2)(x-x_3)$, including shape, intercepts, and behaviour as $x \to \infty$ and $x \to -\infty$.
- Solve cubic equations using technology, and algebraically in cases where the equation is factorised.
- Sketch the graphs of cubic functions, with and without technology.
- Model and solve problems that involve cubic functions, with and without technology.

Topic 3: Functions and relations

Sub-topic: Introduction to functions and relations (5 hours)

- Understand the concept of a relation as a mapping between sets, a graph and as a rule or a formula that defines one variable quantity in terms of another.
- Recognise the distinction between functions and relations and use the vertical line test to determine whether a relation is a function.
- Recognise and use function notation, domain and range, and independent and dependent variables.
- Recognise and use piece-wise functions as a combination of multiple sub-functions with restricted domains.
- Model and solve problems that involve piece-wise functions with and without technology.

Sub-topic: Graphs of relations (4 hours)

- Recognise and determine features of the graphs of $x^2 + y^2 = r^2$ and $(x-h)^2 + (y-k)^2 = r^2$, including their circular shapes, centres and radii.
- Recognise and determine features of the graph of $y^2 = x$, including its parabolic shape and axis of symmetry.
- Recognise and determine features of the graphs of $y = a\sqrt{x-h} + k$, including their shape, intercepts, and behaviour as $x \to \infty$ and $x \to -\infty$.
- Sketch the graphs of relations, with and without technology.
- Model and solve problems that involve relations, with and without technology.

Sub-topic: Reciprocal functions (2 hours)

- Recognise features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{(x-h)} + k$, including their hyperbolic shape, intercepts, asymptotes, and behaviour as $x \to \infty$ and $x \to -\infty$.
- Model and solve problems that involve reciprocal functions, with and without technology.
- Sketch the graphs of reciprocal functions, with and without technology.

Topic 4: Trigonometric functions

Sub-topic: Circular measure and radian measure (2 hours)

- Define and use radian measure and understand its relationship with degree measure.
- Calculate lengths of arcs and areas of sectors in circles.

Sub-topic: Introduction to trigonometric functions (8 hours)

- Understand the unit circle definition of $\cos(\theta)$, $\sin(\theta)$ and $\tan(\theta)$ and periodicity using radians.
- Understand and use the exact values of $\cos(\theta)$, $\sin(\theta)$ and $\tan(\theta)$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$.
- Sketch the graphs of $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$ on extended domains.
- Recognise and determine the effect of the parameters a, b, h and k on the graphs of $y = a\sin\big(b(x-h)\big) + k$, $y = a\cos\big(b(x-h)\big) + k$, with and without technology.
- Sketch the graphs of $y = a\sin\big(b(x-h)\big) + k$, $y = a\cos\big(b(x-h)\big) + k$, with and without technology.
- Solve trigonometric equations, with and without technology, including the use of the Pythagorean identity $\sin^2(A) + \cos^2(A) = 1$.
- Model and solve problems that involve trigonometric functions, with and without technology.

Topic 5: Probability

Sub-topic: Language of events and sets (4 hours)

- Use the concepts and language of outcomes, sample spaces and events as sets of outcomes.
- Use set language and notation for events, including $\overline{A}$ or A' for the complement of an event A, $A \cap B$ for the intersection of event A and event B, and $A \cup B$ for the union of event A and event B, and recognise mutually exclusive events.
- Use everyday occurrences to illustrate set descriptions and representations of events, and set operations, including the use of Venn diagrams.

Sub-topic: Conditional probability and independence (7 hours)

- Use the rules $P\big(\overline{A}\big) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- Understand the notion of a conditional probability and recognise and use language that indicates conditionality.
- Use the notation $P(A|B)$ and the formula $P(A \cap B) = P(A|B)P(B)$ to solve problems.
- Understand and use the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$.
- Use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B.
- Use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events.
- Model and solve problems that involve probability, with and without technology.

Unit 2: Calculus and further functions

In Unit 2, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Exponential functions
- Topic 2: Logarithms and logarithmic functions
- Topic 3: Introduction to differential calculus
- Topic 4: Applications of differential calculus
- Topic 5: Further differentiation.

Exponential graphs are examined and their applications in a wide range of settings are explored. Logarithms are introduced. Logarithmic laws and definitions are developed and used, logarithmic functions are explored graphically and algebraically and the applications of logarithmic functions are studied.

Rates and average rates of change are also introduced, followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically, geometrically as gradients of chords and tangents, and algebraically.

Calculus is developed to study the derivatives of power and polynomial functions, with applications of the derivative to curve sketching, calculating gradients, finding equations of tangents and normals (a link to linear functions is assumed knowledge), determining instantaneous rates of change of displacements as velocities and solving problems using differentiation rules.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Exponential functions

Sub-topic: Indices and index laws (4 hours)

- Use indices (including negative and fractional indices) and the index laws.
- Convert radicals to and from fractional indices.
- Understand and use scientific notation.

Sub-topic: Introduction to exponential functions (6 hours)

- Recognise and determine the qualitative features of the graph of $y = r^x$ (where $r > 0$), including asymptote and intercept.
- Recognise and determine the effect of the parameters h, k and r on the graph of $y = r^{(x-h)} + k$ (where $r > 0$), with and without technology.
- Sketch the graphs of exponential functions, with and without technology.
- Solve equations involving exponential functions, with and without technology.
- Model and solve problems that involve exponential functions, with and without technology.

Topic 2: Logarithms and logarithmic functions

Sub-topic: Logarithms and logarithmic laws (5 hours)

- Define logarithms as indices, where $a^x = b$ is equivalent to $x = \log_a(b)$, and convert between both forms.
- Use logarithmic laws and definitions
  - $\log_a(x) + \log_a(y) = \log_a(xy)$
  - $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$
  - $\log_a(x^n) = n\log_a(x)$
  - $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$
  - $\log_a(a) = 1$
  - $\log_a(1) = 0$
- Solve equations involving indices using logarithms, with and without technology.

Sub-topic: Logarithmic functions (7 hours)

- Recognise and determine the qualitative features of the graph of $y = \log_a(x)$ (where $a > 1$), including asymptote and intercept.
- Recognise and determine the effect of the parameters a, h and k on the graph of $y = \log_a(x - h) + k$ (where $a > 1$), with and without technology.
- Sketch graphs of logarithmic functions, with and without technology.
- Solve equations involving logarithmic functions with and without technology.
- Model and solve problems that involve logarithmic functions, e.g. decibels in acoustics and the Richter scale for earthquake magnitude, with and without technology.

Topic 3: Introduction to differential calculus

Sub-topic: Rates of change and the concept of derivatives (10 hours)

- Determine average rate of change in a variety of practical contexts.
- Use the rule $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to determine the derivative of simple power functions and polynomial functions from first principles.
- Interpret the derivative as the instantaneous rate of change.
- Interpret the derivative as the gradient of a tangent line of the graph of $y = f(x)$.
- Use the rule $\frac{d}{dx}x^n = nx^{n-1}$ for positive integers.
- Understand the concept of the derivative as a function.
- Recognise and use properties of the derivative $\frac{d}{dx}\big(f(x) + g(x)\big) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$.
- Calculate derivatives of power and polynomial functions.

Topic 4: Applications of differential calculus

Sub-topic: Graphical applications of derivatives (12 hours)

- Determine instantaneous rates of change.
- Determine the equation of a tangent and a normal of the graph of $y = f(x)$.
- Construct and interpret displacement-time graphs, with velocity as the slope of the tangent.
- Recognise that velocity is the instantaneous rate of change of displacement with respect to time.
- Use the first derivative of a function to determine and identify the nature of stationary points.
- Sketch curves associated with power functions and polynomials up to degree 4; find stationary points and local and global maxima and minima with and without technology; and examine behaviour as $x \to \infty$ and $x \to -\infty$.

Topic 5: Further differentiation

Sub-topic: Differentiation rules (11 hours)

- Use the chain rule, if $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, to determine the derivative of composite functions involving power and polynomial functions.
- Use the product rule, $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$, to determine the derivative of products of functions involving power and polynomial functions.
- Use the quotient rule, $\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$, to determine the derivative of quotients of functions involving power and polynomial functions.
- Solve problems that involve combinations of the chain rule, product rule and quotient rule to differentiate functions involving power and polynomial functions, expressing derivatives in simplest and factorised form.

Unit 3: Further calculus and introduction to statistics

In Unit 3, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Differentiation of exponential and logarithmic functions
- Topic 2: Differentiation of trigonometric functions and differentiation rules
- Topic 3: Further applications of differentiation
- Topic 4: Introduction to integration
- Topic 5: Discrete random variables.

The study of calculus continues with the derivatives of exponential, logarithmic and trigonometric functions and their applications, together with some differentiation techniques and applications to optimisation problems and graph sketching. Integration, both as a process that reverses differentiation and as a way of determining displacement given velocity or acceleration, is introduced.

Discrete random variables are introduced; this supports the development of a framework for statistical inference. Use of discrete random variables in modelling random processes involving chance and variation are studied.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Differentiation of exponential and logarithmic functions

Sub-topic: Calculus of exponential functions (6 hours)

- Estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$, using technology, for various values of $a > 0$.
- Recognise that e is the unique number a for which the above limit is 1.
- Recognise and determine the qualitative features of the graph of $y = e^x$, including asymptote and intercept.
- Use the rules $\frac{d}{dx}e^x = e^x$ and $\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$.

Sub-topic: Calculus of logarithmic functions (8 hours)

- Recognise and determine the qualitative features of the graph of $y = \ln(x) = \log_e(x)$, including asymptote and intercept.
- Recognise and use the inverse relationship of the functions $y = e^x$ and $y = \ln(x)$.
- Solve equations involving exponential and logarithmic functions with base e, with and without technology.
- Use the rules $\frac{d}{dx}\ln(x) = \frac{1}{x}$ and $\frac{d}{dx}\ln\big(f(x)\big) = \frac{f'(x)}{f(x)}$.
- Model and solve problems that involve derivatives of exponential and logarithmic functions, with and without technology.

Topic 2: Differentiation of trigonometric functions and differentiation rules

Sub-topic: Calculus of trigonometric functions (5 hours)

- Use the rules $\frac{d}{dx}\sin(x) = \cos(x)$ and $\frac{d}{dx}\sin\big(f(x)\big) = f'(x)\cos\big(f(x)\big)$.
- Use the rules $\frac{d}{dx}\cos(x) = -\sin(x)$ and $\frac{d}{dx}\cos\big(f(x)\big) = -f'(x)\sin\big(f(x)\big)$.
- Model and solve problems that involve derivatives of trigonometric functions, with and without technology.

Sub-topic: Differentiation rules (5 hours)

- Use the chain rule to determine the derivative of composite functions involving exponential, logarithmic and trigonometric functions, expressing derivatives in simplest and factorised form.
- Use the product rule to determine the derivative of exponential, logarithmic and trigonometric functions, expressing derivatives in simplest and factorised form.
- Use the quotient rule to determine the derivative of exponential, logarithmic and trigonometric functions, expressing derivatives in simplest and factorised form.
- Solve problems that involve combinations of the chain rule, product rule and quotient rule to differentiate exponential, logarithmic and trigonometric functions.

Topic 3: Further applications of differentiation

Sub-topic: The second derivative and applications of differentiation (10 hours)

- Understand the concept of the second derivative as the rate of change of the first derivative function.
- Recognise acceleration as the second derivative of displacement position with respect to time.
- Understand the concepts of concavity and points of inflection and their relationship with the second derivative.
- Understand and use the second derivative test for finding local maxima and minima.
- Sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection.
- Model and solve optimisation problems from a wide variety of fields using first and second derivatives, where the function to be optimised is either given or to be developed.

Topic 4: Introduction to integration

Sub-topic: Anti-differentiation (9 hours)

- Recognise anti-differentiation as the reverse of differentiation.
- Use the notation $\int f(x)\,dx$ for anti-derivatives or indefinite integrals.
- Use the formula $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ for $n \neq -1$.
- Use the formula $\int e^x\,dx = e^x + c$.
- Use the formula $\int \frac{1}{x}\,dx = \ln(x) + c$, for $x > 0$.
- Use the formulas $\int \sin(x)\,dx = -\cos(x) + c$ and $\int \cos(x)\,dx = \sin(x) + c$.
- Understand and use the formulas $\int \big(f(x) + g(x)\big)dx = \int f(x)dx + \int g(x)dx$ and $\int k\,f(x)dx = k\int f(x)dx$.
- Determine indefinite integrals of the form $\int f(ax+b)dx$.
- Determine $f(x)$ given $f'(x)$ and an initial condition $f(a) = b$.
- Determine displacement given velocity and the initial value of displacement.
- Determine displacement given acceleration and initial values of displacement and velocity.
- Model and solve problems that involve indefinite integrals, with and without technology.

Topic 5: Discrete random variables

Sub-topic: General discrete random variables (5 hours)

- Understand the concepts of a discrete random variable and its associated probability function, and its use in modelling data.
- Use relative frequencies obtained from data to determine point estimates of probabilities associated with a discrete random variable.
- Recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes.
- Recognise non-uniform discrete random variables and use them to model random phenomena.
- Determine and use the mean (expected value) of a discrete random variable as a measurement of centre, $E(X) = \mu = \sum p_i x_i$ where p_i is the probability of outcome x_i occurring.
- Determine and use the variance of a discrete random variable as a measure of spread, $Var(X) = \sum p_i (x_i - \mu)^2$ where p_i is the probability of outcome x_i occurring, μ is the mean.
- Determine and use the standard deviation of a discrete random variable, $\sqrt{Var(X)}$, as a measure of spread.
- Model and solve problems that involve discrete random variables and associated probabilities, with and without technology.

Sub-topic: Bernoulli distributions (2 hours)

- Use a Bernoulli random variable as a model for two-outcome situations.
- Identify contexts suitable for modelling by Bernoulli random variables.
- Recognise and determine the mean p and variance $p(1-p)$ of the Bernoulli distribution with parameter p.
- Model and solve problems that involve Bernoulli random variables and associated probabilities, with and without technology.

Sub-topic: Binomial distributions (5 hours)

- Understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of 'successes', r, in n independent Bernoulli trials, with the same probability of success p in each trial.
- Identify contexts suitable for modelling by binomial random variables.
- Determine and use the probabilities $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$ associated with the binomial distribution with parameters n and p.
- Calculate the mean np and variance $np(1-p)$ of a binomial distribution using technology and algebraic methods.
- Use the language of probability, including *at most*, *at least*, *no more than*, *no less than*, *inclusive* and *between*.
- Model and solve problems that involve binomial distributions and associated probabilities with and without technology.

Unit 4: Further calculus, trigonometry and statistics

In Unit 4, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Further integration
- Topic 2: Trigonometry
- Topic 3: Continuous random variables and the normal distribution
- Topic 4: Sampling and proportions
- Topic 5: Interval estimates for proportions.

The study of integral calculus continues with the introduction of the fundamental theorem of calculus and ways of calculating areas under or between curves.

The cosine and sine rules are established and used.

Continuous random variables and their applications are explored and the normal distribution is used in a variety of contexts. Sample and population proportions are explored. The study of statistical inference in this unit is the culmination of earlier work on probability and random variables. The goal of statistical inference is to estimate an unknown parameter associated with a population using a sample of data drawn from that population.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Further integration

Sub-topic: Fundamental theorem of calculus and definite integrals (3 hours)

- Use sums of the form $\sum_i f(x_i)\,\delta x_i$ to estimate the area under the curve $y = f(x)$.
- Recognise the definite integral $\int_a^b f(x)\ dx$ as a limit of sums of the form $\sum_i f(x_i)\,\delta x_i$.
- Understand the fundamental theorem of calculus, $\int_a^b f(x)\ dx = F(b) - F(a)$, and use it to calculate definite integrals.
- Use the definite integral $\int_a^b f(x)\ dx$ to determine the area under the curve $y = f(x)$ between $x = a$ and $x = b$ if $f(x) > 0$ over this interval.

Sub-topic: Applications of integration (8 hours)

- Calculate the area enclosed by a curve and the x-axis over a given domain, with and without technology.
- Calculate the area between curves, with and without technology.
- Use the trapezoidal rule,
  $\int_a^b f(x)\ dx \approx \frac{w}{2}\left[f(x_0) + 2\big(f(x_1) + f(x_2) + f(x_3) + \ldots f(x_{n-1})\big) + f(x_n)\right]$, where $w = \frac{b-a}{n}$,
  to approximate an area and the value of a definite integral, with and without technology.
- Calculate total change by integrating instantaneous or marginal rates of change, with and without technology.
- Model and solve problems that involve definite integrals, including motion problems, with and without technology.

Topic 2: Trigonometry

Sub-topic: Cosine and sine rules (10 hours)

- Use the sine rule (ambiguous case is required), $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$, where a, b and c are the side lengths of the triangle and A, B and C are the corresponding opposite angles.
- Use the cosine rule, $c^2 = a^2 + b^2 - 2ab\cos(C)$.
- Use the formula $\text{area} = \frac{1}{2} b\, c \sin(A)$ to calculate the area of a triangle.
- Model and solve problems that involve the sine rule, cosine rule and the area formula in two- and three-dimensional contexts (including bearings, directions and angles of elevation and depression), with and without technology.

Topic 3: Continuous random variables and the normal distribution

Sub-topic: General continuous random variables (6 hours)

- Use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable.
- Understand the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts.
- Calculate the expected value, $E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$, of a continuous random variable where $p(x)$ is the probability density function.
- Calculate the variance, $Var(X) = \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 p(x)dx$, and standard deviation σ, of a continuous random variable.
- Understand standardised normal variables (z-values, z-scores) and use these to compare samples.

Sub-topic: Normal distributions (6 hours)

- Identify contexts, e.g. naturally occurring variations, that are suitable for modelling by normal random variables.
- Recognise features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution.
- Recognise and use the link between the normal distribution and the notation $X \sim N(\mu, \sigma^2)$.
- Calculate probabilities and quantiles associated with a given normal distribution, using technology.
- Model and solve problems that involve normal distributions, with and without technology (distribution tables are not required).

Topic 4: Sampling and proportions

Sub-topic: Random sampling (3 hours)

- Understand the concept of a random sample.
- Understand sources of bias in samples, and procedures to ensure randomness.
- Identify and use procedures to ensure randomness.
- Recognise and use graphical displays of real and simulated data of random samples from various types of distributions, including uniform, Bernoulli, binomial and normal.

Sub-topic: Sample proportions (8 hours)

- Understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{p(1-p)/n}$ of the sample proportion $\hat{p}$, where n is the sample size.
- Recognise and use the approximate normality of the distribution of $\hat{p}$ for large samples.
- Use repeated random sampling data, for a variety of values of p and a range of sample sizes, to examine the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{\hat{p}(1-\hat{p})/n}}$, where the closeness of the approximation depends on both n and p.

Topic 5: Interval estimates for proportions

Sub-topic: Confidence intervals for proportions (11 hours)

- Understand the concept of an interval estimate for a parameter associated with a random variable.
- Understand and use the approximate confidence interval, $\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$, as an interval estimate for p, the population proportion, where z is the appropriate quantile for the standard normal distribution.
- Understand and use the approximate margin of error, $z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- Understand and use the relationship between margin of error, level of confidence and sample size.
- Understand that there are variations in confidence intervals between samples and that most, but not all, confidence intervals contain p.
- Model and solve problems that involve interval estimates for proportions, with and without technology.

Assessment

Internal assessment 1: Problem-solving and modelling task (20%)

Students provide a written response to a specific mathematical investigative scenario or context using subject matter from at least one of the topics in Unit 3 or Unit 4. While students may undertake some research, it is not the focus of this task.

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

This assessment may draw on subject matter from Units 3 and 4.

This task requires students to:

- independently respond to a specific mathematical investigative scenario or context that highlights a real-life application of mathematics
- use relevant stimulus material involving the selected subject matter
- address all the stages of the problem-solving and mathematical modelling approach
- respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Conditions

- Students will use 3 hours of class time and their own time out of class to develop their response.
- This is an individual task.
- Data may be provided or collected individually or collected in groups.
- Appendixes can include raw data, repeated calculations, evidence of authentication and student notes (appendixes are not marked).
- Students must use technology, e.g. scientific calculator, graphics calculator, spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation or word processing.

Response requirements

Written: up to 10 A4 pages, up to 2000 words

Mark allocation

Criterion	Assessment objectives	Marks
Formulate	1, 5	4
Solve	1, 2, 6	7
Evaluate	4, 5	5
Communicate	3, 5	4
	Total marks:	20

Instrument-specific marking guide (IA1)

Formulate	Marks
The student response has the following characteristics:	
• justified statements of important assumptions • justified statements of important observations • justified mathematical translation of important aspects of the task	3–4
• statement of a relevant assumption • statement of a relevant observation • mathematical translation of an aspect of the task.	1–2
The student response does not match any of the descriptors above.	0

Solve	Marks
The student response has the following characteristics:	
• accurate use of mathematical knowledge for important aspects of the task • efficient use of technology • a complete solution	6–7
• use of mathematical knowledge for an important aspect of the task • use of technology • substantial progress towards a solution	4–5
• simplistic use of mathematical knowledge relevant to the task • simplistic use of technology • progress towards a solution	2–3
• inappropriate use of mathematical knowledge or technology.	1
The student response does not match any of the descriptors above.	0

Evaluate	Marks
The student response has the following characteristics:	
• verified results • justified statements about the reasonableness of the solution by considering the assumptions • justified statements about the reasonableness of the solution by considering the observations • justified statements of relevant strengths of the solution • justified statements of relevant limitations of the solution	4–5
• a verified result • statement about the reasonableness of the solution by considering an assumption or observation • statement of a relevant strength or relevant limitation of the solution	2–3
• statement about the reasonableness of a result or the solution • statement of a strength or limitation.	1
The student response does not match any of the descriptors above.	0

Communicate	Marks
The student response has the following characteristics:	
• correct use of appropriate mathematical language • logical organisation of the response, which can be read independently of the task sheet • justification of decisions using mathematical reasoning	3–4
• use of some appropriate mathematical language • adequate organisation of the response • statement of a relevant decision.	1–2
The student response does not match any of the descriptors above.	0

Internal assessment 2: Examination — short response (15%)

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

The teacher provides an examination that:

- asks students to respond to a number of unseen short response questions
- representatively samples subject matter from any three of the five topics in Unit 3
- provides opportunities for both technology-free and technology-active responses
- may ask students to respond using single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams
  - use assumed knowledge from Units 1 and 2.

Question specifications

The examination must be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	60%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	20%	These questions can focus on any of the objectives.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	20%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience.

Conditions

- This is an individual supervised task.
- The task may be delivered in two consecutive sessions only if
  - questions in each session are unseen
  - teaching or feedback is not provided between the sessions.
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The teacher must provide the QCAA Mathematical Methods formula book.
- Students
  - are required to use technology
  - must not bring notes into the examination.

Mark allocation

Criterion	Assessment objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5, 6	15
	Total marks:	15

Instrument-specific marking guide (IA2)

Foundational knowledge and problem-solving	Cut-off	Marks
The student response has the following characteristics:		
• consistently correct recall and use of mathematical knowledge; authoritative and accurate communication of mathematical knowledge; astute evaluation of the reasonableness of solutions; use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical knowledge to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations	> 93%	15
	> 87%	14
• correct recall and use of mathematical knowledge; clear communication of mathematical knowledge; considered evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical knowledge to solve problems in simple familiar, complex familiar and complex unfamiliar situations	> 80%	13
	> 73%	12
• thorough recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and application of mathematical knowledge to solve problems in simple familiar and complex familiar situations	> 67%	11
	> 60%	10
• recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of some solutions; some use of mathematical reasoning; and some application of mathematical knowledge to make progress towards solving problems in simple familiar situations	> 53%	9
	> 47%	8
• some recall and use of mathematical knowledge; and basic communication of mathematical knowledge	> 40%	7
	> 33%	6
• infrequent recall and use of mathematical knowledge; and basic communication of some mathematical knowledge	> 27%	5
	> 20%	4
• isolated recall and use of mathematical knowledge; and partial communication of rudimentary mathematical knowledge	> 13%	3
	> 7%	2
• isolated and inaccurate recall and use of mathematical knowledge; and disjointed and unclear communication of mathematical knowledge.	> 0%	1
The student response does not match any of the descriptors above.		0

Internal assessment 3: Examination — short response (15%)

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

The teacher provides an examination that:

- asks students to respond to a number of unseen short response questions
- representatively samples subject matter from any three of the five topics in Unit 4
- provides opportunities for both technology-free and technology-active responses
- may ask students to respond using single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams
  - use assumed knowledge from Units 1, 2 and 3.

Question specifications

The examination must be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	60%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	20%	These questions can focus on any of the objectives.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	20%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience.

Conditions

- This is an individual supervised task.
- The task may be delivered in two consecutive sessions only if
  - questions in each session are unseen
  - teaching or feedback is not provided between the sessions.
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The teacher must provide the QCAA Mathematical Methods formula book.
- Students
  - are required to use technology
  - must not bring notes into the examination.

Mark allocation

Criterion	Assessment objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5, 6	15
	Total marks:	15

Instrument-specific marking guide (IA3)

Foundational knowledge and problem-solving	Cut-off	Marks
The student response has the following characteristics:		
• consistently correct recall and use of mathematical knowledge; authoritative and accurate communication of mathematical knowledge; astute evaluation of the reasonableness of solutions; use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical knowledge to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations	> 93%	15
	> 87%	14
• correct recall and use of mathematical knowledge; clear communication of mathematical knowledge; considered evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical knowledge to solve problems in simple familiar, complex familiar and complex unfamiliar situations	> 80%	13
	> 73%	12
• thorough recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and application of mathematical knowledge to solve problems in simple familiar and complex familiar situations	> 67%	11
	> 60%	10
• recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of some solutions; some use of mathematical reasoning; and some application of mathematical knowledge to make progress towards solving problems in simple familiar situations	> 53%	9
	> 47%	8
• some recall and use of mathematical knowledge; and basic communication of mathematical knowledge	> 40%	7
	> 33%	6
• infrequent recall and use of mathematical knowledge; and basic communication of some mathematical knowledge	> 27%	5
	> 20%	4
• isolated recall and use of mathematical knowledge; and partial communication of rudimentary mathematical knowledge	> 13%	3
	> 7%	2
• isolated and inaccurate recall and use of mathematical knowledge; and disjointed and unclear communication of mathematical knowledge.	> 0%	1
The student response does not match any of the descriptors above.		0

External assessment: Examination — combination response (50%)

External assessment is developed and marked by the QCAA. The external assessment in Mathematical Methods is common to all schools and administered under the same conditions, at the same time, on the same day.

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

This examination:

- consists of two papers: Paper 1 — technology-free, Paper 2 — technology-active
- asks students to respond to a number of unseen questions relating to Units 3 and 4
- may ask students to respond using
  - multiple choice
  - single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams
  - use assumed knowledge from Units 1 and 2.

Paper 1

- Weighted at 25%
- Contains short response and multiple choice questions
- Technology-free

Paper 2

- Weighted at 25%
- Contains short response and multiple choice questions
- Technology-active

Question specifications

The examination will be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	60%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	20%	These questions can focus on any of the objectives.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	20%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience.

Conditions

Paper 1

- Mode: written
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The QCAA provides the QCAA Mathematical Methods formula book.
- Students must not bring notes, calculators, technology or other resources into the examination.

Paper 2

- Mode: written
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The QCAA provides the QCAA Mathematical Methods formula book.
- Students
  - may use a handheld QCAA-approved graphics calculator (no CAS functionality) and/or a handheld QCAA-approved scientific calculator
  - must not bring notes or other resources into the examination.

Glossary

The syllabus glossary is available at www.qcaa.qld.edu.au/downloads/senior-qce/common/snr_glossary_cognitive_verbs.pdf.

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Version history

Version	Date of change	Information
1.0	January 2024	Released for familiarisation and planning (with implementation starting in 2025)
1.1	July 2024	Released for implementation with minor updates
1.2	October 2024	ISBN removed and minor updates