General Mathematics 2025 v1.0

General senior syllabus

January 2024

Queensland Government
Queensland Curriculum & Assessment Authority

For all Queensland schools

ISBN

Electronic version: 978-1-74378-300-9

Queensland Curriculum & Assessment Authority
PO Box 307 Spring Hill QLD 4004 Australia

Phone: (07) 3864 0299
Email: office@qcaa.qld.edu.au
Website: www.qcaa.qld.edu.au

Contents

Queensland syllabuses for senior subjects

In Queensland, a syllabus for a senior subject is an official 'map' of a senior school subject. A syllabus's function is to support schools in delivering the Queensland Certificate of Education (QCE) system through high-quality and high-equity curriculum and assessment.

Syllabuses are based on design principles developed from independent international research about how excellence and equity are promoted in the documents teachers use to develop and enliven the curriculum.

Syllabuses for senior subjects build on student learning in the Prep to Year 10 Australian Curriculum and include General, General (Extension), Senior External Examination (SEE), Applied, Applied (Essential) and Short Course syllabuses.

More information about syllabuses for senior subjects is available at www.qcaa.qld.edu.au/senior/senior-subjects and in the 'Queensland curriculum' section of the *QCE and QCIA policy and procedures handbook*.

Teaching, learning and assessment resources will support the implementation of a syllabus for a senior subject. More information about professional resources for senior syllabuses is available on the QCAA website and via the QCAA Portal.

Course overview

Rationale

Mathematics is a unique and powerful intellectual discipline that is used to investigate patterns, order, generality and uncertainty. It is a way of thinking in which problems are explored and solved through observation, reflection and logical reasoning. It uses a concise system of communication, with written, symbolic, spoken and visual components. Mathematics is creative, requires initiative and promotes curiosity in an increasingly complex and data-driven world. It is the foundation of all quantitative disciplines.

To prepare students with the knowledge, skills and confidence to participate effectively in the community and the economy requires the development of skills that reflect the demands of the 21st century. Students undertaking Mathematics will develop their critical and creative thinking, oral and written communication, information & communication technologies (ICT) capability, ability to collaborate, and sense of personal and social responsibility — ultimately becoming lifelong learners who demonstrate initiative when facing a challenge. The use of technology to make connections between mathematical theory, practice and application has a positive effect on the development of conceptual understanding and student disposition towards mathematics.

Mathematics teaching and learning practices range from practising essential mathematical routines to develop procedural fluency, through to investigating scenarios, modelling the real world, solving problems and explaining reasoning. When students achieve procedural fluency, they carry out procedures flexibly, accurately and efficiently. When factual knowledge and concepts come to mind readily, students are able to make more complex use of knowledge to successfully formulate, represent and solve mathematical problems. Problem-solving helps to develop an ability to transfer mathematical skills and ideas between different contexts. This assists students to make connections between related concepts and adapt what they already know to new and unfamiliar situations. With appropriate effort and experience, through discussion, collaboration and reflection of ideas, students should develop confidence and experience success in their use of mathematics.

The major domains of mathematics in General Mathematics are Number and algebra, Measurement and geometry, Statistics and Networks and matrices, building on the content of the P–10 Australian Curriculum. Learning reinforces prior knowledge and further develops key mathematical ideas, including rates and percentages, concepts from financial mathematics, linear and non-linear expressions, sequences, the use of matrices and networks to model and solve authentic problems, the use of trigonometry to find solutions to practical problems, and the exploration of real-world phenomena in statistics.

General Mathematics is designed for students who want to extend their mathematical skills beyond Year 10 but whose future studies or employment pathways do not require calculus. It incorporates a practical approach that equips learners for their needs as future citizens. Students will learn to ask appropriate questions, map out pathways, reason about complex solutions, set up models and communicate in different forms. They will experience the relevance of mathematics to their daily lives, communities and cultural backgrounds. They will develop the ability to understand, analyse and take action regarding social issues in their world. When students gain skill and self-assurance, when they understand the content and when they evaluate their success by using and transferring their knowledge, they develop a mathematical mindset.

Syllabus objectives

The syllabus objectives outline what students have the opportunity to learn.

1. **Recall mathematical knowledge.**

   When students recall mathematical knowledge, they recognise features of remembered information. They recognise relevant concepts, rules, definitions, techniques and algorithms.

2. **Use mathematical knowledge.**

   When students use mathematical knowledge, they put into effect relevant concepts, rules, definitions, techniques and algorithms. They perform calculations with and without technology.

3. **Communicate mathematical knowledge.**

   When students communicate mathematical knowledge, they use mathematical language (terminology, symbols, conventions and representations) and everyday language. They organise and present information in graphical and symbolic form, and describe and represent mathematical models.

4. **Evaluate the reasonableness of solutions.**

   When students evaluate the reasonableness of solutions, they interpret their mathematical results in the context of the situation and reflect on whether the problem has been solved. They verify results by using estimation skills and checking calculations, with and without technology. They make an appraisal by assessing implications, strengths and limitations of solutions and/or models, and use this to consider if alternative methods or refinements are required.

5. **Justify procedures and decisions.**

   When students justify procedures and decisions, they explain their mathematical reasoning in detail. They make relationships evident, logically organise mathematical arguments, and provide reasons for choices made and conclusions reached.

6. **Solve mathematical problems.**

   When students solve mathematical problems, they analyse the context of the problem to translate information into mathematical forms. They make decisions about the concepts, techniques and technology to be used and apply these to develop a solution. They develop, refine and use mathematical models, where applicable.

Designing a course of study in General Mathematics

Syllabuses are designed for teachers to make professional decisions to tailor curriculum and assessment design and delivery to suit their school context and the goals, aspirations and abilities of their students within the parameters of Queensland's senior phase of learning.

The syllabus is used by teachers to develop curriculum for their school context. The term *course of study* describes the unique curriculum and assessment that students engage with in each school context. A course of study is the product of a series of decisions made by a school to select, organise and contextualise subject matter, integrate complementary and important learning, and create assessment tasks in accordance with syllabus specifications.

It is encouraged that, where possible, a course of study is designed such that teaching, learning and assessment activities are integrated and enlivened in an authentic setting.

Course structure

General Mathematics is a General senior syllabus. It contains four QCAA-developed units from which schools develop their course of study.

Each unit has been developed with a notional time of 55 hours of teaching and learning, including assessment.

Students should complete Unit 1 and Unit 2 before beginning Units 3 and 4. Units 3 and 4 are studied as a pair.

More information about the requirements for administering senior syllabuses is available in the 'Queensland curriculum' section of the *QCE and QCIA policy and procedures handbook*.

Curriculum

Senior syllabuses set out only what is essential while being flexible so teachers can make curriculum decisions to suit their students, school context, resources and expertise.

Within the requirements set out in this syllabus and the *QCE and QCIA policy and procedures handbook*, schools have autonomy to decide:

- how and when subject matter is delivered
- how, when and why learning experiences are developed, and the context in which learning occurs
- how opportunities are provided in the course of study for explicit and integrated teaching and learning of complementary skills.

These decisions allow teachers to develop a course of study that is rich, engaging and relevant for their students.

Assessment

Senior syllabuses set out only what is essential while being flexible so teachers can make assessment decisions to suit their students, school context, resources and expertise.

General senior syllabuses contain assessment specifications and conditions for the assessment instruments that must be implemented with Units 3 and 4. These specifications and conditions ensure comparability, equity and validity in assessment.

Within the requirements set out in this syllabus and the *QCE and QCIA policy and procedures handbook*, schools have autonomy to decide:

- specific assessment task details
- assessment contexts to suit available resources
- how the assessment task will be integrated with teaching and learning activities
- how authentic the task will be.

In Unit 1 and Unit 2, schools:

- develop at least two but no more than four assessments
- complete at least one assessment for each unit
- ensure that each unit objective is assessed at least once.

In Units 3 and 4, schools develop three assessments using the assessment specifications and conditions provided in the syllabus.

More information about assessment in senior syllabuses is available in 'The assessment system' section of the *QCE and QCIA policy and procedures handbook*.

Subject matter

Each unit contains a unit description, unit objectives and subject matter. Subject matter is the body of information, mental procedures and psychomotor procedures (see Marzano & Kendall 2007, 2008) that are necessary for students' learning and engagement with the subject. Subject matter itself is not the specification of learning experiences but provides the basis for the design of student learning experiences.

Subject matter has a direct relationship with the unit objectives and provides statements of learning that have been constructed in a similar way to objectives.

Aboriginal perspectives and Torres Strait Islander perspectives

The QCAA is committed to reconciliation. As part of its commitment, the QCAA affirms that:

- Aboriginal peoples and Torres Strait Islander peoples are the first Australians, and have the oldest living cultures in human history
- Aboriginal peoples and Torres Strait Islander peoples have strong cultural traditions and speak diverse languages and dialects, other than Standard Australian English
- teaching and learning in Queensland schools should provide opportunities for students to deepen their knowledge of Australia by engaging with the perspectives of Aboriginal peoples and Torres Strait Islander peoples
- positive outcomes for Aboriginal students and Torres Strait Islander students are supported by successfully embedding Aboriginal perspectives and Torres Strait Islander perspectives across planning, teaching and assessing student achievement.

Guidelines about Aboriginal perspectives and Torres Strait Islander perspectives and resources for teaching are available at www.qcaa.qld.edu.au/k-12-policies/aboriginal-torres-strait-islander-perspectives.

Where appropriate, Aboriginal perspectives and Torres Strait Islander perspectives have been embedded in the subject matter.

Complementary skills

Opportunities for the development of complementary skills have been embedded throughout subject matter. These skills, which overlap and interact with syllabus subject matter, are derived from current education, industry and community expectations and encompass the knowledge, skills, capabilities, behaviours and dispositions that will help students live and work successfully in the 21st century.

These complementary skills are:

- literacy — the knowledge, skills, behaviours and dispositions about language and texts essential for understanding and conveying English language content
- numeracy — the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations, to recognise and understand the role of mathematics in the world, and to develop the dispositions and capacities to use mathematical knowledge and skills purposefully
- 21st century skills — the attributes and skills students need to prepare them for higher education, work, and engagement in a complex and rapidly changing world. These skills include critical thinking, creative thinking, communication, collaboration and teamwork, personal and social skills, and digital literacy. The explanations of associated skills are available at www.qcaa.qld.edu.au/senior/senior-subjects/general-subjects/21st-century-skills.

It is expected that aspects of literacy, numeracy and 21st century skills will be developed by engaging in the learning outlined in this syllabus. Teachers may choose to create additional explicit and intentional opportunities for the development of these skills as they design the course of study.

Additional subject-specific information

Additional subject-specific information has been included to support and inform the development of a course of study.

Assumed knowledge, prior learning or experience

Assumed knowledge refers to the subject matter that teachers can expect students to know prior to beginning this subject. Emphasis is placed on the mastery of content, ensuring key concepts or procedures are learnt fully so they will not need reteaching.

Developing mastery often involves multiple approaches to teaching and conceptualising the same mathematical concept. When students have a good understanding of a key concept or procedure, they are more easily able to make connections to related new subject matter and apply what they already know to new problems.

Subject matter from previous unit/s is assumed for subsequent unit/s.

The following is a non-exhaustive list of assumed knowledge based on the subject matter in the P–10 Australian Curriculum version 9.

- Solve problems involving percentages, rates, ratios, simple algebraic fractions and duration, including 12- and 24-hour time.
- Recognise irrational numbers in applied contexts (e.g. π).
- Round decimals to a given accuracy appropriate to the context and use appropriate rounding and estimation to check the reasonableness of solutions.
- Recognise the effect of using approximations of real numbers in repeated calculations.
- Solve problems involving very small and very large real numbers expressed in scientific notation.
- Apply the exponent laws with integer exponents and the zero-exponent, using exponent notation with numbers.
- Recognise and use variables to represent everyday formulas algebraically and substitute values into formulas to determine an unknown.
- Expand, factorise, rearrange and simplify algebraic expressions, applying the associative, commutative, identity, distributive and inverse properties.

Problem-solving and mathematical modelling

A key aspect of learning mathematics is to develop strategic competence; that is, to formulate, represent and solve mathematical problems (Kilpatrick, Swafford & Bradford 2001). As such, problem-solving is a focus of mathematics education research, curriculum and teaching (Sullivan 2011). This focus is not to the exclusion of routine exercises, which are necessary for practising, attaining mastery and being able to respond automatically. But mathematics education in the 21st century goes beyond this to include innovative problems that are complex, unfamiliar and non-routine (Mevarech & Kramarski 2014).

Problem-solving in mathematics can be set in purely mathematical contexts or real-world contexts. When set in the real world, problem-solving in mathematics involves mathematical modelling.

Problem-solving

Problem-solving is required when a task or goal has limiting conditions placed upon it or an obstacle blocking the path to a solution (Marzano & Kendall 2007). It involves:

- knowledge of the relevant details
- using generalisations and principles to identify, define and interpret the problem
- mental computation and estimation
- critical, creative and lateral thinking
- creating or choosing a strategy
- making decisions
- testing, monitoring and evaluating solutions.

Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples without understanding. Through problem-solving, students will make connections between mathematics topics, across the curriculum and with the real world, and see the value and usefulness of mathematics. Problems may be real-world or abstract, and presented to students as issues, statements or questions that may require them to use primary or secondary data.

Mathematical modelling

Mathematical modelling begins from an assumption that mathematics is everywhere in the world around us — a challenge is to identify where it is present, access it and apply it productively. Models are developed in order to better understand real-world phenomena, to make predictions and answer questions. A mathematical model depicts a situation by expressing relationships using mathematical concepts and language. It refers to the set of simplifying assumptions (such as the relevant variables or the shape of something); the set of assumed relationships between variables; and the resulting representation (such as a formula) that can be used to generate an answer (Stacey 2015).

Mathematical modelling involves:

- formulating a mathematical representation of a problem derived from within a real-world context
- using mathematics concepts and techniques to obtain results
- interpreting the results by referring back to the original problem context
- revising the model (where necessary) (Geiger, Faragher & Goos 2010).

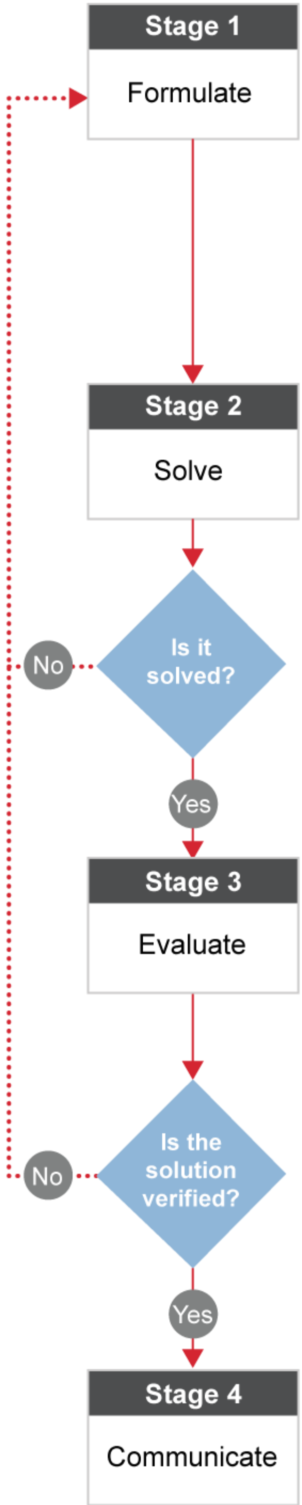
Through developing and applying mathematical models, students cumulatively become real-world problem-solvers. Ultimately, this means that not only can they productively address problems set by others, but also that they develop the ability to identify and address problems and answer questions that matter to them.

The following section outlines an approach to problem-solving and mathematical modelling.[1] Problems must be real-world, and can be presented to students as issues, statements or questions that may require them to use primary or secondary data.

---

[1] A wide variety of frameworks for problem-solving and modelling exist in mathematics education literature. The approach outlined here aligns with and is informed by other approaches, such as Polya (1957) in *How to Solve It: A new aspect of mathematical method* (1957), the Australian Curriculum (ACARA 2015a) *Statistical investigation process*, the OECD/PISA Mathematics framework (OECD 2015, 2003) and *A framework for success in implementing mathematical modelling in the secondary classroom* (Stillman et al. 2007). For further reading see Blum et al. (2007); Kaiser et al. (2011); and Stillman et al. (2013).

Figure 1: An approach to problem-solving and mathematical modelling



Once students understand what the problem is asking, they must design a plan to solve the problem. Students translate the problem into a mathematically purposeful representation by first determining the applicable mathematical knowledge that is required to make progress with the problem. Important assumptions, variables and observations are identified and justified, based on the logic of a proposed solution and/or model.

In mathematical modelling, formulating a model involves the process of mathematisation — moving from the real world to the mathematical world.

Students select and apply mathematical knowledge previously learnt to solve the problem. Possible approaches are wide-ranging and include synthesising and refining existing models, and generating and testing hypotheses with primary or secondary data and information, to produce a complete solution.

Solutions can be found using algebraic, graphic, arithmetic and/or numeric methods, with and/or without technology.

Once a possible solution has been achieved, students need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. They verify their results and evaluate the reasonableness of the solution to the problem in relation to the original issue, statement or question.

This involves exploring the strengths and limitations of the solution and/or model. Where necessary, this will require going back through the process to further refine the solution and/or model. In mathematical modelling, students must check that the output of their model provides a complete solution to the real-world problem it has been designed to address.

This stage emphasises the importance of methodological rigour and the fact that problem-solving and mathematical modelling is not usually linear and involves an iterative process.

The development of solutions and/or models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated and justified clearly and fully. Students communicate findings logically and concisely using mathematical and everyday language. They draw conclusions, discussing the results, strengths and limitations of the solution and/or model. Students could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem.

Approaches to problem-solving and mathematical modelling in the classroom

When teaching problem-solving and mathematical modelling, teachers should consider *teaching for* and *learning through* problem-solving and mathematical modelling. When *teaching for*, students are taught the specific mathematical rules, definitions, procedures, problem-solving strategies and critical elements of the model that are needed to solve a given problem. When *learning through*, students are presented with problems to solve, but must apply the knowledge and skills they have previously been taught to solve it. By solving these problems, students are able to develop new mathematical understanding and skills. This requires an explicit and connected approach to teaching problem-solving and mathematical modelling that necessitates fluency of critical facts and processes at each step.

The following describes three different approaches to teaching problem-solving and mathematical modelling[2] along the continua between *teaching for* and *learning through*:

Approach	Description	*Teaching for or learning through*
Dependent	The teacher explicitly demonstrates and teaches the concepts and techniques required to solve the problem, and/or develop a mathematical model. This usually involves students solving (stage 2) and evaluating and verifying (stage 3).	*Teaching for*
Guided	The teacher influences the choice of concepts and techniques, and/or model that students use to solve the problem. Guidance is provided and all stages of the approach are used.	Moving towards *learning through*
Independent	The teacher cedes control and students work independently, choosing their own solution and/or model, and working at their own level of mathematics. The independent approach is the most challenging.	*Learning through*

These approaches are not mutually exclusive. An independent approach (*learning through*) might be undertaken as an extension of a dependent or guided activity that students have previously undertaken (*teaching for*). Students need to have attained the relevant foundational understanding and skills before working independently during a problem-solving and modelling task. This capacity needs to be built over time through the course of study with teachers closely monitoring student progress.

[2] Based on Galbraith (1989).

Strategies for retaining and recalling information for assessment

The following practices[3] can support preparation for senior assessment in General Mathematics.

The spacing effect

The spacing effect draws on research about forgetting and learning curves. By recalling and revisiting information at intervals, rather than at the end of a study cycle, students remember a greater percentage of the information with a higher level of accuracy. Exposing students to information and materials numerous times over multiple spaced intervals solidifies long-term memory, positively affecting retention and recall.

Teachers should plan teaching and learning sequences that allow time to revisit previously taught information and skills at several intervals. These repeated learning opportunities also provide opportunities for teachers to provide formative feedback to students.

The retrieval effect

The retrieval effect helps students to practise remembering through quick, regular, low-stakes questioning or quizzes that exercise their memories and develop their ability to engage in the deliberate act of recalling information. This has been shown to be more effective at developing long-term memories than activities that require students to search through notes or other resources.

Students may see an inability to remember as an obstacle, but they should be encouraged to understand that this is an opportunity for learning to take place. By trying to recall information, students exercise or strengthen their memory and may also identify gaps in their learning. The more difficult the retrieval practice, the better it can be for long-term learning.

Interleaving

Interleaving involves interspersing the concepts, categories, skills or types of questions that students focus on in class or revision. This is in contrast to blocking, in which these elements are grouped together in a block of time. For example, for concepts A, B and C:

- Blocking A A A A A B B B B B C C C C C
- Interleaving A B C B C A B A C A C B C A B

Studies have found that interleaving in instruction or revision produces better long-term recall of subject matter. Interleaving also ensures that spacing occurs, as instances of practice are spread out over time.

Additionally, because exposure to one concept is interleaved with exposure to another, students have more opportunities to distinguish between related concepts. This highlighting of differences may explain why studies have found that interleaving enhances inductive learning, where participants use exemplars to develop an understanding of broader concepts or categories. Spacing without interleaving does not appear to benefit this type of learning.

Interleaving can seem counterintuitive — even in studies where interleaving enhanced learning, participants often felt that they had learnt more with blocked study. Despite this, their performance in testing indicated greater learning through the interleaving approach.

[3] Based on Agarwal, Roediger, McDaniel & McDermott (2020); Birnbaum, Kornell, Ligon Bjork & Bjork (2013); Carpenter & Agarwal (2020); Chen, Paas & Sweller (2021); Ebbinghaus (1885); Rohrer (2012); Taylor & Rohrer (2010).

Reporting

General information about determining and reporting results for senior syllabuses is provided in the 'Determining and reporting results' section of the *QCE and QCIA policy and procedures handbook*.

Reporting standards

Reporting standards are summary statements that describe typical performance at each of the five levels (A–E).

A
The student recalls, uses and communicates comprehensive mathematical knowledge drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar, complex familiar and complex unfamiliar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar, complex familiar and complex unfamiliar situations.
B
The student recalls, uses and communicates thorough mathematical knowledge drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar and complex familiar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar and complex situations.
C
The student recalls, uses and communicates mathematical knowledge drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar situations.
D
The student recalls, uses and communicates partial mathematical knowledge drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar situations. The student sometimes evaluates the reasonableness of solutions, sometimes justifies procedures and decisions, and solves some mathematical problems in simple familiar situations.
E
The student recalls, uses and communicates isolated mathematical knowledge drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar situations. The student rarely evaluates the reasonableness of solutions, and infrequently justifies procedures and decisions in simple familiar situations.

Determining and reporting results

Unit 1 and Unit 2

Schools make judgments on individual assessment instruments using a method determined by the school. They may use the reporting standards or develop an instrument-specific marking guide (ISMG). Marks are not required for determining a unit result for reporting to the QCAA.

The unit assessment program comprises the assessment instrument/s designed by the school to allow the students to demonstrate the unit objectives. The unit judgment of A–E is made using reporting standards.

Schools report student results for Unit 1 and Unit 2 to the QCAA as satisfactory (S) or unsatisfactory (U). Where appropriate, schools may also report a not rated (NR).

Units 3 and 4

Schools mark each of the three internal assessment instruments implemented in Units 3 and 4 using ISMGs.

Schools report a provisional mark by criterion to the QCAA for each internal assessment.

Once confirmed by the QCAA, these results will be combined with the result of the external assessment developed and marked by the QCAA.

The QCAA uses these results to determine each student's subject result as a mark out of 100 and as an A–E.

Units

Unit 1: Money, measurement, algebra and linear equations

In Unit 1, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Consumer arithmetic
- Topic 2: Shape and measurement
- Topic 3: Similarity and scale
- Topic 4: Algebra
- Topic 5: Linear equations and their graphs.

Consumer arithmetic reviews the concepts of rate and percentage change in the context of earning and managing money and provides an opportunity for the use of spreadsheets. Shape and measurement and Similarity and scale extend the knowledge and skills students developed in the P–10 Australian Curriculum with Pythagoras' theorem, the concept of similarity and problems involving simple and compound geometric shapes. Students apply these skills in a range of practical contexts, including those involving three-dimensional shapes. Algebra further develops students' mastery of algebraic techniques and skills, and Linear equations and their graphs uses linear equations and straight-line graphs, to model and analyse practical situations.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Consumer arithmetic

Sub-topic: Applications of rates, percentages and use of spreadsheets (14 hours)

- Understand the meaning of rates and percentages.
- Calculate weekly, fortnightly or monthly wages from an annual salary, and wages from an hourly rate, including situations involving overtime and other allowances and earnings based on commission or piecework.
- Calculate income support payments based on government allowances and pensions.
- Prepare a personal budget for a given income, taking into account fixed and discretionary spending.
- Compare prices and values using the unit cost method.
- Apply percentage increase or decrease in various contexts, e.g. inflation of costs and wages, percentage mark-ups and discounts, percentage profit and loss, GST, simple and compound interest.
- Use currency exchange rates to convert between the Australian dollar and foreign currencies.
- Calculate the dividend paid on a portfolio of shares, given the percentage dividend or dividend paid per share, and compare share values by calculating a price-to-earnings ratio.
- Use a spreadsheet to display examples of the above computations when multiple or repeated computations are required, e.g. preparing a wage sheet displaying the weekly earnings of workers in an organisation, preparing a budget, investigating the potential cost of owning and operating a car over a year.

Topic 2: Shape and measurement

Sub-topic: Pythagoras' theorem (3 hours)

- Understand and use Pythagoras' theorem to solve practical problems in two dimensions and simple applications in three dimensions.
  - $c^2 = a^2 + b^2$ where c is length of the hypotenuse and a and b are lengths of the two perpendicular sides

Sub-topic: Mensuration (9 hours)

- Calculate perimeters, P, of standard two-dimensional objects in practical situations, including circles, sectors of circles, triangles, rectangles, trapeziums, parallelograms and composites.
  - circle: $C = 2\pi r$, where C is circumference and r is radius
  - sector of circle: $P = 2r + \frac{\theta}{180}\pi r$ where θ is central angle and r is radius
- Calculate areas, A, of standard two-dimensional objects in practical situations, including circles, sectors of circles, triangles, rectangles, parallelograms, trapeziums and composites.
  - circle: $A = \pi r^2$ where r is radius
  - sector of circle: $A = \frac{\theta}{360}\pi r^2$ where θ is central angle and r is radius

- triangle: $A = \frac{1}{2}bh$ where b is base length and h is perpendicular height
- parallelogram: $A = bh$ where b is base length and h is perpendicular height
- trapezium: $A = \frac{1}{2}(a + b)h$ where a and b are parallel lengths and h is perpendicular height

- Calculate surface areas, S, of standard three-dimensional objects in practical situations, including rectangular prisms, cylinders, pyramids, cones, spheres and composites.
  - cylinder: $S = 2\pi rh + 2\pi r^2$ where r is radius and h is perpendicular height
  - cone: $S = \pi rs + \pi r^2$ where r is radius and s is slant height
  - sphere: $S = 4\pi r^2$ where r is radius
- Calculate volumes, V, and capacities of standard three-dimensional objects in practical situations, including rectangular prisms, cylinders, pyramids, cones, spheres and composites.
  - prism: $V = Ah$ where A is base area and h is perpendicular height
  - cylinder: $V = \pi r^2 h$ where r is radius and h is perpendicular height
  - pyramid: $V = \frac{1}{3}Ah$ where A is base area and h is perpendicular height
  - cone: $V = \frac{1}{3}\pi r^2 h$ where r is radius and h is perpendicular height
  - sphere: $V = \frac{4}{3}\pi r^3$ where r is radius
- Solve practical problems involving shape and measurement.

Topic 3: Similarity and scale

Sub-topic: Similar figures and scale factors (8 hours)

- Understand the conditions for similarity of two-dimensional figures, including similar triangles.
- Use the scale factor for two similar figures to solve linear scaling problems.
- Determine measurements from scale drawings (e.g. maps and building plans) to solve problems.
- Determine a scale factor and use it to solve scaling problems, e.g. calculating lengths and areas of similar figures; and calculating surface areas, volumes and capacities of similar solids.

Topic 4: Algebra

Sub-topic: Linear and non-linear relationships (8 hours)

- Substitute numerical values into linear and simple non-linear algebraic expressions, and evaluate.
- Find the value of a pronumeral in linear and simple non-linear equations given the values of the other pronumerals, transposing equations where necessary.
- Use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities.

Topic 5: Linear equations and their graphs

Sub-topic: Linear equations (5 hours)

- Solve linear equations, including equations with variables on both sides and equations with rational solutions.
- Develop a linear equation from a description in words.
- Solve practical problems involving linear equations.

Sub-topic: Straight-line graphs (8 hours)

- Understand and use the slope-intercept form of a linear function, $y = mx + c$ where m is slope (gradient) and c is *y*-intercept.
- Construct a straight-line graph using a linear function of the form, $y = mx + c$ where m is slope (gradient) and c is *y*-intercept.
- Determine the slope (gradient), *x*-intercept and *y*-intercept of a straight line from both its equation and its graph.
- Interpret, in context, the slope (gradient) and intercept of a linear function used to model and analyse a practical situation.
- Construct and analyse a straight-line graph to model a given linear relationship, e.g. modelling the cost of filling a fuel tank of a car against the number of litres of petrol required.

Unit 2: Applications of linear equations and trigonometry, matrices and univariate data analysis

In Unit 2, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Applications of linear equations and their graphs
- Topic 2: Applications of trigonometry
- Topic 3: Matrices
- Topic 4: Univariate data analysis 1
- Topic 5: Univariate data analysis 2.

Applications of linear equations and their graphs uses simultaneous linear equations, piece-wise linear graphs and step graphs to model and solve practical problems. Applications of trigonometry extends students' knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression and bearings in navigation. Matrices introduces the new concept of matrices for storing information and modelling and solving problems. Univariate data analysis develops students' ability to organise and summarise univariate data in the context of conducting a statistical investigation.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Applications of linear equations and their graphs

Sub-topic: Simultaneous linear equations and their applications (6 hours)

- Solve a pair of simultaneous linear equations, algebraically using substitution and elimination, and graphically.
- Solve practical problems involving simultaneous linear equations.

Sub-topic: Piece-wise linear graphs and step graphs (5 hours)

- Sketch piece-wise linear graphs and step graphs.
- Interpret piece-wise linear graphs and step graphs used to model practical situations.

Topic 2: Applications of trigonometry

Sub-topic: Applications of trigonometry (12 hours)

- Understand and use the trigonometric ratios to find the size of an unknown angle, θ , or the length of an unknown side in a right-angled triangle.
  - $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
  - $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
  - $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
- Calculate the area of a non-right-angled triangle and solve related practical problems.
  - $\text{area} = \dfrac{1}{2}bc\sin A$, given two sides, b and c, and an included angle, A
  - Heron's rule: $\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$, given three sides, a, b and c
- Solve two-dimensional problems involving a non-right-angled triangle, $\triangle ABC$, with sides, a, b and c, and corresponding angles, A, B and C.
  - sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ (ambiguous case excluded)
  - cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$
- Solve two-dimensional practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of true bearings.

Topic 3: Matrices

Sub-topic: Matrices and matrix arithmetic (10 hours)

- Use matrices for storing and displaying information that can be presented in rows and columns, e.g. tables, databases, links in social or road networks.
- Recognise different types of matrices, including row matrix, column matrix, square matrix, zero matrix and identity matrix, and determine the size of the matrix.
- Perform matrix addition, subtraction and multiplication by a scalar.
- Perform matrix multiplication manually up to 3×3 matrices but not limited to square matrices.
- Determine the power of a matrix using technology with matrix arithmetic capabilities when appropriate.
- Use matrices, including matrix products and powers of matrices, to model and solve problems, e.g. costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can communicate with each other via a third person.

Topic 4: Univariate data analysis 1

Sub-topic: Making sense of data relating to a single statistical variable (12 hours)

- Understand the meaning of univariate data.
- Classify a statistical variable as categorical or numerical.
- Classify a categorical variable as ordinal or nominal and use tables and pie, bar and column charts to organise and display the data, e.g. ordinal: income level (high, medium, low); nominal: place of birth (Australia, overseas).
- Classify a numerical variable as discrete or continuous, e.g. discrete: the number of people in a room; continuous: the temperature in degrees Celsius.
- Select, construct and justify an appropriate graphical display to describe the distribution of a numerical dataset, including dot plot, stem-and-leaf plot, column chart and histogram.
- Describe a graphical display in terms of the number of modes, shape (symmetric versus positively or negatively skewed), measures of centre and spread, and outliers, and interpret this information in the context of the data.
- Understand and calculate the mean, median, mode, range and interquartile range (IQR) of a dataset, with and without technology.
  - mean: $\bar{x} = \frac{\sum x}{n}$
  - median: $\left(\frac{n+1}{2}\right)^{\text{th}}$ data value
- Understand and calculate the (sample) standard deviation, s_x, of a dataset, using technology only.
- Use statistics as measures of centre and spread of a data distribution, being aware of their limitations.

Topic 5: Univariate data analysis 2

Sub-topic: Comparing data for a single numerical variable across two or more groups (10 hours)

- Construct and use parallel box plots, including identifying possible outliers, to compare datasets in terms of median, spread (range and IQR) and outliers to interpret and communicate the differences observed in the context of the data.
  - outliers (identifying): $Q_1 - 1.5 \times \text{IQR} \leq x \leq Q_3 + 1.5 \times \text{IQR}$ where Q_1 is lower quartile and Q_3 is upper quartile
- Compare datasets in terms of mean, median, range, IQR and standard deviation, interpret the differences observed in the context of the data, and report the findings in a systematic and concise manner.

Unit 3: Bivariate data and time series analysis, sequences and Earth geometry

In Unit 3, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Bivariate data analysis 1
- Topic 2: Bivariate data analysis 2
- Topic 3: Time series analysis
- Topic 4: Growth and decay in sequences
- Topic 5: Earth geometry and time zones.

Bivariate data analysis introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a method for analysing linear associations. Time series analysis continues students' study of statistics by introducing them to the concepts and techniques of time series analysis. Growth and decay in sequences employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4. Earth geometry and time zones offers an opportunity to use contexts relevant to students.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Bivariate data analysis 1

Sub-topic: Identifying and describing associations between two categorical variables (5 hours)

- Understand the meaning of bivariate data.
- Construct two-way frequency tables and determine the associated row and column sums and percentages.
- Use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association.
- Understand an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data.

Sub-topic: Identifying and describing associations between two numerical variables (6 hours)

- Identify the explanatory variable and the response variable.
- Construct and use a scatterplot to identify the association between two numerical variables.
- Describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak).
- Calculate Pearson's correlation coefficient, r, from raw data using technology, and interpret it to quantify the strength of a linear association.
- Calculate the coefficient of determination, R^2, from raw data using technology, and use it to assess the strength of a linear association in terms of the explained variation.
- Use the correlation coefficient, r, to determine the coefficient of determination, R^2, and vice versa.

Topic 2: Bivariate data analysis 2

Sub-topic: Fitting a linear model to numerical data (6 hours)

- Model a linear relationship by using technology to fit a least-squares line to the data, in the form of $y = mx + c$ where m is slope (gradient) and c is *y*-intercept.
- Understand and use $m = r\frac{s_y}{s_x}$ and $c = \bar{y} - m\bar{x}$ to determine the equation of a least-squares line, where m is slope (gradient), r is correlation coefficient, s_y is (sample) standard deviation of y values, s_x is (sample) standard deviation of x values, c is *y*-intercept, $\bar{y}$ is mean of y values and $\bar{x}$ is mean of x values.
- Construct a residual plot and use it to assess the appropriateness of fitting a linear model to the data.
- Interpret the *y*-intercept and slope (gradient) of the fitted line.
- Distinguish between interpolation and extrapolation.
- Use the equation of the least-squares line to make predictions.
- Recognise and explain the potential dangers of extrapolation.

Sub-topic: Association and causation (6 hours)

- Recognise and explain that an observed association between two variables (categorical and/or numerical) does not necessarily mean that there is a causal relationship between them.
- Identify and communicate possible non-causal explanations for an association, including coincidence or the influence of another variable.
- Solve practical problems by identifying, analysing and describing associations between two variables (categorical and/or numerical).

Topic 3: Time series analysis

Sub-topic: Describing and interpreting patterns in time series data (3 hours)

- Construct and use time series plots.
- Describe time series plots by identifying features, including trend (long-term direction, e.g. increasing/decreasing), seasonality (systematic, calendar-related movements) and irregular fluctuations (unsystematic, short-term fluctuations).

Sub-topic: Analysing time series data (8 hours)

- Smooth time series data by calculating a simple moving average using the mean or median for an odd number of data, including the use of spreadsheets.
- Deseasonalise a time series by calculating the seasonal indices using the average percentage method, including the use of spreadsheets.
- Fit a least-squares line to model long-term trends in time series data.
- Solve practical problems that involve the analysis of time series data.

Topic 4: Growth and decay in sequences

Sub-topic: The arithmetic sequence (5 hours)

- Use recursion to generate an arithmetic sequence.
- Display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations.
- Use the rule for the n^{th} term of an arithmetic sequence.
  - $t_n = t_1 + (n-1)d$ where t_n is n^{th} term, t_1 is first term, n is term number and d is common difference
- Use arithmetic sequences to model and analyse practical situations involving linear growth or decay, e.g. analysing a simple interest loan or investment, calculating a taxi fare based on the flag fall and the charge per kilometre, calculating the value of an item using the straight-line method of depreciation.

Sub-topic: The geometric sequence (6 hours)

- Use recursion to generate a geometric sequence.
- Display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations.
- Use the rule for the n^{th} term of a geometric sequence.
  - $t_n = t_1 r^{(n-1)}$ where t_n is n^{th} term, t_1 is first term, n is term number and r is common ratio.
- Use geometric sequences to model and analyse practical situations involving geometric growth and decay (use of logarithms not required), e.g. modelling the growth of a bacterial population that doubles in size each hour, calculating the value of an item using the diminishing-value method of depreciation.

Topic 5: Earth geometry and time zones

Sub-topic: Locations on the Earth (5 hours)

- Understand the meaning of great circles.
- Understand the meaning of angles of latitude and longitude (in decimal degrees, and degrees and minutes) in relation to the equator and the prime meridian respectively.
- Locate positions on Earth's surface given latitude and longitude, e.g. using a globe, map, GPS and other digital technologies.
- State latitude and longitude for positions on Earth's surface, e.g. investigating a map of Australia and locating boundary positions for Aboriginal peoples' and Torres Strait Islander peoples' language groups, Australian landmarks or local land boundaries.
- Calculate angular distance and distance between two places on Earth on the same meridian.
  - $D = 111.2 \times \text{angular distance}$ where D is distance in kilometres
- Calculate angular distance and distance between two places on Earth on the same parallel of latitude.
  - $D = 111.2\cos\theta \times \text{angular distance}$ where D is distance in kilometres and θ is latitude
- Solve practical problems involving latitude, longitude, angular distance and distance.

Sub-topic: Time zones (5 hours)

- Understand the meaning of Greenwich Mean Time (GMT), International Date Line and Coordinated Universal Time (UTC).
- Understand the link between longitude and time.
- Determine the number of degrees of longitude for a given time difference.
- Calculate time differences between two places on Earth.
- Solve practical problems involving time zones, making allowances for daylight saving where necessary, e.g. seasonal time systems used by Aboriginal peoples and Torres Strait Islander peoples, making phone calls, broadcasting events, travelling, preparing an itinerary.

Unit 4: Investing and networking

In Unit 4, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Loans, investments and annuities 1
- Topic 2: Loans, investments and annuities 2
- Topic 3: Graphs and networks
- Topic 4: Networks and decision mathematics 1
- Topic 5: Networks and decision mathematics 2.

Loans, investments and annuities aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments. Graphs and networks introduces students to the language of graphs and the ways in which graphs, represented as a collection of points and interconnecting lines, can be used to model and analyse everyday situations such as a rail or social network. Networks and decision mathematics uses networks to model and aid decision-making in practical situations.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Topic 1: Loans, investments and annuities 1

Sub-topic: Compound interest loans and investments (6 hours)

- Use a recurrence relation to model a compound interest loan or investment.
  - $A_{n+1} = rA_n$ where A_{n+1} is total amount at the beginning of the $(n+1)^{th}$ period, A_n is total amount at the beginning of the n^{th} period, and $r = 1 + i$ where i is interest rate per compounding period.
- Use the compound interest formula to model a compound interest loan or investment.
  - $A = P(1+i)^n$ where A is total amount after n compounding periods, P is principal, i is interest rate per compounding period and n is number of compounding periods.
- Calculate the effective annual rate of interest, $i_{effective}$, and use the results to compare interest paid on loans or investments when interest is paid or charged for different compounding periods, including daily, monthly, quarterly and six-monthly.
  - $i_{effective} = (1+i)^k - 1$ where i is interest rate per compounding period and k is number of compounding periods per year
- Solve practical problems involving compound interest loans or investments, including determining the total amount of the loan or investment, total interest, principal, interest rate per year and per compounding period, and the effect of the interest rate and number of compounding periods on the total amount.

Sub-topic: Present value of ordinary annuities (6 hours)

- Use a recurrence relation to model the present value of an ordinary annuity, e.g. reducing balance loan or retirement pension with periodic payments where interest is calculated before the periodic payment is made.
  - $A_{n+1} = rA_n - d$ where A_{n+1} is total amount at the beginning of the $(n+1)^{th}$ period, A_n is total amount at the beginning of the n^{th} period, d is periodic payment, and $r = n + 1$ where i is interest rate per compounding period.
- Use the present value annuity formula to model the present value of an ordinary annuity, e.g. reducing balance loan or retirement pension where interest is calculated before the periodic payment is made.
  - $A_{PV} = d\left(\frac{1-(1+i)^{-n}}{i}\right)$ where A_{PV} is total amount after n compounding periods, d is periodic payment, i is interest rate per compounding period and n is number of compounding periods.
- Solve practical problems involving the present value of an ordinary annuity, including determining the total amount of the annuity, periodic payment, total payments and total interest.

Topic 2: Loans, investments and annuities 2

Sub-topic: Perpetuities and future value of ordinary annuities (8 hours)

- Use a recurrence relation to model the future value of an ordinary annuity, e.g. compound interest investment with periodic payments where interest is calculated before the periodic payment is made.
  - $A_{n+1} = rA_n + d$ where A_{n+1} is total amount at the beginning of the $(n+1)^{th}$ period, A_n is total amount at the beginning of the n^{th} period, d is periodic payment and $r = i + 1$ where i is interest rate per compounding period.
- Use the future value annuity formula to model the future value of an ordinary annuity, e.g. compound interest investment with periodic payments where interest is calculated before the periodic payment is made.
  - $A_{FV} = d\left(\frac{(1+i)^n - 1}{i}\right)$ where A_{FV} is total amount after n compounding periods, d is periodic payment, i is interest rate per compounding period and n is number of compounding periods.
- Solve practical problems involving the future value of an ordinary annuity, including determining the total amount of the annuity, periodic payment, total payments and total interest.
- Use the perpetuity formula, $A = \frac{d}{i}$ where A is total amount, d is periodic payment and i is interest rate per compounding period.
- Solve practical problems involving perpetuities, including determining the total amount of the perpetuity, periodic payment and interest rate per compounding period.

Topic 3: Graphs and networks

Sub-topic: Graphs, associated terminology and the adjacency matrix (4 hours)

- Understand the meaning of graph, vertex (node), edge (arc), loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph (digraph) weighted graph and network.
- Construct a network diagram to represent practical situations, e.g. tracks connecting camp sites in a national park, a social network, a transport network with one-way streets, the results of a round-robin sporting competition.
- Construct an adjacency matrix from a given graph or digraph.
- Construct a graph or digraph from a given adjacency matrix.

Sub-topic: Planar graphs, paths and cycles (8 hours)

- Understand the meaning of planar graph and face.
- Apply Euler's formula to solve problems relating to planar graphs.
  - $v+f-e=2$ where v is number of vertices, f is number of faces and e is number of edges
- Understand the meaning of walk, trail, path, open walk, open trail, open path, closed walk, closed trail (circuit), closed path (cycle), connected graph and bridge.
- Solve practical problems to determine the shortest path between two vertices in a weighted graph (by trial-and-error methods only).
- Understand the meaning of Eulerian trail, semi-Eulerian graph, Eulerian circuit and Eulerian graph, and the conditions for their existence.
- Solve practical problems involving semi-Eulerian graphs and Eulerian graphs.
- Understand the meaning of Hamiltonian path, semi-Hamiltonian graph, Hamiltonian cycle and Hamiltonian graph.
- Solve practical problems involving semi-Hamiltonian graphs and Hamiltonian graphs (by trial-and-error methods only).

Topic 4: Networks and decision mathematics 1

Sub-topic: Trees and minimum connector problems (4 hours)

- Understand the meaning of tree, spanning tree and minimum spanning tree.
- Determine a minimum spanning tree in a weighted connected graph.
- Solve practical problems involving minimum spanning trees, e.g. minimising the length of cable needed to provide power from a single power station to substations in several towns.

Sub-topic: Project planning and scheduling using critical path analysis (CPA) (8 hours)

- Construct a project network diagram (activity on arc) to represent the durations and interdependencies of activities that must be completed during the project (excluding dummy activities).
- Use forward and backward scanning to determine the earliest starting time (EST) and latest starting time (LST) for each activity in the project.
- Use ESTs and LSTs to locate the critical path/s for a project.
- Use the critical path to determine the minimum time for a project to be completed.
- Calculate float times for non-critical activities.
- Solve small-scale practical problems involving critical path analysis.

Topic 5: Networks and decision mathematics 2

Sub-topic: Flow networks (4 hours)

- Understand the meaning of source node, sink node, cut, minimum cut and maximum flow.
- Use a flow network diagram to identify a cut.
- Determine the capacity of a cut.
- Solve small-scale practical problems involving flow networks (up to 8 possible cuts), including determining the minimum cut and the maximum flow.

Sub-topic: Assigning order and the Hungarian algorithm (7 hours)

- Use a bipartite graph and its tabular or matrix form to represent possible assignments for an allocation problem.
- Determine the optimum (minimum and maximum) assignment/s for small-scale practical problems by inspection.
- Use the Hungarian algorithm (3×3 up to 5×5 square matrices) to determine the optimum (minimum and maximum) assignment/s for larger practical problems.

Assessment

Internal assessment 1: Problem-solving and modelling task (20%)

Students provide a written response to a specific mathematical investigative scenario or context using subject matter from at least one of the topics in Unit 3 or Unit 4. While students may undertake some research, it is not the focus of this task.

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

This task requires students to:

- independently respond to a specific mathematical investigative scenario or context that highlights a real-life application of mathematics
- use relevant stimulus material involving the selected subject matter
- address all the stages of the problem-solving and mathematical modelling approach
- respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Conditions

- Students will use 3 hours of class time and their own time out of class to develop their response.
- This is an individual task.
- Data may be provided or collected individually or collected in groups.
- Appendixes can include raw data, repeated calculations, evidence of authentication and student notes (appendixes are not marked).
- Students must use technology, e.g. scientific calculator, graphics calculator, spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation or word processing.

Response requirements

Written: up to 10 A4 pages, up to 2000 words

Mark allocation

Criterion	Assessment objectives	Marks
Formulate	1, 5	4
Solve	1, 2, 6	7
Evaluate	4, 5	5
Communicate	3, 5	4
	Total marks:	20

Instrument-specific marking guide

Formulate	Marks
The student response has the following characteristics:	
• justified statements of important assumptions • justified statements of important observations • justified mathematical translation of important aspects of the task	3–4
• statement of a relevant assumption • statement of a relevant observation • mathematical translation of an aspect of the task.	1–2
The student response does not match any of the descriptors above.	0

Solve	Marks
The student response has the following characteristics:	
• accurate use of mathematical knowledge for important aspects of the task • efficient use of technology • a complete solution	6–7
• use of mathematical knowledge for an important aspect of the task • use of technology • substantial progress towards a solution	4–5
• simplistic use of mathematical knowledge relevant to the task • simplistic use of technology • progress towards a solution	2–3
• inappropriate use of mathematical knowledge or technology.	1
The student response does not match any of the descriptors above.	0

Evaluate	Marks
The student response has the following characteristics:	
• verified results • justified statements about the reasonableness of the solution by considering the assumptions • justified statements about the reasonableness of the solution by considering the observations • justified statements of relevant strengths of the solution • justified statements of relevant limitations of the solution	4–5
• a verified result • statement about the reasonableness of the solution by considering an assumption or observation • statement of a relevant strength or relevant limitation of the solution	2–3
• statement about the reasonableness of a result or the solution • statement of a strength or limitation.	1
The student response does not match any of the descriptors above.	0

Communicate	Marks
The student response has the following characteristics:	
• correct use of appropriate mathematical language • logical organisation of the response, which can be read independently of the task sheet • justification of decisions using mathematical reasoning	3–4
• use of some appropriate mathematical language • adequate organisation of the response • statement of a decision using mathematical reasoning.	1–2
The student response does not match any of the descriptors above.	0

Internal assessment 2: Examination — short response (15%)

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

The teacher provides an examination that:

- asks students to respond to a number of unseen short response questions
- representatively samples subject matter from any three of the five topics in Unit 3
- may ask students to respond using single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams
  - use assumed knowledge from Units 1 and 2.

Question specifications

The examination must be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	60%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	20%	These questions can focus on any of the objectives.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	20%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience.

Conditions

- This is an individual supervised task.
- The task may be delivered in two consecutive sessions only if
  - questions in each session are unseen
  - teaching or feedback is not provided between the sessions.
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The teacher must provide the QCAA General Mathematics formula book.
- Students
  - are required to use technology
  - must not bring notes into the examination.

Mark allocation

Criterion	Assessment objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5, 6	15
	Total marks:	15

Instrument-specific marking guide

Foundational knowledge and problem-solving	Cut-off	Marks
The student response has the following characteristics:		
• consistently correct recall and use of mathematical knowledge; authoritative and accurate communication of mathematical knowledge; astute evaluation of the reasonableness of solutions; use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical knowledge to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations	> 93%	15
	> 87%	14
• correct recall and use of mathematical knowledge; clear communication of mathematical knowledge; considered evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical knowledge to solve problems in simple familiar, complex familiar and complex unfamiliar situations	> 80%	13
	> 73%	12
• thorough recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and application of mathematical knowledge to solve problems in simple familiar and complex familiar situations	> 67%	11
	> 60%	10
• recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of some solutions; some use of mathematical reasoning; and some application of mathematical knowledge to make progress towards solving problems in simple familiar situations	> 53%	9
	> 47%	8
• some recall and use of mathematical knowledge; and basic communication of mathematical knowledge	> 40%	7
	> 33%	6
• infrequent recall and use of mathematical knowledge; and basic communication of some mathematical knowledge	> 27%	5
	> 20%	4
• isolated recall and use of mathematical knowledge; and partial communication of rudimentary mathematical knowledge	> 13%	3
	> 7%	2
• isolated and inaccurate recall and use of mathematical knowledge; and disjointed and unclear communication of mathematical knowledge.	> 0%	1
The student response does not match any of the descriptors above.		0

Internal assessment 3: Examination — short response (15%)

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

The teacher provides an examination that:

- asks students to respond to a number of unseen short response questions
- representatively samples subject matter from any three of the five topics in Unit 4
- may ask students to respond using single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams
  - use assumed knowledge from Units 1, 2 and 3.

Question specifications

The examination must be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	60%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	20%	These questions can focus on any of the objectives.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	20%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience.

Conditions

- This is an individual supervised task.
- The task may be delivered in two consecutive sessions only if
  - questions in each session are unseen
  - teaching or feedback is not provided between the sessions.
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The teacher must provide the QCAA General Mathematics formula book.
- Students
  - are required to use technology
  - must not bring notes into the examination.

Mark allocation

Criterion	Assessment objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5, 6	15
	Total marks:	15

Instrument-specific marking guide

Foundational knowledge and problem-solving	Cut-off	Marks
The student response has the following characteristics:		
• consistently correct recall and use of mathematical knowledge; authoritative and accurate communication of mathematical knowledge; astute evaluation of the reasonableness of solutions; use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical knowledge to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations	> 93%	15
	> 87%	14
• correct recall and use of mathematical knowledge; clear communication of mathematical knowledge; considered evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical knowledge to solve problems in simple familiar, complex familiar and complex unfamiliar situations	> 80%	13
	> 73%	12
• thorough recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and application of mathematical knowledge to solve problems in simple familiar and complex familiar situations	> 67%	11
	> 60%	10
• recall and use of mathematical knowledge; communication of mathematical knowledge; evaluation of the reasonableness of some solutions; some use of mathematical reasoning; and some application of mathematical knowledge to make progress towards solving problems in simple familiar situations	> 53%	9
	> 47%	8
• some recall and use of mathematical knowledge; and basic communication of mathematical knowledge	> 40%	7
	> 33%	6
• infrequent recall and use of mathematical knowledge; and basic communication of some mathematical knowledge	> 27%	5
	> 20%	4
• isolated recall and use of mathematical knowledge; and partial communication of rudimentary mathematical knowledge	> 13%	3
	> 7%	2
• isolated and inaccurate recall and use of mathematical knowledge; and disjointed and unclear communication of mathematical knowledge.	> 0%	1
The student response does not match any of the descriptors above.		0

External assessment: Examination — combination response (50%)

External assessment is developed and marked by the QCAA. The external assessment in General Mathematics is common to all schools and administered under the same conditions, at the same time, on the same day.

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

This examination:

- consists of two papers: Paper 1 — simple familiar, Paper 2 — complex familiar and complex unfamiliar
- asks students to respond to a number of unseen short response questions relating to Units 3 and 4
- may ask students to respond using
  - multiple choice
  - single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams
  - use assumed knowledge from Units 1 and 2.

Paper 1

- Weighted at 30%
- Contains short response simple familiar questions, including multiple choice

Paper 2

- Weighted at 20%
- Contains short response complex familiar and complex unfamiliar questions

This examination will be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	60%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	20%	These questions can focus on any of the objectives.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	20%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience.

Conditions

Paper 1

- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The QCAA provides the QCAA General Mathematics formula book.
- Students
  - may use a handheld QCAA-approved scientific calculator
  - must not bring notes or other resources into the examination.

Paper 2

- Time allowed
  - Perusal time: 5 minutes
  - Working time: 90 minutes
- The QCAA provides the QCAA General Mathematics formula book.
- Students
  - may use a handheld QCAA-approved scientific calculator
  - must not bring notes or other resources into the examination.

Glossary

The syllabus glossary is available at www.qcaa.qld.edu.au/downloads/senior-qce/common/snr_glossary_cognitive_verbs.pdf.

References

Agarwal, PK, Roediger, HL, McDaniel, MA & McDermott, KB 2020, 'How to use retrieval practice to improve learning', *Retrieval Practice*, http://pdf.retrievalpractice.org/RetrievalPracticeGuide.pdf.

Australian Curriculum and Assessment Authority (ACARA) 2015, *Australian Curriculum Senior Secondary Curriculum: General Mathematic*s, version 7.5, v7-5.australiancurriculum.edu.au/seniorsecondary/mathematics/general-mathematics/curriculum/seniorsecondary.

—— 2015a, *Australian Curriculum Senior Secondary Curriculum: General Mathematics Glossary*, version 7.5, https://www.australiancurriculum.edu.au/senior-secondary-curriculum/mathematics/general-mathematics/glossary/.

Birnbaum, MS, Kornell, N, Ligon Bjork, E & Bjork, RA 2013, 'Why interleaving enhances inductive learning: The roles of discrimination and retrieval', *Memory & Cognition*, vol. 41, pp. 392–402, https://doi.org/10.3758/s13421-012-0272-7.

Blum, W, Galbraith, PL, Henn, HW & Niss, M 2007, *Modelling and applications in mathematics education,* Springer, New York.

Carpenter, SK & Agarwal, PK 2020, 'How to use spaced retrieval practice to boost learning', *Retrieval Practice,* http://pdf.retrievalpractice.org/SpacingGuide.pdf.

Chen, O, Paas, F, & Sweller, J 2021, 'Spacing and interleaving effects require distinct theoretical bases: A systematic review testing the cognitive load and discriminative-contrast hypotheses', *Educational Psychology Review*, vol. 33, pp. 1499–1522, https://doi.org/10.1007/s10648-021-09613-w.

Ebbinghaus, H 1885, *Memory: A contribution to experimental psychology*, HA Ruger & CE Bussenius (trans.), Columbia University, New York, 1913, https://psychclassics.yorku.ca/Ebbinghaus/index.htm.

Galbraith, P 1989, 'From applications to modelling', in D Blane & M Evans (eds), *Mathematical modelling for the senior years*, The Mathematical Association of Victoria, Parkville, pp. 78–86.

Geiger, V, Faragher, R & Goos, M 2010, 'CAS-enabled technologies as "agents provocateurs" in teaching and learning mathematical modelling in secondary school classrooms', *Mathematics Education Research Journal*, vol. 22, no. 2, pp. 48–68, doi.org/10.1007/BF03217565.

Goos, M 2014, 'Mathematics classroom assessment', *Encyclopedia of Mathematics Education*, Springer, Dordrecht, pp. 413–417.

Goos, M Geiger, V & Dole, S 2012, 'Auditing the numeracy demands of the middle years curriculum', *Mathematics Education: Expanding horizons — Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia*, Mathematics Education Research Group of Australasia, Singapore, pp. 314–321.

Grønmo, LS, Lindquist, M, Arora, A & Mullis, IVS 2015, 'TIMSS 2015 Mathematics Framework', *TIMSS 2015 Assessment Frameworks*, International Study Center, Boston, timssandpirls.bc.edu/timss2015/frameworks.html#.

Kaiser, G, Blum, W, Ferri, RB & Stillman, G (eds) 2011, *Trends in teaching and learning of mathematical modelling: ICTMA14,* International perspectives on the teaching and learning of mathematical modelling, vol. 1, Springer, Vancouver.

Kilpatrick, J, Swafford, J, & Bradford, F (eds) 2001, *Adding It Up: Helping children learn mathematics*, National Academies Press, Washington, DC.

Marzano, RJ & Kendall, JS 2008, *Designing and Assessing Educational Objectives: Applying the new taxonomy*, Corwin Press, USA.

—— 2007, *The New Taxonomy of Educational Objectives*, 2nd edn, Corwin Press, USA.

Mevarech, Z, & Kramarski, B 2014, *Critical Maths for Innovative Societies: The role of metacognitive pedagogies*, OECD Publishing, Paris.

Norton, S, & O'Connor, BR 2016, *Literature review for senior syllabus revisions: Mathematics*, Queensland Curriculum and Assessment Authority, Brisbane.

OECD 2015, *PISA 2015 Mathematics Framework*, OECD Publishing, Paris.

—— 2012, 'Numeracy', in *Literacy, Numeracy and Problem Solving in Technology-Rich Environments — Framework for the OECD Survey of Adult Skills*, OECD, doi.org/10.1787/9789264128859-en.

—— 2003, *PISA 2003 Assessment Framework*, OECD Publishing, Paris.

Polya, G 1957, *How to Solve It: A new aspect of mathematical method*, 2nd edn, Princeton University Press, NJ.

Rohrer, D 2012, 'Interleaving helps students distinguish among similar concepts', *Educational Psychology Review*, vol. 24, pp. 355–367, http://dx.doi.org/10.1007/s10648-012-9201-3.

Shafer, MC & Foster, S 1997, 'What's up on the web: The changing face of assessment', *Principled Practice in Mathematics and Science Education: Fall 1997,* vol. 1, no. 2, pp. 1–8, ncisla.wceruw.org/publications/index.html#newsletters.

Stacey, K. 2015, 'The Real World and the Mathematical World', in Stacey, K & Turner, R (eds), *Assessing Mathematical Literacy: The PISA experience*, Springer, Switzerland, pp. 57–84, doi.org/10.1007/978-3-319-10121-7_3.

Steen, LA 2001, 'The case for quantitative literacy', in *Mathematics and Democracy: The case for quantitative literacy*, National Council on Education and the Disciplines, Princeton, NJ, pp. 1–22.

Stillman, G, Galbraith, P, Brown, J & Edwards, I 2007, 'A framework for success in implementing mathematical modelling in the secondary classroom', *Mathematics: Essential research, essential practice*, vol. 2, pp. 688–697.

Stillman, G, Kaiser, G, Blum, W and Brown, JP (eds) 2013, *Teaching Mathematical Modelling: Connecting to research and practice,* Springer, Vancouver.

Sullivan, P 2011, *Teaching Mathematics: Using research-informed strategies*, ACER Press, Camberwell, Vic.

Taylor, K & Rohrer, D 2010, 'The effects of interleaved practice', *Applied Cognitive Psychology*, vol. 24, issue 6, pp. 837–848, https://psycnet.apa.org/doi/10.1002/acp.1598.

Verhage, H, & de Lange, J 1997, 'Mathematics education and assessment', *Pythagoras*, vol. 42, pp. 14–20.

Webb, DC 2009, 'Designing professional development for assessment', *Educational Designer*, vol. 1, no. 2, pp. 1–26, www.educationaldesigner.org/ed/volume1/issue2/article6/.

White, P, Sullivan, P, Warren, E & Quinlan, C 2000, 'To investigate or not to investigate? The use of content-specific open-ended tasks', *The Australian Mathematics Teacher*, vol. 56, no. 2, pp. 6–9, aamt.edu.au/Journals/Journals-Index/The-Australian-Mathematics-Teacher/AMT-56-2-6.

Version history

Version	Date of change	Information
1.0	January 2024	Released for familiarisation and planning (with implementation starting in 2025)