Mathematical Methods 2019 v1.2

General Senior Syllabus

This syllabus is for implementation with Year 11 students in 2019.

Queensland Government
Queensland Curriculum & Assessment Authority
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1 Course overview

1.1 Introduction

1.1.1 Rationale

Mathematics is a unique and powerful intellectual discipline that is used to investigate patterns, order, generality and uncertainty. It is a way of thinking in which problems are explored and solved through observation, reflection and logical reasoning. It uses a concise system of communication, with written, symbolic, spoken and visual components. Mathematics is creative, requires initiative and promotes curiosity in an increasingly complex and data-driven world. It is the foundation of all quantitative disciplines.

To prepare students with the knowledge, skills and confidence to participate effectively in the community and the economy requires the development of skills that reflect the demands of the 21st century. Students undertaking Mathematics will develop their critical and creative thinking, oral and written communication, information & communication technologies (ICT) capability, ability to collaborate, and sense of personal and social responsibility — ultimately becoming lifelong learners who demonstrate initiative when facing a challenge. The use of technology to make connections between mathematical theory, practice and application has a positive effect on the development of conceptual understanding and student disposition towards mathematics.

Mathematics teaching and learning practices range from practising essential mathematical routines to develop procedural fluency, through to investigating scenarios, modelling the real world, solving problems and explaining reasoning. When students achieve procedural fluency, they carry out procedures flexibly, accurately and efficiently. When factual knowledge and concepts come to mind readily, students are able to make more complex use of knowledge to successfully formulate, represent and solve mathematical problems. Problem-solving helps to develop an ability to transfer mathematical skills and ideas between different contexts. This assists students to make connections between related concepts and adapt what they already know to new and unfamiliar situations. With appropriate effort and experience, through discussion, collaboration and reflection of ideas, students should develop confidence and experience success in their use of mathematics.

The major domains of mathematics in Mathematical Methods are Algebra, Functions, relations and their graphs, Calculus and Statistics. Topics are developed systematically, with increasing levels of sophistication, complexity and connection, and build on algebra, functions and their graphs, and probability from the P–10 Australian Curriculum. Calculus is essential for developing an understanding of the physical world. The domain Statistics is used to describe and analyse phenomena involving uncertainty and variation. Both are the basis for developing effective models of the world and solving complex and abstract mathematical problems. The ability to translate written, numerical, algebraic, symbolic and graphical information from one representation to another is a vital part of learning in Mathematical Methods.

Students who undertake Mathematical Methods will see the connections between mathematics and other areas of the curriculum and apply their mathematical skills to real-world problems, becoming critical thinkers, innovators and problem-solvers. Through solving problems and developing models, they will appreciate that mathematics and statistics are dynamic tools that are critically important in the 21st century.

Assumed knowledge, prior learning or experience

Assumed knowledge refers to the subject matter that teachers can expect students to know prior to beginning this subject. Emphasis is placed on the mastery of content, ensuring key concepts or procedures are learnt fully so they will not need reteaching.

Developing mastery often involves multiple approaches to teaching and conceptualising the same mathematical concept. When students have a good understanding of a key concept or procedure, they are more easily able to make connections to related new subject matter and apply what they already know to new problems.

Subject matter from previous unit/s is assumed for subsequent unit/s.

The following is a non-exhaustive list of assumed knowledge from the P–10 Australian Curriculum that must be learnt or revised and maintained as required:

- factorise, expand and simplify algebraic expressions including monic quadratic expressions using a variety of strategies
- apply the four operations to simple algebraic fractions with numerical denominators
- substitute values into formulas to determine an unknown
- solve problems involving linear equations, including those derived from formulas and those that involve simple algebraic fractions
- recall the equation of a line in the form $y = mx + c$
- determine if lines are parallel or perpendicular lines, including $m_1 = m_2$ and $m_1 m_2 = -1$
- explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate
- solve simple quadratic equations using a range of strategies
- solve linear simultaneous equations, using algebraic and graphical techniques, including using digital technology
- solve linear inequalities and graph their solutions on a number line
- solve right-angled triangle problems using trigonometric skills
- describe the results of two- and three-step chance experiments to determine probabilities of events and investigating the concept of independence and conditional probability
- obtain simple statistics from discrete and continuous data, including mean, median, mode, quartiles, range and interquartile range
- use scatterplots to investigate and comment on relationships between two numerical variables
- investigate and describe bivariate numerical data where the independent variable is time
- translate word problems to mathematical form.

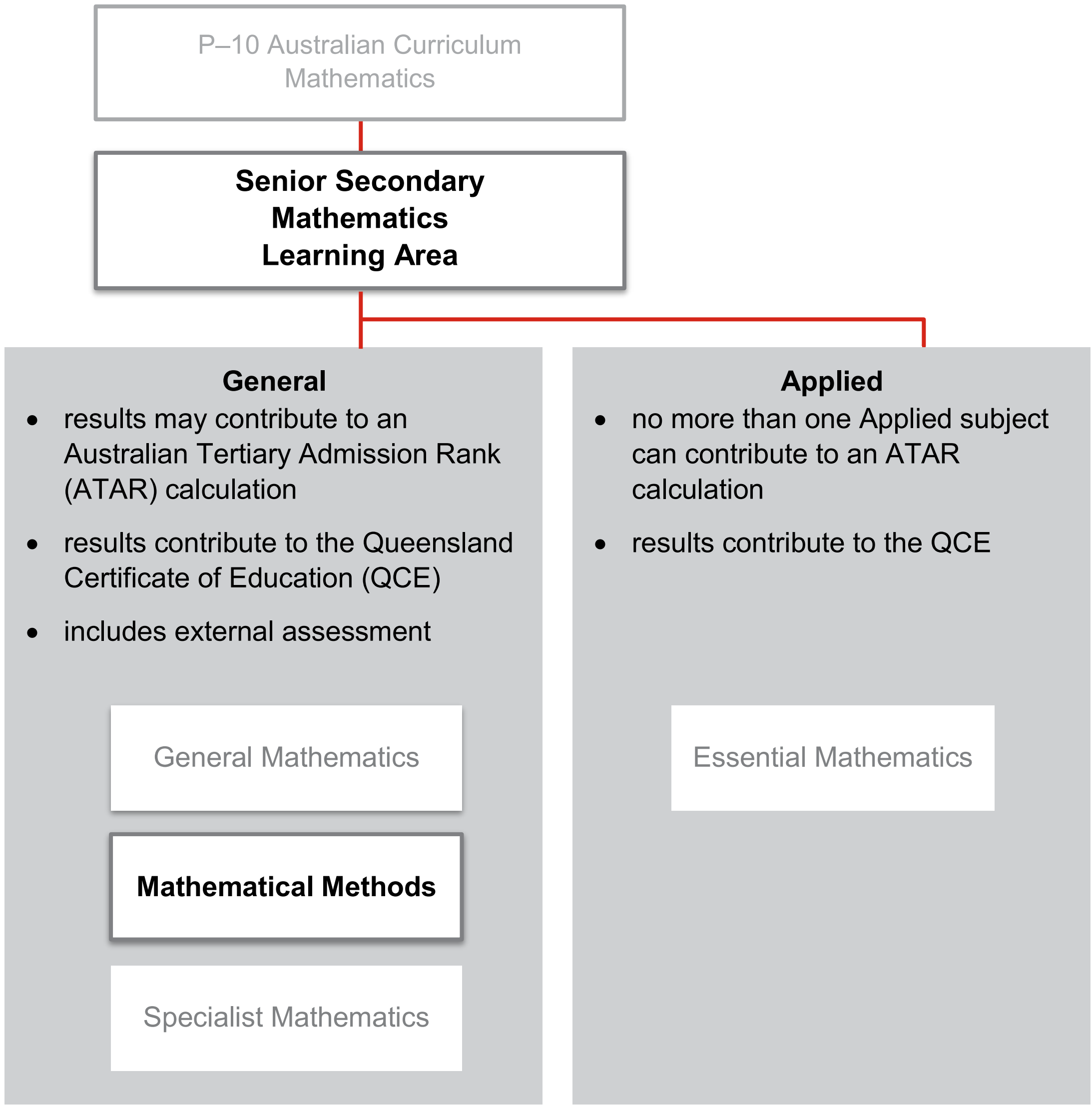
Pathways

Mathematical Methods is a General subject suited to students who are interested in pathways beyond school that lead to tertiary studies, vocational education or work. A course of study in Mathematical Methods can establish a basis for further education and employment in the fields of natural and physical sciences (especially physics and chemistry), mathematics and science education, medical and health sciences (including human biology, biomedical science, nanoscience and forensics), engineering (including chemical, civil, electrical and mechanical engineering, avionics, communications and mining), computer science (including electronics and software design), psychology and business.

1.1.2 Learning area structure

All learning areas build on the P–10 Australian Curriculum.

Figure 1: Learning area structure



1.1.3 Course structure

Mathematical Methods is a course of study consisting of four units. Subject matter, learning experiences and assessment increase in complexity from Units 1 and 2 to Units 3 and 4 as students develop greater independence as learners.

Units 1 and 2 provide foundational learning, which allows students to experience all syllabus objectives and begin engaging with the course subject matter. Students should complete Units 1 and 2 before beginning Unit 3. It is recommended that Unit 3 be completed before Unit 4.

Units 3 and 4 consolidate student learning. Only the results from Units 3 and 4 will contribute to ATAR calculations.

Figure 2 outlines the structure of this course of study.

Each unit has been developed with a notional time of 55 hours of teaching and learning, including assessment.

Figure 2: Course structure

Mathematical Methods

Unit 1
Algebra, statistics and functions

- Topic 1: Arithmetic and geometric sequences and series 1
- Topic 2: Functions and graphs
- Topic 3: Counting and probability
- Topic 4: Exponential functions 1
- Topic 5: Arithmetic and geometric sequences and series 2

Assessment
Formative internal assessment/s

Unit 2
Calculus and further functions

- Topic 1: Exponential functions 2
- Topic 2: The logarithmic function 1
- Topic 3: Trigonometric functions 1
- Topic 4: Introduction to differential calculus
- Topic 5: Further differentiation and applications 1
- Topic 6: Discrete random variables 1

Assessment
Formative internal assessment/s

Unit 3
Further calculus

- Topic 1: The logarithmic function 2
- Topic 2: Further differentiation and applications 2
- Topic 3: Integrals

Assessment
Summative internal assessment 1: Problem-solving and modelling task (20%)
Summative internal assessment 2: Examination (15%)

Unit 4
Further functions and statistics

- Topic 1: Further differentiation and applications 3
- Topic 2: Trigonometric functions 2
- Topic 3: Discrete random variables 2
- Topic 4: Continuous random variables and the normal distribution
- Topic 5: Interval estimates for proportions

Assessment
Summative internal assessment 3: Examination (15%)

Students should have opportunities in Units 1 and 2 to experience and respond to the types of assessment they will encounter in Units 3 and 4.

For reporting purposes, schools should develop at least *one* assessment per unit, with a maximum of *four* assessments across Units 1 and 2.

Summative external assessment: Examination (50%)

1.2 Teaching and learning

1.2.1 Syllabus objectives

The syllabus objectives outline what students have the opportunity to learn. Assessment provides evidence of how well students have achieved the objectives.

Syllabus objectives inform unit objectives, which are contextualised for the subject matter and requirements of the unit. Unit objectives, in turn, inform the assessment objectives, which are further contextualised for the requirements of the assessment instruments. The number of each objective remains constant at all levels, i.e. Syllabus objective 1 relates to Unit objective 1 and to Assessment objective 1 in each assessment instrument.

Syllabus objectives are described in terms of actions that operate on the subject matter. Students are required to use a range of cognitive processes in order to demonstrate and meet the syllabus objectives. These cognitive processes are described in the explanatory paragraph following each objective in terms of four levels: retrieval, comprehension, analytical processes (analysis), and knowledge utilisation, with each process building on the previous processes (see Marzano & Kendall 2007, 2008). That is, comprehension requires retrieval, and knowledge utilisation requires retrieval, comprehension and analytical processes (analysis).

By the conclusion of the course of study, students will:

Syllabus objective	Unit 1	Unit 2	Unit 3	Unit 4
1. select, recall and use facts, rules, definitions and procedures drawn from ‘Algebra, Functions, relations and their graphs, Calculus and Statistics	●	●	●	●
2. comprehend mathematical concepts and techniques drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics	●	●	●	●
3. communicate using mathematical, statistical and everyday language and conventions	●	●	●	●
4. evaluate the reasonableness of solutions	●	●	●	●
5. justify procedures and decisions by explaining mathematical reasoning	●	●	●	●
6. solve problems by applying mathematical concepts and techniques drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics	●	●	●	●

1. select, recall and use facts, rules, definitions and procedures drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics

When students select, recall and use facts, rules, definitions and procedures, they recognise particular features of remembered information and consider its accuracy and relevance. They present facts, rules, definitions and procedures and put them into effect, performing calculations with and without the use of technology.

2. comprehend mathematical concepts and techniques drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics

When students comprehend, they understand the meaning, nature and purpose of the mathematics they are learning. They identify, articulate and symbolise the critical elements of the relevant concepts and techniques, making connections between topics and between the ‘why’ and the ‘how’ of mathematics.

3. **communicate using mathematical, statistical and everyday language and conventions**

   When students communicate, they use mathematical and statistical terminology, symbols, conventions and everyday language to organise and present information in graphical and symbolic form, and describe and represent mathematical and statistical models.

4. **evaluate the reasonableness of solutions**

   When students evaluate the reasonableness of solutions, they interpret their mathematical results in the context of the situation. They reflect on whether the problem has been solved by using estimation skills and checking calculations using their knowledge of relevant facts, rules, definitions and procedures. They make an appraisal by assessing strengths, implications and limitations of solutions and/or models with and without technology, and use this to consider if alternative methods or refinements are required.

5. **justify procedures and decisions by explaining mathematical reasoning**

   When students justify procedures and decisions by explaining mathematical reasoning, they describe their mathematical thinking in detail, identifying causes and making relationships evident, constructing mathematical arguments and providing reasons for choices made and conclusions reached. Students use their conceptual understanding to connect what they already know to new information. Mathematical reasoning is rigorous and requires clarity, precision, completeness and due regard to the order of statements.

6. **solve problems by applying mathematical concepts and techniques drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics**

   When students solve problems by applying mathematical concepts and techniques, they analyse the context of the problem and make decisions about the concepts, techniques and technology that must be used to develop a solution. They analyse, generalise and translate information into a mathematically workable format, synthesise and refine models, and generate and test hypotheses with primary or secondary data and information.

1.2.2 Underpinning factors

There are three skill sets that underpin senior syllabuses and are essential for defining the distinctive nature of subjects:

- literacy — the set of knowledge and skills about language and texts essential for understanding and conveying Mathematical Methods content
- numeracy — the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations, to recognise and understand the role of mathematics in the world, and to develop the dispositions and capacities to use mathematical knowledge and skills purposefully
- 21st century skills — the attributes and skills students need to prepare them for higher education, work and engagement in a complex and rapidly changing world.

These skill sets, which overlap and interact, are derived from current education, industry and community expectations. They encompass the knowledge, skills, capabilities, behaviours and dispositions that will help students live and work successfully in the 21st century.

Together these three skill sets shape the development of senior subject syllabuses. Although coverage of each skill set may vary from syllabus to syllabus, students should be provided with opportunities to learn through and about these skills over the course of study. Each skill set contains identifiable knowledge and skills that can be directly assessed.

Literacy in Mathematical Methods

Literacy skills and strategies enable students to express, interpret and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their abilities to read, write, visualise and talk about complex situations involving a range of mathematical ideas.

Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

To understand and use Mathematical Methods content, teaching and learning strategies include:

- breaking the language code to make meaning of Mathematical Methods language and texts
- comprehending language and texts to make literal and inferred meanings about Mathematical Methods content
- using Mathematical Methods ideas and information in classroom, real-world and/or lifelike contexts to progress students' learning.

To analyse and evaluate Mathematical Methods content, teaching and learning strategies include:

- making conclusions about the purpose and audience of Mathematical Methods language and texts
- analysing the ways language is used to convey ideas and information in Mathematical Methods texts
- transforming language and texts to convey Mathematical Methods ideas and information in particular ways to suit audience and purpose.

These aspects of literacy knowledge and skills are embedded in the syllabus objectives, unit objectives and subject matter, and instrument-specific marking guides (ISMGs) for Mathematical Methods.

Numeracy in Mathematical Methods

Numeracy relates to the capacity to deal with quantitative aspects of life (Goos, Geiger & Dole 2012). It involves accessing, using, interpreting and communicating mathematical information and ideas when engaging with and managing the mathematical demands of real contexts — everyday and civic life, the world of work, and opportunities for further learning (OECD 2012). Numerate citizens who are constructive, engaged and reflective are able to use mathematics to help make credible judgments and reasoned decisions (OECD 2015).

Unlike mathematics, numeracy must be understood as inseparable from context:

> Mathematics climbs the ladder of abstraction to see, from sufficient height, common patterns in seemingly different things. Abstraction is what gives mathematics its power; it is what enables methods derived in one context to be applied in others. But abstraction is not the focus of numeracy. Instead, numeracy clings to specifics, marshalling all relevant aspects of setting and context to reach conclusions.
>
> To enable students to become numerate, teachers must encourage them to see and use mathematics in everything they do. Numeracy is driven by issues that are important to people in their lives and work, not by future needs of the few who may make professional use of mathematics or statistics (Steen 2001, pp. 17–18).

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in the P–10 years. For example, this subject contains financial applications of mathematics that will assist students to become informed consumers of investments. It also contains statistics topics that will equip students for the ever-increasing demands of the information age. Students will also learn about the probability of certain events occurring and will therefore be well equipped to make informed decisions.

These aspects of numeracy knowledge and skills are embedded in the syllabus objectives, unit objectives and subject matter, and ISMGs for Mathematical Methods.

21st century skills

The 21st century skills identified in the following table reflect a common agreement, both in Australia and internationally, on the skills and attributes students need to prepare them for higher education, work and engagement in a complex and rapidly changing world.

21st century skills	Associated skills	21st century skills	Associated skills
critical thinking	• analytical thinking • problem-solving • decision-making • reasoning • reflecting and evaluating • intellectual flexibility	creative thinking	• innovation • initiative and enterprise • curiosity and imagination • creativity • generating and applying new ideas • identifying alternatives • seeing or making new links
communication	• effective oral and written communication • using language, symbols and texts • communicating ideas effectively with diverse audiences	collaboration and teamwork	• relating to others (interacting with others) • recognising and using diverse perspectives • participating and contributing • community connections
personal and social skills	• adaptability/flexibility • management (self, career, time, planning and organising) • character (resilience, mindfulness, open- and fair-mindedness, self-awareness) • leadership • citizenship • cultural awareness • ethical (and moral) understanding	information & communication technologies (ICT) skills	• operations and concepts • accessing and analysing information • being productive users of technology • digital citizenship (being safe, positive and responsible online)

Mathematical Methods helps develop the following 21st century skills:

- critical thinking
- creative thinking
- communication
- information & communication technologies (ICT) skills.

These elements of 21st century skills are embedded in the syllabus objectives, unit objectives and subject matter, and ISMGs for Mathematical Methods.

Use of digital technology

An important aspect of teaching and learning in the 21st century is to embed digital technologies so that they are not seen as optional tools. Digital technologies allow new approaches to explaining and presenting mathematics, and can assist in connecting representations and deepening understanding. They can make previously inaccessible mathematics accessible and increase the opportunities for teachers to make mathematics interesting to a wider range of students. The computational and graphing capabilities of digital technologies enable students to engage in active learning through exploratory work and experiments using realistic data. The ability to visualise solutions can give problems more meaning. Digital technologies can support the development of conceptual understanding that can lead to enhanced procedural fluency.

To meet the requirements of this syllabus, students must make use of a range of digital technologies, such as:

- general-purpose computer software that can be used for mathematics teaching and learning, e.g. spreadsheet software, applications
- computer software designed for mathematics teaching and learning, e.g. dynamic graphing software, dynamic geometry software
- handheld (calculator) technologies designed for mathematics teaching and learning, e.g. scientific, graphics (non-CAS or CAS) calculators, smartphone and tablet apps.

Students must make choices about various forms of technology and develop the ability to work with these flexibly. Technology use must go beyond simple computation or word processing.

Access to a handheld graphics calculator (no CAS functionality) is a requirement for Paper 2 of the external assessment. Scientific calculators may also be used.

1.2.3 Aboriginal perspectives and Torres Strait Islander perspectives

The QCAA is committed to reconciliation in Australia. As part of its commitment, the QCAA affirms that:

- Aboriginal peoples and Torres Strait Islander peoples are the first Australians, and have the oldest living cultures in human history
- Aboriginal peoples and Torres Strait Islander peoples have strong cultural traditions and speak diverse languages and dialects, other than Standard Australian English
- teaching and learning in Queensland schools should provide opportunities for students to deepen their knowledge of Australia by engaging with the perspectives of Aboriginal peoples and Torres Strait Islander peoples

- positive outcomes for Aboriginal students and Torres Strait Islander students are supported by successfully embedding Aboriginal perspectives and Torres Strait Islander perspectives across planning, teaching and assessing student achievement.

Guidelines about Aboriginal perspectives and Torres Strait Islander perspectives and resources for teaching are available at www.qcaa.qld.edu.au/k-12-policies/aboriginal-torres-strait-islander-perspectives.

Where appropriate, Aboriginal perspectives and Torres Strait Islander perspectives have been embedded in the subject matter.

To understand and use mathematics content, teaching and learning strategies may include:

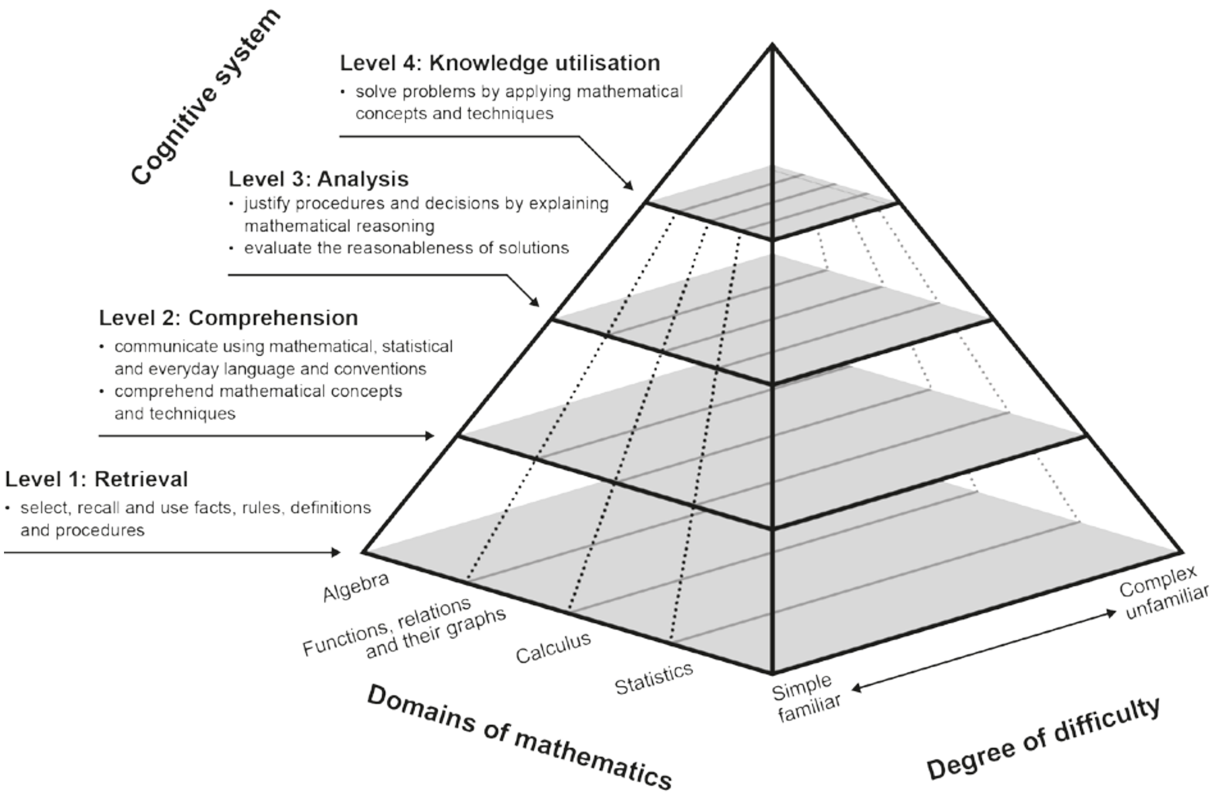
- using pedagogies such as Maths as Storytelling (MAST)
- using mathematics subject matter in real-world Aboriginal contexts and Torres Strait Islander contexts
- identifying the specific issues that may affect Aboriginal peoples and Torres Strait Islander peoples that are relevant to the mathematics topics being covered
- providing learning experiences and opportunities that support the application of students' general mathematical knowledge and problem-solving processes in an Aboriginal context and Torres Strait Islander context.

1.2.4 Pedagogical and conceptual frameworks

The relationship between foundational knowledge and problem-solving

To succeed in mathematics assessment, students must understand the subject matter (organised in domains of mathematics), draw on a range of cognitive skills, and apply these to problems of varying degrees of difficulty, from simple and routine, through to unfamiliar situations, complex contexts, and multi-step solutions (Grønmo et al. 2015). The relationship between the domains of mathematics in Mathematics Methods, level of cognitive skill required (syllabus objective) and degree of difficulty is represented in three dimensions for mathematics problems in the following diagram.

Figure 3: Assessment pyramid



Adapted from Verhage & de Lange (1997) and Marzano & Kendall (2007).

Principles of developing mathematics problems

This representation, known as the 'assessment pyramid', shows the relative distribution of thinking and range of difficulty of mathematics problems.[1] It places an emphasis on building up from the basics. Success in mathematics is built on knowledge of basic facts and proficiency with foundational processes (Norton & O'Connor 2016). With a solid foundation, students can then be asked to apply higher level cognitive processes in more complex and unfamiliar situations that require the application of a wider range of concepts and skills.

The degree of difficulty

The difficulty of a problem is defined by its complexity and a student's familiarity with it, not the level of cognitive process required to solve it. The complexity of a particular type of problem doesn't change, but familiarity does. With practice, students become more familiar with a process and can execute it more quickly and easily (Marzano & Kendall 2007).

The cognitive system

To solve a full range of mathematics problems, students are required to engage the cognitive system at all four levels of processing knowledge: retrieval, comprehension, analysis and knowledge utilisation (Marzano & Kendall 2007). The syllabus objectives are represented in the pyramid model through their alignment to these levels.

[1] In an assessment instrument for Mathematics, a 'problem' is synonymous with 'assessment item' (a question, task or command that forms part of an assessment technique).

Using a full range of questions

The pyramid model shows that problems requiring Level 1 processes to solve them can be hard and relatively complex, even though they are based on 'retrieval' and therefore might seem easy and straightforward (Shafer & Foster 1997). Problems requiring higher level processes to solve them are not necessarily more difficult than those in Level 1. There are some students who find Level 1 processes more challenging and have more success in solving problems requiring Levels 2, 3 and 4 (Webb 2009).

The distance along the domains of mathematics dimension and the degree of difficulty dimension decreases for higher levels. Problems requiring Level 1 processes can more easily be based on distinct subject matter and the difference between easy and hard can be great. Problems that require students to use more levels of cognition tend to also involve making connections with subject matter within and across the domains of mathematics. They are often placed in contexts that require strategic mathematical decisions and making representations according to situation and purpose. At higher levels the difference between easy and hard is smaller (Shafer & Foster 1997; Webb 2009). Students should master basic facts and processes through practising simple familiar problems, before moving on to those that are more complex and unfamiliar, at any level.[2]

The assessment pyramid helps visualise what is necessary for a complete assessment program. Problems in a complete mathematics program need to assess a student's growth and achievement in all domains of mathematics and across the full range of objectives. Over time, through a teaching and learning period, students will be exposed to problems that 'fill the pyramid'. Each assessment instrument will reflect this for the relevant subject matter, providing students with the opportunity to demonstrate what they know and can do at all levels of thinking and varying degrees of difficulty (Shafer & Foster 1997).

Problem-solving and mathematical modelling

A key aspect of learning mathematics is to develop strategic competence; that is, to formulate, represent and solve mathematical problems (Kilpatrick, Swafford & Bradford 2001). As such, problem-solving is a focus of mathematics education research, curriculum and teaching (Sullivan 2011). This focus is not to the exclusion of routine exercises, which are necessary for practising, attaining mastery and being able to respond automatically. But mathematics education in the 21st century goes beyond this to include innovative problems that are complex, unfamiliar and non-routine (Mevarech & Kramarski 2014).

Problem-solving in mathematics can be set in purely mathematical contexts or real-world contexts. When set in the real world, problem-solving in mathematics involves mathematical modelling.

[2] Complex unfamiliar questions that require more levels of cognitive skills should not be equated with elaborate problem-solving tasks and modelling questions only. A single-answer, conventional question, such as: 'Find the equation of the line passing through the points (2,1) and (1,3)' can be adapted to a more open-ended question, such as: 'Write the equations of at least five lines passing through the point (2,1)' (Goos 2014). This revised question targets the identical subject matter but provides the possibility of easily identifying diverse student understanding and skills by moving it towards complex unfamiliar questions and assessing more cognitive skills. For further examples, see White et al. (2000).

Problem-solving

Problem-solving is required when a task or goal has limiting conditions placed upon it or an obstacle blocking the path to a solution (Marzano & Kendall 2007). It involves:

- knowledge of the relevant details
- using generalisations and principles to identify, define and interpret the problem
- mental computation and estimation
- critical, creative and lateral thinking
- creating or choosing a strategy
- making decisions
- testing, monitoring and evaluating solutions.

Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples without understanding. Through problem-solving, students will make connections between mathematics topics, across the curriculum and with the real world, and see the value and usefulness of mathematics. Problems may be real-world or abstract, and presented to students as issues, statements or questions that may require them to use primary or secondary data.

Mathematical modelling

Mathematical modelling begins from an assumption that mathematics is everywhere in the world around us — a challenge is to identify where it is present, access it and apply it productively. Models are developed in order to better understand real-world phenomena, to make predictions and answer questions. A mathematical model depicts a situation by expressing relationships using mathematical concepts and language. It refers to the set of simplifying assumptions (such as the relevant variables or the shape of something); the set of assumed relationships between variables; and the resulting representation (such as a formula) that can be used to generate an answer (Stacey 2015).

Mathematical modelling involves:

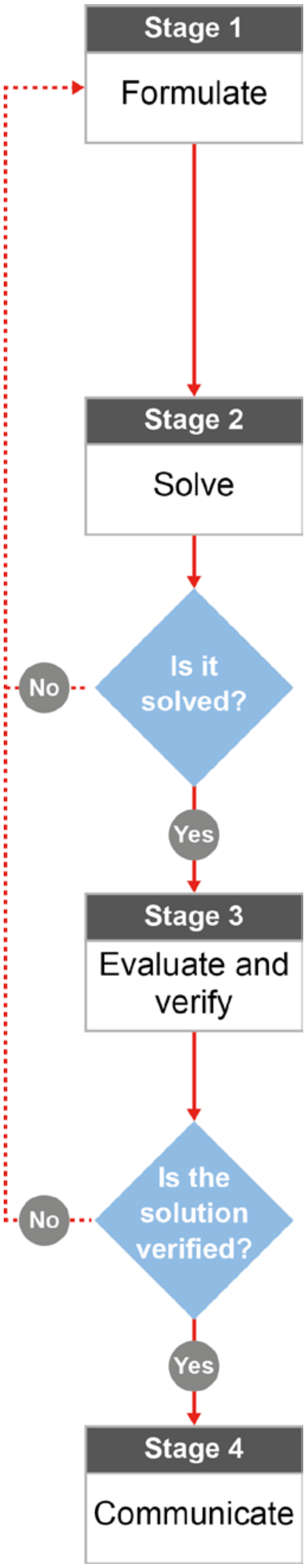
- formulating a mathematical representation of a problem derived from within a real-world context
- using mathematics concepts and techniques to obtain results
- interpreting the results by referring back to the original problem context
- revising the model (where necessary) (Geiger, Faragher & Goos 2010).

Through developing and applying mathematical models, students cumulatively become real-world problem-solvers. Ultimately, this means that not only can they productively address problems set by others, but also that they develop the ability to identify and address problems and answer questions that matter to them.

The following section outlines an approach to problem-solving and mathematical modelling.[3] Problems must be real-world, and can be presented to students as issues, statements or questions that may require them to use primary or secondary data.

[3] A wide variety of frameworks for problem-solving and modelling exist in mathematics education literature. The approach outlined here aligns with and is informed by other approaches, such as Polya (1957) in *How to Solve It: A new aspect of mathematical method* (1957), the Australian Curriculum (ACARA 2015a) *Statistical investigation process*, the OECD/PISA Mathematics framework (OECD 2015, 2003) and *A framework for success in implementing mathematical modelling in the secondary classroom* (Stillman et al. 2007). For further reading see Blum et al. (2007); Kaiser et al. (2011); and Stillman et al. (2013).

Figure 4: An approach to problem-solving and mathematical modelling



Once students understand what the problem is asking, they must design a plan to solve the problem. Students translate the problem into a mathematically purposeful representation by first determining the applicable mathematical and/or statistical principles, concepts, techniques and technology that are required to make progress with the problem. Appropriate assumptions, variables and observations are identified and documented, based on the logic of a proposed solution and/or model.

In mathematical modelling, formulating a model involves the process of mathematisation — moving from the real world to the mathematical world.

Students select and apply mathematical and/or statistical procedures, concepts and techniques previously learnt to solve the mathematical problem to be addressed through their model. Possible approaches are wide-ranging and include synthesising and refining existing models, and generating and testing hypotheses with primary or secondary data and information, as well as using standard mathematical techniques to produce a valid solution.

Solutions can be found using algebraic, graphic, arithmetic and/or numeric methods, with and/or without technology.

Once a possible solution has been achieved, students need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. They evaluate their results and make a judgment about the solution/s to the problem in relation to the original issue, statement or question.

This involves exploring the strengths and limitations of the solution and/or model. Where necessary, this will require going back through the process to further refine the solution and/or model. In mathematical modelling, students must check that the output of their model provides a valid solution to the real-world problem it has been designed to address.

This stage emphasises the importance of methodological rigour and the fact that problem-solving and mathematical modelling is not usually linear and involves an iterative process.

The development of solutions and models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated clearly and fully. Students communicate findings systematically and concisely using mathematical, statistical and everyday language. They draw conclusions, discussing the key results and the strengths and limitations of the solution and/or model. Students could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem.

Approaches to problem-solving and mathematical modelling in the classroom

When teaching problem-solving and mathematical modelling, teachers should consider *teaching for* and *learning through* problem-solving and mathematical modelling. When *teaching for*, students are taught the specific mathematical rules, definitions, procedures, problem-solving strategies and critical elements of the model that are needed to solve a given problem. When *learning through*, students are presented with problems to solve, but must apply the knowledge and skills they have previously been taught to solve it. By solving these problems, students are able to develop new mathematical understanding and skills. This requires an explicit and connected approach to teaching problem-solving and mathematical modelling that necessitates fluency of critical facts and processes at each step.

The following describes three different approaches to teaching problem-solving and mathematical modelling[4] along the continua between *teaching for* and *learning through*:

Approach	Description	*Teaching for* or *learning through*
Dependent	The teacher explicitly demonstrates and teaches the concepts and techniques required to solve the problem, and/or develop a mathematical model. This usually involves students solving (stage 2), and evaluating and verifying (stage 3).	*Teaching for*
Guided	The teacher influences the choice of concepts and techniques, and/or model that students use to solve the problem. Guidance is provided and all stages of the approach are used.	Moving towards *learning through*
Independent	The teacher cedes control and students work independently, choosing their own solution and/or model, and working at their own level of mathematics. The independent approach is the most challenging.	*Learning through*

These approaches are not mutually exclusive. An independent approach (*learning through*) might be undertaken as an extension of a dependent or guided activity that students have previously undertaken (*teaching for*). Students need to have attained the relevant foundational understanding and skills before working independently during a problem-solving and modelling task. This capacity needs to be built over time through the course of study with teachers closely monitoring student progress.

1.2.5 Subject matter

Subject matter is the body of information, mental procedures and psychomotor procedures (see Marzano & Kendall 2007, 2008) that are necessary for students' learning and engagement with Mathematical Methods. It is particular to each unit in the course of study and provides the basis for student learning experiences.

Subject matter has a direct relationship to the unit objectives, but is of a finer granularity and is more specific. These statements of learning are constructed in a similar way to objectives. Each statement:

- describes an action (or combination of actions) — what the student is expected to do
- describes the element — expressed as information, mental procedures and/or psychomotor procedures
- is contextualised for the topic or circumstance particular to the unit.

[4] Based on Galbraith (1989).

Subject matter in Mathematical Methods is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

1.3 Assessment — general information

Assessments are formative in Units 1 and 2, and summative in Units 3 and 4.

Assessment	Unit 1	Unit 2	Unit 3	Unit 4
Formative assessments	●	●		
Summative internal assessment 1			●	
Summative internal assessment 2			●	
Summative internal assessment 3				●
Summative external assessment*			●	●

* Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of the supervised examination.

1.3.1 Formative assessments — Units 1 and 2

Formative assessments provide feedback to both students and teachers about each student's progress in the course of study.

Schools develop internal assessments for each senior subject, based on the learning described in Units 1 and 2 of the subject syllabus. Each unit objective must be assessed at least once.

For reporting purposes, schools should devise at least *two* but no more than *four* assessments for Units 1 and 2 of this subject. At least *one* assessment must be completed for *each* unit.

The sequencing, scope and scale of assessments for Units 1 and 2 are matters for each school to decide and should reflect the local context.

Teachers are encouraged to use the A–E descriptors in the reporting standards (Section 1.4) to provide formative feedback to students and to report on progress.

1.3.2 Summative assessments — Units 3 and 4

Students will complete a total of *four* summative assessments — three internal and one external — that count towards their final mark in each subject.

Schools develop *three* internal assessments for each senior subject, based on the learning described in Units 3 and 4 of the syllabus.

The three summative internal assessments will be endorsed and the results confirmed by the QCAA. These results will be combined with a single external assessment developed and marked by the QCAA. The external assessment results for Mathematical Methods will contribute 50% towards a student's result.

Summative internal assessment — instrument-specific marking guides

This syllabus provides ISMGs for the three summative internal assessments in Units 3 and 4.

The ISMGs describe the characteristics evident in student responses and align with the identified assessment objectives. Assessment objectives are drawn from the unit objectives and are contextualised for the requirements of the assessment instrument.

Criteria

Each ISMG groups assessment objectives into criteria. An assessment objective may appear in multiple criteria, or in a single criterion of an assessment.

Making judgments

Assessment evidence of student performance in each criterion is matched to a performance-level descriptor, which describes the typical characteristics of student work.

Where a student response has characteristics from more than one performance level, a best-fit approach is used. Where a performance level has a two-mark range, it must be decided if the best fit is the higher or lower mark of the range.

Authentication

Schools and teachers must have strategies in place for ensuring that work submitted for internal summative assessment is the student's own. Authentication strategies outlined in QCAA guidelines, which include guidance for drafting, scaffolding and teacher feedback, must be adhered to.

Summative external assessment

The summative external assessment adds valuable evidence of achievement to a student's profile. External assessment is:

- common to all schools
- administered under the same conditions at the same time and on the same day
- developed and marked by the QCAA according to a commonly applied marking scheme.

The external assessment contributes 50% to the student's result in Mathematical Methods. It is not privileged over the school-based assessment.

1.4 Reporting standards

Reporting standards are summary statements that succinctly describe typical performance at each of the five levels (A–E). They reflect the cognitive taxonomy and objectives of the course of study.

The primary purpose of reporting standards is for twice-yearly reporting on student progress. These descriptors can also be used to help teachers provide formative feedback to students and to align ISMGs.

Reporting standards

A
The student demonstrates a comprehensive knowledge and understanding of the subject matter; recognises, recalls and uses facts, rules, definitions and procedures; and comprehends and applies mathematical concepts and techniques to solve problems drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar, complex familiar and complex unfamiliar situations. The student explains mathematical reasoning to justify procedures and decisions; evaluates the reasonableness of solutions; communicates using mathematical, statistical and everyday language and conventions; and makes decisions about the choice of technology, and uses the technology, to solve problems in simple familiar, complex familiar and complex unfamiliar situations.
B
The student demonstrates a thorough knowledge and understanding of the subject matter; recognises, recalls and uses facts, rules, definitions and procedures; and comprehends and applies mathematical concepts and techniques to solve problems drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar and complex familiar situations. The student explains mathematical reasoning to justify procedures and decisions; evaluates the reasonableness of solutions; communicates using mathematical, statistical and everyday language and conventions; and makes decisions about the choice of technology, and uses the technology, to solve problems in simple familiar and complex familiar situations.
C
The student demonstrates knowledge and understanding of the subject matter; recognises, recalls and uses facts, rules, definitions and procedures; and comprehends and applies mathematical concepts and techniques to solve problems drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar situations. The student explains mathematical reasoning to justify procedures and decisions; evaluates the reasonableness of solutions; communicates using mathematical, statistical and everyday language and conventions; and uses technology to solve problems in simple familiar situations.
D
The student demonstrates partial knowledge and understanding of the subject matter; recognises, recalls and uses some facts, rules, definitions and procedures; and comprehends and applies aspects of mathematical concepts and techniques to solve some problems drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar situations. The student explains some mathematical reasoning to justify procedures and decisions; sometimes evaluates the reasonableness of solutions; communicates using some mathematical, statistical and everyday language and conventions; and uses technology to solve some problems in simple familiar situations.
E
The student demonstrates isolated knowledge and understanding of the subject matter; infrequently recognises, recalls and uses some facts, rules, definitions and procedures; and infrequently comprehends and applies aspects of mathematical concepts and techniques drawn from Algebra, Functions, relations and their graphs, Calculus and Statistics in simple familiar situations. The student infrequently describes aspects of mathematical reasoning relevant to procedures and decisions; rarely evaluates the reasonableness of solutions; infrequently communicates using some aspects of mathematical, statistical and everyday language and conventions; and uses aspects of technology in simple familiar situations.

2 Unit 1: Algebra, statistics and functions

2.1 Unit description

In Unit 1, students will develop mathematical understandings and skills to solve problems relating to the topics:

- Topic 1: Arithmetic and geometric sequences and series 1
- Topic 2: Functions and graphs
- Topic 3: Counting and probability
- Topic 4: Exponential functions 1
- Topic 5: Arithmetic and geometric sequences and series 2.

Arithmetic and geometric sequences are introduced and their applications are studied. Simple relationships between variable quantities are reviewed and these are used to introduce the key concepts of a function and its graph. Quadratic functions and index rules are revised. The study of inferential statistics begins in this unit with a review of the fundamentals of probability and the introduction of the concepts of conditional probability and independence. The algebraic expansion of powers of a binomial are found using the binomial theorem.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 1. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

2.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 1 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 1 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 1 topics.

2.3 Topic 1: Arithmetic and geometric sequences and series 1

Subject matter
Arithmetic sequences (4 hours) In this sub-topic, students will: • recognise and use the recursive definition of an arithmetic sequence: $t_{n+1} = t_n + d$ • use the formula $t_n = t_1 + (n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature • use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest • establish and use the formula for the sum of the first n terms of an arithmetic sequence: $$S_n = \frac{n}{2}(2t_1 + (n-1)d) = \frac{n}{2}(t_1 + t_n)$$

2.4 Topic 2: Functions and graphs

Subject matter
Functions (4 hours) In this sub-topic, students will: • understand the concept of a relation as a mapping between sets, a graph and as a rule or a formula that defines one variable quantity in terms of another • recognise the distinction between functions and relations and use the vertical line test to determine whether a relation is a function • use function notation, domain and range, and independent and dependent variables • examine transformations of the graphs of $f(x)$, including dilations and reflections, and the graphs of $y = af(x)$ and $y = f(bx)$, translations, and the graphs of $y = f(x+c)$ and $y = f(x) + d$; $a, b, c, d \in R$ • recognise and use piece-wise functions as a combination of multiple sub-functions with restricted domains • identify contexts suitable for modelling piece-wise functions and use them to solve practical problems (taxation, taxis, the changing velocity of a parachutist).
Review of quadratic relationships (7 hours) In this sub-topic, students will: • examine examples of quadratically related variables • recognise and determine features of the graphs of $y = x^2, y = ax^2 + bx + c,\ y = a(x-b)^2 + c,$ and $y = a(x-b)(x-c)$, including their parabolic nature, turning points, axes of symmetry and intercepts • solve quadratic equations algebraically using factorisation, the quadratic formula (both exact and approximate solutions), and completing the square and using technology • identify contexts suitable for modelling with quadratic functions and use models to solve problems with and without technology; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis • understand the role of the discriminant to determine the number of solutions to a quadratic equation • determine turning points and zeros of quadratic functions with and without technology.
Inverse proportions (3 hours) In this sub-topic, students will: • examine examples of inverse proportion • recognise features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{(x-b)}$, including their hyperbolic shapes, their intercepts, their asymptotes and behaviour as $x \to \infty$ and $x \to -\infty$.

Subject matter
Powers and polynomials (9 hours) In this sub-topic, students will: • identify the coefficients and the degree of a polynomial • expand quadratic and cubic polynomials from factors • recognise and determine features of the graphs of $y = x^3, y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x \rightarrow \infty$ and $x \rightarrow -\infty$ • use the factor theorem to factorise cubic polynomials in cases where a linear factor is easily obtained • solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained • recognise and determine features of the graphs $y = a(x-b)^4 + c$, including shape and behaviour • solve equations involving combinations of the functions above, using technology where appropriate.
Graphs of relations (3 hours) In this sub-topic, students will: • recognise and determine features of the graphs of $x^2 + y^2 = r^2$ and $(x-a)^2+(y-b)^2 = r^2$, including their circular shapes, centres and radii • recognise and determine features of the graph of $y^2 = x$, including its parabolic shape and axis of symmetry.

2.5 Topic 3: Counting and probability

Subject matter
Language of events and sets (4 hours) In this sub-topic, students will: • recall the concepts and language of outcomes, sample spaces and events as sets of outcomes • use set language and notation for events, including $\bar{A}$ or A' for the complement of an event, $A, A \cap B$ for the intersection of events A and B, and $A \cup B$ for the union, and recognise mutually exclusive events • use everyday occurrences to illustrate set descriptions and representations of events, and set operations, including the use of Venn diagrams.
Review of the fundamentals of probability (3 hours) In this sub-topic, students will: • recall probability as a measure of 'the likelihood of occurrence' of an event • recall the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty • recall the rules $P(\bar{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ • use relative frequencies obtained from data as point estimates of probabilities.
Conditional probability and independence (7 hours) In this sub-topic, students will: • understand the notion of a conditional probability, and recognise and use language that indicates conditionality • use the notation $P(A\|B)$ and the formula $P(A \cap B) = P(A\|B)P(B)$ to solve problems • understand and use the notion of independence of an event A from an event B, as defined by $P(A\|B) = P(A)$ • establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B • use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events.

Subject matter
Binomial expansion (3 hours) In this sub-topic students will: • understand the notion of a combination as an unordered set of r objects taken from a set of n distinct objects • recognise and use the link between Pascal's triangle and the notation $\binom{n}{r}$ • expand $(x+y)^n$ for small positive integers n.

2.6 Topic 4: Exponential functions 1

Subject matter
Indices and the index laws (2 hours) In this sub-topic, students will: • recall indices (including negative and fractional indices) and the index laws • convert radicals to and from fractional indices • understand and use scientific notation.

2.7 Topic 5: Arithmetic and geometric sequences and series 2

Subject matter
Geometric sequences (6 hours) In this sub-topic, students will: • recognise and use the recursive definition of a geometric sequence: $t_{n+1} = rt_n$ • use the formula $t_n = t_1 r^{(n-1)}$ for the general term of a geometric sequence and recognise its exponential nature • understand the limiting behaviour as $n \to \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r • establish and use the formula $S_n = t_1 \frac{(r^n-1)}{(r-1)}$ for the sum of the first n terms of a geometric sequence • establish and use the formula $S_\infty = \frac{t_1}{(1-r)}$, $\lvert r \rvert < 1$ for the sum to infinity of a geometric progression • use geometric sequences in contexts involving geometric growth or decay, including compound interest and annuities.

2.8 Assessment guidance

In constructing assessment instruments for Unit 1, schools should ensure that the objectives cover, or are chosen from, the unit objectives. If one assessment instrument is developed for a unit, it must assess all the unit objectives; if more than one assessment instrument is developed, the unit objectives must be covered across those instruments.

It is suggested that schools develop:

- a problem-solving and modelling task that assesses Unit 1 Topic 2, and
- an internal examination that representatively samples subject matter from Unit 1 not assessed in the problem-solving and modelling task.

3 Unit 2: Calculus and further functions

3.1 Unit description

In Unit 2, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Exponential functions 2
- Topic 2: The logarithmic function 1
- Topic 3: Trigonometric functions 1
- Topic 4: Introduction to differential calculus
- Topic 5: Further differentiation and applications 1
- Topic 6: Discrete random variables 1.

Exponential graphs are examined and their applications in a wide range of settings are explored. Logarithms are introduced and the basic trigonometric functions are studied. Rates and average rates of change are also introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically by calculating difference quotients both geometrically, as gradients of chords and tangents, and algebraically.

Calculus is developed to study the derivatives of polynomial and power functions, with applications of the derivative to curve sketching, calculating gradients and equations of tangents (a link to linear function assumed knowledge), determining instantaneous velocities and solving optimisation problems. Discrete random variables are introduced; this supports the development of a framework for statistical inference.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 2. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

3.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 2 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 2 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 2 topics.

3.3 Topic 1: Exponential functions 2

Subject matter
Introduction to exponential functions (6 hours) In this sub-topic, students will: • recognise and determine the qualitative features of the graph of $y = a^x$ $(a > 0)$, including asymptotes, and of its translations ($y = a^x + b$ and $y = a^{x+c}$) • recognise and determine the features of the graphs of $y = b.a^x$ and $y = a^{kx}$ $(k \neq 0)$ • identify contexts suitable for modelling by exponential functions and use models to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis • solve equations involving exponential functions with and without technology.

3.4 Topic 2: The logarithmic function 1

Subject matter
Introduction to logs (4 hours) In this sub-topic, students will: • define logarithms as indices: $a^x = b$ is equivalent to $x = \log_a(b)$ • recognise the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a(y)$ • solve equations involving indices with and without technology.

3.5 Topic 3: Trigonometric functions 1

Subject matter
Circular measure and radian measure (2 hours) In this sub-topic, students will: • define and use radian measure and understand its relationship with degree measure • calculate lengths of arcs and areas of sectors in circles.
Introduction to trigonometric functions (7 hours) In this sub-topic, students will: • understand the unit circle definition of $\cos(\theta), \sin(\theta)$ and $\tan(\theta)$ and periodicity using radians • recall the exact values of $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ • sketch the graphs of $y = \sin(x), y = \cos(x)$, and $y = \tan(x)$ on extended domains • investigate the effect of the parameters A, B, C and D on the graphs of $y = A\sin(B(x + C)) + D, y = A\cos(B(x + C)) + D$ with and without technology • sketch the graphs of $y = A\sin(B(x + C)) + D, y = A\cos(B(x + C)) + D$ with and without technology • identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis • solve equations involving trigonometric functions with and without technology; including use of the Pythagorean identity $\sin^2(A) + \cos^2(A) = 1$.

3.6 Topic 4: Introduction to differential calculus

Subject matter
Rates of change and the concept of derivatives (8 hours) In this sub-topic, students will: • explore average and instantaneous rate of change in a variety of practical contexts • use a numerical technique to estimate a limit or an average rate of change • examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit • differentiate simple power functions and polynomial functions from first principles • interpret the derivative as the instantaneous rate of change • interpret the derivative as the gradient of a tangent line of the graph of $y = f(x)$.
Properties and computation of derivatives (7 hours) In this sub-topic, students will: • examine examples of variable rates of change of non-linear functions • establish the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for positive integers • understand the concept of the derivative as a function • recognise and use properties of the derivative $\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$ • calculate derivatives of power and polynomial functions.
Applications of derivatives (10 hours) In this sub-topic, students will: • determine instantaneous rates of change • determine the gradient of a tangent and the equation of the tangent • construct and interpret displacement-time graphs, with velocity as the slope of the tangent • sketch curves associated with power functions and polynomials up to and including degree 4; find stationary points and local and global maxima and minima with and without technology; and examine behaviour as $x \to \infty$ and $x \to -\infty$ • identify contexts suitable for modelling optimisation problems involving polynomials up to and including degree 4 and power functions on finite interval domains, and use models to solve practical problems with and without technology; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.

3.7 Topic 5: Further differentiation and applications 1

Subject matter
Differentiation rules (6 hours) In this sub-topic, students will: • understand and apply the product rule and quotient rule for power and polynomial functions • understand the notion of composition of power and polynomial functions and use the chain rule for determining the derivatives of composite functions • select and apply the product rule, quotient rule and chain rule to differentiate power and polynomial functions; express derivative in simplest and factorised form.

3.8 Topic 6: Discrete random variables 1

Subject matter
General discrete random variables (5 hours) In this sub-topic, students will: • understand the concepts of a discrete random variable and its associated probability function, and its use in modelling data • use relative frequencies obtained from data to determine point estimates of probabilities associated with a discrete random variable • recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes • examine simple examples of non-uniform discrete random variables • recognise the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases • recognise the variance and standard deviation of a discrete random variable as a measure of spread, and evaluate these in simple cases • use discrete random variables and associated probabilities to solve practical problems.

3.9 Assessment guidance

In constructing assessment instruments for Unit 2, schools should ensure that the objectives cover, or are chosen from, the unit objectives. If one assessment instrument is developed for a unit, it must assess all the unit objectives; if more than one assessment instrument is developed, the unit objectives must be covered across those instruments.

It is suggested that schools develop:

- an internal examination that representatively samples subject matter from all Unit 2 topics and/or
- an internal examination that representatively samples subject matter from Units 1 and 2.

4 Unit 3: Further calculus

4.1 Unit description

In Unit 3, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: The logarithmic function 2
- Topic 2: Further differentiation and applications 2
- Topic 3: Integrals.

Logarithmic laws and definitions are developed and used. Logarithmic functions are explored graphically and algebraically. The study of calculus continues with the derivatives of exponential, logarithmic and trigonometric functions and their applications, together with some differentiation techniques and applications to optimisation problems and graph sketching. Integration, both as a process that reverses differentiation and as a way of calculating areas and the fundamental theorem of calculus, is introduced.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 3. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

4.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

Unit objective	IA1	IA2	EA
1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics	●	●	●
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics	●	●	●
3. communicate using mathematical, statistical and everyday language and conventions	●	●	●
4. evaluate the reasonableness of solutions	●	●	●
5. justify procedures and decisions by explaining mathematical reasoning	●	●	●
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.	●	●	●

4.3 Topic 1: The logarithmic function 2

Subject matter
Logarithmic laws and logarithmic functions (8 hours) In this sub-topic, students will: • establish and use logarithmic laws and definitions • interpret and use logarithmic scales such as decibels in acoustics, the Richter scale for earthquake magnitude, octaves in music, pH in chemistry • solve equations involving indices with and without technology • recognise the qualitative features of the graph of $y = \log_a(x)$ ($a > 1$), including asymptotes, and of its translations $y = \log_a(x) + b$ and $y = \log_a(x + c)$ • solve equations involving logarithmic functions with and without technology • identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.

4.4 Topic 2: Further differentiation and applications 2

Subject matter
Calculus of exponential functions (8 hours) In this sub-topic, students will: • estimate the limit of $\frac{a^h - 1}{h}$ as $h \to 0$ using technology, for various values of $a > 0$ • recognise that e is the unique number a for which the above limit is 1 • define the exponential function e^x • establish and use the formula $\frac{d}{dx}(e^x) = e^x$ and $\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$ • identify contexts suitable for mathematical modelling by exponential functions and their derivatives and use the model to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.
Calculus of logarithmic functions (8 hours) In this sub-topic, students will: • define the natural logarithm $\ln(x) = \log_e(x)$ • recognise and use the inverse relationship of the functions $y = e^x$ and $y = \ln(x)$ • establish and use the formulas $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$ and $\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$ • use logarithmic functions and their derivatives to solve practical problems.
Calculus of trigonometric functions (8 hours) In this sub-topic, students will: • establish the formulas $\frac{d}{dx}\sin(x) = \cos(x)$, and $\frac{d}{dx}\cos(x) = -\sin(x)$ by numerical estimations of the limits and informal proofs based on geometric constructions • identify contexts suitable for modelling by trigonometric functions and their derivatives and use the model to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis • use trigonometric functions and their derivatives to solve practical problems; including trigonometric functions of the form $y = \sin(f(x))$ and $y = \cos(f(x))$.
Differentiation rules (5 hours) In this sub-topic, students will: • select and apply the product rule, quotient rule and chain rule to differentiate functions; express derivatives in simplest and factorised form.

4.5 Topic 3: Integrals

Subject matter
Anti-differentiation (9 hours) In this sub-topic, students will: • recognise anti-differentiation as the reverse of differentiation • use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals • establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$ • establish and use the formula $\int e^x dx = e^x + c$ • establish and use the formulas $\int \frac{1}{x}dx = \ln(x) + c$, for $x > 0$ and $\int \frac{1}{ax+b}dx = \frac{1}{a}\ln(ax+b) + c$ • establish and use the formulas $\int \sin(x)\,dx = -\cos(x) + c$ and $\int \cos(x)dx = \sin(x) + c$ • understand and use the formula for indefinite integrals of the form $\int \big(f(x) + g(x)\big)dx = \int f(x)dx + \int g(x)dx$ • determine indefinite integrals of the form $\int f(ax+b)dx$ • determine $f(x)$, given $f'(x)$ and an initial condition $f(a) = b$ • determine the integral of a function using information about the derivative of the given function (integration by recognition) • determine displacement given velocity in linear motion problems.
Fundamental theorem of calculus and definite integrals (3 hours) In this sub-topic, students will: • examine the area problem, and use sums of the form $\sum_i f(x_i)\,\delta x_i$, to estimate the area under the curve $y = f(x)$ • use the trapezoidal rule for the approximation of the value of a definite integral numerically • interpret the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$ • recognise the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\,\delta x_i$ • understand the formula $\int_a^b f(x)dx = F(b) - F(a)$ and use it to calculate definite integrals.
Applications of integration (6 hours) In this sub-topic, students will: • calculate the area under a curve • calculate total change by integrating instantaneous or marginal rate of change • calculate the area between curves with and without technology • determine displacements given acceleration and initial values of displacement and velocity.

4.6 Assessment

4.6.1 Summative internal assessment 1 (IA1): Problem-solving and modelling task (20%)

Description

This assessment focuses on the interpretation, analysis and evaluation of ideas and information. It is an independent task responding to a particular situation or stimuli. While students may undertake some research in the writing of the problem-solving and modelling task, it is not the focus of this technique. This assessment occurs over an extended and defined period of time. Students will use class time and their own time to develop a response.

The problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Further differentiation and applications 2
- Topic 3: Integrals.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3.

Specifications

Description

A problem-solving and modelling task is an assessment instrument developed in response to a mathematical investigative scenario or context. It requires students to respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Students must provide a response to a specific task or issue that is set in a context that highlights a real-life application of mathematics. The task requires students to use relevant stimulus material involving the selected subject matter and must have sufficient scope to allow students to address all the stages of problem-solving and modelling approach (see Section 1.2.4). Technology must be used.

The response is written and must be able to be read and interpreted independently of the instrument task sheet.

Conditions

- Write:
  - up to 10 pages (including tables, figures and diagrams)
  - maximum of 2000 words
  - appendixes can include raw data, repeated calculations, evidence of authentication and student notes (appendixes are not to be marked).
- Duration: 4 weeks (including 3 hours of class time).
- Other:
  - opportunity may be provided for group work, but unique responses must be developed by each student
  - use of technology is required; schools must specify the technology used, e.g. scientific calculator, graphics calculator (CAS or non-CAS), spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation or word processing
  - the teacher provides the mathematical investigative scenario or context for the problem-solving and modelling task.

Task examples

Examples of problem-solving and modelling tasks include:

- a report that investigates motion sickness on a Ferris wheel using the up and down velocity and acceleration of the Ferris wheel car
- a report that determines the functions that model a waterslide, using the functions to investigate the velocity and acceleration on the waterslide at different points
- a report that determines the shaded region of a logo, represented by a number of curves, by first finding the functions that model the logo design and then using calculus techniques to determine the shaded area
- a magazine article that investigates the spread of a disease using a developed disease model, predicts the number of people who will be affected by the disease in the future and determines when the rate of spread of the disease is at its maximum.

Summary of the instrument-specific marking guide

The following table summarises the criteria, assessment objectives and mark allocation for the problem-solving and modelling task.

Criterion	Objectives	Marks
Formulate	1, 2 and 5	4
Solve	1 and 6	7
Evaluate and verify	4 and 5	5
Communicate	3	4
Total		**20**

Instrument-specific marking guide

Criterion: Formulate

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques.	3–4
• statement of some assumptions • statement of some observations • translation of simple aspects of the problem by identifying mathematical concepts and techniques.	1–2
• does not satisfy any of the descriptors above.	0

Criterion: Solve

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3

The student work has the following characteristics:	Marks
• accurate use of complex procedures to reach a valid solution • discerning application of mathematical concepts and techniques relevant to the task • accurate and appropriate use of technology.	6–7
• use of complex procedures to reach a reasonable solution • application of mathematical concepts and techniques relevant to the task • use of technology.	4–5
• use of simple procedures to make some progress towards a solution • simplistic application of mathematical concepts and techniques relevant to the task • superficial use of technology.	2–3
• inappropriate use of technology or procedures.	1
• does not satisfy any of the descriptors above.	0

Criterion: Evaluate and verify

Assessment objectives

4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	4–5
• statements about the reasonableness of solutions by considering the context of the task • statements about relevant strengths and limitations of the solution and/or model • statements about decisions made relevant to the context of the task.	2–3
• statement about a decision and/or the reasonableness of a solution.	1
• does not satisfy any of the descriptors above.	0

Criterion: Communicate

Assessment objective

3. communicate using mathematical, statistical and everyday language and conventions

The student work has the following characteristics:	Marks
• correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4
• use of some appropriate language and conventions to develop the response • adequate organisation of the response.	1–2
• does not satisfy any of the descriptors above.	0

4.6.2 Summative internal assessment 2 (IA2): Examination (15%)

Description

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 3 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.

Specifications

Description

The examination representatively samples subject matter from all Unit 3 topics. Where relevant, the focus of this assessment should be on subject matter not assessed in the problem-solving and modelling task.

Subject matter from Units 1 and 2 is considered assumed knowledge.

The examination must ensure that all assessment objectives are assessed. The examination should be designed using the principles of developing mathematics problems in Section 1.2.4. The total number of marks used in an examination marking scheme is a school decision. However, in order to correctly apply the ISMG, the percentage allocation of marks must match the following specifications.

Mark allocations

Percentage of marks	Degree of difficulty
~ 20%	**Complex unfamiliar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable; that is – the required procedure is not clear from the way the problem is posed, and – in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
~ 20%	**Complex familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, *or* – in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
~ 60%	**Simple familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, *or* – in a context that has been a focus of prior learning. Students are *not* required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.

Conditions

- Time: 120 minutes plus 5 minutes perusal.
- Length: the number of short-response items should allow students to complete the response in the set time.
- Short-response format, consisting of a number of items that ask students to respond to the following activities
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams
  - short items requiring single-word, term, sentence or short-paragraph responses
  - justifying solutions using appropriate mathematical language where applicable

  - responding to seen or unseen stimulus materials
  - interpreting ideas and information.
- Other
  - the instrument must be designed in such a way as to ensure that items provide for a balance of both technology-free and technology-active responses
  - seen stimulus — teachers must ensure the purpose of the technique is not compromised
  - unseen stimulus — materials or questions must not be copied from information or texts that students have previously been exposed to or have used directly in class
  - when stimulus materials are used, they will be succinct enough to allow students sufficient time to engage with them; for stimulus materials that are lengthy, complex or large in number, they will be shared with students prior to the administration of the assessment instrument
  - only the QCAA formula sheet must be provided
  - notes are not permitted
  - use of technology is required; schools must specify the technology used, e.g. scientific calculator, graphics calculator (CAS or non-CAS), spreadsheet program, and/or other mathematical software; use of technology must go beyond simple computation.

Summary of the instrument-specific marking guide

The following table summarises the mark allocation for the objectives assessed in the examination.

Criterion	Objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5 and 6	15
Total		**15**

Instrument-specific marking guide

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and	> 93%	15

The student work has the following characteristics:	Cut-off	Marks
techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0

4.6.3 Summative external assessment (EA): Examination (50%)

General information

Summative external assessment is developed and marked by the QCAA. In Mathematical Methods it contributes 50% to a student's overall subject result.

Summative external assessment assesses learning from Units 3 and 4. Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of the supervised examination.

The external assessment in Mathematical Methods is common to all schools and administered under the same conditions, at the same time, on the same day.

See Section 5.8.2.

5 Unit 4: Further functions and statistics

5.1 Unit description

In Unit 4, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Further differentiation and application 3
- Topic 2: Trigonometric functions 2
- Topic 3: Discrete random variables 2
- Topic 4: Continuous random variables and the normal distribution
- Topic 5: Interval estimates for proportions.

The study of calculus continues with some differentiation techniques and applications to optimisation problems and graph sketching. The cosine and sine rules are established and used. Use of discrete random variables in modelling random processes involving chance and variation are studied. Continuous random variables and their applications are explored and the normal distribution is used in a variety of contexts. The study of statistical inference in this unit is the culmination of earlier work on probability and random variables. The goal of statistical inference is to estimate an unknown parameter associated with a population using a sample of data drawn from that population. In Mathematical Methods, statistical inference is restricted to estimating proportions in two-outcome populations.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 4. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

5.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

Unit objective	IA3	EA
1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics	●	●
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics	●	●
3. communicate using mathematical, statistical and everyday language and conventions	●	●
4. evaluate the reasonableness of solutions	●	●
5. justify procedures and decisions by explaining mathematical reasoning	●	●
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.	●	●

5.3 Topic 1: Further differentiation and applications 3

Subject matter

The second derivative and applications of differentiation (9 hours)

In this sub-topic, students will:

- understand the concept of the second derivative as the rate of change of the first derivative function
- recognise acceleration as the second derivative of displacement position with respect to time
- understand the concepts of concavity and points of inflection and their relationship with the second derivative
- understand and use the second derivative test for finding local maxima and minima
- sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection
- solve optimisation problems from a wide variety of fields using first and second derivatives, where the function to be optimised is both given and developed.

5.4 Topic 2: Trigonometric functions 2

Subject matter

Cosine and sine rules (9 hours)

In this sub-topic, students will:

- recall sine, cosine and tangent as ratios of side lengths in right-angled triangles
- understand the unit circle definition of $\cos(\theta)$, $\sin(\theta)$ and $\tan(\theta)$ and periodicity using degrees and radians
- establish and use the sine (ambiguous case is required) and cosine rules and the formula $\text{area} = \frac{1}{2}bc\sin(A)$ for the area of a triangle
- construct mathematical models using the sine and cosine rules in two- and three-dimensional contexts (including bearings in two-dimensional context) and use the model to solve problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.

5.5 Topic 3: Discrete random variables 2

Subject matter

Bernoulli distributions (3 hours)

In this sub-topic, students will:

- use a Bernoulli random variable as a model for two-outcome situations
- identify contexts suitable for modelling by Bernoulli random variables
- recognise and determine the mean p and variance $p(1-p)$ of the Bernoulli distribution with parameter p
- use Bernoulli random variables and associated probabilities to model data and solve practical problems.

Subject matter
Binomial distributions (5 hours) In this sub-topic, students will: • understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial • identify contexts suitable for modelling by binomial random variables • determine and use the probabilities $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$ associated with the binomial distribution with parameters n and p • calculate the mean np and variance $np(1-p)$ of a binomial distribution using technology and algebraic methods • identify contexts suitable to model binomial distributions and associated probabilities to solve practical problems, including the language of 'at most' and 'at least'.

5.6 Topic 4: Continuous random variables and the normal distribution

Subject matter
General continuous random variables (6 hours) In this sub-topic, students will: • use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable • understand the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts • calculate the expected value, variance and standard deviation of a continuous random variable in simple cases • understand standardised normal variables (z-values, z-scores) and use these to compare samples.
Normal distributions (6 hours) In this sub-topic, students will: • identify contexts, such as naturally occurring variations, that are suitable for modelling by normal random variables • recognise features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution • calculate probabilities and quantiles associated with a given normal distribution using technology and use these to solve practical problems.

5.7 Topic 5: Interval estimates for proportions

Subject matter
Random sampling (3 hours) In this sub-topic, students will: • understand the concept of a random sample • discuss sources of bias in samples, and procedures to ensure randomness • investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli, using graphical displays of real and simulated data.

Subject matter
Sample proportions (6 hours) In this sub-topic, students will: • understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{(p(1-p)/n}$ of the sample proportion $\hat{p}$ • consider the approximate normality of the distribution of $\hat{p}$ for large samples • simulate repeated random sampling, for a variety of values of p and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{(\hat{p}(1-\hat{p})/n)}}$ where the closeness of the approximation depends on both n and p.
Confidence intervals for proportions (8 hours) In this sub-topic, students will: • understand the concept of an interval estimate for a parameter associated with a random variable • use the approximate confidence interval $\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$, as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution • define the approximate margin of error $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and understand the trade-off between margin of error and level of confidence • use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain p.

5.8 Assessment

5.8.1 Summative internal assessment 3 (IA3): Examination (15%)

Description

This examination assesses the application of a range of cognitions to a number of items, drawn from Unit 4 Topics 1–5. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

Specifications

Description

The examination representatively samples subject matter from all Unit 4 topics.

Subject matter from Units 1, 2 and 3 is considered assumed knowledge.

The examination must ensure that all assessment objectives are assessed. The examination should be designed using the principles of developing mathematics problems in Section 1.2.4. The total number of marks used in an examination marking scheme is a school decision. However, in order to correctly apply the ISMG, the percentage allocation of marks must match the specifications below.

Mark allocations

Percentage of marks	Degree of difficulty
~ 20%	**Complex unfamiliar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable; that is − the required procedure is not clear from the way the problem is posed, and − in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
~ 20%	**Complex familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable; that is − the required procedure is clear from the way the problem is posed, *or* − in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
~ 60%	**Simple familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable; that is − the required procedure is clear from the way the problem is posed, *or* − in a context that has been a focus of prior learning. Students are *not* required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.

Conditions

- Time: 120 minutes plus 5 minutes perusal.
- Length: the number of short-response items should allow students to complete the response in the set time.
- Short-response format, consisting of a number of items that ask students to respond to the following activities
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams
  - short items requiring single-word, sentence or short-paragraph responses
  - justifying solutions using appropriate mathematical language where applicable

- responding to seen or unseen stimulus materials
- interpreting ideas and information.

- Other
  - the instrument must be designed in such a way as to ensure that items provide a balance of both technology-free and technology-active responses
  - seen stimulus — teachers must ensure the purpose of the technique is not compromised
  - unseen stimulus — materials or questions must not be copied from information or texts that students have previously been exposed to or have used directly in class
  - when stimulus materials are used, they will be succinct enough to allow students sufficient time to engage with them; for stimulus materials that are lengthy, complex or large in number, they will be shared with students prior to the administration of the assessment instrument
  - only the QCAA formula sheet must be provided
  - notes are not permitted
  - use of technology is required; schools must specify the technology used, e.g. scientific calculator, graphics calculator (CAS or non-CAS), spreadsheet program, and/or other mathematical software; use of technology must go beyond simple computation.

Summary of the instrument-specific marking guide

The following table summarises the mark allocation for the objectives assessed in the examination.

Criterion	Objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5 and 6	15
Total		**15**

Instrument-specific marking guide

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0

5.8.2 Summative external assessment (EA): Examination (50%)

General information

Summative external assessment is developed and marked by the QCAA. In Mathematical Methods it contributes 50% to a student's overall subject result.

Summative external assessment assesses learning from Units 3 and 4. Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of the examination.

The external assessment in Mathematical Methods is common to all schools and administered under the same conditions, at the same time, on the same day.

Description

This examination consists of two papers: technology-free (Paper 1) and technology-active (Paper 2). The examination assesses the application of a range of cognitions to a number of items drawn from Units 3 and 4. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decision by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Specifications

Description

The external assessment for Mathematical Methods will representatively sample subject matter from Units 3 and 4.

The percentage allocation of marks for each paper of the external examination will match the specifications below.

Mark allocations

Percentage of marks	Degree of difficulty
~ 20%	**Complex unfamiliar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable; that is – the required procedure is not clear from the way the problem is posed, and – in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
~ 20%	**Complex familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, *or* – in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
~ 60%	**Simple familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, *or* – in a context that has been a focus of prior learning. Students are *not* required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.

Conditions

- Time
  - Paper 1 (technology-free, 25%); 90 minutes plus 5 minutes perusal
  - Paper 2 (technology-active, 25%); 90 minutes plus 5 minutes perusal.
- Length: the number of short-response items should allow students to complete the response in the set time.
- Short-response format, consisting of a number of items that ask students to respond to the following activities
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams

- short items requiring multiple-choice, single-word, term, sentence or short-paragraph responses
- justifying solutions using appropriate mathematical language where applicable
- responding to seen or unseen stimulus materials
- interpreting ideas and information.

• Other
  - the QCAA formula sheet will be provided for both papers
  - no calculator or technology of any type is permitted in Paper 1 (technology-free); access to a QCAA-approved handheld graphics calculator (no CAS functionality) is a requirement for Paper 2 (technology-active) of the external assessment, and scientific calculators may also be used.

Instrument-specific marking guide

No ISMG is provided for the external assessment.

6 Glossary

Term	Explanation
A	
accomplished	highly trained or skilled in a particular activity; perfected in knowledge or training; expert
accuracy	the condition or quality of being true, correct or exact; freedom from error or defect; precision or exactness; correctness; in science, the extent to which a measurement result represents the quantity it purports to measure; an accurate measurement result includes an estimate of the true value and an estimate of the uncertainty
accurate	precise and exact; to the point; consistent with or exactly conforming to a truth, standard, rule, model, convention or known facts; free from error or defect; meticulous; correct in all details
adept	very/highly skilled or proficient at something; expert
adequate	satisfactory or acceptable in quality or quantity equal to the requirement or occasion
algorithm	a precisely defined procedure that can be applied and systematically followed through to a conclusion
analyse	dissect to ascertain and examine constituent parts and/or their relationships; break down or examine in order to identify the essential elements, features, components or structure; determine the logic and reasonableness of information; examine or consider something in order to explain and interpret it, for the purpose of finding meaning or relationships and identifying patterns, similarities and differences
anti-differentiation	the process of solving for anti-derivatives; an anti-derivative, primitive or indefinite integral of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'(x) = f(x)$, anti-derivatives are not unique; if $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number; $\int f(x)dx = F(x) + c$ denotes the set of all anti-derivatives of $f(x)$; the number c is called the constant of integration, e.g. since $\frac{d}{dx}(x^3) = 3x^2$, we can write $\int 3x^2\,dx = x^3 + c$
applied learning	the acquisition and application of knowledge, understanding and skills in real-world or lifelike contexts that may encompass workplace, industry and community situations; it emphasises learning through doing and includes both theory and the application of theory, connecting subject knowledge and understanding with the development of practical skills

Term	Explanation
Applied subject	a subject whose primary pathway is work and vocational education; it emphasises applied learning and community connections; a subject for which a syllabus has been developed by the QCAA with the following characteristics: results from courses developed from Applied syllabuses contribute to the QCE; results may contribute to ATAR calculations
apply	use knowledge and understanding in response to a given situation or circumstance; carry out or use a procedure in a given or particular situation
appraise	evaluate the worth, significance or status of something; judge or consider a text or piece of work
appreciate	recognise or make a judgment about the value or worth of something; understand fully; grasp the full implications of
appropriate	acceptable; suitable or fitting for a particular purpose, circumstance, context, etc.
apt	suitable to the purpose or occasion; fitting, appropriate
area of study	a division of, or a section within a unit
argue	give reasons for or against something; challenge or debate an issue or idea; persuade, prove or try to prove by giving reasons
arithmetic sequence	a sequence of numbers such that the difference of any two successive numbers in the sequence is a constant, e.g. the sequence 2, 5, 8, 11, 14, 17, … is an arithmetic sequence with common difference 3, if the initial term of an arithmetic sequence is t_1 and the common difference of successive members is d, then the n^{th} term t_n of the sequence is given by: $t_n = t_1 + (n-1)d$ for $n \geq 1$ a recursive definition is $t_{n+1} = t_n + d$, where d is the common difference and $n \geq 1$
aspect	a particular part of a feature of something; a facet, phase or part of a whole
assess	measure, determine, evaluate, estimate or make a judgment about the value, quality, outcomes, results, size, significance, nature or extent of something
assessment	purposeful and systematic collection of information about students' achievements
assessment instrument	a tool or device used to gather information about student achievement
assessment objectives	drawn from the unit objectives and contextualised for the requirements of the assessment instrument (see also 'syllabus objectives', 'unit objectives')
assessment technique	the method used to gather evidence about student achievement, (e.g. examination, project, investigation)
assumptions	conditions that are stated to be true when beginning to solve a problem

Term	Explanation
astute	showing an ability to accurately assess situations or people; of keen discernment
asymptote	a line is an asymptote to a curve if the distance between the line and the curve approaches zero as they 'tend to infinity'. For example, the line with equation $x = \pi/2$ is a vertical asymptote to the graph of $y = \tan x$, and the line with equation $y = 0$ is a horizontal asymptote to the graph of $y = 1/x$.
ATAR	Australian Tertiary Admission Rank
authoritative	able to be trusted as being accurate or true; reliable; commanding and self-confident; likely to be respected and obeyed
B	
balanced	keeping or showing a balance; not biased; fairly judged or presented; taking everything into account in a fair, well-judged way
basic	fundamental
Bernoulli random variable	a variable with two possible values, 0 and 1; the parameter associated with such a random variable is the probability p of obtaining a 1
Bernoulli trial	a chance experiment with possible outcomes, typically labelled *success* and *failure*
binomial distribution	a distribution giving the probability of obtaining a specified number of successes in a set of trials where each trial can end in either a success or a failure
binomial theorem	the expansion $(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + y^n$
C	
calculate	determine or find (e.g. a number, answer) by using mathematical processes; obtain a numerical answer showing the relevant stages in the working; ascertain/determine from given facts, figures or information
categorise	place in or assign to a particular class or group; arrange or order by classes or categories; classify, sort out, sort, separate
chain rule	relates the derivative of the composite of two functions to the functions and their derivatives: if $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$ and in Leibniz notation: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
challenging	difficult but interesting; testing one's abilities; demanding and thought-provoking; usually involving unfamiliar or less familiar elements
characteristic	a typical feature or quality
clarify	make clear or intelligible; explain; make a statement or situation less confused and more comprehensible

Term	Explanation
clarity	clearness of thought or expression; the quality of being coherent and intelligible; free from obscurity of sense; without ambiguity; explicit; easy to perceive, understand or interpret
classify	arrange, distribute or order in classes or categories according to shared qualities or characteristics
clear	free from confusion, uncertainty, or doubt; easily seen, heard or understood
clearly	in a clear manner; plainly and openly, without ambiguity
coherent	having a natural or due agreement of parts; connected; consistent; logical, orderly; well-structured and makes sense; rational, with parts that are harmonious; having an internally consistent relation of parts
cohesive	characterised by being united, bound together or having integrated meaning; forming a united whole
comment	express an opinion, observation or reaction in speech or writing; give a judgment based on a given statement or result of a calculation
communicate	convey knowledge and/or understandings to others; make known; transmit
compare	display recognition of similarities and differences and recognise the significance of these similarities and differences
competent	having suitable or sufficient skills, knowledge, experience, etc. for some purpose; adequate but not exceptional; capable; suitable or sufficient for the purpose; having the necessary ability, knowledge or skill to do something successfully; efficient and capable (of a person); acceptable and satisfactory, though not outstanding
competently	in an efficient and capable way; in an acceptable and satisfactory, though not outstanding, way
completing the square	rewriting the quadratic expression $ax^2 + bx + c$ as $a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ is called completing the square
complex	composed or consisting of many different and interconnected parts or factors; compound; composite; characterised by an involved combination of parts; complicated; intricate; a complex whole or system; a complicated assembly of particulars

Term	Explanation
complex familiar	problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, *or* – in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
complex unfamiliar	problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable; that is – the required procedure is not clear from the way the problem is posed, and – in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
composite functions	if $y = g(x)$ and $z = f(y)$ for functions f and g, then z is a composite function of x; write $z = f \circ g(x) = f(g(x))$ e.g. $z = \sqrt{x^2 + 3}$ expresses z as a composite of the functions $f(y) = \sqrt{y}$ and $g(x) = x^2 + 3$
comprehend	understand the meaning or nature of; grasp mentally
comprehensive	inclusive; of large content or scope; including or dealing with all or nearly all elements or aspects of something; wide-ranging; detailed and thorough, including all that is relevant
concavity	a description of whether a graph is bending upwards or downwards, a function is concave downwards when the second derivative is negative; a function is concave upwards when the second derivative is positive
concise	expressing much in few words; giving a lot of information clearly and in a few words; brief, comprehensive and to the point; succinct, clear, without repetition of information
concisely	in a way that is brief but comprehensive; expressing much in few words; clearly and succinctly

Term	Explanation
conditional probability	the probability that an event A occurs can change if it becomes known that another event B occurs; the new probability is known as a conditional probability and is written as $P(A\|B)$. If B has occurred, the sample space is reduced by discarding all outcomes that are not in the event B; the new sample space, called the reduced sample space, is B; the conditional probability of the event A is given by $$P(A\|B) = \frac{P(A \cap B)}{P(B)}$$
conduct	direct in action or course; manage; organise; carry out
confidence interval	provides a range of values that describe the uncertainty surrounding an estimate
consider	think deliberately or carefully about something, typically before making a decision; take something into account when making a judgment; view attentively or scrutinise; reflect on
considerable	fairly large or great; thought about deliberately and with a purpose
considered	formed after careful and deliberate thought
consistent	agreeing or accordant; compatible; not self-opposed or self-contradictory, constantly adhering to the same principles; acting in the same way over time, especially so as to be fair or accurate; unchanging in nature, standard, or effect over time; not containing any logical contradictions (of an argument); constant in achievement or effect over a period of time
construct	create or put together (e.g. an argument) by arranging ideas or items; display information in a diagrammatic or logical form; make; build
continuous random variable	a variable whose set of possible values are all of the real numbers in some interval
contrast	display recognition of differences by deliberate juxtaposition of contrary elements; show how things are different or opposite; give an account of the differences between two or more items or situations, referring to both or all of them throughout
controlled	shows the exercise of restraint or direction over; held in check; restrained, managed or kept within certain bounds
convention	the generally agreed upon way in which something is done; in a mathematical context this refers to notation symbols, abbreviations, usage and setting out
convincing	persuaded by argument or proof; leaving no margin of doubt; clear; capable of causing someone to believe that something is true or real; persuading or assuring by argument or evidence; appearing worthy of belief; credible or plausible
cosine rule	for a triangle of side lengths a, b and c, and corresponding angles A, B and C, the cosine rule states that $$c^2 = a^2+b^2 - 2ab\cos C$$
course	a defined amount of learning developed from a subject syllabus

Term	Explanation
create	bring something into being or existence; produce or evolve from one's own thought or imagination; reorganise or put elements together into a new pattern or structure or to form a coherent or functional whole
creative	resulting from originality of thought or expression; relating to or involving the use of the imagination or original ideas to create something; having good imagination or original ideas
credible	capable or worthy of being believed; believable; convincing
criterion	the property or characteristic by which something is judged or appraised
critical	involving skilful judgment as to truth, merit, etc.; involving the objective analysis and evaluation of an issue in order to form a judgment; expressing or involving an analysis of the merits and faults of a work of literature, music, or art; incorporating a detailed and scholarly analysis and commentary (of a text); rationally appraising for logical consistency and merit
critique	review (e.g. a theory, practice, performance) in a detailed, analytical and critical way
cursory	hasty, and therefore not thorough or detailed; performed with little attention to detail; going rapidly over something, without noticing details; hasty; superficial
D	
decide	reach a resolution as a result of consideration; make a choice from a number of alternatives
deduce	reach a conclusion that is necessarily true, provided a given set of assumptions is true; arrive at, reach or draw a logical conclusion from reasoning and the information given
defensible	justifiable by argument; capable of being defended in argument
define	give the meaning of a word, phrase, concept or physical quantity; state meaning and identify or describe qualities
demonstrate	prove or make clear by argument, reasoning or evidence, illustrating with practical example; show by example; give a practical exhibition
derive	arrive at by reasoning; manipulate a mathematical relationship to give a new equation or relationship; in mathematics, obtain the derivative of a function
describe	give an account (written or spoken) of a situation, event, pattern or process, or of the characteristics or features of something
design	produce a plan, simulation, model or similar; plan, form or conceive in the mind; in English, select, organise and use particular elements in the process of text construction for particular purposes; these elements may be linguistic (words), visual (images), audio (sounds), gestural (body language), spatial (arrangement on the page or screen) and multimodal (a combination of more than one)

Term	Explanation
detailed	executed with great attention to the fine points; meticulous; including many of the parts or facts
determine	establish, conclude or ascertain after consideration, observation, investigation or calculation; decide or come to a resolution
develop	elaborate, expand or enlarge in detail; add detail and fullness to; cause to become more complex or intricate
devise	think out; plan; contrive; invent
differentiate	identify the difference/s in or between two or more things; distinguish, discriminate; recognise or ascertain what makes something distinct from similar things; in mathematics, obtain the derivative of a function
discerning	discriminating; showing intellectual perception; showing good judgment; making thoughtful and astute choices; selected for value or relevance
discrete random variable	a variable whose possible values are the counting numbers 0, 1, 2, 3,…, or that form a finite set, e.g. the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin
discriminant	the discriminant of the quadratic expression $ax^2 + bx + c$ is the quantity $b^2 - 4ac$
discriminate	note, observe or recognise a difference; make or constitute a distinction in or between; differentiate; note or distinguish as different
discriminating	differentiating; distinctive; perceiving differences or distinctions with nicety; possessing discrimination; perceptive and judicious; making judgments about quality; having or showing refined taste or good judgment
discuss	examine by argument; sift the considerations for and against; debate; talk or write about a topic, including a range of arguments, factors or hypotheses; consider, taking into account different issues and ideas, points for and/or against, and supporting opinions or conclusions with evidence
disjointed	disconnected; incoherent; lacking a coherent order/sequence or connection
distinguish	recognise as distinct or different; note points of difference between; discriminate; discern; make clear a difference/s between two or more concepts or items
diverse	of various kinds or forms; different from each other
document	support (e.g. an assertion, claim, statement) with evidence (e.g. decisive information, written references, citations)
draw conclusions	make a judgment based on reasoning and evidence

Term	Explanation
E	
e	Euler's number (e) is an irrational number whose decimal expansion begins $$e = 2.7182818284590452353602874713527 \ldots$$ it is the base of the natural logarithms, and can be defined in various ways, including $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ and $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$
effective	successful in producing the intended, desired or expected result; meeting the assigned purpose
efficient	working in a well-organised and competent way; maximum productivity with minimal expenditure of effort; acting or producing effectively with a minimum of waste, expense or unnecessary effort
element	a component or constituent part of a complex whole; a fundamental, essential or irreducible part of a composite entity
elementary	simple or uncompounded; relating to or dealing with elements, rudiments or first principles (of a subject); of the most basic kind; straightforward and uncomplicated
erroneous	based on or containing error; mistaken; incorrect
essential	absolutely necessary; indispensable; of critical importance for achieving something
evaluate	make an appraisal by weighing up or assessing strengths, implications and limitations; make judgments about ideas, works, solutions or methods in relation to selected criteria; examine and determine the merit, value or significance of something, based on criteria
examination	a supervised test that assesses the application of a range of cognitions to one or more provided items such as questions, scenarios and/or problems; student responses are completed individually, under supervised conditions, and in a set timeframe
examine	investigate, inspect or scrutinise; inquire or search into; consider or discuss an argument or concept in a way that uncovers the assumptions and interrelationships of the issue
expected value	the expected value $E(X)$ of a random variable X is a measure of the central tendency of its distribution; if X is discrete, $E(X) = \mu = \sum_i p_i x_i$, where the x_i are the possible values of X and $p_i = P(X = x_i)$; if X is continuous, $E(X) = \mu = \int_{-\infty}^{\infty} xp(x)dx$, where $p(x)$ is the probability density function of X
experiment	try out or test new ideas or methods, especially in order to discover or prove something; undertake or perform a scientific procedure to test a hypothesis, make a discovery or demonstrate a known fact
explain	make an idea or situation plain or clear by describing it in more detail or revealing relevant facts; give an account; provide additional information

Term	Explanation
explicit	clearly and distinctly expressing all that is meant; unequivocal; clearly developed or formulated; leaving nothing merely implied or suggested
explore	look into both closely and broadly; scrutinise; inquire into or discuss something in detail
express	convey, show or communicate (e.g. a thought, opinion, feeling, emotion, idea or viewpoint); in words, art, music or movement, convey or suggest a representation of; depict
extended response	an open-ended assessment technique that focuses on the interpretation, analysis, examination and/or evaluation of ideas and information in response to a particular situation or stimulus; while students may undertake some research when writing of the extended response, it is not the focus of this technique; an extended response occurs over an extended and defined period of time
Extension subject	a two-unit subject (Units 3 and 4) for which a syllabus has been developed by QCAA, that is an extension of one or more General subject/s, studied concurrently with, Units 3 and 4 of that subject or after completion of, Units 3 and 4 of that subject
extensive	of great extent; wide; broad; far-reaching; comprehensive; lengthy; detailed; large in amount or scale
external assessment	summative assessment that occurs towards the end of a course of study and is common to all schools; developed and marked by the QCAA according to a commonly applied marking scheme
external examination	a supervised test, developed and marked by the QCAA, that assesses the application of a range of cognitions to multiple provided items such as questions, scenarios and/or problems; student responses are completed individually, under supervised conditions, and in a set timeframe
extrapolate	infer or estimate by extending or projecting known information; conjecture; infer from what is known; extend the application of something (e.g. a method or conclusion) to an unknown situation by assuming that existing trends will continue or similar methods will be applicable
F	
factor theorem	a theorem linking factors and zeros of a polynomial
factorise	convert the sum of terms in an extended from to a product
factual	relating to or based on facts; concerned with what is actually the case; actually occurring; having verified existence
familiar	well-acquainted; thoroughly conversant with; well known from long or close association; often encountered or experienced; common; (of materials, texts, skills or circumstances) having been the focus of learning experiences or previously encountered in prior learning activities
feasible	capable of being achieved, accomplished or put into effect; reasonable enough to be believed or accepted; probable; likely

Term	Explanation
fluent	spoken or written with ease; able to speak or write smoothly, easily or readily; articulate; eloquent; in artistic performance, characteristic of a highly developed and excellently controlled technique; flowing; polished; flowing smoothly, easily and effortlessly
fluently	in a graceful and seemingly effortless manner; in a way that progresses smoothly and readily
formative assessment	assessment whose major purpose is to improve teaching and student achievement
fragmented	disorganised; broken down; disjointed or isolated
frequent	happening or occurring often at short intervals; constant, habitual, or regular
function	a function f is a rule that associates with each element x in a set S a unique element $f(x)$ in a set T; we write $x \mapsto f(x)$ to indicate the mapping of x to $f(x)$; the set S is called the domain of f and the set T is called the codomain; the subset of T consisting of all the elements $f(x): x \in S$ is called the range of f; if we write $y = f(x)$, we say that x is the independent variable and y is the dependent variable
fundamental	forming a necessary base or core; of central importance; affecting or relating to the essential nature of something; part of a foundation or basis
fundamental theorem of calculus	relates differentiation and definite integrals; it has two forms: $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$ and $\int_a^b f'(x)dx = f(b) - f(a)$
G	
General subject	a subject for which a syllabus has been developed by the QCAA with the following characteristics: results from courses developed from General syllabuses contribute to the QCE; General subjects have an external assessment component; results may contribute to ATAR calculations
generate	produce; create; bring into existence
geometric sequence	a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number, called the common ratio, e.g. the sequence 3, 6, 12, 24, ... is a geometric sequence with common ratio 2, similarly, the sequence 40, 20, 10, 5, 2.5, … is a geometric sequence with common ratio $\frac{1}{2}$; if the initial term of a geometric sequence is t_1 and the common ratio of successive members is r, then the n^{th} term t_n, of the sequence is given by $t_n = t_1 r^{(n-1)}$ for $n \geq 1$
gradient	the gradient of the straight line passing through points (x_1, y_1) and (x_2, y_2) is the ratio $\frac{y_2 - y_1}{x_2 - x_1}$; slope is a synonym for gradient
graph of a function	the graph of a function f is the set of all points (x, y) in the Cartesian plane where x is in the domain of f and $y = f(x)$

Term	Explanation
H	
hypothesise	formulate a supposition to account for known facts or observed occurrences; conjecture, theorise, speculate; especially on uncertain or tentative grounds
I	
identify	distinguish; locate, recognise and name; establish or indicate who or what someone or something is; provide an answer from a number of possibilities; recognise and state a distinguishing factor or feature
illogical	lacking sense or sound reasoning; contrary to or disregardful of the rules of logic; unreasonable
implement	put something into effect, e.g. a plan or proposal
implicit	implied, rather than expressly stated; not plainly expressed; capable of being inferred from something else
improbable	not probable; unlikely to be true or to happen; not easy to believe
inaccurate	not accurate
inappropriate	not suitable or proper in the circumstances
inconsistent	lacking agreement, as one thing with another, or two or more things in relation to each other; at variance; not consistent; not in keeping; not in accordance; incompatible, incongruous
independent	thinking or acting for oneself, not influenced by others
independent events	two events are independent if knowing that one occurs tells us nothing about the other; the concept can be defined formally using probabilities in various ways, e.g. events A and B are independent if $P(A \cap B) = P(A)P(B)$, if $P(A\|B) = P(A)$, or if $P(B) = P(B\|A)$, for events A and B with non-zero probabilities, any one of these equations implies any other
in-depth	comprehensive and with thorough coverage; extensive or profound; well-balanced or fully developed
index laws	the rules $a^x a^y = a^{x+y}, a^{-x} = \frac{1}{a^x}, (a^x)^y = a^{xy}, a^0 = 1$, and $(ab)^x = a^x b^x$, for any real numbers x, y, a and b, with $a > 0$ and $b > 0$
infer	derive or conclude something from evidence and reasoning, rather than from explicit statements; listen or read beyond what has been literally expressed; imply or hint at
informed	knowledgeable; learned; having relevant knowledge; being conversant with the topic; based on an understanding of the facts of the situation (of a decision or judgment)
innovative	new and original; introducing new ideas; original and creative in thinking
insightful	showing understanding of a situation or process; understanding relationships in complex situations; informed by observation and deduction

Term	Explanation
instrument-specific marking guide	ISMG; a tool for marking that describes the characteristics evident in student responses and aligns with the identified objectives for the assessment (see 'assessment objectives')
integral	*adjective* necessary for the completeness of the whole; essential or fundamental; *noun* in mathematics, the result of integration; an expression from which a given function, equation, or system of equations is derived by differentiation
intended	designed; meant; done on purpose; intentional
internal assessment	assessments that are developed by schools; summative internal assessments are endorsed by the QCAA before use in schools and results externally confirmed contribute towards a student's final result
interpret	use knowledge and understanding to recognise trends and draw conclusions from given information; make clear or explicit; elucidate or understand in a particular way; bring out the meaning of, e.g. a dramatic or musical work, by performance or execution; bring out the meaning of an artwork by artistic representation or performance; give one's own interpretation of; identify or draw meaning from, or give meaning to, information presented in various forms, such as words, symbols, pictures or graphs
interval estimate	in statistics estimation, the use of information derived from a sample to produce an estimate of an unknown probability or population parameter; an interval derived from the sample that, in some sense, is likely to contain the parameter; an example of an interval estimate for p is a confidence interval centred on the relative frequency f
investigate	carry out an examination or formal inquiry in order to establish or obtain facts and reach new conclusions; search, inquire into, interpret and draw conclusions about data and information
investigation	an assessment technique that requires students to research a specific problem, question, issue, design challenge or hypothesis through the collection, analysis and synthesis of primary and/or secondary data; it uses research or investigative practices to assess a range of cognitions in a particular context; an investigation occurs over an extended and defined period of time
irrelevant	not relevant; not applicable or pertinent; not connected with or relevant to something
ISMG	instrument-specific marking guide; a tool for marking that describes the characteristics evident in student responses and aligns with the identified objectives for the assessment (see 'assessment objectives')
isolated	detached, separate, or unconnected with other things; one-off; something set apart or characterised as different in some way

Term	Explanation
J	
judge	form an opinion or conclusion about; apply both procedural and deliberative operations to make a determination
justified	sound reasons or evidence are provided to support an argument, statement or conclusion
justify	give reasons or evidence to support an answer, response or conclusion; show or prove how an argument, statement or conclusion is right or reasonable
L	
learning area	a grouping of subjects, with related characteristics, within a broad field of learning, e.g. the Arts, sciences, languages
level of confidence	the level of confidence associated with a confidence interval for an unknown population parameter is the probability that a random confidence interval will contain the parameter
local and global maxima and minima	a stationary point on the graph $y = f(x)$ of a differentiable function is a point where $f'(x) = 0$; $f(x_0)$ is a local maximum of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x near x_0; $f(x_0)$ is a global maximum of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x in the domain of f; $f(x_0)$ is a local minimum of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x near x_0; $f(x_0)$ is a global maximum of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x in the domain of f
logarithmic laws	the algebraic properties of logarithms include the rules $\log_a(xy) = \log_a(x) + \log_a(y)$, $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$, $\log_a(x^n) = n\log_a(x)$ and $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$, for any positive real numbers x, y and a
logical	rational and valid; internally consistent; reasonable; reasoning in accordance with the principles/rules of logic or formal argument; characterised by or capable of clear, sound reasoning; (of an action, decision, etc.) expected or sensible under the circumstances
logically	according to the rules of logic or formal argument; in a way that shows clear, sound reasoning; in a way that is expected or sensible
M	
make decisions	select from available options; weigh up positives and negatives of each option and consider all the alternatives to arrive at a position
manipulate	adapt or change to suit one's purpose
margin of error	the margin of error of a confidence interval of the form $f - E < p < f + E$ is E, the half-width of the confidence interval; if p is actually in the confidence interval it is the maximum difference between f and p
mathematical model	a depiction of a situation that expresses relationships using mathematical concepts and language, usually as an algebraic, diagrammatic, graphical or tabular representation

Term	Explanation
mathematical modelling	involves: • formulating a mathematical representation of a problem derived from within a real-world context • using mathematics concepts and techniques to obtain results • interpreting the results by referring back to the original problem context • revising the model (where necessary)
mental procedures	a domain of knowledge in Marzano's taxonomy, and acted upon by the cognitive, metacognitive and self-systems; sometimes referred to as 'procedural knowledge' there are three distinct phases to the acquisition of mental procedures — the cognitive stage, the associative stage, and the autonomous stage; the two categories of mental procedures are skills (single rules, algorithms and tactics) and processes (macroprocedures)
methodical	performed, disposed or acting in a systematic way; orderly; characterised by method or order; performed or carried out systematically
minimal	least possible; small, the least amount; negligible
modify	change the form or qualities of; make partial or minor changes to something
multimodal	uses a combination of at least two modes (e.g. spoken, written), delivered at the same time, to communicate ideas and information to a live or virtual audience, for a particular purpose; the selected modes are integrated so that each mode contributes significantly to the response
mutually exclusive events	two events, A and B, are mutually exclusive if there is no outcome in which both events occur; for mutually exclusive events $P(A \cup B) = P(A) + P(B)$
N	
narrow	limited in range or scope; lacking breadth of view; limited in amount; barely sufficient or adequate; restricted
nuanced	showing a subtle difference or distinction in expression, meaning, response, etc.; finely differentiated; characterised by subtle shades of meaning or expression; a subtle distinction, variation or quality; sensibility to, awareness of, or ability to express delicate shadings, as of meaning, feeling, or value
O	
objectives	see 'syllabus objectives', 'unit objectives', 'assessment objectives'
observations	data or information required to solve a mathematical problem and/or develop a mathematical model; empirical evidence
obvious	clearly perceptible or evident; easily seen, recognised or understood
optimal	best, most favourable, under a particular set of circumstances

Term	Explanation
organise	arrange, order; form as or into a whole consisting of interdependent or coordinated parts, especially for harmonious or united action
organised	systematically ordered and arranged; having a formal organisational structure to arrange, coordinate and carry out activities
outstanding	exceptionally good; clearly noticeable; prominent; conspicuous; striking
P	
parameter	a characteristic value of a particular population, such as the mean; remains constant for a particular analysis, while the values assigned to variables change; the values that allow a model to define a particular situation, e.g. m and c in the function $y = mx + c$
partial	not total or general; existing only in part; attempted, but incomplete
particular	distinguished or different from others or from the ordinary; noteworthy
Pascal's triangle	a triangular arrangement of binomial coefficients in which the n^{th} row consists of the binomial coefficients $\binom{n}{r}$; for $0 \leq r \leq n$, each interior entry is the sum of the two entries above it, and the sum of the entries in the n^{th} row is 2^n
perceptive	having or showing insight and the ability to perceive or understand; discerning (see also 'discriminating')
performance	an assessment technique that requires students to demonstrate a range of cognitive, technical, creative and/or expressive skills and to apply theoretical and conceptual understandings, through the psychomotor domain; it involves student application of identified skills when responding to a task that involves solving a problem, providing a solution or conveying meaning or intent; a performance is developed over an extended and defined period of time
persuasive	capable of changing someone's ideas, opinions or beliefs; appearing worthy of approval or acceptance; (of an argument or statement) communicating reasonably or credibly (see also 'convincing')
perusal time	time allocated in an assessment to reading items and tasks and associated assessment materials; no writing is allowed; students may not make notes and may not commence responding to the assessment in the response space/book
piece-wise function	a function which is defined by more than one formula where each formula applies to a certain interval of the main function's domain
planning time	time allocated in an assessment to planning how to respond to items and tasks and associated assessment materials; students may make notes but may not commence responding to the assessment in the response space/book; notes made during planning are not collected, nor are they graded or used as evidence of achievement

Term	Explanation
point estimates of probabilities	in statistics estimation, the use of information derived from a sample to produce an estimate of an unknown probability or population parameter; if the estimate is a single number, this number is called a point estimate; e.g. the relative frequency of a specified event in a large number of Bernoulli trials
points of inflection	a point P on the graph of $y = f(x)$ is a point of inflection if the concavity changes at P, i.e. points near P on one side of P lie above the tangent at P and points near P on the other side of P lie below the tangent at P
polished	flawless or excellent; performed with skilful ease
polynomial	an expression consisting of the sum of two or more terms, each of which is the product of a constant and a variable raised to an integral power: $ax^2 + bx + c$ is a polynomial, where a, b, and c are constants and x is a variable.
power function	a function of the form $f(x) = ax^p$ where a is any constant other than zero and p is a real number
precise	definite or exact; definitely or strictly stated, defined or fixed; characterised by definite or exact expression or execution
precision	accuracy; exactness; exact observance of forms in conduct or actions
predict	give an expected result of an upcoming action or event; suggest what may happen based on available information
probability density function	the probability density function of a continuous random variable is a function that describes the relative likelihood that the random variable takes a particular value; formally, if $p(x)$ is the probability density of the continuous random variable X, then the probability that X takes a value in some interval $[a, b]$ is given by $\int_a^b p(x)\,dx$
procedural vocabulary	instructional terms used in a mathematical context (e.g. calculate, convert, determine, identify, justify, show, sketch, solve, state).
product	an assessment technique that focuses on the output or result of a process requiring the application of a range of cognitive, physical, technical, creative and/or expressive skills, and theoretical and conceptual understandings; a product is developed over an extended and defined period of time
product rule	relates the derivative of the product of two functions to the functions and their derivatives if $h(x) = f(x)g(x)$, then $h'(x) = f(x)g'(x) + f'(x)g(x)$, and in Leibniz notation: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$
proficient	well advanced or expert in any art, science or subject; competent, skilled or adept in doing or using something

Term	Explanation
project	an assessment technique that focuses on a problem-solving process requiring the application of a range of cognitive, technical and creative skills and theoretical understandings; the response is a coherent work that documents the iterative process undertaken to develop a solution and includes written paragraphs and annotations, diagrams, sketches, drawings, photographs, video, spoken presentations, physical prototypes and/or models; a project is developed over an extended and defined period of time
propose	put forward (e.g. a point of view, idea, argument, suggestion) for consideration or action
prove	use a sequence of steps to obtain the required result in a formal way
psychomotor procedures	a domain of knowledge in Marzano's taxonomy, and acted upon by the cognitive, metacognitive and self-systems; these are physical procedures used to negotiate daily life and to engage in complex physical activities; the two categories of psychomotor procedures are skills (foundational procedures and simple combination procedures) and processes (complex combination procedures)
purposeful	having an intended or desired result; having a useful purpose; determined; resolute; full of meaning; significant; intentional
Q	
QCE	Queensland Certificate of Education
quadratic formula	if $ax^2 + bx + c = 0$ with $a \neq 0$, then $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ this formula for the roots is called the quadratic formula
qualitative statements	statements relating to the quality or qualities; statements of a non-numerical nature
quantile	a quantile t_α for a continuous random variable X is defined by $P(X > t_\alpha) = \alpha$, where $0 < \alpha < 1$; the median m of X is the quantile corresponding to $\alpha = 0.5$: $P(X > m) = 0.5$
quantitative analysis	use of mathematical measurements and calculations, including statistics, to analyse the relationships between variables; may include use of the correlation coefficient, coefficient of determination, simple residual analysis or outlier analysis
Queensland Certificate of Education	QCE
quotient rule	relates the derivative of the quotient of two functions to the functions and their derivatives; if $h(x) = \frac{f(x)}{g(x)}$, then $h'(x) = \frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$, and in Leibniz notation: $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Term	Explanation
R	
radian measure	the radian measure θ of an angle in a sector of a circle is defined by $\theta = \frac{\ell}{r}$ where r is the radius and ℓ is the arc length, thus an angle whose degree measure is 180 has radian measure π
random variable	a numerical quantity whose value depends on the outcome of a chance experiment, e.g. the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin, the number of tonnes of wheat produced in Australia in a year
realise	create or make (e.g. a musical, artistic or dramatic work); actualise; make real or concrete; give reality or substance to
reasonable	endowed with reason; having sound judgment; fair and sensible; based on good sense; average; appropriate, moderate
reasonableness of solutions	to justify solutions obtained with or without technology using everyday language, mathematical language or a combination of both; may be applied to calculations to check working, or to questions that require a relationship back to the context
reasoned	logical and sound; based on logic or good sense; logically thought out and presented with justification; guided by reason; well-grounded; considered
recall	remember; present remembered ideas, facts or experiences; bring something back into thought, attention or into one's mind
recognise	identify or recall particular features of information from knowledge; identify that an item, characteristic or quality exists; perceive as existing or true; be aware of or acknowledge
refined	developed or improved so as to be precise, exact or subtle
reflect on	think about deeply and carefully
rehearsed	practised; previously experienced; practised extensively
related	associated with or linked to
relevance	being related to the matter at hand
relevant	bearing upon or connected with the matter in hand; to the purpose; applicable and pertinent; having a direct bearing on
repetitive	containing or characterised by repetition, especially when unnecessary or tiresome
reporting	providing information that succinctly describes student performance at different junctures throughout a course of study
representatively sample	in this syllabus, a selection of subject matter that accurately reflects the intended learning of a topic
resolve	in the Arts, consolidate and communicate intent through a synthesis of ideas and application of media to express meaning

Term	Explanation
routine	often encountered, previously experienced; commonplace; customary and regular; well-practised; performed as part of a regular procedure, rather than for a special reason
rudimentary	relating to rudiments or first principles; elementary; undeveloped; involving or limited to basic principles; relating to an immature, undeveloped or basic form
S	
safe	secure; not risky
sample proportion	the fraction of samples which are successes i.e. $\hat{p} = \frac{x}{n}$ where x is the number of successes and n is the number of trials
second derivative test	according to the second derivative test, if $f'(x) = 0$, then $f(x)$ is a local maximum of f if $f''(x) < 0$ and $f(x)$ is a local minimum if $f''(x) > 0$
secure	sure; certain; able to be counted on; self-confident; poised; dependable; confident; assured; not liable to fail
select	choose in preference to another or others; pick out
sensitive	capable of perceiving with a sense or senses; aware of the attitudes, feelings or circumstances of others; having acute mental or emotional sensibility; relating to or connected with the senses or sensation
sequence	place in a continuous or connected series; arrange in a particular order
show	provide the relevant reasoning to support a response
significant	important; of consequence; expressing a meaning; indicative; includes all that is important; sufficiently great or important to be worthy of attention; noteworthy; having a particular meaning; indicative of something
simple	easy to understand, deal with and use; not complex or complicated; plain; not elaborate or artificial; may concern a single or basic aspect; involving few elements, components or steps
simple familiar	problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, *or* – in a context that has been a focus of prior learning. Students are not required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.

Term	Explanation
simplistic	characterised by extreme simplification, especially if misleading; oversimplified
sine rule	for a triangle of side lengths, *a*, *b* and *c* and angles *A*, *B* and *C*, the sine rule states that: $$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$
sketch	execute a drawing or painting in simple form, giving essential features but not necessarily with detail or accuracy; in mathematics, represent by means of a diagram or graph; the sketch should give a general idea of the required shape or relationship and should include features
skilful	having technical facility or practical ability; possessing, showing, involving or requiring skill; expert, dexterous; demonstrating the knowledge, ability or training to perform a certain activity or task well; trained, practised or experienced
skilled	having or showing the knowledge, ability or training to perform a certain activity or task well; having skill; trained or experienced; showing, involving or requiring skill
solve	find an answer to, explanation for, or means of dealing with (e.g. a problem); work out the answer or solution to (e.g. a mathematical problem); obtain the answer/s using algebraic, numerical and/or graphical methods
sophisticated	of intellectual complexity; reflecting a high degree of skill, intelligence, etc.; employing advanced or refined methods or concepts; highly developed or complicated
specific	clearly defined or identified; precise and clear in making statements or issuing instructions; having a special application or reference; explicit, or definite; peculiar or proper to something, as qualities, characteristics, effects, etc.
sporadic	happening now and again or at intervals; irregular or occasional; appearing in scattered or isolated instances
standard deviation	a measure of the variability or spread of a dataset; it indicates the degree to which the individual data values are spread around their mean; the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is: $$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$
statement	a sentence or assertion
straightforward	without difficulty; uncomplicated; direct; easy to do or understand

Term	Explanation
structure	*verb* give a pattern, organisation or arrangement to; construct or arrange according to a plan; *noun* in languages, arrangement of words into larger units, e.g. phrases, clauses, sentences, paragraphs and whole texts, in line with cultural, intercultural and textual conventions
structured	organised or arranged so as to produce a desired result
subject	a branch or area of knowledge or learning defined by a syllabus; school subjects are usually based in a discipline or field of study (see also 'course')
subject matter	the subject-specific body of information, mental procedures and psychomotor procedures that are necessary for students' learning and engagement within that subject
substantial	of ample or considerable amount, quantity, size, etc.; of real worth or value; firmly or solidly established; of real significance; reliable; important, worthwhile
substantiated	established by proof or competent evidence
subtle	fine or delicate in meaning or intent; making use of indirect methods; not straightforward or obvious
successful	achieving or having achieved success; accomplishing a desired aim or result
succinct	expressed in few words; concise; terse; characterised by conciseness or brevity; brief and clear
sufficient	enough or adequate for the purpose
suitable	appropriate; fitting; conforming or agreeing in nature, condition, or action
summarise	give a brief statement of a general theme or major point/s; present ideas and information in fewer words and in sequence
summative assessment	assessment whose major purpose is to indicate student achievement; summative assessments contribute towards a student's subject result
superficial	concerned with or comprehending only what is on the surface or obvious; shallow; not profound, thorough, deep or complete; existing or occurring at or on the surface; cursory; lacking depth of character or understanding; apparent and sometimes trivial
supported	corroborated; given greater credibility by providing evidence
sustained	carried on continuously, without interruption, or without any diminishing of intensity or extent
syllabus	a document that prescribes the curriculum for a course of study

Term	Explanation
syllabus objectives	outline what the school is required to teach and what students have the opportunity to learn; described in terms of actions that operate on the subject matter; the overarching objectives for a course of study (see also ‘unit objectives’, ‘assessment objectives’)
symbolise	represent or identify by a symbol or symbols
synthesise	combine different parts or elements (e.g. information, ideas, components) into a whole, in order to create new understanding
systematic	done or acting according to a fixed plan or system; methodical; organised and logical; having, showing, or involving a system, method, or plan; characterised by system or method; methodical; arranged in, or comprising an ordered system
T	
technical vocabulary	terms that have a precise mathematical meaning (e.g. categorical data, chain rule, decimal fraction, imaginary number, log laws, linear regression, sine rule, whole number); may include everyday words used in a mathematical context (e.g. capacity, differentiate, evaluate, integrate, order, property, sample, union)
test	take measures to check the quality, performance or reliability of something
thorough	carried out through, or applied to the whole of something; carried out completely and carefully; including all that is required; complete with attention to every detail; not superficial or partial; performed or written with care and completeness; taking pains to do something carefully and completely
thoughtful	occupied with, or given to thought; contemplative; meditative; reflective; characterised by or manifesting thought
topic	a division of, or sub-section within a unit; all topics/sub-topics within a unit are interrelated
U	
unclear	not clear or distinct; not easy to understand; obscure
understand	perceive what is meant by something; grasp; be familiar with (e.g. an idea); construct meaning from messages, including oral, written and graphic communication
uneven	unequal; not properly corresponding or agreeing; irregular; varying; not uniform; not equally balanced
unfamiliar	not previously encountered; situations or materials that have not been the focus of prior learning experiences or activities
uniform discrete random variable	a variable whose possible values have equal probability of occurrence; if there are n possible values, the probability of occurrence of any one of them is $\frac{1}{n}$
unit	a defined amount of subject matter delivered in a specific context or with a particular focus; it includes unit objectives particular to the unit, subject matter and assessment direction

Term	Explanation
unit objectives	drawn from the syllabus objectives and contextualised for the subject matter and requirements of a particular unit; they are assessed at least once in the unit (see also 'syllabus objectives', 'assessment objectives')
unrelated	having no relationship; unconnected
use	operate or put into effect; apply knowledge or rules to put theory into practice
V	
vague	not definite in statement or meaning; not explicit or precise; not definitely fixed, determined or known; of uncertain, indefinite or unclear character or meaning; not clear in thought or understanding; couched in general or indefinite terms; not definitely or precisely expressed; deficient in details or particulars; thinking or communicating in an unfocused or imprecise way
valid	sound, just or well-founded; authoritative; having a sound basis in logic or fact (of an argument or point); reasonable or cogent; able to be supported; legitimate and defensible; applicable
variable	*adjective* apt or liable to vary or change; changeable; inconsistent; (readily) susceptible or capable of variation; fluctuating, uncertain; *noun* in mathematics, a symbol, or the quantity it signifies, that may represent any one of a given set of number and other objects
variance	the variance $Var(X)$ of a random variable X is a measure of the spread of its distribution; if X is discrete, $Var(X) = \sum_i p_i(x_i - \mu)^2$, where $\mu = E(X)$ is the expected value; if X is continuous, $Var(X) = \int_{-\infty}^{\infty}(x-\mu)^2 p(x)dx$
variety	a number or range of things of different kinds, or the same general class, that are distinct in character or quality; (of sources) a number of different modes or references
vertical line test	a test to determine whether a relation is a function; a relation between two real variables x and y is a function and $y = f(x)$ for some function f, if (and only if) each vertical line, i.e. each line parallel to the y-axis, intersects the graph of the relation in at most one point
W	
wide	of great range or scope; embracing a great number or variety of subjects, cases, etc.; of full extent
with expression	in words, art, music or movement, conveying or indicating feeling, spirit, character, etc.; a way of expressing or representing something; vivid, effective or persuasive communication

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8 Version history

Version	Date of change	Update
1.1	June 2017	Minor amendments to ISMGs Unit 4: Topic 5 formula updated to $\frac{\hat{p}-p}{\sqrt{(\hat{p}(1-\hat{p})/n)}}$
1.2	July 2018	Minor editorial amendments to assumed knowledge, prior learning or experience
		Minor amendment to syllabus objective 6 explanatory paragraph
		Minor amendments to pedagogical and conceptual frameworks
		Unit 1 — minor amendments to: • subject matter • assessment guidance
		• Minor amendments to subject matter across Units 2 and 3
		Summative internal assessment 1 (IA1) — minor amendments to: • description • conditions • ISMG
		Summative internal assessments 2 and 3 (IA2 & IA3) — minor amendments to: • description • degree of difficulty definitions
		Summative external assessment (EA): Examination (50%) — minor amendments to: • degree of difficulty definitions • conditions
		Glossary updates

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