General Mathematics 2019 v1.2

General Senior Syllabus

This syllabus is for implementation with Year 11 students in 2019.

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For all Queensland schools

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1 Course overview

1.1 Introduction

1.1.1 Rationale

Mathematics is a unique and powerful intellectual discipline that is used to investigate patterns, order, generality and uncertainty. It is a way of thinking in which problems are explored and solved through observation, reflection and logical reasoning. It uses a concise system of communication, with written, symbolic, spoken and visual components. Mathematics is creative, requires initiative and promotes curiosity in an increasingly complex and data-driven world. It is the foundation of all quantitative disciplines.

To prepare students with the knowledge, skills and confidence to participate effectively in the community and the economy requires the development of skills that reflect the demands of the 21st century. Students undertaking Mathematics will develop their critical and creative thinking, oral and written communication, information & communication technologies (ICT) capability, ability to collaborate, and sense of personal and social responsibility — ultimately becoming lifelong learners who demonstrate initiative when facing a challenge. The use of technology to make connections between mathematical theory, practice and application has a positive effect on the development of conceptual understanding and student disposition towards mathematics.

Mathematics teaching and learning practices range from practising essential mathematical routines to develop procedural fluency, through to investigating scenarios, modelling the real world, solving problems and explaining reasoning. When students achieve procedural fluency, they carry out procedures flexibly, accurately and efficiently. When factual knowledge and concepts come to mind readily, students are able to make more complex use of knowledge to successfully formulate, represent and solve mathematical problems. Problem-solving helps to develop an ability to transfer mathematical skills and ideas between different contexts. This assists students to make connections between related concepts and adapt what they already know to new and unfamiliar situations. With appropriate effort and experience, through discussion, collaboration and reflection of ideas, students should develop confidence and experience success in their use of mathematics.

The major domains of mathematics in General Mathematics are Number and algebra, Measurement and geometry, Statistics and Networks and matrices, building on the content of the P–10 Australian Curriculum. Learning reinforces prior knowledge and further develops key mathematical ideas, including rates and percentages, concepts from financial mathematics, linear and non-linear expressions, sequences, the use of matrices and networks to model and solve authentic problems, the use of trigonometry to find solutions to practical problems, and the exploration of real-world phenomena in statistics.

General Mathematics is designed for students who want to extend their mathematical skills beyond Year 10 but whose future studies or employment pathways do not require calculus. It incorporates a practical approach that equips learners for their needs as future citizens. Students will learn to ask appropriate questions, map out pathways, reason about complex solutions, set up models and communicate in different forms. They will experience the relevance of mathematics to their daily lives, communities and cultural backgrounds. They will develop the ability to understand, analyse and take action regarding social issues in their world. When students gain skill and self-assurance, when they understand the content and when they evaluate their success by using and transferring their knowledge, they develop a mathematical mindset.

Assumed knowledge, prior learning or experience

Assumed knowledge refers to the subject matter that teachers can expect students to know prior to beginning each unit. Emphasis is placed on the mastery of content, ensuring key concepts or procedures are learnt fully so they will not need reteaching.

Developing mastery often involves multiple approaches to teaching and conceptualising the same mathematical concept. When students have a good understanding of a key concept or procedure, they are more easily able to make connections to related new subject matter and apply what they already know to new problems.

Subject matter from previous unit/s is assumed for subsequent unit/s.

The following is a non-exhaustive list of assumed knowledge from the P–10 Australian Curriculum that must be learnt or revised and maintained as required:

- solve a range of problems using percentages, rates and ratios, surface area and volume, Pythagoras' theorem, simple algebraic fractions, linear and quadratic equations
- understand the connection between algebraic and graphical representations, using appropriate technology when necessary
- calculate and compare measures of central tendency (mean, median and mode) and measures of spread; determine quartiles, interquartile range (IQR) and range
- construct and interpret box plots and use them to compare datasets; compare shapes of box plots to corresponding histograms and dot plots
- use scatter plots to investigate and comment on relationships between two numerical variables
- understand bivariate numerical data where the independent variable is time
- solve right-angled triangle problems, using trigonometric ratios
- solve simultaneous equations
- construct back-to-back stem-and-leaf plots and histograms
- solve linear equations
- understand the difference between numerical and categorical variables
- solve basic problems involving simple and compound interest.

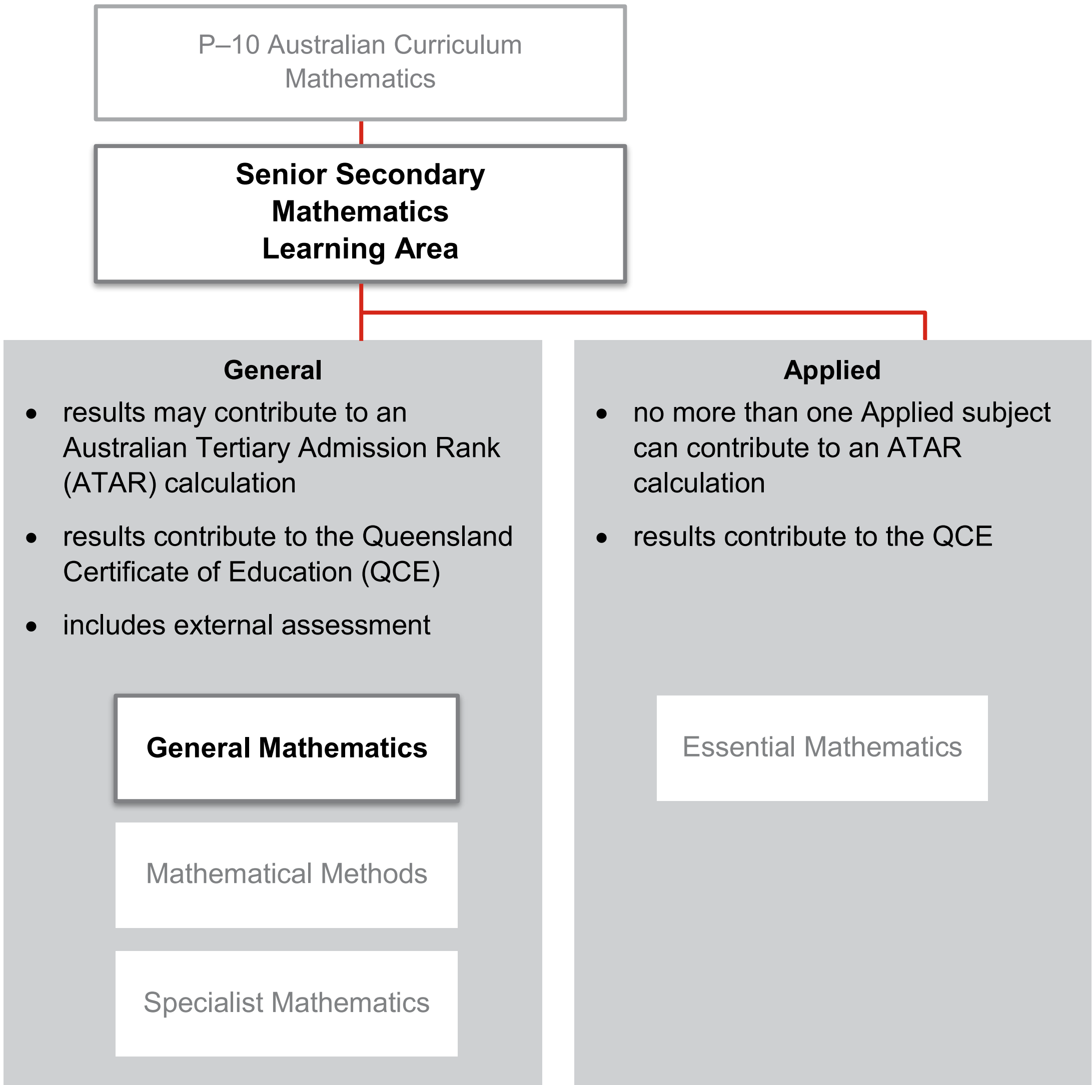
Pathways

General Mathematics is a General subject suited to students who are interested in pathways beyond school that lead to tertiary studies, vocational education or work. A course of study in General Mathematics can establish a basis for further education and employment in the fields of business, commerce, education, finance, IT, social science and the arts.

1.1.2 Learning area structure

All learning areas build on the P–10 Australian Curriculum.

Figure 1: Learning area structure



1.1.3 Course structure

General Mathematics is a course of study consisting of four units. Subject matter, learning experiences and assessment increase in complexity from Units 1 and 2 to Units 3 and 4 as students develop greater independence as learners.

Units 1 and 2 provide foundational learning, which allows students to experience all syllabus objectives and begin engaging with the course subject matter. Students should complete Units 1 and 2 before beginning Unit 3. It is recommended that Unit 3 be completed before Unit 4.

Units 3 and 4 consolidate student learning. Only the results from Units 3 and 4 will contribute to ATAR calculations.

Figure 2 outlines the structure of this course of study.

Each unit has been developed with a notional time of 55 hours of teaching and learning, including assessment.

Figure 2: Course structure

General Mathematics

Unit 1
Money, measurement and relations

- Topic 1: Consumer arithmetic
- Topic 2: Shape and measurement
- Topic 3: Linear equations and their graphs

Assessment
Formative internal assessment/s

Unit 2
Applied trigonometry, algebra, matrices and univariate data

- Topic 1: Applications of trigonometry
- Topic 2: Algebra and matrices
- Topic 3: Univariate data analysis

Assessment
Formative internal assessment/s

Students should have opportunities in Units 1 and 2 to experience and respond to the types of assessment they will encounter in Units 3 and 4.

For reporting purposes, schools should develop at least *one* assessment per unit, with a maximum of *four* assessments across Units 1 and 2.

Unit 3
Bivariate data, sequences and change, and Earth geometry

- Topic 1: Bivariate data analysis
- Topic 2: Time series analysis
- Topic 3: Growth and decay in sequences
- Topic 4: Earth geometry and time zones

Assessment
Summative internal assessment 1:
Problem-solving and modelling task (20%)

Summative internal assessment 2:
Examination (15%)

Unit 4
Investing and networking

- Topic 1: Loans, investments and annuities
- Topic 2: Graphs and networks
- Topic 3: Networks and decision mathematics

Assessment
Summative internal assessment 3:
Examination (15%)

Summative external assessment:
Examination (50%)

1.2 Teaching and learning

1.2.1 Syllabus objectives

The syllabus objectives outline what students have the opportunity to learn. Assessment provides evidence of how well students have achieved the objectives.

Syllabus objectives inform unit objectives, which are contextualised for the subject matter and requirements of the unit. Unit objectives, in turn, inform the assessment objectives, which are further contextualised for the requirements of the assessment instruments. The number of each objective remains constant at all levels, i.e. Syllabus objective 1 relates to Unit objective 1 and to Assessment objective 1 in each assessment instrument.

Syllabus objectives are described in terms of actions that operate on the subject matter. Students are required to use a range of cognitive processes in order to demonstrate and meet the syllabus objectives. These cognitive processes are described in the explanatory paragraph following each objective in terms of four levels: retrieval, comprehension, analytical processes (analysis), and knowledge utilisation, with each process building on the previous processes (see Marzano & Kendall 2007, 2008). That is, comprehension requires retrieval, and knowledge utilisation requires retrieval, comprehension and analytical processes (analysis).

By the conclusion of the course of study, students will:

Syllabus objective	Unit 1	Unit 2	Unit 3	Unit 4
1. select, recall and use facts, rules, definitions and procedures drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices	●	●	●	●
2. comprehend mathematical concepts and techniques drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices	●	●	●	●
3. communicate using mathematical, statistical and everyday language and conventions	●	●	●	●
4. evaluate the reasonableness of solutions	●	●	●	●
5. justify procedures and decisions by explaining mathematical reasoning	●	●	●	●
6. solve problems by applying mathematical concepts and techniques drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices	●	●	●	●

1. select, recall and use facts, rules, definitions and procedures drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices

When students select, recall and use facts, rules, definitions and procedures, they recognise particular features of remembered information and consider its accuracy and relevance. They present facts, rules, definitions and procedures and put them into effect, performing calculations with and without technology.

2. **comprehend mathematical concepts and techniques drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices**

   When students comprehend, they understand the meaning, nature and purpose of the mathematics they are learning. They identify, articulate and symbolise the critical elements of the relevant concepts and techniques, making connections between topics and between the 'why' and the 'how' of mathematics.

3. **communicate using mathematical, statistical and everyday language and conventions**

   When students communicate, they use mathematical and statistical terminology, symbols, conventions and everyday language to organise and present information in graphical and symbolic form, and describe and represent mathematical and statistical models.

4. **evaluate the reasonableness of solutions**

   When students evaluate the reasonableness of solutions, they interpret their mathematical results in the context of the situation. They reflect on whether the problem has been solved by using estimation skills and checking calculations using their knowledge of relevant facts, rules, definitions and procedures. They make an appraisal by assessing strengths, implications and limitations of solutions and/or models, with and without technology, and use this to consider if alternative methods are required.

5. **justify procedures and decisions by explaining mathematical reasoning**

   When students justify procedures and decisions by explaining mathematical reasoning, they describe their mathematical thinking in detail, identifying causes and making relationships evident, constructing mathematical arguments and providing reasons for choices made and conclusions reached. Students use their conceptual understanding to connect what they already know to new information. Mathematical reasoning is rigorous and requires clarity, precision, completeness and due regard to the order of statements.

6. **solve problems by applying mathematical concepts and techniques drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices**

   When students solve problems by applying mathematical concepts and techniques, they analyse the context of the problem and make decisions about the concepts, techniques and technology that must be used to develop a solution. They analyse, generalise and translate information into a mathematically workable format, synthesise and refine models, and generate and test hypotheses with primary or secondary data and information.

1.2.2 Underpinning factors

There are three skill sets that underpin senior syllabuses and are essential for defining the distinctive nature of subjects:

- literacy — the set of knowledge and skills about language and texts essential for understanding and conveying General Mathematics content
- numeracy — the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations, to recognise and understand the role of mathematics in the world, and to develop the dispositions and capacities to use mathematical knowledge and skills purposefully
- 21st century skills — the attributes and skills students need to prepare them for higher education, work and engagement in a complex and rapidly changing world.

These skill sets, which overlap and interact, are derived from current education, industry and community expectations. They encompass the knowledge, skills, capabilities, behaviours and dispositions that will help students live and work successfully in the 21st century.

Together these three skill sets shape the development of senior subject syllabuses. Although coverage of each skill set may vary from syllabus to syllabus, students should be provided with opportunities to learn through and about these skills over the course of study. Each skill set contains identifiable knowledge and skills that can be directly assessed.

Literacy in General Mathematics

Literacy skills and strategies enable students to express, interpret and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their abilities to read, write, visualise and talk about complex situations involving a range of mathematical ideas.

Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

To understand and use General Mathematics content, teaching and learning strategies include:

- breaking the language code to make meaning of General Mathematics language and texts
- comprehending language and texts to make literal and inferred meanings about General Mathematics content
- using General Mathematics ideas and information in classroom, real-world and/or lifelike contexts to progress students' learning.

To analyse and evaluate General Mathematics content, teaching and learning strategies include:

- making conclusions about the purpose and audience of General Mathematics language and texts
- analysing the ways language is used to convey ideas and information in General Mathematics texts
- transforming language and texts to convey General Mathematics ideas and information in particular ways to suit audience and purpose.

These aspects of literacy knowledge and skills are embedded in the syllabus objectives, unit objectives and subject matter, and instrument-specific marking guides (ISMGs) for General Mathematics.

Numeracy in General Mathematics

Numeracy relates to the capacity to deal with quantitative aspects of life (Goos, Geiger & Dole 2012). It involves accessing, using, interpreting and communicating mathematical information and ideas when engaging with and managing the mathematical demands of real contexts — everyday and civic life, the world of work, and opportunities for further learning (OECD 2012). Numerate citizens who are constructive, engaged and reflective are able to use mathematics to help make credible judgments and reasoned decisions (OECD 2015).

Unlike mathematics, numeracy must be understood as inseparable from context:

> Mathematics climbs the ladder of abstraction to see, from sufficient height, common patterns in seemingly different things. Abstraction is what gives mathematics its power; it is what enables methods derived in one context to be applied in others. But abstraction is not the focus of numeracy. Instead, numeracy clings to specifics, marshalling all relevant aspects of setting and context to reach conclusions.
>
> To enable students to become numerate, teachers must encourage them to see and use mathematics in everything they do. Numeracy is driven by issues that are important to people in their lives and work, not by future needs of the few who may make professional use of mathematics or statistics (Steen 2001, pp. 17–18).

The students who undertake General Mathematics will develop their numeracy skills at a more sophisticated level than in the P–10 years. For example, this subject contains financial applications of mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age.

These aspects of numeracy knowledge and skills are embedded in the syllabus objectives, unit objectives and subject matter, and ISMGs for General Mathematics.

21st century skills

The 21st century skills identified in the following table reflect a common agreement, both in Australia and internationally, on the skills and attributes students need to prepare them for higher education, work and engagement in a complex and rapidly changing world.

21st century skills	Associated skills	21st century skills	Associated skills
critical thinking	• analytical thinking • problem-solving • decision-making • reasoning • reflecting and evaluating • intellectual flexibility	creative thinking	• innovation • initiative and enterprise • curiosity and imagination • creativity • generating and applying new ideas • identifying alternatives • seeing or making new links
communication	• effective oral and written communication • using language, symbols and texts • communicating ideas effectively with diverse audiences	collaboration and teamwork	• relating to others (interacting with others) • recognising and using diverse perspectives • participating and contributing • community connections

21st century skills	Associated skills	21st century skills	Associated skills
personal and social skills	• adaptability/flexibility • management (self, career, time, planning and organising) • character (resilience, mindfulness, open- and fair-mindedness, self-awareness) • leadership • citizenship • cultural awareness • ethical (and moral) understanding	information & communication technologies (ICT) skills	• operations and concepts • accessing and analysing information • being productive users of technology • digital citizenship (being safe, positive and responsible online)

General Mathematics helps develop the following 21st century skills:

- critical thinking
- creative thinking
- communication
- information & communication technologies (ICT) skills.

These elements of 21st century skills are embedded in the syllabus objectives, unit objectives and subject matter, and ISMGs for General Mathematics.

Use of digital technology

An important aspect of teaching and learning in the 21st century is to embed digital technologies so that they are not seen as optional tools. Digital technologies allow new approaches to explaining and presenting mathematics, and can assist in connecting representations and deepening understanding. They can make previously inaccessible mathematics accessible and increase the opportunities for teachers to make mathematics interesting to a wider range of students. The computational and graphing capabilities of digital technologies enable students to engage in active learning through exploratory work and experiments using realistic data. The ability to visualise solutions can give problems more meaning. Digital technologies can support the development of conceptual understanding that can lead to enhanced procedural fluency.

To meet the requirements of this syllabus, students must make use of a range of digital technologies, such as:

- general-purpose computer software that can be used for mathematics teaching and learning, e.g. spreadsheet software, applications
- computer software designed for mathematics teaching and learning, e.g. dynamic graphing software, dynamic geometry software
- handheld (calculator) technologies designed for mathematics teaching and learning, e.g. scientific, graphics (non-CAS or CAS) calculators, smartphone and tablet apps.

Students must make choices about various forms of technology and develop the ability to work with these flexibly. Technology use must go beyond simple computation or word processing.

Access to a scientific calculator is a requirement for the external examination. Graphics calculators and other technologies are not permitted in the external examination.

1.2.3 Aboriginal perspectives and Torres Strait Islander perspectives

The QCAA is committed to reconciliation in Australia. As part of its commitment, the QCAA affirms that:

- Aboriginal peoples and Torres Strait Islander peoples are the first Australians, and have the oldest living cultures in human history
- Aboriginal peoples and Torres Strait Islander peoples have strong cultural traditions and speak diverse languages and dialects, other than Standard Australian English
- teaching and learning in Queensland schools should provide opportunities for students to deepen their knowledge of Australia by engaging with the perspectives of Aboriginal peoples and Torres Strait Islander peoples
- positive outcomes for Aboriginal students and Torres Strait Islander students are supported by successfully embedding Aboriginal perspectives and Torres Strait Islander perspectives across planning, teaching and assessing student achievement.

Guidelines about Aboriginal perspectives and Torres Strait Islander perspectives and resources for teaching are available at www.qcaa.qld.edu.au/k-12-policies/aboriginal-torres-strait-islander-perspectives.

Where appropriate, Aboriginal perspectives and Torres Strait Islander perspectives have been embedded in the subject matter.

In Mathematics, students have the opportunity to gain an awareness of the contributions of Aboriginal peoples and Torres Strait Islander peoples at local, regional, national and global levels through contextualisation of the subject matter.

To understand and use mathematics content, teaching and learning strategies may include:

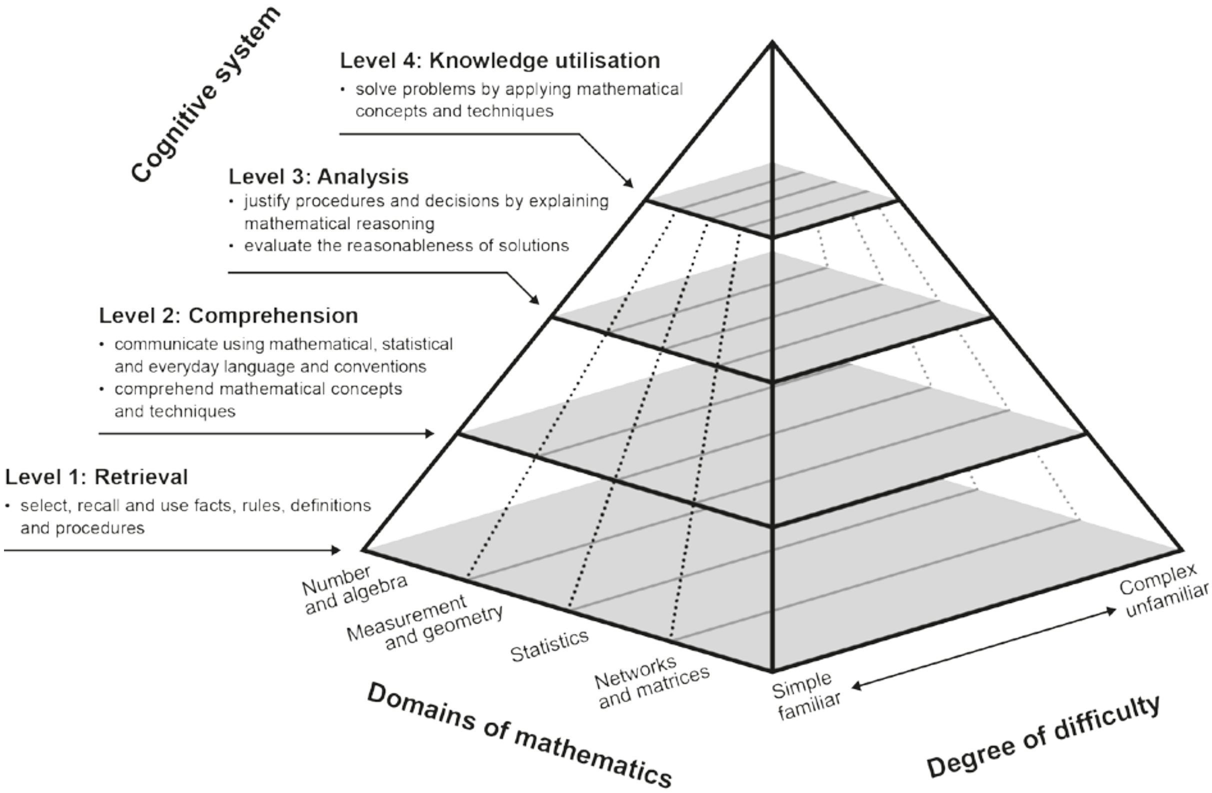
- using pedagogies such as Maths as Storytelling (MAST)
- using mathematics subject matter in real-world Aboriginal contexts and Torres Strait Islander contexts
- identifying the specific issues that may affect Aboriginal peoples and Torres Strait Islander peoples that are relevant to the mathematics topics being covered
- providing learning experiences and opportunities that support the application of students' general mathematical knowledge and problem-solving processes in an Aboriginal and Torres Strait Islander context.

1.2.4 Pedagogical and conceptual frameworks

The relationship between foundational knowledge and problem-solving

To succeed in mathematics assessment, students must understand the subject matter (organised in domains of mathematics), draw on a range of cognitive skills, and apply these to problems of varying degrees of difficulty, from simple and routine, through to unfamiliar situations, complex contexts, and multi-step solutions (Grønmo et al. 2015). The relationship between the domains of mathematics in General Mathematics, level of cognitive skill required (syllabus objective) and degree of difficulty is represented in three dimensions for mathematics problems in the following diagram.

Figure 3: Assessment pyramid



Adapted from Verhage & de Lange (1997) and Marzano & Kendall (2007).

Principles of developing mathematics problems

This representation, known as the 'assessment pyramid', shows the relative distribution of thinking and range of difficulty of mathematics problems.[1] It places an emphasis on building up from the basics. Success in mathematics is built on knowledge of basic facts and proficiency with foundational processes (Norton & O'Connor 2016). With a solid foundation, students can then be asked to apply higher level cognitive processes in more complex and unfamiliar situations that require the application of a wider range of concepts and skills.

The degree of difficulty

The difficulty of a problem is defined by its complexity and a student's familiarity with it, not the level of cognitive process required to solve it. The complexity of a particular type of problem doesn't change, but familiarity does. With practice, students become more familiar with a process and can execute it more quickly and easily (Marzano & Kendall 2007).

The cognitive system

To solve a full range of mathematics problems, students are required to engage the cognitive system at all four levels of processing knowledge: retrieval, comprehension, analysis and knowledge utilisation (Marzano & Kendall 2007). The syllabus objectives are represented in the pyramid model through their alignment to these levels.

[1] In an assessment instrument for Mathematics, a 'problem' is synonymous with 'assessment item' (a question, task or command that forms part of an assessment technique).

Using a full range of questions

The pyramid model shows that problems requiring Level 1 processes to solve them can be hard and relatively complex, even though they are based on 'retrieval' and therefore might seem easy and straightforward (Shafer & Foster 1997). Problems requiring higher level processes to solve them are not necessarily more difficult than those in Level 1. There are some students who find Level 1 processes more challenging and have more success in solving problems requiring Levels 2, 3 and 4 (Webb 2009).

The distance along the domains of mathematics dimension and the degree of difficulty dimension decreases for higher levels. Problems requiring Level 1 processes can more easily be based on distinct subject matter and the difference between easy and hard can be great. Problems that require students to use more levels of cognition tend to also involve making connections with subject matter within and across the domains of mathematics. They are often placed in contexts that require strategic mathematical decisions and making representations according to situation and purpose. At higher levels the difference between easy and hard is smaller (Shafer & Foster 1997; Webb 2009). Students should master basic facts and processes through practising simple familiar problems before moving on to those that are more complex and unfamiliar, at any level.[2]

The assessment pyramid helps visualise what is necessary for a complete assessment program. Problems in a complete mathematics program need to assess a student's growth and achievement in all domains of mathematics and across the full range of objectives. Over time, through a teaching and learning period, students will be exposed to problems that 'fill the pyramid'. Each assessment instrument will reflect this for the relevant subject matter, providing students with the opportunity to demonstrate what they know and can do at all levels of thinking and varying degrees of difficulty (Shafer & Foster 1997).

Problem-solving and mathematical modelling

A key aspect of learning mathematics is to develop strategic competence; that is, to formulate, represent and solve mathematical problems (Kilpatrick, Swafford & Bradford 2001). As such, problem-solving is a focus of mathematics education research, curriculum and teaching (Sullivan 2011). This focus is not to the exclusion of routine exercises, which are necessary for practising, attaining mastery and being able to respond automatically. But mathematics education in the 21st century goes beyond this to include innovative problems that are complex, unfamiliar and non-routine (Mevarech & Kramarski 2014).

Problem-solving in mathematics can be set in purely mathematical contexts or real-world contexts. When set in the real world, problem-solving in mathematics involves mathematical modelling.

[2] Complex unfamiliar questions that require more levels of cognitive skills should not be equated with elaborate problem-solving tasks and modelling questions only. A single-answer, conventional question, such as: 'Find the equation of the line passing through the points (2,1) and (1,3)' can be adapted to a more open-ended question, such as: 'Write the equations of at least five lines passing through the point (2,1)' (Goos 2014). This revised question targets the identical subject matter but provides the possibility of easily identifying diverse student understanding and skills by moving it towards complex unfamiliar questions and assessing more cognitive skills. For further examples, see White et al. (2000).

Problem-solving

Problem-solving is required when a task or goal has limiting conditions placed upon it or an obstacle blocking the path to a solution (Marzano & Kendall 2007). It involves:

- knowledge of the relevant details
- using generalisations and principles to identify, define and interpret the problem
- mental computation and estimation
- critical, creative and lateral thinking
- creating or choosing a strategy
- making decisions
- testing, monitoring and evaluating solutions.

Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples without understanding. Through problem-solving, students will make connections between mathematics topics, across the curriculum and with the real world, and see the value and usefulness of mathematics. Problems may be real-world or abstract, and presented to students as issues, statements or questions that may require them to use primary or secondary data.

Mathematical modelling

Mathematical modelling begins from an assumption that mathematics is everywhere in the world around us — a challenge is to identify where it is present, access it and apply it productively. Models are developed in order to better understand real-world phenomena, to make predictions and answer questions. A mathematical model depicts a situation by expressing relationships using mathematical concepts and language. It refers to the set of simplifying assumptions (such as the relevant variables or the shape of something); the set of assumed relationships between variables; and the resulting representation (such as a formula) that can be used to generate an answer (Stacey 2015).

Mathematical modelling involves:

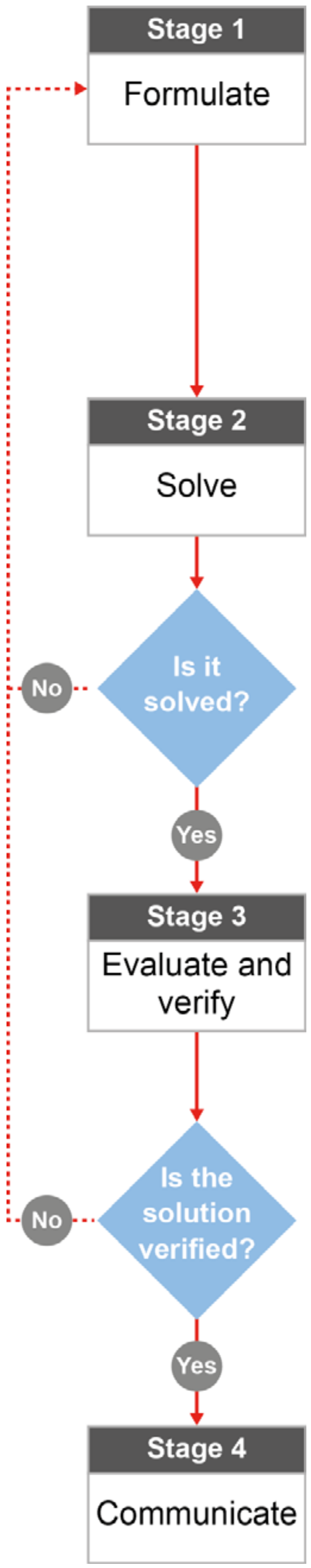
- formulating a mathematical representation of a problem derived from within a real-world context
- using mathematics concepts and techniques to obtain results
- interpreting the results by referring back to the original problem context
- revising the model (where necessary) (Geiger, Faragher & Goos 2010).

Through developing and applying mathematical models, students cumulatively become real-world problem-solvers. Ultimately, this means that not only can they productively address problems set by others, but also that they develop the ability to identify and address problems and answer questions that matter to them.

The following section outlines an approach to problem-solving and mathematical modelling.[3] Problems must be real-world, and can be presented to students as issues, statements or questions that may require them to use primary or secondary data.

[3] A wide variety of frameworks for problem-solving and modelling exist in mathematics education literature. The approach outlined here aligns with and is informed by other approaches, such as Polya in *How to Solve It: A new aspect of mathematical method* (1957), the Australian Curriculum (2015a) *Statistical investigation process*, the OECD/PISA Mathematics framework (OECD 2015, 2003) and 'A framework for success in implementing mathematical modelling in the secondary classroom' (Stillman et al. 2007). For further reading see Blum et al. (2007); Kaiser et al. (2011); and Stillman et al. (2013).

Figure 4: An approach to problem-solving and mathematical modelling



Once students understand what the problem is asking, they must design a plan to solve the problem. Students translate the problem into a mathematically purposeful representation by first determining the applicable mathematical and/or statistical principles, concepts, techniques and technology that are required to make progress with the problem. Appropriate assumptions, variables and observations are identified and documented, based on the logic of a proposed solution and/or model.

In mathematical modelling, formulating a model involves the process of mathematisation — moving from the real world to the mathematical world.

Students select and apply mathematical and/or statistical procedures, concepts and techniques previously learnt to solve the mathematical problem to be addressed through their model. Possible approaches are wide-ranging and include synthesising and refining existing models, and generating and testing hypotheses with primary or secondary data and information, as well as using standard mathematical techniques to produce a valid solution.

Solutions can be found using algebraic, graphic, arithmetic and/or numeric methods, with and/or without technology.

Once a possible solution has been achieved, students need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. They evaluate their results and make a judgment about the solution/s to the problem in relation to the original issue, statement or question.

This involves exploring the strengths and limitations of the solution and/or model. Where necessary, this will require going back through the process to further refine the solution and/or model. In mathematical modelling, students must check that the output of their model provides a valid solution to the real-world problem it has been designed to address.

This stage emphasises the importance of methodological rigour and the fact that problem-solving and mathematical modelling is not usually linear and involves an iterative process.

The development of solutions and models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated clearly and fully. Students communicate findings systematically and concisely using mathematical, statistical and everyday language. They draw conclusions, discussing the key results and the strengths and limitations of the solution and/or model. Students could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem.

Approaches to problem-solving and mathematical modelling in the classroom

When teaching problem-solving and mathematical modelling, teachers should consider t*eaching for* and *learning through* problem-solving and mathematical modelling. When *teaching for*, students are taught the specific mathematical rules, definitions, procedures, problem-solving strategies and critical elements of the model that are needed to solve a given problem. When *learning through*, students are presented with problems to solve, but must apply the knowledge and skills they have previously been taught to solve it. By solving these problems, students are able to develop new mathematical understanding and skills. This requires an explicit and connected approach to teaching problem-solving and mathematical modelling that necessitates fluency of critical facts and processes at each step.

The following describes three different approaches to teaching problem-solving and mathematical modelling[4] along the continua between *teaching for* and *learning through*:

Approach	Description	*Teaching for* or *learning through*
Dependent	The teacher explicitly demonstrates and teaches the concepts and techniques required to solve the problem, and/or develop a mathematical model. This usually involves students solving (stage 2), and evaluating and verifying (stage 3).	*Teaching for*
Guided	The teacher influences the choice of concepts and techniques, and/or model that students use to solve the problem. Guidance is provided and all stages of the approach are used.	Moving towards *learning through*
Independent	The teacher cedes control and students work independently, choosing their own solution and/or model, and working at their own level of mathematics. The independent approach is the most challenging.	*Learning through*

These approaches are not mutually exclusive. An independent approach (*learning through*) might be undertaken as an extension of a dependent or guided activity that students have previously undertaken (*teaching for*). Students need to have attained the relevant foundational understanding and skills before working independently during a problem-solving and modelling task. This capacity needs to be built over time through the course of study with teachers closely monitoring student progress.

1.2.5 Subject matter

Subject matter is the body of information, mental procedures and psychomotor procedures (see Marzano & Kendall 2007, 2008) that are necessary for students' learning and engagement with General Mathematics. It is particular to each unit in the course of study and provides the basis for student learning experiences.

Subject matter has a direct relationship to the unit objectives, but is of a finer granularity and is more specific. These statements of learning are constructed in a similar way to objectives. Each statement:

- describes an action (or combination of actions) — what the student is expected to do
- describes the element — expressed as information, mental procedures and/or psychomotor procedures
- is contextualised for the topic or circumstance particular to the unit.

Subject matter in General Mathematics is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

[4] Based on Galbraith (1989).

1.3 Assessment — general information

Assessments are formative in Units 1 and 2, and summative in Units 3 and 4.

Assessment	Unit 1	Unit 2	Unit 3	Unit 4
Formative assessments	●	●		
Summative internal assessment 1			●	
Summative internal assessment 2			●	
Summative internal assessment 3				●
Summative external assessment*			●	●

* Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of the supervised examination.

1.3.1 Formative assessments — Units 1 and 2

Formative assessments provide feedback to both students and teachers about each student's progress in the course of study.

Schools develop internal assessments for each senior subject, based on the learning described in Units 1 and 2 of the subject syllabus. Each unit objective must be assessed at least once.

For reporting purposes, schools should devise at least *two* but no more than *four* assessments for Units 1 and 2 of this subject. At least *one* assessment must be completed for *each* unit.

The sequencing, scope and scale of assessments for Units 1 and 2 are matters for each school to decide and should reflect the local context.

Teachers are encouraged to use the A–E descriptors in the reporting standards (Section 1.4) to provide formative feedback to students and to report on progress.

1.3.2 Summative assessments — Units 3 and 4

Students will complete a total of *four* summative assessments — three internal and one external — that count towards their final mark in each subject.

Schools develop *three* internal assessments for each senior subject, based on the learning described in Units 3 and 4 of the syllabus.

The three summative internal assessments will be endorsed and the results confirmed by the QCAA. These results will be combined with a single external assessment developed and marked by the QCAA. The external assessment results for General Mathematics will contribute 50% towards a student's result.

Summative internal assessment — instrument-specific marking guides

This syllabus provides ISMGs for the three summative internal assessments in Units 3 and 4.

The ISMGs describe the characteristics evident in student responses and align with the identified assessment objectives. Assessment objectives are drawn from the unit objectives and are contextualised for the requirements of the assessment instrument.

Criteria

Each ISMG groups assessment objectives into criteria. An assessment objective may appear in multiple criteria, or in a single criterion of an assessment.

Making judgments

Assessment evidence of student performance in each criterion is matched to a performance-level descriptor, which describes the typical characteristics of student work.

Where a student response has characteristics from more than one performance level, a best-fit approach is used. Where a performance level has a two-mark range, it must be decided if the best fit is the higher or lower mark of the range.

Authentication

Schools and teachers must have strategies in place for ensuring that work submitted for internal summative assessment is the student's own. Authentication strategies outlined in QCAA guidelines, which include guidance for drafting, scaffolding and teacher feedback, must be adhered to.

Summative external assessment

The summative external assessment adds valuable evidence of achievement to a student's profile. External assessment is:

- common to all schools
- administered under the same conditions at the same time and on the same day
- developed and marked by the QCAA according to a commonly applied marking scheme.

The external assessment contributes 50% to the student's result in General Mathematics. It is not privileged over the school-based assessment.

1.4 Reporting standards

Reporting standards are summary statements that succinctly describe typical performance at each of the five levels (A–E). They reflect the cognitive taxonomy and objectives of the course of study.

The primary purpose of reporting standards is for twice-yearly reporting on student progress. These descriptors can also be used to help teachers provide formative feedback to students and to align ISMGs.

Reporting standards

A
The student demonstrates a comprehensive knowledge and understanding of the subject matter; recognises, recalls and uses facts, rules, definitions and procedures; and comprehends and applies mathematical concepts and techniques to solve problems drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar, complex familiar and complex unfamiliar situations. The student explains mathematical reasoning to justify procedures and decisions; evaluates the reasonableness of solutions; communicates using mathematical, statistical and everyday language and conventions; and makes decisions about the choice of technology, and uses the technology, to solve problems in simple familiar, complex familiar and complex unfamiliar situations.
B
The student demonstrates a thorough knowledge and understanding of the subject matter; recognises, recalls and uses facts, rules, definitions and procedures; and comprehends and applies mathematical concepts and techniques to solve problems drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar and complex familiar situations. The student explains mathematical reasoning to justify procedures and decisions; evaluates the reasonableness of solutions; communicates using mathematical, statistical and everyday language and conventions; and makes decisions about the choice of technology, and uses the technology, to solve problems in simple familiar and complex familiar situations.
C
The student demonstrates knowledge and understanding of the subject matter; recognises, recalls and uses facts, rules, definitions and procedures; and comprehends and applies mathematical concepts and techniques to solve problems drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar situations. The student explains mathematical reasoning to justify procedures and decisions; evaluates the reasonableness of solutions; communicates using mathematical, statistical and everyday language and conventions; and uses technology to solve problems in simple familiar situations.
D
The student demonstrates partial knowledge and understanding of the subject matter; recognises, recalls and uses some facts, rules, definitions and procedures; and comprehends and applies aspects of mathematical concepts and techniques to solve some problems drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar situations. The student explains some mathematical reasoning to justify procedures and decisions; sometimes evaluates the reasonableness of solutions; communicates using some mathematical, statistical and everyday language and conventions; and uses technology to solve some problems in simple familiar situations.
E
The student demonstrates isolated knowledge and understanding of the subject matter; infrequently recognises, recalls and uses some facts, rules, definitions and procedures; and infrequently comprehends and applies aspects of mathematical concepts and techniques drawn from Number and algebra, Measurement and geometry, Statistics and Networks and matrices in simple familiar situations. The student infrequently describes aspects of mathematical reasoning relevant to procedures and decisions; rarely evaluates the reasonableness of solutions; infrequently communicates using some aspects of mathematical, statistical and everyday language and conventions; and uses aspects of technology in simple familiar situations.

2 Unit 1: Money, measurement and relations

2.1 Unit description

In Unit 1, students will develop mathematical understandings and skills to solve problems relating to the topics:

- Topic 1: Consumer arithmetic
- Topic 2: Shape and measurement
- Topic 3: Linear equations and their graphs.

Consumer arithmetic reviews the concepts of rate and percentage change in the context of earning and managing money, and provides an opportunity for the use of spreadsheets. Shape and measurement builds on and extends the knowledge and skills students developed in the P–10 Australian Curriculum with the concept of similarity and problems involving simple and compound geometric shapes. Students apply these skills in a range of practical contexts, including those involving three-dimensional shapes. Linear equations and their graphs uses linear equations and straight-line graphs, as well as piece-wise linear graphs and step graphs, to model and analyse practical situations.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 1. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

2.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 1 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 1 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 1 topics.

2.4 Topic 1: Consumer arithmetic

Subject matter
Applications of rates, percentages and use of spreadsheets (14 hours) In this sub-topic, students will: • review definitions of rates and percentages • calculate weekly or monthly wages from an annual salary, and wages from an hourly rate, including situations involving overtime and other allowances and earnings based on commission or piecework • calculate payments based on government allowances and pensions, such as youth allowances, unemployment, disability and study • prepare a personal budget for a given income, taking into account fixed and discretionary spending • compare prices and values using the unit cost method • apply percentage increase or decrease in various contexts, e.g. determining the impact of inflation on costs and wages over time, calculating percentage mark-ups and discounts, calculating GST, calculating profit or loss in absolute and percentage terms, and calculating simple and compound interest • use currency exchange rates to determine the cost in Australian dollars of purchasing a given amount of a foreign currency, such as US$1500, or the value of a given amount of foreign currency when converted to Australian dollars, such as the value of €2050 in Australian dollars • calculate the dividend paid on a portfolio of shares, given the percentage dividend or dividend paid per share, for each share; and compare share values by calculating a price-to-earnings ratio • use a spreadsheet to display examples of the above computations when multiple or repeated computations are required, e.g. preparing a wage sheet displaying the weekly earnings of workers in a fast-food store where hours of employment and hourly rates of pay may differ, preparing a budget or investigating the potential cost of owning and operating a car over a year.

2.5 Topic 2: Shape and measurement

Subject matter
Pythagoras' theorem (3 hours) In this sub-topic, students will: • review Pythagoras' theorem and use it to solve practical problems in two dimensions and simple applications in three dimensions.
Mensuration (8 hours) In this sub-topic, students will: • solve practical problems requiring the calculation of perimeters and areas of circles, sectors of circles, triangles, rectangles, trapeziums, parallelograms and composites • calculate the volumes and capacities of standard three-dimensional objects, including spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations, such as the volume of water contained in a swimming pool • calculate the surface areas of standard three-dimensional objects, e.g. spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations, such as the surface area of a cylindrical food container.
Similar figures and scale factors (7 hours) In this sub-topic, students will: • review the conditions for similarity of two-dimensional figures, including similar triangles • use the scale factor for two similar figures to solve linear scaling problems • obtain measurements from scale drawings, such as maps or building plans, to solve problems • obtain a scale factor and use it to solve scaling problems involving the calculation of the areas of similar figures, including the use of shadow sticks, calculating the height of trees, use of a clinometer • obtain a scale factor and use it to solve scaling problems involving the calculation of surface areas and volumes of similar solids.

2.6 Topic 3: Linear equations and their graphs

Subject matter
Linear equations (5 hours) In this sub-topic, students will: • identify and solve linear equations, including variables on both sides, fractions, non-integer solutions • develop a linear equation from a description in words.
Straight-line graphs and their applications (7 hours) In this sub-topic, students will: • construct straight-line graphs using $y = a + bx$ both with and without the aid of technology • determine the slope and intercepts of a straight-line graph from both its equation and its plot • interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation • construct and analyse a straight-line graph to model a given linear relationship, such as modelling the cost of filling a fuel tank of a car against the number of litres of petrol required.
Simultaneous linear equations and their applications (6 hours) In this sub-topic, students will: • solve a pair of simultaneous linear equations in the format $y = mx + c$, using technology when appropriate; they must solve equations algebraically, graphically, by substitution and by the elimination method • solve practical problems that involve finding the point of intersection of two straight-line graphs, such as determining the break-even point where cost and revenue are represented by linear equations.
Piece-wise linear graphs and step graphs (5 hours) In this sub-topic, students will: • sketch piece-wise linear graphs and step graphs, using technology where appropriate • interpret piece-wise linear and step graphs used to model practical situations.

2.7 Assessment guidance

In constructing assessment instruments for Unit 1, schools should ensure that the objectives cover, or are chosen from, the unit objectives. If one assessment instrument is developed for a unit, it must assess all the unit objectives; if more than one assessment instrument is developed, the unit objectives must be covered across those instruments.

It is suggested that schools develop:

- a problem-solving and modelling task that assesses Unit 1 Topic 1: Consumer arithmetic, and
- an internal examination that representatively samples subject matter from Unit 1 not assessed in the problem-solving and modelling task.

3 Unit 2: Applied trigonometry, algebra, matrices and univariate data

3.1 Unit description

In Unit 2, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Applications of trigonometry
- Topic 2: Algebra and matrices
- Topic 3: Univariate data analysis.

Applications of trigonometry extends students' knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression and bearings in navigation. Algebra and matrices continues the study of algebra and introduces the new topic of matrices. Univariate data analysis develops students' ability to organise and summarise univariate data in the context of conducting a statistical investigation.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 2. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

3.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 2 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 2 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 2 topics.

3.3 Topic 1: Applications of trigonometry

Subject matter

Applications of trigonometry (11 hours)

In this sub-topic, students will:

- review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle
- determine the area of a triangle given two sides and an included angle by using the rule $\text{area} = \frac{1}{2}bc\sin A$, or given three sides by using Heron's rule $A = \sqrt{s(s-a)(s-b)(s-c)}$, *where* $s = \frac{a+b+c}{2}$, and solve related practical problems
- solve two-dimensional problems involving non-right-angled triangles using the sine rule (ambiguous case excluded) and the cosine rule
- solve two-dimensional practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of true bearings.

3.4 Topic 2: Algebra and matrices

Subject matter

Linear and non-linear relationships (8 hours)

In this sub-topic, students will:

- substitute numerical values into linear algebraic and simple non-linear algebraic expressions, and evaluate, e.g. order two polynomials, proportional, inversely proportional
- find the value of the subject of the formula, given the values of the other pronumerals in the formula
- transpose linear equations and simple non-linear algebraic equations, e.g. order two polynomials, proportional, inversely proportional
- use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities, e.g. a table displaying the body mass index (BMI) of people with different weights and heights.

Matrices and matrix arithmetic (10 hours)

In this sub-topic, students will:

- use matrices for storing and displaying information that can be presented in rows and columns, e.g. tables, databases, links in social or road networks
- recognise different types of matrices (row matrix, column matrix (or vector matrix), square matrix, zero matrix, identity matrix) and determine the size of the matrix
- perform matrix addition, subtraction, and multiplication by a scalar
- perform matrix multiplication (manually up to a 3 x 3 but not limited to square matrices)
- determining the power of a matrix using technology with matrix arithmetic capabilities when appropriate
- use matrices, including matrix products and powers of matrices, to model and solve problems, e.g. costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can communicate with each other via a third person.

3.6 Topic 3: Univariate data analysis

Subject matter
Making sense of data relating to a single statistical variable (14 hours) In this sub-topic, students will: • define univariate data • classify statistical variables as categorical or numerical • classify a categorical variable as ordinal or nominal and use tables and pie, bar and column charts to organise and display the data, e.g. ordinal: income level (high, medium, low); or nominal: place of birth (Australia, overseas) • classify a numerical variable as discrete or continuous, e.g. discrete: the number of rooms in a house; or continuous: the temperature in degrees Celsius • select, construct and justify an appropriate graphical display to describe the distribution of a numerical dataset, including dot plot, stem-and-leaf plot, column chart or histogram • describe the graphical displays in terms of the number of modes, shape (symmetric versus positively or negatively skewed), measures of centre and spread, and outliers and interpret this information in the context of the data • determine the mean, $\bar{x}$, and standard deviation (using technology) of a dataset and use statistics as measures of location and spread of a data distribution, being aware of the significance of the size of the standard deviation.
Comparing data for a numerical variable across two or more groups (12 hours) In this sub-topic, students will: • construct and use parallel box plots (including the use of the $Q_1 - 1.5 \times \mathrm{IQR} \leq x \leq Q_3 + 1.5 \times \mathrm{IQR}$ criteria for identifying possible outliers) to compare datasets in terms of median, spread (IQR and range) and outliers to interpret and communicate the differences observed in the context of the data • compare datasets using medians, means, IQRs, ranges or standard deviations for a single numerical variable, interpret the differences observed in the context of the data and report the findings in a systematic and concise manner.

3.7 Assessment guidance

In constructing assessment instruments for Unit 2, schools should ensure that the objectives cover, or are chosen from, the unit objectives. If one assessment instrument is developed for a unit, it must assess all the unit objectives; if more than one assessment instrument is developed, the unit objectives must be covered across those instruments.

It is suggested that schools develop:

- an internal examination that representatively samples subject matter from all Unit 2 topics, and/or
- an internal examination that representatively samples subject matter from Units 1 and 2.

4 Unit 3: Bivariate data, sequences and change, and Earth geometry

4.1 Unit description

In Unit 3, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Bivariate data analysis
- Topic 2: Time series analysis
- Topic 3: Growth and decay in sequences
- Topic 4: Earth geometry and time zones.

Bivariate data analysis introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a method for analysing linear associations. Time series analysis continues students' study of statistics by introducing them to the concepts and techniques of time series analysis. Growth and decay in sequences employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4. Earth geometry and time zones offers an opportunity to use contexts relevant to students.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 3. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

4.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

Unit objective	IA1	IA2	EA
1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics	•	•	•
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics	•	•	•
3. communicate using mathematical, statistical and everyday language and conventions	•	•	•
4. evaluate the reasonableness of solutions	•	•	•
5. justify procedures and decisions by explaining mathematical reasoning	•	•	•
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.	•	•	•

4.3 Topic 1: Bivariate data analysis

Subject matter
Identifying and describing associations between two categorical variables (4 hours) In this sub-topic, students will: • define bivariate data • construct two-way frequency tables and determine the associated row and column sums and percentages • use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association • understand an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data.
Identifying and describing associations between two numerical variables (6 hours) In this sub-topic, students will: • construct a scatterplot to identify patterns in the data suggesting the presence of an association • understand an association between two numerical variables in terms of direction (positive/negative), form (linear) and strength (strong/moderate/weak) • calculate and interpret the correlation coefficient (r) to quantify the strength of a linear association using Pearson's correlation coefficient.
Fitting a linear model to numerical data (7 hours) In this sub-topic, students will: • identify the response variable and the explanatory variable • use a scatterplot to identify the nature of the relationship between variables • model a linear relationship by fitting a least-squares line to the data • use a residual plot to assess the appropriateness of fitting a linear model to the data • interpret the intercept and slope of the fitted line • use, not calculate, the coefficient of determination (R^2) to assess the strength of a linear association in terms of the explained variation • use the equation of a fitted line to make predictions • distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation.
Association and causation (7 hours) In this sub-topic, students will: • recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between them • identify and communicate possible non-causal explanations for an association, including coincidence and confounding due to a common response to another variable • solve practical problems by identifying, analysing and describing associations between two categorical variables or between two numerical variables.

4.4 Topic 2: Time series analysis

Subject matter
Describing and interpreting patterns in time series data (4 hours) In this sub-topic, students will: • construct time series plots • describe time series plots by identifying features such as trend (long-term direction), seasonality (systematic, calendar-related movements) and irregular fluctuations (unsystematic, short-term fluctuations), and recognise when there are outliers, e.g. one-off unanticipated events.
Analysing time series data (10 hours) In this sub-topic, students will: • smooth time series data by using a simple moving average, including the use of spreadsheets to implement this process • calculate seasonal indices by using the average percentage method • deseasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process • fit a least-squares line to model long-term trends in time series data, using appropriate technology • solve practical problems that involve the analysis of time series data.

4.5 Topic 3: Growth and decay in sequences

Subject matter
The arithmetic sequence (4 hours) In this sub-topic, students will: • use recursion to generate an arithmetic sequence • display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations • use the rule for the n^{th} term using $t_n = t_1 + (n-1)d$, where t_n represents the n^{th} term of the sequence, $t_1 =$ first term, $n =$ term number and $d =$ common difference of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions • use arithmetic sequences to model and analyse practical situations involving linear growth or decay, such as analysing a simple interest loan or investment, calculating a taxi fare based on the flag fall and the charge per kilometre, or calculating the value of an office photocopier at the end of each year using the straight-line method or the unit cost method of depreciation.
The geometric sequence (5 hours) In this sub-topic, students will: • use recursion to generate a geometric sequence • display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations • use the rule for the n^{th} term using $t_n = t_1 r^{(n-1)}$ where t_n represents the n^{th} term of the sequence, $t_1 =$ first term, $n =$ term number and $r =$ common ratio of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions • use geometric sequences to model and analyse (numerically or graphically only) practical problems involving geometric growth and decay (logarithmic solutions not required), such as analysing a compound interest loan or investment, the growth of a bacterial population that doubles in size each hour or the decreasing height of the bounce of a ball at each bounce; or calculating the value of office furniture at the end of each year using the declining (reducing) balance method to depreciate.

4.6 Topic 4: Earth geometry and time zones

Subject matter
Locations on the Earth (3 hours) In this sub-topic, students will: • define the meaning of great circles • define the meaning of angles of latitude and longitude in relation to the equator and the prime meridian • locate positions on Earth's surface given latitude and longitude, e.g. using a globe, an atlas, GPS and other digital technologies • state latitude and longitude for positions on Earth's surface and world maps (in degrees only) • use a local area map to state the position of a given place in degrees and minutes, e.g. investigating the map of Australia and locating boundary positions for Aboriginal language groups, such as the Three Sisters in the Blue Mountains or the local area's Aboriginal land and the positions of boundaries • calculate angular distance (in degrees and minutes) and distance (in kilometres) between two places on Earth on the same meridian using $D = 111.2 \times \text{angular distance}$ • calculate angular distance (in degrees and minutes) and distance (in kilometres) between two places on Earth on the same parallel of latitude using $D = 111.2 \cos \theta \times \text{angular distance}$ • calculate distances between two places on Earth, using appropriate technology.
Time zones (5 hours) In this sub-topic, students will: • define Greenwich Mean Time (GMT), International Date Line and Coordinated Universal Time (UTC) • understand the link between longitude and time • determine the number of degrees of longitude for a time difference of one hour • solve problems involving time zones in Australia and in neighbouring nations, making any necessary allowances for daylight saving, including seasonal time systems used by Aboriginal peoples and Torres Strait Islander peoples • solve problems involving GMT, International Date Line and UTC • calculate time differences between two places on Earth • solve problems associated with time zones, such as online purchasing, making phone calls overseas and broadcasting international events • solve problems relating to travelling east and west incorporating time zone changes, such as preparing an itinerary for an overseas holiday with corresponding times.

4.7 Assessment

4.7.1 Summative internal assessment 1 (IA1): Problem-solving and modelling task (20%)

Description

This assessment focuses on the interpretation, analysis and evaluation of ideas and information. It is an independent task responding to a particular situation or stimuli. While students may undertake some research in the writing of the problem-solving and modelling task, it is not the focus of this technique. This assessment occurs over an extended and defined period of time. Students will use class time and their own time to develop a response.

The problem-solving and modelling task must use subject matter from at least one of the following topics in Unit 3:

- Topic 1: Bivariate data analysis
- Topic 2: Time series analysis
- Topic 3: Growth and decay in sequences.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 1, 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 1, 2 and/or 3
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 1, 2 and/or 3.

Specifications

Description

A problem-solving and modelling task is an assessment instrument developed in response to a mathematical investigative scenario or context. It requires students to respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Students must provide a response to a specific task or issue that is set in a context that highlights a real-life application of mathematics. The task requires students to use relevant stimulus material involving the selected subject matter and must have sufficient scope to allow students to address all the stages of the problem-solving and modelling approach (see Section 1.2.4). Technology must be used.

The response is written and must be able to be read and interpreted independently of the instrument task sheet.

Conditions

- Written
  - up to 10 pages (including tables, figures and diagrams)
  - maximum of 2000 words
  - appendixes can include raw data, repeated calculations, evidence of authentication and student notes (appendixes are not to be marked).
- Duration: 4 weeks (including 3 hours of class time)
- Other
  - opportunity may be provided for group work, but unique responses must be developed by each student
  - use of technology is required; schools must specify the technology used, e.g. scientific calculator, graphics calculator, spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation or word processing
  - the teacher provides the mathematical investigative scenario or context for the problem-solving and modelling task.

Task examples

Examples of problem-solving and modelling tasks include:

- a written persuasive argument using mathematical logic to present and justify a point of view, e.g. causation and association — is there a correlation between height and dominant hand reaction time?
- a journal article, e.g. investigate attitudes to environmental sustainability issues, including pollution — compare attitudes of students today to those of a community in the 1990s
- a report using time series analysis and spreadsheets, e.g. analyse sales figures over a one-month period compared to previous years; identify trends, seasonality, irregular fluctuations and outliers; deseasonalise or make a seasonal adjustment using a seasonal index.

Summary of the instrument-specific marking guide

The following table summarises the criteria, assessment objectives and mark allocation for the problem-solving and modelling task.

Criterion	Objectives	Marks
Formulate	1, 2 and 5	4
Solve	1 and 6	7
Evaluate and verify	4 and 5	5
Communicate	3	4
Total		**20**

Instrument-specific marking guide

Criterion: Formulate

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures, drawn from Unit 3 Topics 1, 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 1, 2 and/or 3
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques.	3–4
• statement of some assumptions • statement of some observations • translation of simple aspects of the problem by identifying mathematical concepts and techniques.	1–2
• does not satisfy any of the descriptors above.	0

Criterion: Solve

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 1, 2 and/or 3
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 1, 2 and/or 3

The student work has the following characteristics:	Marks
• accurate use of complex procedures to reach a valid solution • discerning application of mathematical concepts and techniques relevant to the task • accurate and appropriate use of technology.	6–7
• use of complex procedures to reach a reasonable solution • application of mathematical concepts and techniques relevant to the task • use of technology.	4–5
• use of simple procedures to make some progress towards a solution • simplistic application of mathematical concepts and techniques relevant to the task • superficial use of technology.	2–3
• inappropriate use of technology or procedures.	1
• does not satisfy any of the descriptors above.	0

Criterion: Evaluate and verify

Assessment objectives

4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	4–5
• statements about the reasonableness of solutions by considering the context of the task • statements about relevant strengths and limitations of the solution and/or model • statements about decisions made relevant to the context of the task.	2–3
• statement about a decision and/or the reasonableness of a solution.	1
• does not satisfy any of the descriptors above.	0

Criterion: Communicate

Assessment objective

3. communicate using mathematical, statistical and everyday language and conventions

The student work has the following characteristics:	Marks
• correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4
• use of some appropriate language and conventions to develop the response • adequate organisation of the response.	1–2
• does not satisfy any of the descriptors above.	0

4.7.2 Summative internal assessment 2 (IA2): Examination (15%)

Description

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 3 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.

Specifications

Description

The examination representatively samples subject matter from all Unit 3 topics. Where relevant, the focus of this assessment should be on subject matter not assessed in the problem-solving and modelling task.

Subject matter from Units 1 and 2 is considered assumed knowledge.

The examination must ensure that all assessment objectives are assessed. The examination should be designed using the principles of developing mathematics problems in Section 1.2.4. The total number of marks used in an examination marking scheme is a school decision. However, in order to correctly apply the ISMG, the percentage allocation of marks must match the following specifications.

Mark allocations

Percentage of marks	Degree of difficulty
~ 20%	**Complex unfamiliar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable, that is – the required procedure is not clear from the way the problem is posed; and – in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
~ 20%	**Complex familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, *or* – in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
~ 60%	**Simple familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable, that is – the required procedure is clear from the way the problem is posed, *or* – is in a context that has been a focus of prior learning. Students are *not* required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.

Conditions

- Time: 120 minutes plus 5 minutes perusal.
- Length: the number of short-response items should allow students to complete the response in the set time.
- Short-response format, consisting of a number of items that ask students to respond to the following activities
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams
  - short items requiring single-word, term, sentence or short-paragraph responses
  - justifying solutions using appropriate mathematical language where applicable
  - responding to seen or unseen stimulus materials
  - interpreting ideas and information.

- Other
  - seen stimulus — teachers must ensure the purpose of the technique is not compromised
  - unseen stimulus — materials or questions must not be copied from information or texts that students have previously been exposed to or have used directly in class
  - when stimulus materials are used, they will be succinct enough to allow students sufficient time to engage with them; for stimulus materials that are lengthy, complex or large in number, they will be shared with students prior to the administration of the assessment instrument
  - only the QCAA formula sheet must be provided
  - notes are not permitted
  - use of technology is required; schools must specify the technology used, e.g. calculator, spreadsheet program, scientific calculator.

Summary of the instrument-specific marking guide

The following table summarises the criteria, assessment objectives and mark allocation for the examination.

Criterion	Objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5 and 6	15
Total		**15**

Instrument-specific marking guide

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0

4.7.3 Summative external assessment (EA): Examination (50%)

General information

Summative external assessment is developed and marked by the QCAA. In General Mathematics it contributes 50% to a student's overall subject result.

Summative external assessment assesses learning from Units 3 and 4. Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of the supervised examination.

The external assessment in General Mathematics is common to all schools and administered under the same conditions, at the same time, on the same day.

See Section 5.6.2.

5 Unit 4: Investing and networking

5.1 Unit description

In Unit 4, students will develop mathematical understandings and skills to solve problems relating to:

- Topic 1: Loans, investments and annuities
- Topic 2: Graphs and networks
- Topic 3: Networks and decision mathematics.

Loans, investments and annuities aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments. Graphs and networks introduces students to the language of graphs and the ways in which graphs, represented as a collection of points and interconnecting lines, can be used to model and analyse everyday situations such as a rail or social network. Networks and decision mathematics uses networks to model and aid decision-making in practical situations.

Unit requirements

Subject matter describes the concepts, ideas, knowledge, understanding and skills that students are to learn in Unit 4. It is organised into topics and sub-topics. Notional time allocations have been provided for each sub-topic.

5.2 Unit objectives

Unit objectives are drawn from the syllabus objectives and are contextualised for the subject matter and requirements of the unit. Each unit objective must be assessed at least once.

Students will:

Unit objective	IA3	EA
1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics	•	•
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics	•	•
3. communicate using mathematical, statistical and everyday language and conventions	•	•
4. evaluate the reasonableness of solutions	•	•
5. justify procedures and decisions by explaining mathematical reasoning	•	•
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.	•	•

5.3 Topic 1: Loans, investments and annuities

Subject matter
Compound interest loans and investments (6 hours) In this sub-topic, students will: • use a recurrence relation $A_{n+1} = rA_n$ to model a compound interest loan or investment, and investigate (numerically and graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment, e.g. payday loan • calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly • solve problems involving compound interest loans or investments, e.g. determining the future value of a loan, the number of compounding periods for an investment to exceed a given value, the interest rate needed for an investment to exceed a given value.
Reducing balance loans (compound interest loans with periodic repayments) (6 hours) In this sub-topic, students will: • use a recurrence relation, $A_{n+1} = rA_n - R$ (where $R =$ monthly repayment) to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan • with the aid of appropriate technology, solve problems involving reducing balance loans, e.g. determining the monthly repayments required to pay off a housing loan.
Annuities and perpetuities (compound interest investments with periodic payments made from the investment) (8 hours) In this sub-topic, students will: • use a recurrence relation $A_{n+1} = rA_n + d$ to model an annuity and investigate (numerically or graphically) the effect of the amount invested, the interest rate, and the payment amount on the duration of the annuity • solve problems involving annuities, including perpetuities as a special case, e.g. determining the amount to be invested in an annuity to provide a regular monthly income of a certain amount.

5.4 Topic 2: Graphs and networks

Subject matter
Graphs, associated terminology and the adjacency matrix (4 hours) In this sub-topic, students will: • understand the meanings of the terms graph, edge, vertex, loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph (digraph), arc, weighted graph and network • identify practical situations that can be represented by a network and construct such networks, e.g. trails connecting camp sites in a national park, a social network, a transport network with one-way streets, a food web, the results of a round-robin sporting competition • construct an adjacency matrix from a given graph or digraph.

Subject matter
Planar graphs, paths and cycles (8 hours) In this sub-topic, students will: • understand the meaning of the terms planar graph and face • apply Euler's formula, $v + f - e = 2$, to solve problems relating to planar graphs • understand the meaning of the terms walk, trail, path, closed walk, closed trail, cycle, connected graph and bridge • investigate and solve practical problems to determine the shortest path between two vertices in a weighted graph (by trial-and-error methods only) • understand the meaning of the terms Eulerian graph, Eulerian trail, semi-Eulerian graph, semi-Eulerian trail and the conditions for their existence, and use these concepts to investigate and solve practical problems, e.g. the Königsberg bridge problem, planning a garbage bin collection route • understand the meaning of the terms Hamiltonian graph and semi-Hamiltonian graph and use these concepts to investigate and solve practical problems (by trial-and-error methods only), e.g. planning a sightseeing tourist route around a city, the travelling-salesman problem.

5.5 Topic 3: Networks and decision mathematics

Subject matter
Trees and minimum connector problems (4 hours) In this sub-topic, students will: • understand the meaning of the terms tree and spanning tree • identify practical examples • identify a minimum spanning tree in a weighted connected graph, e.g. using Prim's algorithm • use minimal spanning trees to solve minimal connector problems, e.g. minimising the length of cable needed to provide power from a single power station to substations in several towns.
Project planning and scheduling using critical path analysis (CPA) (8 hours) In this sub-topic, students will: • construct a network diagram to represent the durations and interdependencies of activities that must be completed during the project, e.g. preparing a meal • use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project • use ESTs and LSTs to locate the critical path/s for the project • use the critical path to determine the minimum time for a project to be completed • calculate float times for non-critical activities.
Flow networks (3 hours) In this sub-topic, students will: • solve small-scale network flow problems including the use of the 'maximum-flow minimum-cut' theorem, e.g. determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank to a terminal.
Assigning order and the Hungarian algorithm (8 hours) In this sub-topic, students will: • use a bipartite graph and its tabular or matrix form to represent an assignment/allocation problem, e.g. assigning four swimmers to the four places in a medley relay team to maximise the team's chances of winning • determine the optimum assignment/s for small-scale problems by inspection, or by use of the Hungarian algorithm (3×3) for larger problems.

5.6 Assessment

5.6.1 Summative internal assessment 3 (IA3): Examination (15%)

Description

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 4 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

Specifications

Description

This examination representatively samples subject matter from all Unit 4 topics.

Subject matter from Units 1, 2 and 3 is considered assumed knowledge.

The examination must ensure that a balance of all assessment objectives are assessed. The examination should be designed using the principles of developing mathematics problems in Section 1.2.4. The total number of marks used in an examination marking scheme is a school decision. However, in order to correctly apply the ISMG, the percentage allocation of marks must match the specifications below.

Mark allocations

Percentage of marks	Degree of difficulty

~ 20%	**Complex unfamiliar**
	Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where:
	• relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and
	• all the information to solve the problem is not immediately identifiable; that is
	  − the required procedure is not clear from the way the problem is posed, and
	  − in a context in which students have had limited prior experience.
	Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.

~ 20%	**Complex familiar**
	Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where:
	• relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and
	• all of the information to solve the problem is identifiable; that is
	  − the required procedure is clear from the way the problem is posed, *or*
	  − in a context that has been a focus of prior learning.
	Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.

~ 60%	**Simple familiar**
	Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where:
	• relationships and interactions are obvious and have few elements; and
	• all of the information to solve the problem is identifiable; that is
	  − the required procedure is clear from the way the problem is posed, *or*
	  − in a context that has been a focus of prior learning.
	Students are *not* required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.

Conditions

- Time: 120 minutes plus 5 minutes perusal.
- Length: the number of short-response items should allow students to complete the response in the set time.
- Short-response format, consisting of a number of items that ask students to respond to the following activities
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams
  - short items requiring single-word, sentence or short-paragraph responses
  - justifying solutions using appropriate mathematical language where applicable

- responding to seen or unseen stimulus materials
- interpreting ideas and information.

- Other
  - seen stimulus — teachers must ensure the purpose of the technique is not compromised
  - unseen stimulus — materials or questions must not be copied from information or texts that students have previously been exposed to or have used directly in class
  - when stimulus materials are used, they will be succinct enough to allow students sufficient time to engage with them; for stimulus materials that are lengthy, complex or large in number, they will be shared with students prior to the administration of the assessment instrument
  - only the QCAA formula sheet must be provided
  - notes are not permitted
  - use of technology is required; schools must specify the technology used, e.g. scientific calculator, graphics calculator, spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation.

Summary of the instrument-specific marking guide

The following table summarises the mark allocation for the objectives assessed in the examination.

Criterion	Objectives	Marks
Foundational knowledge and problem-solving	1, 2, 3, 4, 5 and 6	15
Total		**15**

Instrument-specific marking guide

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0

5.6.2 Summative external assessment (EA): Examination (50%)

General information

Summative external assessment is developed and marked by the QCAA. In General Mathematics it contributes 50% to a student's overall subject result.

Summative external assessment assesses learning from Units 3 and 4. Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of the examination.

The external assessment in General Mathematics is common to all schools and administered under the same conditions, at the same time, on the same day.

Description

This examination consists of two papers. It assesses the application of a range of cognitions to a number of items, drawn from Units 3 and 4. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment objectives

This assessment technique is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Specifications

Description

The external assessment for General Mathematics representatively samples subject matter from all topics in Units 3 and 4.

The percentage allocation of marks for each paper of the external examination will match the specifications below.

Mark allocations

Percentage of marks	Degree of difficulty
~ 20%	**Complex unfamiliar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable; that is − the required procedure is not clear from the way the problem is posed, and − in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
~ 20%	**Complex familiar** Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable; that is − the required procedure is clear from the way the problem is posed, *or* − in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
~ 60%	Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable; that is − the required procedure is clear from the way the problem is posed, *or* − in a context that has been a focus of prior learning. Students are *not* required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.

Conditions

- Time
  - Paper 1 (30%): 90 minutes plus 5 minutes perusal
    - multiple choice and short response, simple familiar questions, QCAA-approved scientific calculator only
  - Paper 2 (20%): 90 minutes plus 5 minutes perusal
    - short response, complex familiar and complex unfamiliar questions, QCAA-approved scientific calculator only.
- Length: the number of short-response items should allow students to complete the response in the set time.

- Short-response format, consisting of a number of items that ask students to respond to the following activities
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams
  - short items requiring multiple-choice, single-word, term, sentence or short-paragraph responses
  - justifying solutions using appropriate mathematical language where applicable
  - responding to seen or unseen stimulus materials
  - interpreting ideas and information.
- Other
  - the QCAA formula sheet will be provided for both papers
  - notes are not permitted
  - access to a handheld QCAA-approved scientific calculator is required for papers 1 and 2 (no other form of technology is permitted).

Instrument-specific marking guide

No ISMG is provided for the external assessment.

6 Glossary

Term	Explanation
A	
accomplished	highly trained or skilled in a particular activity; perfected in knowledge or training; expert
accuracy	the condition or quality of being true, correct or exact; freedom from error or defect; precision or exactness; correctness; in science, the extent to which a measurement result represents the quantity it purports to measure; an accurate measurement result includes an estimate of the true value and an estimate of the uncertainty
accurate	precise and exact; to the point; consistent with or exactly conforming to a truth, standard, rule, model, convention or known facts; free from error or defect; meticulous; correct in all details
adept	very/highly skilled or proficient at something; expert
adequate	satisfactory or acceptable in quality or quantity equal to the requirement or occasion
adjacency matrix	an adjacency matrix for a non-directed graph with n vertices is an $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j; in an adjacency matrix, a loop is counted as one edge; for a directed graph, the entry in row i and column j is the number of directed edges (arcs) joining the vertex i and j in the direction i to j
algorithm	a precisely defined procedure that can be applied and systematically followed through to a conclusion
analyse	dissect to ascertain and examine constituent parts and/or their relationships; break down or examine in order to identify the essential elements, features, components or structure; determine the logic and reasonableness of information; examine or consider something in order to explain and interpret it, for the purpose of finding meaning or relationships and identifying patterns, similarities and differences
analytic procedures	using algebraic and/or numerical techniques as the primary approach to solving a problem
annuity	a compound interest investment from which payments are made on a regular basis for a defined period of time
applied learning	the acquisition and application of knowledge, understanding and skills in real-world or lifelike contexts that may encompass workplace, industry and community situations; it emphasises learning through doing and includes both theory and the application of theory, connecting subject knowledge and understanding with the development of practical skills

Term	Explanation
Applied subject	a subject whose primary pathway is work and vocational education; it emphasises applied learning and community connections; a subject for which a syllabus has been developed by the QCAA with the following characteristics: results from courses developed from Applied syllabuses contribute to the QCE; results may contribute to ATAR calculations
apply	use knowledge and understanding in response to a given situation or circumstance; carry out or use a procedure in a given or particular situation
appraise	evaluate the worth, significance or status of something; judge or consider a text or piece of work
appreciate	recognise or make a judgment about the value or worth of something; understand fully; grasp the full implications of
appropriate	acceptable; suitable or fitting for a particular purpose, circumstance, context, etc.
apt	suitable to the purpose or occasion; fitting, appropriate
area of study	a division of, or a section within a unit
argue	give reasons for or against something; challenge or debate an issue or idea; persuade, prove or try to prove by giving reasons
arithmetic sequence	a sequence of numbers such that the difference between any two successive members of the sequence is constant, e.g. the sequence 2, 5, 8, 11, 14, 17, … is an arithmetic sequence with first term 2 and common difference 3, by inspection of the sequence, the rule for the nth term t_n of this sequence is $t_n = 2 + (n - 1)3 = 3n - 1,\ n \geq 1,$ if t_n is used to denote the n^{th} term in the sequence, then a recursion relation that will generate this sequence is $t_1 = 2, t_{n+1} = t_n + 3, n \geq 1$
aspect	a particular part of a feature of something; a facet, phase or part of a whole
assess	measure, determine, evaluate, estimate or make a judgment about the value, quality, outcomes, results, size, significance, nature or extent of something
assessment	purposeful and systematic collection of information about students' achievements
assessment instrument	a tool or device used to gather information about student achievement
assessment objectives	drawn from the unit objectives and contextualised for the requirements of the assessment instrument (see also 'syllabus objectives', 'unit objectives')
assessment technique	the method used to gather evidence about student achievement, (e.g. examination, project, investigation)

Term	Explanation
association	a general term used to describe the relationship between two (or more) variables; often used interchangeably with ‘correlation’; association does not imply causation
assumptions	conditions that are stated to be true when beginning to solve a problem
astute	showing an ability to accurately assess situations or people; of keen discernment
ATAR	Australian Tertiary Admission Rank
authoritative	able to be trusted as being accurate or true; reliable; commanding and self-confident; likely to be respected and obeyed
average percentage method	used for calculating a seasonal index, the data for each season are expressed as percentages of the average for the year; the percentages for the corresponding seasons for different years are then averaged using a mean or median to arrive at a seasonal index
B	
balanced	keeping or showing a balance; not biased; fairly judged or presented; taking everything into account in a fair, well-judged way
basic	fundamental
bearing	the direction of a fixed point, or the path of an object, from the point of observation
bipartite graph	a graph whose set of vertices can be split into two distinct groups in such a way that each edge of the graph joins a vertex in the first group to a vertex in the second group
break-even point	the point at which revenue begins to exceed costs
bridge	a bridge is an edge in a connected graph that, if removed, leaves a graph disconnected
C	
calculate	determine or find (e.g. a number, answer) by using mathematical processes; obtain a numerical answer showing the relevant stages in the working; ascertain/determine from given facts, figures or information
categorical variable	a variable whose values are categories, e.g. favourite colour, type of pet
categorise	place in or assign to a particular class or group; arrange or order by classes or categories; classify, sort out, sort, separate
causal relationship	a relationship between an explanatory variable and a response variable is said to be causal if the change in the explanatory variable causes a change in the response variable

Term	Explanation
challenging	difficult but interesting; testing one's abilities; demanding and thought-provoking; usually involving unfamiliar or less familiar elements
characteristic	a typical feature or quality
clarify	make clear or intelligible; explain; make a statement or situation less confused and more comprehensible
clarity	clearness of thought or expression; the quality of being coherent and intelligible; free from obscurity of sense; without ambiguity; explicit; easy to perceive, understand or interpret
classify	arrange, distribute or order in classes or categories according to shared qualities or characteristics
clear	free from confusion, uncertainty, or doubt; easily seen, heard or understood
clearly	in a clear manner; plainly and openly, without ambiguity
coefficient of determination	in a linear model between two variables, the coefficient of determination (R^2) is the proportion of the total variation that can be explained by the linear relationship existing between the two variables, usually expressed as a percentage
coherent	having a natural or due agreement of parts; connected; consistent; logical, orderly; well-structured and makes sense; rational, with parts that are harmonious; having an internally consistent relation of parts
cohesive	characterised by being united, bound together or having integrated meaning; forming a united whole
column matrix	a matrix that has only one column
comment	express an opinion, observation or reaction in speech or writing; give a judgment based on a given statement or result of a calculation
communicate	convey knowledge and/or understandings to others; make known; transmit
compare	display recognition of similarities and differences and recognise the significance of these similarities and differences
competent	having suitable or sufficient skills, knowledge, experience, etc. for some purpose; adequate but not exceptional; capable; suitable or sufficient for the purpose; having the necessary ability, knowledge or skill to do something successfully; efficient and capable (of a person); acceptable and satisfactory, though not outstanding
competently	in an efficient and capable way; in an acceptable and satisfactory, though not outstanding, way
complete graph	a simple graph in which every vertex is joined to every other vertex by an edge; a complete graph with n vertices is denoted K_n

Term	Explanation
complex	composed or consisting of many different and interconnected parts or factors; compound; composite; characterised by an involved combination of parts; complicated; intricate; a complex whole or system; a complicated assembly of particulars
complex familiar	problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning. Some interpretation, clarification and analysis will be required to develop responses. These problems can focus on any of the objectives.
complex unfamiliar	problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and • all the information to solve the problem is not immediately identifiable; that is – the required procedure is not clear from the way the problem is posed, and – in a context in which students have had limited prior experience. Students interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 4, 5 and 6.
compound interest	the interest earned when each successive interest payment is added to the principal for the purpose of calculating the next interest payment, e.g. if the principal P earns compound interest A at the rate of $i\%$ per period, then after n periods the total amount accrued is $A = P(1+i)^n$ when plotted on a graph, the total amount accrued is shown to grow exponentially
comprehend	understand the meaning or nature of; grasp mentally
comprehensive	inclusive; of large content or scope; including or dealing with all or nearly all elements or aspects of something; wide-ranging; detailed and thorough, including all that is relevant
concise	expressing much in few words; giving a lot of information clearly and in a few words; brief, comprehensive and to the point; succinct, clear, without repetition of information
concisely	in a way that is brief but comprehensive; expressing much in few words; clearly and succinctly

Term	Explanation
conduct	direct in action or course; manage; organise; carry out
connected graph	a graph is connected if there is a path between each pair of vertices
consider	think deliberately or carefully about something, typically before making a decision; take something into account when making a judgment; view attentively or scrutinise; reflect on
considerable	fairly large or great; thought about deliberately and with a purpose
considered	formed after careful and deliberate thought
consistent	agreeing or accordant; compatible; not self-opposed or self-contradictory, constantly adhering to the same principles; acting in the same way over time, especially so as to be fair or accurate; unchanging in nature, standard, or effect over time; not containing any logical contradictions (of an argument); constant in achievement or effect over a period of time
construct	create or put together (e.g. an argument) by arranging ideas or items; display information in a diagrammatic or logical form; make; build
continuous variable	a numerical variable that can take any value that lies within an interval, e.g. height or reaction time
contrast	display recognition of differences by deliberate juxtaposition of contrary elements; show how things are different or opposite; give an account of the differences between two or more items or situations, referring to both or all of them throughout
controlled	shows the exercise of restraint or direction over; held in check; restrained, managed or kept within certain bounds
convention	the generally agreed upon way in which something is done; in a mathematical context this refers to notation symbols, abbreviations, usage and setting out
convincing	persuaded by argument or proof; leaving no margin of doubt; clear; capable of causing someone to believe that something is true or real; persuading or assuring by argument or evidence; appearing worthy of belief; credible or plausible
Coordinated Universal Time	UTC; a system of time measurement used to regulate time and clocks around the world, based on International Atomic Time with leap seconds added at irregular intervals to compensate for the earth's slowing rotation; except in highly technical situations, regarded as equivalent to Greenwich Mean Time; initiated in 1961 and adopted internationally in 1972
correlation coefficient	the correlation coefficient (r) is a measure of the strength of the linear relationship between a pair of variables; for variables x and y, and computed for n cases, the formula for r is $r = \frac{1}{n-1}\sum\left(\frac{x_i - \bar{x}}{s_x} \times \frac{y_i - \bar{y}}{s_y}\right)$

Term	Explanation
cosine rule	for a triangle of side lengths a, b and c, and corresponding angles A, B and C, the cosine rule states that $$c^2 = a^2 + b^2 - 2ab\cos C$$
course	a defined amount of learning developed from a subject syllabus
create	bring something into being or existence; produce or evolve from one's own thought or imagination; reorganise or put elements together into a new pattern or structure or to form a coherent or functional whole
creative	resulting from originality of thought or expression; relating to or involving the use of the imagination or original ideas to create something; having good imagination or original ideas
credible	capable or worthy of being believed; believable; convincing
criterion	the property or characteristic by which something is judged or appraised
critical	involving skilful judgment as to truth, merit, etc.; involving the objective analysis and evaluation of an issue in order to form a judgment; expressing or involving an analysis of the merits and faults of a work of literature, music, or art; incorporating a detailed and scholarly analysis and commentary (of a text); rationally appraising for logical consistency and merit
critical path analysis	a project often involves many related activities, some of which cannot be started until one or more earlier tasks have been completed, one way of scheduling such activities that takes this into account is to construct a network diagram
critique	review (e.g. a theory, practice, performance) in a detailed, analytical and critical way
cursory	hasty, and therefore not thorough or detailed; performed with little attention to detail; going rapidly over something, without noticing details; hasty; superficial
D	
decide	reach a resolution as a result of consideration; make a choice from a number of alternatives
deduce	reach a conclusion that is necessarily true, provided a given set of assumptions is true; arrive at, reach or draw a logical conclusion from reasoning and the information given
defensible	justifiable by argument; capable of being defended in argument
define	give the meaning of a word, phrase, concept or physical quantity; state meaning and identify or describe qualities
degree of a vertex	in a graph, the degree of a vertex is the number of edges incident with the vertex, with loops counted twice
demonstrate	prove or make clear by argument, reasoning or evidence, illustrating with practical example; show by example; give a practical exhibition

Term	Explanation
derive	arrive at by reasoning; manipulate a mathematical relationship to give a new equation or relationship; in mathematics, obtain the derivative of a function
describe	give an account (written or spoken) of a situation, event, pattern or process, or of the characteristics or features of something
design	produce a plan, simulation, model or similar; plan, form or conceive in the mind; in English, select, organise and use particular elements in the process of text construction for particular purposes; these elements may be linguistic (words), visual (images), audio (sounds), gestural (body language), spatial (arrangement on the page or screen) and multimodal (a combination of more than one)
detailed	executed with great attention to the fine points; meticulous; including many of the parts or facts
determine	establish, conclude or ascertain after consideration, observation, investigation or calculation; decide or come to a resolution
develop	elaborate, expand or enlarge in detail; add detail and fullness to; cause to become more complex or intricate
devise	think out; plan; contrive; invent
differentiate	identify the difference/s in or between two or more things; distinguish, discriminate; recognise or ascertain what makes something distinct from similar things; in mathematics, obtain the derivative of a function
directed graph	a diagram comprising points, called vertices, joined by directed lines called arcs; directed graphs are commonly called digraphs
discerning	discriminating; showing intellectual perception; showing good judgment; making thoughtful and astute choices; selected for value or relevance
discrete variable	a numerical variable that can take only certain values, e.g. number of people in a car, shoe size
discriminate	note, observe or recognise a difference; make or constitute a distinction in or between; differentiate; note or distinguish as different
discriminating	differentiating; distinctive; perceiving differences or distinctions with nicety; possessing discrimination; perceptive and judicious; making judgments about quality; having or showing refined taste or good judgment
discuss	examine by argument; sift the considerations for and against; debate; talk or write about a topic, including a range of arguments, factors or hypotheses; consider, taking into account different issues and ideas, points for and/or against, and supporting opinions or conclusions with evidence
disjointed	disconnected; incoherent; lacking a coherent order/sequence or connection

Term	Explanation
distinguish	recognise as distinct or different; note points of difference between; discriminate; discern; make clear a difference/s between two or more concepts or items
diverse	of various kinds or forms; different from each other
document	support (e.g. an assertion, claim, statement) with evidence (e.g. decisive information, written references, citations)
draw conclusions	make a judgment based on reasoning and evidence
E	
earliest starting time	using a forward scan of a network diagram to determine the earliest time an activity can begin
effective	successful in producing the intended, desired or expected result; meeting the assigned purpose
effective annual rate of interest	the effective annual rate of interest $i_{effective}$ is used to compare the interest paid on loans (or investments) with the same nominal annual interest rate i but with different compounding periods (daily, monthly, quarterly, annually, other); if the number of compounding periods per annum is n, then $i_{effective} = \left(1 + \frac{i}{n}\right)^n - 1$
efficient	working in a well-organised and competent way; maximum productivity with minimal expenditure of effort; acting or producing effectively with a minimum of waste, expense or unnecessary effort
element	a component or constituent part of a complex whole; a fundamental, essential or irreducible part of a composite entity
elementary	simple or uncompounded; relating to or dealing with elements, rudiments or first principles (of a subject); of the most basic kind; straightforward and uncomplicated
erroneous	based on or containing error; mistaken; incorrect
essential	absolutely necessary; indispensable; of critical importance for achieving something
Euler's formula	for a connected planar graph, Euler's rule states that $v + f - e = 2$ where v = number of vertices, e = number of edges and f = number of faces
Eulerian graph	a connected graph is Eulerian if it has a closed trail (starts and ends at the same vertex), that is, includes every edge once only, such a trail is called a Eulerian trail; a Eulerian trail may include repeated vertices; a connected graph is semi-Eulerian if there is an open trail that includes every edge once only

Term	Explanation
evaluate	make an appraisal by weighing up or assessing strengths, implications and limitations; make judgments about ideas, works, solutions or methods in relation to selected criteria; examine and determine the merit, value or significance of something, based on criteria
examination	a supervised test that assesses the application of a range of cognitions to one or more provided items such as questions, scenarios and/or problems; student responses are completed individually, under supervised conditions, and in a set timeframe
examine	investigate, inspect or scrutinise; inquire or search into; consider or discuss an argument or concept in a way that uncovers the assumptions and interrelationships of the issue
experiment	try out or test new ideas or methods, especially in order to discover or prove something; undertake or perform a scientific procedure to test a hypothesis, make a discovery or demonstrate a known fact
explain	make an idea or situation plain or clear by describing it in more detail or revealing relevant facts; give an account; provide additional information
explanatory variable	when investigating relationships in bivariate data, the explanatory variable (independent variable) is the variable used to explain or predict a difference in the response variable (dependent variable), e.g. when investigating the relationship between the temperature of a loaf of bread and the time it has spent in a hot oven, temperature is the response variable and time is the explanatory variable
explicit	clearly and distinctly expressing all that is meant; unequivocal; clearly developed or formulated; leaving nothing merely implied or suggested
explore	look into both closely and broadly; scrutinise; inquire into or discuss something in detail
express	convey, show or communicate (e.g. a thought, opinion, feeling, emotion, idea or viewpoint); in words, art, music or movement, convey or suggest a representation of; depict
extended response	an open-ended assessment technique that focuses on the interpretation, analysis, examination and/or evaluation of ideas and information in response to a particular situation or stimulus; while students may undertake some research when writing of the extended response, it is not the focus of this technique; an extended response occurs over an extended and defined period of time
Extension subject	a two-unit subject (Units 3 and 4) for which a syllabus has been developed by QCAA, that is an extension of one or more General subject/s, studied concurrently with, Units 3 and 4 of that subject or after completion of, Units 3 and 4 of that subject
extensive	of great extent; wide; broad; far-reaching; comprehensive; lengthy; detailed; large in amount or scale

Term	Explanation
external assessment	summative assessment that occurs towards the end of a course of study and is common to all schools; developed and marked by the QCAA according to a commonly applied marking scheme
external examination	a supervised test, developed and marked by the QCAA, that assesses the application of a range of cognitions to multiple provided items such as questions, scenarios and/or problems; student responses are completed individually, under supervised conditions, and in a set timeframe
extrapolate	infer or estimate by extending or projecting known information; conjecture; infer from what is known; extend the application of something (e.g. a method or conclusion) to an unknown situation by assuming that existing trends will continue or similar methods will be applicable
F	
face	the faces of a planar graph are the regions bounded by the edges including the outer infinitely large region
factual	relating to or based on facts; concerned with what is actually the case; actually occurring; having verified existence
familiar	well-acquainted; thoroughly conversant with; well known from long or close association; often encountered or experienced; common; (of materials, texts, skills or circumstances) having been the focus of learning experiences or previously encountered in prior learning activities
feasible	capable of being achieved, accomplished or put into effect; reasonable enough to be believed or accepted; probable; likely
float time	the amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks; also called *slack time*
flow network	a directed graph where each edge has a capacity and each edge receives a flow, the amount of flow on an edge cannot exceed the capacity of the edge; a flow must satisfy the restriction that the amount of flow into a node equals the amount of flow out of it, except when it is a source, which has more outgoing flow, or a sink, which has more incoming flow
fluent	spoken or written with ease; able to speak or write smoothly, easily or readily; articulate; eloquent; in artistic performance, characteristic of a highly developed and excellently controlled technique; flowing; polished; flowing smoothly, easily and effortlessly
fluently	in a graceful and seemingly effortless manner; in a way that progresses smoothly and readily
formative assessment	assessment whose major purpose is to improve teaching and student achievement
fragmented	disorganised; broken down; disjointed or isolated

Term	Explanation
frequent	happening or occurring often at short intervals; constant, habitual, or regular
fundamental	forming a necessary base or core; of central importance; affecting or relating to the essential nature of something; part of a foundation or basis
G	
General subject	a subject for which a syllabus has been developed by the QCAA with the following characteristics: results from courses developed from General syllabuses contribute to the QCE; General subjects have an external assessment component; results may contribute to ATAR calculations
generate	produce; create; bring into existence
geometric growth or decay	a sequence displays geometric growth or decay when each term is some constant multiple (greater or less than one) of the preceding term: a multiple greater than one corresponds to growth, a multiple less than one corresponds to decay, e.g. 1, 2, 4, … displays geometric growth because each term is double the previous term, 100, 10, 0.1, … displays geometric decay because each term is one tenth of the previous term
geometric sequence	a sequence of numbers where each term after the first is found by multiplying the previous term by a fixed non-zero number (excluding ±1) called the common ratio, e.g. 2, 6, 18, ... is a geometric sequence with first term 2 and common ratio 3; by inspection of the sequence, the rule for the n^{th} term of this sequence is $t_n = 2 \times 3^{(n-1)},\ n \geq 1$ if t_n is used to denote the n^{th} term in the sequence, then a recursion relation that will generate this sequence is $t_1 = 2, t_n + 1 = 3t_n,\ n \geq 1$
global positioning system	GPS; a navigational system that relies on information received from a network of satellites to provide the latitude and longitude of an object
GMT	Greenwich Mean Time, the mean solar time of the meridian through Greenwich, England, widely used throughout the world (from 1884 to 1972) as a basis for calculating local time; equivalent to UTC
goods and services tax	GST; a broad sales tax of 10% on most goods and services transactions in Australia
GPS	global positioning system, a navigational system that relies on information received from a network of satellites to provide the latitude and longitude of an object
great circle	in earth geometry, a circle that cuts through the centre of the Earth; a circle that has a radius the same as the earth, e.g. the equator and the meridians of longitude

Term	Explanation
Greenwich Mean Time	GMT; the mean solar time of the meridian through Greenwich, England, widely used throughout the world (from 1884 to 1972) as a basis for calculating local time; equivalent to UTC
GST	goods and services tax, a broad sales tax of 10% on most goods and services transactions in Australia
H	
Hamiltonian graph	a network diagram that begins and ends at the same vertex and connects every vertex only once
Heron's rule	a rule for determining the area of a triangle given the lengths of its sides: the area A of a triangle of side lengths a, b and c is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$
hypothesise	formulate a supposition to account for known facts or observed occurrences; conjecture, theorise, speculate; especially on uncertain or tentative grounds
I	
identify	distinguish; locate, recognise and name; establish or indicate who or what someone or something is; provide an answer from a number of possibilities; recognise and state a distinguishing factor or feature
identity matrix	a square matrix in which all of the elements in the leading diagonal are 1s and the remaining elements are 0s; identity matrices are designated by the letter I e.g. $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ there is an identity matrix for each size (or order) of a square matrix
illogical	lacking sense or sound reasoning; contrary to or disregardful of the rules of logic; unreasonable
implement	put something into effect, e.g. a plan or proposal
implicit	implied, rather than expressly stated; not plainly expressed; capable of being inferred from something else
improbable	not probable; unlikely to be true or to happen; not easy to believe
inaccurate	not accurate
inappropriate	not suitable or proper in the circumstances
inconsistent	lacking agreement, as one thing with another, or two or more things in relation to each other; at variance; not consistent; not in keeping; not in accordance; incompatible, incongruous
independent	thinking or acting for oneself, not influenced by others
in-depth	comprehensive and with thorough coverage; extensive or profound; well-balanced or fully developed

Term	Explanation
infer	derive or conclude something from evidence and reasoning, rather than from explicit statements; listen or read beyond what has been literally expressed; imply or hint at
informed	knowledgeable; learned; having relevant knowledge; being conversant with the topic; based on an understanding of the facts of the situation (of a decision or judgment)
innovative	new and original; introducing new ideas; original and creative in thinking
insightful	showing understanding of a situation or process; understanding relationships in complex situations; informed by observation and deduction
instrument-specific marking guide	ISMG; a tool for marking that describes the characteristics evident in student responses and aligns with the identified objectives for the assessment (see 'assessment objectives')
integral	*adjective* necessary for the completeness of the whole; essential or fundamental; *noun* in mathematics, the result of integration; an expression from which a given function, equation, or system of equations is derived by differentiation
intended	designed; meant; done on purpose; intentional
internal assessment	assessments that are developed by schools; summative internal assessments are endorsed by the QCAA before use in schools and results externally confirmed contribute towards a student's final result
International Date Line	a line, theoretically coinciding with the meridian of 180° from Greenwich, England, the regions on either side of which are counted as differing by one day in their calendar dates
interpolation	in the context of fitting a linear relationship between two variables, interpolation occurs when the fitted model is used to make predictions using values of the explanatory variable that lie within the range of the original data
interpret	use knowledge and understanding to recognise trends and draw conclusions from given information; make clear or explicit; elucidate or understand in a particular way; bring out the meaning of, e.g. a dramatic or musical work, by performance or execution; bring out the meaning of an artwork by artistic representation or performance; give one's own interpretation of; identify or draw meaning from, or give meaning to, information presented in various forms, such as words, symbols, pictures or graphs

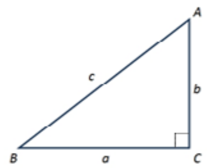
Term	Explanation
interquartile range	a measure of the spread within a numerical dataset; it is equal to the upper quartile (Q_3) minus the lower quartile (Q_1), that is, $IQR = Q_3 - Q_1$ • Q_1 is the median of the lower half of the data (excluding the median, Q_2, of the dataset) • Q_3 is the median of the upper half of the data (excluding the median, Q_2, of the dataset) hence IQR is the width of an interval that contains the middle 50% (approximately) of the data values; to be exactly 50%, the sample size must be a multiple of four
investigate	carry out an examination or formal inquiry in order to establish or obtain facts and reach new conclusions; search, inquire into, interpret and draw conclusions about data and information
investigation	an assessment technique that requires students to research a specific problem, question, issue, design challenge or hypothesis through the collection, analysis and synthesis of primary and/or secondary data; it uses research or investigative practices to assess a range of cognitions in a particular context; an investigation occurs over an extended and defined period of time
irrelevant	not relevant; not applicable or pertinent; not connected with or relevant to something
ISMG	instrument-specific marking guide; a tool for marking that describes the characteristics evident in student responses and aligns with the identified objectives for the assessment (see ‘assessment objectives’)
isolated	detached, separate, or unconnected with other things; one-off; something set apart or characterised as different in some way
J	
judge	form an opinion or conclusion about; apply both procedural and deliberative operations to make a determination
justified	sound reasons or evidence are provided to support an argument, statement or conclusion
justify	give reasons or evidence to support an answer, response or conclusion; show or prove how an argument, statement or conclusion is right or reasonable
L	
latitude	the angular distance north or south from the equator of a point on the Earth’s surface, usually measured in degrees, minutes and seconds, e.g. Canberra has a latitude of 35°17′ S

Term	Explanation
learning area	a grouping of subjects, with related characteristics, within a broad field of learning, e.g. the Arts, sciences, languages
least-squares line	the line of best fit for which the sum of the squared residuals is the smallest; the general form of this line is: $$y = a + bx$$ where the slope of the line is $b = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = r\frac{s_y}{s_x}$ and the y-intercept is $a = \bar{y} - b\bar{x}$
least-squares method	in fitting a straight line $y = a + bx$ to the relationship between a response variable y and an explanatory variable x, the least-squares line is the line for which the sum of the squared residuals is the smallest
length	in graphs and networks, the length of a walk is the number of edges it includes
linear equation	a linear equation in one variable x is an equation of the form $ax + b = 0$, e.g. $3x + 1 = 0$; a linear equation in two variables x and y is an equation of the form $ax + by + c = 0$, e.g. $2x - 3y + 5 = 0$
linear graph	a graph of a linear equation with two variables; if the linear equation is written in the form $y = a + bx$, then a represents the y-intercept and b represents the slope (or gradient) of the linear graph
linear growth or decay	a sequence displays linear growth or decay when the difference between successive terms is constant; a positive constant difference corresponds to linear growth while a negative constant difference corresponds to decay e.g. the sequence, 1, 4, 7, … displays linear growth because the difference between successive terms is 3; the sequence, 100, 90, 80, … displays linear decay because the difference between successive terms is –10; by definition, arithmetic sequences display linear growth or decay
logical	rational and valid; internally consistent; reasonable; reasoning in accordance with the principles/rules of logic or formal argument; characterised by or capable of clear, sound reasoning; (of an action, decision, etc.) expected or sensible under the circumstances
logically	according to the rules of logic or formal argument; in a way that shows clear, sound reasoning; in a way that is expected or sensible
longitude	angular distance east or west on the Earth's surface, measured along the equator by the angle contained between the meridian of a particular place and the prime meridian, usually in degrees, minutes and seconds, e.g. Canberra has a longitude of 149°08′ E
M	
make decisions	select from available options; weigh up positives and negatives of each option and consider all the alternatives to arrive at a position

Term	Explanation
manipulate	adapt or change to suit one's purpose
mathematical model	a depiction of a situation that expresses relationships using mathematical concepts and language, usually as an algebraic, diagrammatic, graphical or tabular representation
mathematical modelling	involves: • formulating a mathematical representation of a problem derived from within a real-world context • using mathematics concepts and techniques to obtain results • interpreting the results by referring back to the original problem context • revising the model (where necessary)
matrix	a rectangular array of elements or entities, usually numbers, displayed in rows and columns
matrix addition	if **A** and **B** are matrices of the same size (order) and the elements of **A** are a_{ij} and the elements of **B** are b_{ij}, then the elements of **A** + **B** are $a_{ij} + b_{ij}$ e.g. $\begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 0 & 12 \end{bmatrix}$
matrix multiplication	the process of multiplying a matrix by another matrix
mean	the arithmetic mean, $\bar{x}$, of a list of numbers is the sum of the data values divided by the number of values in the list
median	the median is the value in an ordered set of data values that divides the data into two parts of equal size; when there are an odd number of data values, the median is the middle value; when there is an even number of data values, the median is the mean of the two central values
mental procedures	a domain of knowledge in Marzano's taxonomy, and acted upon by the cognitive, metacognitive and self-systems; sometimes referred to as 'procedural knowledge' there are three distinct phases to the acquisition of mental procedures — the cognitive stage, the associative stage, and the autonomous stage; the two categories of mental procedures are skills (single rules, algorithms and tactics) and processes (macroprocedures)
meridian	the half of a great circle of the Earth ending at the poles, used in measuring east–west position
methodical	performed, disposed or acting in a systematic way; orderly; characterised by method or order; performed or carried out systematically
minimal	least possible; small, the least amount; negligible
minimum spanning tree	for a given connected weighted graph, the edges that connect all vertices together with the minimum total edge
mode	in mathematics, the most frequently occurring value in a dataset; a dataset can have more than one mode

Term	Explanation
modify	change the form or qualities of; make partial or minor changes to something
moving average	a process that reduces the effect of non-typical data and makes the overall trend easier to see; in a time series, a simple moving average is a method used to smooth the time series by replacing each observation with a simple average of the observation and its near neighbours
multimodal	uses a combination of at least two modes (e.g. spoken, written), delivered at the same time, to communicate ideas and information to a live or virtual audience, for a particular purpose; the selected modes are integrated so that each mode contributes significantly to the response
N	
narrow	limited in range or scope; lacking breadth of view; limited in amount; barely sufficient or adequate; restricted
network diagram	a set of points, called vertices, that are joined by a set of lines called edges; each edge joins two vertices
nuanced	showing a subtle difference or distinction in expression, meaning, response, etc.; finely differentiated; characterised by subtle shades of meaning or expression; a subtle distinction, variation or quality; sensibility to, awareness of, or ability to express delicate shadings, as of meaning, feeling, or value
O	
objectives	see 'syllabus objectives', 'unit objectives', 'assessment objectives'
observation	data or information required to solve a mathematical problem and/or develop a mathematical model; empirical evidence
obvious	clearly perceptible or evident; easily seen, recognised or understood
optimal	best, most favourable, under a particular set of circumstances
organise	arrange, order; form as or into a whole consisting of interdependent or coordinated parts, especially for harmonious or united action
organised	systematically ordered and arranged; having a formal organisational structure to arrange, coordinate and carry out activities
outlier	an outlier in a set of data is an observation that appears to be inconsistent with the other data $Q_1 - 1.5 \times \mathrm{IQR} \leq x \leq Q_3 + 1.5 \times \mathrm{IQR}$
outstanding	exceptionally good; clearly noticeable; prominent; conspicuous; striking

Term	Explanation
P	
partial	not total or general; existing only in part; attempted, but incomplete
particular	distinguished or different from others or from the ordinary; noteworthy
path	in a graph, a walk in which all the edges and all the vertices are different; a path that starts and finishes at different vertices is said to be open, while a path that starts and finishes at the same vertex is said to be closed; a cycle is a closed path
perceptive	having or showing insight and the ability to perceive or understand; discerning (see also 'discriminating')
performance	an assessment technique that requires students to demonstrate a range of cognitive, technical, creative and/or expressive skills and to apply theoretical and conceptual understandings, through the psychomotor domain; it involves student application of identified skills when responding to a task that involves solving a problem, providing a solution or conveying meaning or intent; a performance is developed over an extended and defined period of time
persuasive	capable of changing someone's ideas, opinions or beliefs; appearing worthy of approval or acceptance; (of an argument or statement) communicating reasonably or credibly (see also 'convincing')
perusal time	time allocated in an assessment to reading items and tasks and associated assessment materials; no writing is allowed; students may not make notes and may not commence responding to the assessment in the response space/book
piece-wise linear graph	a graph consisting of one or more overlapping line segments, sometimes called a line segment graph
planar graph	a graph that can be drawn in the plane; a planar graph can always be drawn so that no two edges cross
planning time	time allocated in an assessment to planning how to respond to items and tasks and associated assessment materials; students may make notes but may not commence responding to the assessment in the response space/book; notes made during planning are not collected, nor are they graded or used as evidence of achievement
polished	flawless or excellent; performed with skilful ease
polynomial	an expression consisting of the sum of two or more terms, each of which is the product of a constant and a variable raised to an integral power: $ax^2 + bx + c$ is a polynomial, where a, b, and c are constants and x is a variable.
precise	definite or exact; definitely or strictly stated, defined or fixed; characterised by definite or exact expression or execution
precision	accuracy; exactness; exact observance of forms in conduct or actions

Term	Explanation
predict	give an expected result of an upcoming action or event; suggest what may happen based on available information
price-to-earnings ratio	the price to earnings ratio of a share (P/E ratio) is defined as: $$\text{P/E ratio} = \frac{\text{market price per share}}{\text{annual earnings per share}}$$
Prim's algorithm	an algorithm for determining a minimum spanning tree in a connected weighted graph
prime meridian	the meridian from which longitude east and west is reckoned, usually that of Greenwich, England
procedural vocabulary	instructional terms used in a mathematical context (e.g. calculate, convert, determine, identify, justify, show, sketch, solve, state)
product	an assessment technique that focuses on the output or result of a process requiring the application of a range of cognitive, physical, technical, creative and/or expressive skills, and theoretical and conceptual understandings; a product is developed over an extended and defined period of time
proficient	well advanced or expert in any art, science or subject; competent, skilled or adept in doing or using something
project	an assessment technique that focuses on a problem-solving process requiring the application of a range of cognitive, technical and creative skills and theoretical understandings; the response is a coherent work that documents the iterative process undertaken to develop a solution and includes written paragraphs and annotations, diagrams, sketches, drawings, photographs, video, spoken presentations, physical prototypes and/or models; a project is developed over an extended and defined period of time
propose	put forward (e.g. a point of view, idea, argument, suggestion) for consideration or action
prove	use a sequence of steps to obtain the required result in a formal way
psychomotor procedures	a domain of knowledge in Marzano's taxonomy, and acted upon by the cognitive, metacognitive and self-systems; these are physical procedures used to negotiate daily life and to engage in complex physical activities; the two categories of psychomotor procedures are skills (foundational procedures and simple combination procedures) and processes (complex combination procedures)
purposeful	having an intended or desired result; having a useful purpose; determined; resolute; full of meaning; significant; intentional
Pythagoras' theorem	the square of the hypotenuse of a right-angled triangle equals the sum of the squares of the lengths of the other two sides; in symbols, $a^2 + b^2 = c^2$ 

Term	Explanation
Q	
QCE	Queensland Certificate of Education
qualitative statements	statements relating to a quality or qualities; statements of a non-numerical nature
quantitative analysis	use of mathematical measurements and calculations, including statistics, to analyse the relationships between variables; may include use of the correlation coefficient, coefficient of determination, simple residual analysis or outlier analysis
Queensland Certificate of Education	QCE
R	
realise	create or make (e.g. a musical, artistic or dramatic work); actualise; make real or concrete; give reality or substance to
reasonable	endowed with reason; having sound judgment; fair and sensible; based on good sense; average; appropriate, moderate
reasonableness of solutions	to justify solutions obtained with or without technology using everyday language, mathematical language or a combination of both; may be applied to calculations to check working, or to questions that require a relationship back to the context
reasoned	logical and sound; based on logic or good sense; logically thought out and presented with justification; guided by reason; well-grounded; considered
recall	remember; present remembered ideas, facts or experiences; bring something back into thought, attention or into one's mind
recognise	identify or recall particular features of information from knowledge; identify that an item, characteristic or quality exists; perceive as existing or true; be aware of or acknowledge
recurrence relation	an equation that recursively defines a sequence, that is, once one or more initial terms are given, each further term of the sequence is defined as a function of the preceding terms
reducing balance loan	a type of compound interest loan where the loan is repaid in regular payments and the interest paid is calculated on the amount still owing (the reducing balance of the loan) after each payment is made
refined	developed or improved so as to be precise, exact or subtle
reflect on	think about deeply and carefully
rehearsed	practised; previously experienced; practised extensively
related	associated with or linked to
relevance	being related to the matter at hand
relevant	bearing upon or connected with the matter in hand; to the purpose; applicable and pertinent; having a direct bearing on

Term	Explanation
repetitive	containing or characterised by repetition, especially when unnecessary or tiresome
reporting	providing information that succinctly describes student performance at different junctures throughout a course of study
representatively sample	in this syllabus, a selection of subject matter that accurately reflects the intended learning of a topic
resolve	in the Arts, consolidate and communicate intent through a synthesis of ideas and application of media to express meaning
response variable	also known as the dependent variable; its value is dependent on the value of the explanatory (or independent) variable
routine	often encountered, previously experienced; commonplace; customary and regular; well-practised; performed as part of a regular procedure, rather than for a special reason
row matrix	a matrix that has only one row
rudimentary	relating to rudiments or first principles; elementary; undeveloped; involving or limited to basic principles; relating to an immature, undeveloped or basic form
S	
safe	secure; not risky
scale factor	a number that scales, or multiplies, some quantity; in the equation $y = kx, k$ is the scale factor for x; if two or more figures are similar, their sizes can be compared; the scale factor is the ratio of the length of one side on one figure to the length of the corresponding side on the other figure; a measure of magnification; the change of size
scatterplot	a two-dimensional data plot using Cartesian co-ordinates to display the values of two variables in a bivariate dataset
seasonal adjustment	adjusting for seasonality; a term used to describe a time series from which periodic variations due to seasonal effects have been removed
seasonal index	the seasonal index can be used to remove seasonality from data; an index value is attached to each period of the time series within a year; for the seasons of the year (summer, autumn, winter, spring) there are four separate seasonal indices; for months, there are 12 separate seasonal indices, one for each month, and so on; there are several methods for determining seasonal indices
seasonal variation	a regular rise and fall in the time series that recurs each year; seasonal variation is measured in terms of a seasonal index
secure	sure; certain; able to be counted on; self-confident; poised; dependable; confident; assured; not liable to fail
select	choose in preference to another or others; pick out

Term	Explanation
sensitive	capable of perceiving with a sense or senses; aware of the attitudes, feelings or circumstances of others; having acute mental or emotional sensibility; relating to or connected with the senses or sensation
sequence	place in a continuous or connected series; arrange in a particular order
show	provide the relevant reasoning to support a response
significant	important; of consequence; expressing a meaning; indicative; includes all that is important; sufficiently great or important to be worthy of attention; noteworthy; having a particular meaning; indicative of something
similar figures	two geometric figures are similar if they are of the same shape but not necessarily of the same size
simple	easy to understand, deal with and use; not complex or complicated; plain; not elaborate or artificial; may concern a single or basic aspect; involving few elements, components or steps
simple familiar	problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where: • relationships and interactions are obvious and have few elements; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning. Students are not required to interpret, clarify and analyse problems to develop responses. Typically, these problems focus on objectives 1, 2 and 3.
simple graph	a simple graph has no loops or multiple edges
simple interest	simple interest is the interest accumulated when the interest payment in each period is a fixed fraction of the principal, e.g. if the principle P earns simple interest at the rate of $i\%$ per period, then after n periods the accumulated simple interest is: $$I = Pin$$ when plotted on a graph, the total amount accrued is shown to grow linearly
simplistic	characterised by extreme simplification, especially if misleading; oversimplified
sine rule	for a triangle of side lengths, a, b and c and angles A, B and C, the sine rule states that: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
size of the matrix	two matrixes are said to have the same size (or order) if they have the same number of rows and columns, a matrix with m rows and n columns is an $m \times n$ matrix

Term	Explanation
sketch	execute a drawing or painting in simple form, giving essential features but not necessarily with detail or accuracy; in mathematics, represent by means of a diagram or graph; the sketch should give a general idea of the required shape or relationship and should include features
skilful	having technical facility or practical ability; possessing, showing, involving or requiring skill; expert, dexterous; demonstrating the knowledge, ability or training to perform a certain activity or task well; trained, practised or experienced
skilled	having or showing the knowledge, ability or training to perform a certain activity or task well; having skill; trained or experienced; showing, involving or requiring skill
slope	the steepness, incline, or grade of a line; slope is normally described by the ratio of the 'rise' divided by the 'run' between two points on a line; also 'gradient'
solve	find an answer to, explanation for, or means of dealing with (e.g. a problem); work out the answer or solution to (e.g. a mathematical problem); obtain the answer/s using algebraic, numerical and/or graphical methods
sophisticated	of intellectual complexity; reflecting a high degree of skill, intelligence, etc.; employing advanced or refined methods or concepts; highly developed or complicated
specific	clearly defined or identified; precise and clear in making statements or issuing instructions; having a special application or reference; explicit, or definite; peculiar or proper to something, as qualities, characteristics, effects, etc.
sporadic	happening now and again or at intervals; irregular or occasional; appearing in scattered or isolated instances
square matrix	a square matrix has the same number of rows and columns e.g. $\begin{bmatrix} 4 & 0 \\ 7 & -2 \end{bmatrix}$
standard deviation	a measure of the variability or spread of a dataset; it indicates the degree to which the individual data values are spread around their mean; the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is: $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$
step graph	a graph consisting of one or more non-overlapping horizontal line segments that follow a step-like pattern
statement	a sentence or assertion
straightforward	without difficulty; uncomplicated; direct; easy to do or understand

Term	Explanation
structure	*verb* give a pattern, organisation or arrangement to; construct or arrange according to a plan; *noun* in languages, arrangement of words into larger units, e.g. phrases, clauses, sentences, paragraphs and whole texts, in line with cultural, intercultural and textual conventions
structured	organised or arranged so as to produce a desired result
subject	a branch or area of knowledge or learning defined by a syllabus; school subjects are usually based in a discipline or field of study (see also 'course')
subject matter	the subject-specific body of information, mental procedures and psychomotor procedures that are necessary for students' learning and engagement within that subject
substantial	of ample or considerable amount, quantity, size, etc.; of real worth or value; firmly or solidly established; of real significance; reliable; important, worthwhile
substantiated	established by proof or competent evidence
subtle	fine or delicate in meaning or intent; making use of indirect methods; not straightforward or obvious
successful	achieving or having achieved success; accomplishing a desired aim or result
succinct	expressed in few words; concise; terse; characterised by conciseness or brevity; brief and clear
sufficient	enough or adequate for the purpose
suitable	appropriate; fitting; conforming or agreeing in nature, condition, or action
summarise	give a brief statement of a general theme or major point/s; present ideas and information in fewer words and in sequence
summative assessment	assessment whose major purpose is to indicate student achievement; summative assessments contribute towards a student's subject result
superficial	concerned with or comprehending only what is on the surface or obvious; shallow; not profound, thorough, deep or complete; existing or occurring at or on the surface; cursory; lacking depth of character or understanding; apparent and sometimes trivial
supported	corroborated; given greater credibility by providing evidence
sustained	carried on continuously, without interruption, or without any diminishing of intensity or extent
syllabus	a document that prescribes the curriculum for a course of study

Term	Explanation
syllabus objectives	outline what the school is required to teach and what students have the opportunity to learn; described in terms of actions that operate on the subject matter; the overarching objectives for a course of study (see also ‘unit objectives’, ‘assessment objectives’)
symbolise	represent or identify by a symbol or symbols
synthesise	combine different parts or elements (e.g. information, ideas, components) into a whole, in order to create new understanding
systematic	done or acting according to a fixed plan or system; methodical; organised and logical; having, showing, or involving a system, method, or plan; characterised by system or method; methodical; arranged in, or comprising an ordered system
T	
technical vocabulary	terms that have a precise mathematical meaning (e.g. categorical data, chain rule, decimal fraction, imaginary number, log laws, linear regression, sine rule, whole number); may include everyday words used in a mathematical context (e.g. capacity, differentiate, evaluate, integrate, order, property, sample, union)
test	take measures to check the quality, performance or reliability of something
thorough	carried out through, or applied to the whole of something; carried out completely and carefully; including all that is required; complete with attention to every detail; not superficial or partial; performed or written with care and completeness; taking pains to do something carefully and completely
thoughtful	occupied with, or given to thought; contemplative; meditative; reflective; characterised by or manifesting thought
time series	values of a variable recorded, usually at regular intervals, over a period of time; the observed movement and fluctuations of many such series comprise long-term trend, seasonal variation, and irregular variation or noise
time series plot	the graph of a time series with time plotted on the horizontal axis
time zone	one of the 24 regions or divisions of the globe approximately coinciding with meridians at successive hours from the observatory at Greenwich
topic	a division of, or sub-section within a unit; all topics/sub-topics within a unit are interrelated
trail	in a graph, a trail is a walk in which no edge is repeated; a closed trail must start and finish at the same vertex, whereas an open trail does not
transpose	to rearrange a formula, or equation, usually for the purpose of changing the subject of the formula
tree	in a network, a connected graph with no cycles

Term	Explanation
trend	in a time series, the general direction of the series (increasing/decreasing) over a long period of time
true bearings	true (or three-figure) bearings are measured clockwise in degrees from the north line; three figures are used to specify the direction
two-way frequency table	used for displaying the two-way frequency distribution that arises when a group of individuals or objects are categorised according to two criteria
U	
unclear	not clear or distinct; not easy to understand; obscure
understand	perceive what is meant by something; grasp; be familiar with (e.g. an idea); construct meaning from messages, including oral, written and graphic communication
uneven	unequal; not properly corresponding or agreeing; irregular; varying; not uniform; not equally balanced
unfamiliar	not previously encountered; situations or materials that have not been the focus of prior learning experiences or activities
unit	a defined amount of subject matter delivered in a specific context or with a particular focus; it includes unit objectives particular to the unit, subject matter and assessment direction
unit objectives	drawn from the syllabus objectives and contextualised for the subject matter and requirements of a particular unit; they are assessed at least once in the unit (see also 'syllabus objectives', 'assessment objectives')
unrelated	having no relationship; unconnected
use	operate or put into effect; apply knowledge or rules to put theory into practice
UTC	Coordinated Universal Time; a system of time measurement used to regulate time and clocks around the world, based on International Atomic Time with leap seconds added at irregular intervals to compensate for the earth's slowing rotation; except in highly technical situations, regarded as equivalent to Greenwich Mean Time; initiated in 1961 and adopted internationally in 1972
V	
vague	not definite in statement or meaning; not explicit or precise; not definitely fixed, determined or known; of uncertain, indefinite or unclear character or meaning; not clear in thought or understanding; couched in general or indefinite terms; not definitely or precisely expressed; deficient in details or particulars; thinking or communicating in an unfocused or imprecise way
valid	sound, just or well-founded; authoritative; having a sound basis in logic or fact (of an argument or point); reasonable or cogent; able to be supported; legitimate and defensible; applicable

Term	Explanation
variable	*adjective* apt or liable to vary or change; changeable; inconsistent; (readily) susceptible or capable of variation; fluctuating, uncertain; *noun* in mathematics, a symbol, or the quantity it signifies, that may represent any one of a given set of number and other objects
variety	a number or range of things of different kinds, or the same general class, that are distinct in character or quality; (of sources) a number of different modes or references
W	
walk	in a graph, a walk is sequence of vertices such that from each vertex there is an edge to the next vertex in the sequence; a walk that starts and finishes at different vertices is said to be an open walk; a walk that starts and finishes at the same vertex is said to be closed walk
weighted graph	a graph in which each edge is labelled with a number used to represent some quantity associated with the edge
wide	of great range or scope; embracing a great number or variety of subjects, cases, etc.; of full extent
with expression	in words, art, music or movement, conveying or indicating feeling, spirit, character, etc.; a way of expressing or representing something; vivid, effective or persuasive communication
Z	
zero matrix	a matrix with all of its entries being zero e.g. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

7 References

Australian Curriculum and Assessment Authority (ACARA) 2015, *Australian Curriculum Senior Secondary Curriculum: General Mathematics*, version 7.5, v7-5.australiancurriculum.edu.au/seniorsecondary/mathematics/general-mathematics/curriculum/seniorsecondary.

—— 2015a, *Australian Curriculum Senior Secondary Curriculum: General Mathematics Glossary*, version 7.5, https://www.australiancurriculum.edu.au/senior-secondary-curriculum/mathematics/general-mathematics/glossary/.

Blum, W, Galbraith, PL, Henn, HW & Niss, M 2007, *Modelling and applications in mathematics education,* Springer, New York.

Galbraith, P 1989, 'From applications to modelling', in D Blane & M Evans (eds), *Mathematical modelling for the senior years*, The Mathematical Association of Victoria, Parkville, pp. 78–86.

Geiger, V, Faragher, R & Goos, M 2010, 'CAS-enabled technologies as "agents provocateurs" in teaching and learning mathematical modelling in secondary school classrooms', *Mathematics Education Research Journal*, vol. 22, no. 2, pp. 48–68, doi.org/10.1007/BF03217565.

Goos, M 2014, 'Mathematics classroom assessment', *Encyclopedia of Mathematics Education*, Springer, Dordrecht, pp. 413–417.

Goos, M Geiger, V & Dole, S 2012, 'Auditing the numeracy demands of the middle years curriculum', *Mathematics Education: Expanding horizons — Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia*, Mathematics Education Research Group of Australasia, Singapore, pp. 314–321.

Grønmo, LS, Lindquist, M, Arora, A & Mullis, IVS 2015, 'TIMSS 2015 Mathematics Framework', *TIMSS 2015 Assessment Frameworks*, International Study Center, Boston, timssandpirls.bc.edu/timss2015/frameworks.html#.

Kaiser, G, Blum, W, Ferri, RB & Stillman, G (eds) 2011, *Trends in teaching and learning of mathematical modelling: ICTMA14,* International perspectives on the teaching and learning of mathematical modelling, vol. 1, Springer, Vancouver.

Kilpatrick, J, Swafford, J, & Bradford, F (eds) 2001, *Adding It Up: Helping children learn mathematics*, National Academies Press, Washington, DC.

Marzano, RJ & Kendall, JS 2008, *Designing and Assessing Educational Objectives: Applying the new taxonomy*, Corwin Press, USA.

—— 2007, *The New Taxonomy of Educational Objectives*, 2nd edn, Corwin Press, USA.

Mevarech, Z, & Kramarski, B 2014, *Critical Maths for Innovative Societies: The role of metacognitive pedagogies*, OECD Publishing, Paris.

Norton, S, & O'Connor, BR 2016, *Literature review for senior syllabus revisions: Mathematics*, Queensland Curriculum and Assessment Authority, Brisbane.

OECD 2015, *PISA 2015 Mathematics Framework*, OECD Publishing, Paris.

—— 2012, 'Numeracy', in *Literacy, Numeracy and Problem Solving in Technology-Rich Environments — Framework for the OECD Survey of Adult Skills*, OECD, doi.org/10.1787/9789264128859-en.

—— 2003, *PISA 2003 Assessment Framework*, OECD Publishing, Paris.

Polya, G 1957, *How to Solve It: A new aspect of mathematical method*, 2nd edn, Princeton University Press, NJ.

Shafer, MC & Foster, S 1997, 'What's up on the web: The changing face of assessment', *Principled Practice in Mathematics and Science Education: Fall 1997,* vol. 1, no. 2, pp. 1–8, ncisla.wceruw.org/publications/index.html#newsletters.

Stacey, K. 2015, 'The Real World and the Mathematical World', in Stacey, K & Turner, R (eds), *Assessing Mathematical Literacy: The PISA experience*, Springer, Switzerland, pp. 57–84, doi.org/10.1007/978-3-319-10121-7_3.

Steen, LA 2001, 'The case for quantitative literacy', in *Mathematics and Democracy: The case for quantitative literacy*, National Council on Education and the Disciplines, Princeton, NJ, pp. 1–22.

Stillman, G, Galbraith, P, Brown, J & Edwards, I 2007, 'A framework for success in implementing mathematical modelling in the secondary classroom', *Mathematics: Essential research, essential practice*, vol. 2, pp. 688–697.

Stillman, G, Kaiser, G, Blum, W and Brown, JP (eds) 2013, *Teaching Mathematical Modelling: Connecting to research and practice,* Springer, Vancouver.

Sullivan, P 2011, *Teaching Mathematics: Using research-informed strategies*, ACER Press, Camberwell, Vic.

Verhage, H, & de Lange, J 1997, 'Mathematics education and assessment', *Pythagoras*, vol. 42, pp. 14–20.

Webb, DC 2009, 'Designing professional development for assessment', *Educational Designer*, vol. 1, no. 2, pp. 1–26, www.educationaldesigner.org/ed/volume1/issue2/article6/.

White, P, Sullivan, P, Warren, E & Quinlan, C 2000, 'To investigate or not to investigate? The use of content-specific open-ended tasks', *The Australian Mathematics Teacher*, vol. 56, no. 2, pp. 6–9, aamt.edu.au/Journals/Journals-Index/The-Australian-Mathematics-Teacher/AMT-56-2-6.

8 Version history

<table>
<tr><th>Version</th><th>Date of change</th><th>Update</th></tr>
<tr><td>1.1</td><td>June 2017</td><td>Minor amendments to ISMGs</td></tr>
<tr><td rowspan="8">1.2</td><td rowspan="8">July 2018</td><td>Minor amendment to syllabus objective 6 explanatory paragraph</td></tr>
<tr><td>Minor amendments to pedagogical and conceptual frameworks</td></tr>
<tr><td>Unit 1 — minor amendments to:
• unit description
• subject matter
• assessment guidance</td></tr>
<tr><td>Minor amendments to subject matter across Units 2, 3 and 4</td></tr>
<tr><td>Summative internal assessment 1 (IA1) — minor amendments to:
• description
• conditions
• ISMG</td></tr>
<tr><td>Summative internal assessments 2 and 3 (IA2 & IA3) — minor amendments to:
• description
• degree of difficulty definitions</td></tr>
<tr><td>Summative external assessment (EA) — minor amendments to:
• degree of difficulty definitions
• conditions</td></tr>
<tr><td>Glossary updates</td></tr>
</table>

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General Mathematics General Senior Syllabus 2019

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