Essential Mathematics 2025 v1.0

Applied (Essential) senior syllabus

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QCAA
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For all Queensland schools

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Queensland syllabuses for senior subjects

In Queensland, a syllabus for a senior subject is an official 'map' of a senior school subject. A syllabus's function is to support schools in delivering the Queensland Certificate of Education (QCE) system through high-quality and high-equity curriculum and assessment.

Syllabuses are based on design principles developed from independent international research about how excellence and equity are promoted in the documents teachers use to develop and enliven the curriculum.

Syllabuses for senior subjects build on student learning in the Prep to Year 10 Australian Curriculum and include General, General (Extension), Senior External Examination (SEE), Applied, Applied (Essential) and Short Course syllabuses.

More information about syllabuses for senior subjects is available at www.qcaa.qld.edu.au/senior/senior-subjects and in the 'Queensland curriculum' section of the *QCE and QCIA policy and procedures handbook*.

Teaching, learning and assessment resources will support the implementation of a syllabus for a senior subject. More information about professional resources for senior syllabuses is available on the QCAA website and via the QCAA Portal.

Course overview

Rationale

Mathematics is a unique and powerful intellectual discipline that is used to investigate patterns, order, generality and uncertainty. It is a way of thinking in which problems are explored and solved through observation, reflection and logical reasoning. It uses a concise system of communication, with written, symbolic, spoken and visual components. Mathematics is creative, requires initiative and promotes curiosity in an increasingly complex and data-driven world. It is the foundation of all quantitative disciplines.

To prepare students with the knowledge, skills and confidence to participate effectively in the community and the economy requires the development of skills that reflect the demands of the 21st century. Students undertaking Mathematics will develop their critical and creative thinking, oral and written communication, information & communication technologies (ICT) capability, ability to collaborate, and sense of personal and social responsibility — ultimately becoming lifelong learners who demonstrate initiative when facing a challenge. The use of technology to make connections between mathematical theory, practice and application has a positive effect on the development of conceptual understanding and student disposition towards mathematics.

Mathematics teaching and learning practices range from practising essential mathematical routines to develop procedural fluency, through to investigating scenarios, modelling the real world, solving problems and explaining reasoning. When students achieve procedural fluency, they carry out procedures flexibly, accurately and efficiently. When factual knowledge and concepts come to mind readily, students are able to make more complex use of knowledge to successfully formulate, represent and solve mathematical problems. Problem-solving helps to develop an ability to transfer mathematical skills and ideas between different contexts. This assists students to make connections between related concepts and adapt what they already know to new and unfamiliar situations. With appropriate effort and experience, through discussion, collaboration and reflection of ideas, students should develop confidence and experience success in their use of mathematics.

The major domains of mathematics in Essential Mathematics are Number, Data, Location and time, Measurement and Finance. Teaching and learning builds on the proficiency strands of the P–10 Australian Curriculum. Students develop their conceptual understanding when they undertake tasks that require them to connect mathematical concepts, operations and relations. They will learn to recognise definitions, rules and facts from everyday mathematics and data, and to calculate using appropriate mathematical processes.

Students will benefit from studies in Essential Mathematics because they will develop skills that go beyond the traditional ideas of numeracy. This is achieved through a greater emphasis on estimation, problem-solving and reasoning, which develops students into thinking citizens who interpret and use mathematics to make informed predictions and decisions about personal and financial priorities. Students will see mathematics as applicable to their employability and lifestyles, and develop leadership skills through self-direction and productive engagement in their learning. They will show curiosity and imagination, and appreciate the benefits of technology. Students will gain an appreciation that there is rarely one way of doing things and that real-world mathematics requires adaptability and flexibility.

Syllabus objectives

The syllabus objectives outline what students have the opportunity to learn.

1. **Recall mathematical knowledge.**

   When students recall mathematical knowledge, they recognise features of remembered information. They recognise relevant concepts, rules, definitions, techniques and algorithms.

2. **Use mathematical knowledge.**

   When students use mathematical knowledge, they put into effect relevant concepts, rules, definitions, techniques and algorithms. They perform calculations with and without technology.

3. **Communicate mathematical knowledge.**

   When students communicate mathematical knowledge, they use mathematical language (terminology, symbols, conventions and representations) and everyday language. They organise and present information in graphical and symbolic form, and describe and represent mathematical models.

4. **Evaluate the reasonableness of solutions.**

   When students evaluate the reasonableness of solutions, they interpret their mathematical results in the context of the situation and reflect on whether the problem has been solved. They verify results by using estimation skills and checking calculations, with and without technology. They make an appraisal by assessing implications, strengths and limitations of solutions and/or models, and use this to consider if alternative methods or refinements are required.

5. **Justify procedures and decisions.**

   When students justify procedures and decisions, they explain their mathematical reasoning in detail. They make relationships evident, logically organise mathematical arguments, and provide reasons for choices made and conclusions reached.

6. **Solve mathematical problems.**

   When students solve mathematical problems, they analyse the context of the problem to translate information into mathematical forms. They make decisions about the concepts, techniques and technology to be used and apply these to develop a solution. They develop, refine and use mathematical models, where applicable.

Designing a course of study in Essential Mathematics

Syllabuses are designed for teachers to make professional decisions to tailor curriculum and assessment design and delivery to suit their school context and the goals, aspirations and abilities of their students within the parameters of Queensland's senior phase of learning.

The syllabus is used by teachers to develop curriculum for their school context. The term *course of study* describes the unique curriculum and assessment that students engage with in each school context. A course of study is the product of a series of decisions made by a school to select, organise and contextualise subject matter, integrate complementary and important learning, and create assessment tasks in accordance with syllabus specifications.

It is encouraged that, where possible, a course of study is designed such that teaching, learning and assessment activities are integrated and enlivened in an authentic setting.

Course structure

Essential Mathematics is an Applied (Essential) senior syllabus. It contains four QCAA-developed units from which schools develop their course of study.

Each unit has been developed with a notional time of 55 hours of teaching and learning, including assessment.

Students should complete Unit 1 and Unit 2 before beginning Units 3 and 4. Units 3 and 4 are studied as a pair.

More information about the requirements for administering senior syllabuses is available in the 'Queensland curriculum' section of the *QCE and QCIA policy and procedures handbook*.

Curriculum

Senior syllabuses set out only what is essential while being flexible so teachers can make curriculum decisions to suit their students, school context, resources and expertise.

Within the requirements set out in this syllabus and the *QCE and QCIA policy and procedures handbook*, schools have autonomy to decide:

- how and when subject matter is delivered
- how, when and why learning experiences are developed, and the context in which learning occurs
- how opportunities are provided in the course of study for explicit and integrated teaching and learning of complementary skills.

These decisions allow teachers to develop a course of study that is rich, engaging and relevant for their students.

Assessment

Senior syllabuses set out only what is essential while being flexible so teachers can make assessment decisions to suit their students, school context, resources and expertise.

Applied (Essential) senior syllabuses contain assessment specifications and conditions for the assessment instruments that must be implemented with Units 3 and 4. These specifications and conditions ensure comparability, equity and validity in assessment.

Within the requirements set out in this syllabus and the *QCE and QCIA policy and procedures handbook*, schools have autonomy to decide:

- specific assessment task details
- assessment contexts to suit available resources
- how the assessment task will be integrated with teaching and learning activities
- how authentic the task will be.

In Unit 1 and Unit 2, schools:

- develop at least two but no more than four assessments
- complete at least one assessment for each unit
- ensure that each unit objective is assessed at least once.

In Units 3 and 4, schools develop three assessments using the assessment specifications and conditions provided in the syllabus.

More information about assessment in senior syllabuses is available in 'The assessment system' section of the *QCE and QCIA policy and procedures handbook*.

Subject matter

Each unit contains a unit description, unit objectives and subject matter. Subject matter is the body of information, mental procedures and psychomotor procedures (see Marzano & Kendall 2007, 2008) that are necessary for students' learning and engagement with the subject. Subject matter itself is not the specification of learning experiences but provides the basis for the design of student learning experiences.

Subject matter has a direct relationship with the unit objectives and provides statements of learning that have been constructed in a similar way to objectives.

Aboriginal perspectives and Torres Strait Islander perspectives

The QCAA is committed to reconciliation. As part of its commitment, the QCAA affirms that:

- Aboriginal peoples and Torres Strait Islander peoples are the first Australians, and have the oldest living cultures in human history
- Aboriginal peoples and Torres Strait Islander peoples have strong cultural traditions and speak diverse languages and dialects, other than Standard Australian English
- teaching and learning in Queensland schools should provide opportunities for students to deepen their knowledge of Australia by engaging with the perspectives of Aboriginal peoples and Torres Strait Islander peoples
- positive outcomes for Aboriginal students and Torres Strait Islander students are supported by successfully embedding Aboriginal perspectives and Torres Strait Islander perspectives across planning, teaching and assessing student achievement.

Guidelines about Aboriginal perspectives and Torres Strait Islander perspectives and resources for teaching are available at www.qcaa.qld.edu.au/k-12-policies/aboriginal-torres-strait-islander-perspectives.

Where appropriate, Aboriginal perspectives and Torres Strait Islander perspectives have been embedded in the subject matter.

Complementary skills

Opportunities for the development of complementary skills have been embedded throughout subject matter. These skills, which overlap and interact with syllabus subject matter, are derived from current education, industry and community expectations and encompass the knowledge, skills, capabilities, behaviours and dispositions that will help students live and work successfully in the 21st century.

These complementary skills are:

- literacy — the knowledge, skills, behaviours and dispositions about language and texts essential for understanding and conveying English language content
- numeracy — the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations, to recognise and understand the role of mathematics in the world, and to develop the dispositions and capacities to use mathematical knowledge and skills purposefully
- 21st century skills — the attributes and skills students need to prepare them for higher education, work, and engagement in a complex and rapidly changing world. These skills include critical thinking, creative thinking, communication, collaboration and teamwork, personal and social skills, and digital literacy. The explanations of associated skills are available at www.qcaa.qld.edu.au/senior/senior-subjects/general-subjects/21st-century-skills.

It is expected that aspects of literacy, numeracy and 21st century skills will be developed by engaging in the learning outlined in this syllabus. Teachers may choose to create additional explicit and intentional opportunities for the development of these skills as they design the course of study.

Additional subject-specific information

Additional subject-specific information has been included to support and inform the development of a course of study.

Assumed knowledge, prior learning or experience

Assumed knowledge refers to the subject matter that teachers can expect students to know prior to beginning this subject. Emphasis is placed on the mastery of content, ensuring key concepts or procedures are learnt fully so they will not need reteaching.

Developing mastery often involves multiple approaches to teaching and conceptualising the same mathematical concept. When students have a good understanding of a key concept or procedure, they are more easily able to make connections to related new subject matter and apply what they already know to new problems.

Subject matter from previous unit/s is assumed for subsequent unit/s.

The following is a non-exhaustive list of assumed knowledge based on the subject matter in the P–10 Australian Curriculum version 9.

- Recall concepts of number, percentages and money.
- Read and use graphs and scales.
- Recall concepts of probability, data collection and statistical data representations.
- Use a scientific calculator and other technology, where appropriate.
- Substitute numbers into formulas.
- Translate word problems to mathematical form.

Problem-solving and mathematical modelling

A key aspect of learning mathematics is to develop strategic competence; that is, to formulate, represent and solve mathematical problems (Kilpatrick, Swafford & Bradford 2001). As such, problem-solving is a focus of mathematics education research, curriculum and teaching (Sullivan 2011). This focus is not to the exclusion of routine exercises, which are necessary for practising, attaining mastery and being able to respond automatically. But mathematics education in the 21st century goes beyond this to include innovative problems that are complex, unfamiliar and non-routine (Mevarech & Kramarski 2014).

Problem-solving in mathematics can be set in purely mathematical contexts or real-world contexts. When set in the real world, problem-solving in mathematics involves mathematical modelling.

Problem-solving

Problem-solving is required when a task or goal has limiting conditions placed upon it or an obstacle blocking the path to a solution (Marzano & Kendall 2007). It involves:

- knowledge of the relevant details
- using generalisations and principles to identify, define and interpret the problem
- mental computation and estimation
- critical, creative and lateral thinking
- creating or choosing a strategy
- making decisions
- testing, monitoring and evaluating solutions.

Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples without understanding. Through problem-solving, students will make connections between mathematics topics, across the curriculum and with the real world, and see the value and usefulness of mathematics. Problems may be real-world or abstract, and presented to students as issues, statements or questions that may require them to use primary or secondary data.

Mathematical modelling

Mathematical modelling begins from an assumption that mathematics is everywhere in the world around us — a challenge is to identify where it is present, access it and apply it productively. Models are developed in order to better understand real-world phenomena, to make predictions and answer questions. A mathematical model depicts a situation by expressing relationships using mathematical concepts and language. It refers to the set of simplifying assumptions (such as the relevant variables or the shape of something); the set of assumed relationships between variables; and the resulting representation (such as a formula) that can be used to generate an answer (Stacey 2015).

Mathematical modelling involves:

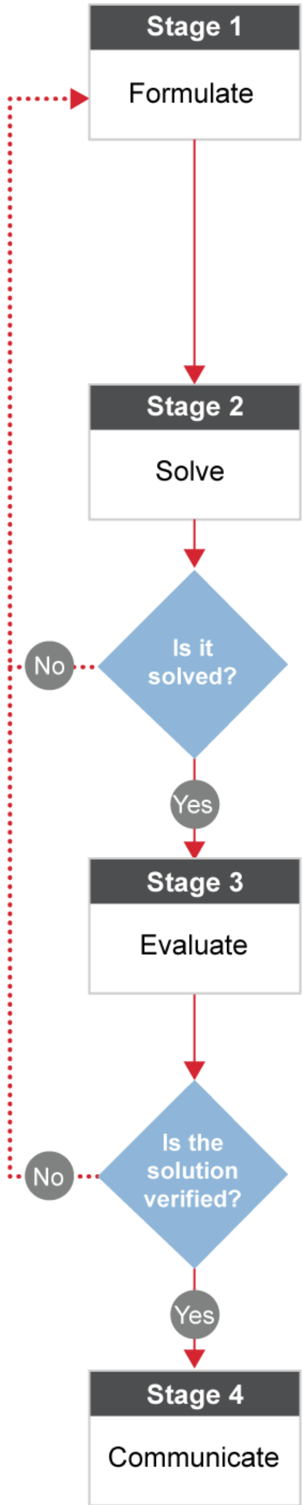
- formulating a mathematical representation of a problem derived from within a real-world context
- using mathematics concepts and techniques to obtain results
- interpreting the results by referring back to the original problem context
- revising the model (where necessary) (Geiger, Faragher & Goos 2010).

Through developing and applying mathematical models, students cumulatively become real-world problem-solvers. Ultimately, this means that not only can they productively address problems set by others, but also that they develop the ability to identify and address problems and answer questions that matter to them.

The following section outlines an approach to problem-solving and mathematical modelling.[1] Problems must be real-world, and can be presented to students as issues, statements or questions that may require them to use primary or secondary data.

[1] A wide variety of frameworks for problem-solving and modelling exist in mathematics education literature. The approach outlined here aligns with and is informed by other approaches, such as Polya (1957) in *How to Solve It: A new aspect of mathematical method* (1957), the Australian Curriculum (ACARA 2015a) *Statistical investigation process*, the OECD/PISA Mathematics framework (OECD 2015, 2003) and *A framework for success in implementing mathematical modelling in the secondary classroom* (Stillman et al. 2007). For further reading see Blum et al. (2007); Kaiser et al. (2011); and Stillman et al. (2013).

Figure 1: An approach to problem-solving and mathematical modelling



Stage 1 – Formulate

Once students understand what the problem is asking, they must design a plan to solve the problem. Students translate the problem into a mathematically purposeful representation by first determining the applicable mathematical knowledge that is required to make progress with the problem. Important assumptions, variables and observations are identified and justified, based on the logic of a proposed solution and/or model.

In mathematical modelling, formulating a model involves the process of mathematisation — moving from the real world to the mathematical world.

Stage 2 – Solve

Students select and apply mathematical knowledge previously learnt to solve the problem. Possible approaches are wide-ranging and include synthesising and refining existing models, and generating and testing hypotheses with primary or secondary data and information, to produce a complete solution.

Solutions can be found using algebraic, graphic, arithmetic and/or numeric methods, with and/or without technology.

Stage 3 – Evaluate

Once a possible solution has been achieved, students need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. They verify their results and evaluate the reasonableness of the solution to the problem in relation to the original issue, statement or question.

This involves exploring the strengths and limitations of the solution and/or model. Where necessary, this will require going back through the process to further refine the solution and/or model. In mathematical modelling, students must check that the output of their model provides a complete solution to the real-world problem it has been designed to address.

This stage emphasises the importance of methodological rigour and the fact that problem-solving and mathematical modelling is not usually linear and involves an iterative process.

Stage 4 – Communicate

The development of solutions and/or models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated and justified clearly and fully. Students communicate findings logically and concisely using mathematical and everyday language. They draw conclusions, discussing the results, strengths and limitations of the solution and/or model. Students could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem.

Approaches to problem-solving and mathematical modelling in the classroom

When teaching problem-solving and mathematical modelling, teachers should consider *teaching for* and *learning through* problem-solving and mathematical modelling. When *teaching for*, students are taught the specific mathematical rules, definitions, procedures, problem-solving strategies and critical elements of the model that are needed to solve a given problem. When *learning through*, students are presented with problems to solve, but must apply the knowledge and skills they have previously been taught to solve it. By solving these problems, students are able to develop new mathematical understanding and skills. This requires an explicit and connected approach to teaching problem-solving and mathematical modelling that necessitates fluency of critical facts and processes at each step.

The following describes three different approaches to teaching problem-solving and mathematical modelling[2] along the continua between *teaching for* and *learning through*:

Approach	Description	*Teaching for* or *learning through*
Dependent	The teacher explicitly demonstrates and teaches the concepts and techniques required to solve the problem, and/or develop a mathematical model. This usually involves students solving (stage 2) and evaluating and verifying (stage 3).	*Teaching for*
Guided	The teacher influences the choice of concepts and techniques, and/or model that students use to solve the problem. Guidance is provided and all stages of the approach are used.	Moving towards *learning through*
Independent	The teacher cedes control and students work independently, choosing their own solution and/or model, and working at their own level of mathematics. The independent approach is the most challenging.	*Learning through*

These approaches are not mutually exclusive. An independent approach (*learning through*) might be undertaken as an extension of a dependent or guided activity that students have previously undertaken (*teaching for*). Students need to have attained the relevant foundational understanding and skills before working independently during a problem-solving and modelling task. This capacity needs to be built over time through the course of study with teachers closely monitoring student progress.

[2] Based on Galbraith (1989).

Strategies for retaining and recalling information for assessment

The following practices[3] can support preparation for senior assessment in Essential Mathematics.

The spacing effect

The spacing effect draws on research about forgetting and learning curves. By recalling and revisiting information at intervals, rather than at the end of a study cycle, students remember a greater percentage of the information with a higher level of accuracy. Exposing students to information and materials numerous times over multiple spaced intervals solidifies long-term memory, positively affecting retention and recall.

Teachers should plan teaching and learning sequences that allow time to revisit previously taught information and skills at several intervals. These repeated learning opportunities also provide opportunities for teachers to provide formative feedback to students.

The retrieval effect

The retrieval effect helps students to practise remembering through quick, regular, low-stakes questioning or quizzes that exercise their memories and develop their ability to engage in the deliberate act of recalling information. This has been shown to be more effective at developing long-term memories than activities that require students to search through notes or other resources.

Students may see an inability to remember as an obstacle, but they should be encouraged to understand that this is an opportunity for learning to take place. By trying to recall information, students exercise or strengthen their memory and may also identify gaps in their learning. The more difficult the retrieval practice, the better it can be for long-term learning.

Interleaving

Interleaving involves interspersing the concepts, categories, skills or types of questions that students focus on in class or revision. This is in contrast to blocking, in which these elements are grouped together in a block of time. For example, for concepts A, B and C:

- Blocking A A A A A B B B B B C C C C C
- Interleaving A B C B C A B A C A C B C A B

Studies have found that interleaving in instruction or revision produces better long-term recall of subject matter. Interleaving also ensures that spacing occurs, as instances of practice are spread out over time.

Additionally, because exposure to one concept is interleaved with exposure to another, students have more opportunities to distinguish between related concepts. This highlighting of differences may explain why studies have found that interleaving enhances inductive learning, where participants use exemplars to develop an understanding of broader concepts or categories. Spacing without interleaving does not appear to benefit this type of learning.

Interleaving can seem counterintuitive — even in studies where interleaving enhanced learning, participants often felt that they had learnt more with blocked study. Despite this, their performance in testing indicated greater learning through the interleaving approach.

[3] Based on Agarwal, Roediger, McDaniel & McDermott (2020); Birnbaum, Kornell, Ligon Bjork & Bjork (2013); Carpenter & Agarwal (2020); Chen, Paas & Sweller (2021); Ebbinghaus (1885); Rohrer (2012); Taylor & Rohrer (2010).

Reporting

General information about determining and reporting results for senior syllabuses is provided in the 'Determining and reporting results' section of the *QCE and QCIA policy and procedures handbook*.

Reporting standards

Reporting standards are summary statements that describe typical performance at each of the five levels (A–E).

A
The student recalls, uses and communicates comprehensive mathematical knowledge drawn from Number, Data, Location and time, Measurement and Finance in simple familiar, complex familiar and complex unfamiliar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar, complex familiar and complex unfamiliar situations.
B
The student recalls, uses and communicates thorough mathematical knowledge drawn from Number, Data, Location and time, Measurement and Finance in simple familiar and complex familiar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar and complex familiar situations.
C
The student recalls, uses and communicates mathematical knowledge drawn from Number, Data, Location and time, Measurement and Finance in simple familiar situations. The student evaluates the reasonableness of solutions, justifies procedures and decisions, and solves mathematical problems in simple familiar situations.
D
The student recalls, uses and communicates partial mathematical knowledge drawn from Number, Data, Location and time, Measurement and Finance in simple familiar situations. The student sometimes evaluates the reasonableness of solutions, sometimes justifies procedures and decisions, and solves some mathematical problems in simple familiar situations.
E
The student recalls, uses and communicates isolated mathematical knowledge drawn from Number, Data, Location and time, Measurement and Finance in simple familiar situations. The student rarely evaluates the reasonableness of solutions, and infrequently justifies procedures and decisions in simple familiar situations.

Determining and reporting results

Unit 1 and Unit 2

Schools make A–E judgments on individual assessment instruments implemented in Unit 1 and Unit 2 using reporting standards.

Schools report results to the QCAA for students who complete Unit 1 and/or Unit 2. Results are reported as satisfactory (S) or unsatisfactory (U). Where appropriate, schools may also report a not rated (NR).

Units 3 and 4

Schools make A–E judgments on each of the four assessment instruments implemented in Units 3 and 4 using instrument-specific standards (ISS).

Schools report instrument results to the QCAA for students enrolled in Units 3 and 4 for each of the four assessments implemented. Where appropriate, schools may also report a not rated (NR).

Schools are also responsible for determining and reporting an A–E final subject result to the QCAA. The subject result is an on-balance judgment about how the pattern of evidence across the four assessments in Units 3 and 4 best matches the characteristics of the reporting standards at one of five levels (A–E).

Units

Unit 1: Number, data and money

In Unit 1, students will develop the mathematical understandings and skills to solve problems relating to:

- Fundamental topic: Calculations
- Topic 1: Number
- Topic 2: Representing data
- Topic 3: Managing money.

The subject matter of the topics in this unit should be applied in contexts that are meaningful and of interest to students. A variety of approaches can be used to achieve this purpose. Two possible contexts that may be used are 'Mathematics of foods' and 'Mathematics of independent living'. However, these contexts may not be relevant to all students. Suitable contexts relevant to the particular student cohort should be chosen.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Fundamental topic: Calculations

Calculations should be integrated throughout Unit 1.

Sub-topic: Calculations

- Solve practical problems requiring basic number operations.
- Apply arithmetic operations according to their correct order.
- Ascertain the reasonableness of answers to arithmetic calculations.
- Use leading-digit approximation to obtain estimates of calculations.
- Use a calculator for multi-step calculations.
- Check results of calculations for accuracy.
- Recognise the significance of place value after the decimal point.
- Evaluate decimal fractions to the required number of decimal places.
- Round up or round down numbers to the required number of decimal places.
- Apply approximation strategies for calculations.

Topic 1: Number

Sub-topic: Ratios (7 hours)

- Demonstrate an understanding of the fundamental ideas and notation of ratio.
- Understand the relationship between fractions and ratios.
- Express a ratio in simplest form using whole numbers.
- Find the ratio of two quantities in its simplest form.
- Divide a quantity in a given ratio [complex].
- Use ratio to describe simple scales [complex].

Sub-topic: Rates (6 hours)

- Identify examples of rates in the real world, e.g. rate of pay, cost per kilogram, currency exchange rates, flow rate from a tap.
- Convert between units for rates.
- Complete calculations with rates, including solving problems involving direct proportion in terms of rate [complex].
- Use rates to make comparisons.
- Use rates to determine costs.

Sub-topic: Percentages (7 hours)

- Calculate a percentage of a given amount.
- Determine one amount expressed as a percentage of another for same units.
- Determine one amount expressed as a percentage of another for different units [complex].

- Apply percentage increases and decreases in real-world contexts, including mark-ups, discounts and GST [complex].
- Determine the overall change in a quantity following repeated percentage changes [complex].

Sub-topic: Units of energy (4 hours)

- Use units of energy to describe consumption of electricity, e.g. kilowatt hours.
- Use units of energy to describe the amount of energy for foods and in activities, including calories and kilojoules.
- Convert from one unit of energy to another including between calories and kilojoules.

Topic 2: Representing data

Sub-topic: Classifying data (2 hours)

- Identify examples of categorical data.
- Identify examples of numerical data.

Sub-topic: Data presentation and interpretation (8 hours)

- Display categorical data in tables and column graphs.
- Display numerical data as frequency distribution tables, dot plots, stem plots and histograms.
- Recognise and identify outliers from a dataset.
- Compare the suitability of different methods of data presentation in real-world contexts [complex].

Topic 3: Managing money

Sub-topic: Earning money (14 hours)

- Find earnings, including salary, wages, overtime, piecework and commission.
- Convert between annual, monthly, fortnightly, weekly and hourly rates of earning [complex].
- Understand the purpose of superannuation.
- Interpret entries on a selection of wage or salary pay slips and timesheets.
- Understand the purpose of taxation and the use of tax file numbers.
- Use tax tables to determine PAYG tax for periodic (weekly/fortnightly/monthly) earnings [complex].
- Interpret entries on a simple PAYG summary.
- Apply the concepts of taxable income, gross income, allowable deductions and levies in simple contexts [complex].
- Calculate a simple income tax return and net income using current income tax rates [complex].

Sub-topic: Budgeting (7 hours)

- Investigate the costs involved in independent living [complex].
- Prepare a personal budget plan [complex].

Unit 2: Data and travel

In Unit 2, students will develop the mathematical understandings and skills to solve problems relating to:

- Fundamental topic: Calculations
- Topic 1: Data collection
- Topic 2: Graphs
- Topic 3: Time and motion.

The subject matter of the topics in this unit should be applied in contexts that are meaningful and of interest to students. A variety of approaches can be used to achieve this purpose. Two possible contexts that may be used are ‘Mathematics of sports’ and ‘Mathematics of travel’. However, these contexts may not be relevant to all students. Suitable contexts relevant to the particular student cohort should be chosen.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Fundamental topic: Calculations

Calculations should be integrated throughout Unit 2.

Sub-topic: Calculations

- Solve practical problems requiring basic number operations.
- Apply arithmetic operations according to their correct order.
- Ascertain the reasonableness of answers to arithmetic calculations.
- Use leading-digit approximation to obtain estimates of calculations.
- Use a calculator for multi-step calculations.
- Check results of calculations for accuracy.
- Recognise the significance of place value after the decimal point.
- Evaluate decimal fractions to the required number of decimal places.
- Round up or round down numbers to the required number of decimal places.
- Apply approximation strategies for calculations.

Topic 1: Data collection

Sub-topic: Census (3 hours)

- Investigate the procedure for conducting a census.
- Investigate the advantages and disadvantages of conducting a census [complex].

Sub-topic: Surveys (5 hours)

- Understand the purpose of sampling to provide an estimate of population values when a census is not used.
- Investigate the different kinds of samples [complex].
- Investigate the advantages and disadvantages of these kinds of samples [complex].

Sub-topic: Simple survey procedure (4 hours)

- Identify the target population to be surveyed.
- Investigate questionnaire design principles, including simple language, unambiguous questions, consideration of number of choices, issues of privacy and ethics, and freedom from bias [complex].

Sub-topic: Sources of bias (3 hours)

- Describe the faults in the process of collecting data.
- Describe sources of error in surveys, including sampling error and measurement error.
- Investigate the possible misrepresentation of the results of a survey due to misunderstanding the procedure or the reliability of generalising the survey findings to the entire population [complex].
- Investigate errors and misrepresentation in surveys, including examples of media misrepresentations of surveys [complex].

Topic 2: Graphs

Sub-topic: Reading and interpreting graphs (7 hours)

- Interpret information presented in graphs, e.g. step graphs, column graphs, pie graphs, picture graphs, line graphs.
- Interpret information presented in two-way tables.
- Discuss and interpret tables and graphs, including misleading graphs found in the media and in factual texts [complex].

Sub-topic: Drawing graphs (8 hours)

- Determine which type of graph is best used to display a dataset.
- Use spreadsheeting software to tabulate and graph data [complex].
- Draw a line graph to represent any data that demonstrates a continuous change [complex].

Sub-topic: Using graphs (7 hours)

- Use graphs in practical situations.
- Interpret graphs in practical situations [complex].
- Draw graphs from given data to represent practical situations [complex].
- Interpret the point of intersection and other important features (x- and y-intercepts) of given graphs of two linear functions drawn from practical contexts [complex].

Topic 3: Time and motion

Sub-topic: Time (6 hours)

- Represent time using 12-hour and 24-hour clocks.
- Calculate time intervals, including time between, time ahead, time behind.
- Interpret timetables for buses, trains and/or ferries.
- Use several timetables and/or electronic technologies to plan the most time-efficient routes [complex].
- Interpret complex timetables, e.g. tide charts, sunrise charts and moon phases [complex].
- Compare the time taken to travel a specific distance with various modes of transport.

Sub-topic: Distance (5 hours)

- Use scales to find distances.
- Investigate distances through trial and error or systematic methods [complex].
- Apply directions to distances calculated on maps including the eight compass points: N, NE, E, SE, S, SW, W, NW [complex].

Sub-topic: Speed (7 hours)

- Identify the appropriate units for different activities.
- Calculate speed, distance and time where s is speed, d is distance and t is time, using the formulas
  - $s = \frac{d}{t}$
  - $d = s \times t$
  - $t = \frac{d}{s}$
- Calculate the time and costs for a journey from distances estimated from maps, given a travelling speed [complex].
- Interpret distance-versus-time graphs, including reference to the steepness of the slope (or average speed) [complex].

Unit 3: Measurement, scales and chance

In Unit 3, students will develop the mathematical understandings and skills to solve problems relating to:

- Fundamental topic: Calculations
- Topic 1: Measurement
- Topic 2: Scales, plans and models
- Topic 3: Probability and relative frequencies.

The subject matter of the topics in this unit should be applied in a context that is meaningful and of interest to students. A variety of approaches can be used to achieve this purpose. Two possible contexts that may be used in this unit are ‘Mathematics of designs’ and ‘Mathematics of the media’. However, these contexts may not be relevant to all students. Suitable contexts relevant to the particular student cohort should be chosen.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Fundamental topic: Calculations

Calculations should be integrated throughout Unit 3.

Sub-topic: Calculations

- Solve practical problems requiring basic number operations.
- Apply arithmetic operations according to their correct order.
- Ascertain the reasonableness of answers to arithmetic calculations.
- Use leading-digit approximation to obtain estimates of calculations.
- Use a calculator for multi-step calculations.
- Check results of calculations for accuracy.
- Recognise the significance of place value after the decimal point.
- Evaluate decimal fractions to the required number of decimal places.
- Round up or round down numbers to the required number of decimal places.
- Apply approximation strategies for calculations.

Topic 1: Measurement

Sub-topic: Converting units of measure (4 hours)

- Use metric units of length (millimetres, centimetres, metres, kilometres), their abbreviations (mm, cm, m, km), conversions between them, and appropriate levels of accuracy and choice of units.
- Use metric units of area (square millimetres, square centimetres, square metres, square kilometres, hectares), their abbreviations (mm^2, cm^2, m^2, km^2, ha), conversions between them and appropriate choices of units.
- Use metric units of volume (cubic millimetres, cubic centimetres, cubic metres), their abbreviations (mm^3, cm^3, m^3), conversions between them and appropriate choices of units.
- Understand and use the relationship between volume and capacity, recognising that 1 cm^3 = 1 mL (millilitre), 1000 cm^3 = 1 L (litre), 1 m^3 = 1 kL (kilolitre), 1000 kL = 1ML (megalitre).
- Use units of time and convert between fractional, decimal and digital representations.
- Use metric units of mass (milligrams, grams, kilograms, metric tonnes), their abbreviations (mg, g, kg, t), conversions between them and appropriate choices of units.

Sub-topic: Geometry (3 hours)

- Recognise the properties of regular polygons, e.g. number of vertices, straight edges and angles.
- Recognise the properties of prisms and pyramids, e.g. number of vertices, straight edges and flat faces.
- Interpret different forms of two-dimensional representations of three-dimensional objects, including nets of prisms and pyramids [complex].

Sub-topic: Linear measure (5 hours)

- Estimate lengths.
- Calculate perimeters of regular polygons, circles and arc lengths.
  - circle: $C = 2\pi r$ where C is circumference and r is radius.
  - arc length: $l = \frac{\theta}{180}\pi r$ where l is arc length, θ is central angle and r is radius.
- Calculate perimeters of composite shapes [complex].

Sub-topic: Area measure (9 hours)

- Estimate areas of different shapes.
- Calculate areas of regular shapes, triangles, parallelograms and circles.
  - triangle: $A = \frac{1}{2}bh$ where b is base length and h is perpendicular height.
  - parallelogram: $A = bh$ where b is base length and h is perpendicular height.
  - circle: $A = \pi r^2$ where r is radius.
- Calculate areas of trapeziums and sectors [complex].
  - trapezium: $A = \frac{1}{2}(a + b)h$ where a and b are parallel lengths and h is perpendicular height.
  - sector: $A = \frac{\theta}{360}\pi r^2$ where θ is central angle and r is radius.
- Calculate areas of composite figures by decomposing them into regular shapes [complex].
- Calculate surface areas of prisms and cylinders [complex].
  - cylinder: $S = 2\pi rh + 2\pi r^2$ where r is radius and h is perpendicular height.
- Calculate surface areas of pyramids and cones [complex].
  - cone: $S = \pi rs + \pi r^2$ where r is radius of a circle and s is slant height.
- Calculate surface areas of composite shapes and spheres [complex].
  - sphere: $S = 4\pi r^2$ where r is radius.

Sub-topic: Volume and capacity (6 hours)

- Estimate volumes and capacities of various objects.
- Calculate volumes and capacities of prisms and cylinders.
  - prism: $V = Ah$ where A is base area and h is perpendicular height.
  - cylinder: $V = \pi r^2 h$ where r is radius and h is perpendicular height.
- Calculate volumes and capacities of pyramids and cones.
  - pyramid: $V = \frac{1}{3}Ah$ where A is base area and h is perpendicular height.
  - cone: $V = \frac{1}{3}\pi r^2 h$ where r is radius and h is perpendicular height.
- Calculate volumes and capacities of composite shapes and spheres [complex].
  - sphere: $V = \frac{4}{3}\pi r^3$ where r is radius.

Topic 2: Scales, plans and models

Sub-topic: Interpret scale drawings (6 hours)

- Interpret commonly used symbols and abbreviations in scale drawings.
- Find actual measurements from scale drawings, including lengths, perimeters and areas.
- Estimate and compare quantities, materials and costs using actual measurements from scale drawings [complex].

Sub-topic: Creating scale drawings (4 hours)

- Understand and apply drawing conventions of scale drawings, including scales in ratio, clear indications of dimensions and clear labelling [complex].
- Construct scale drawings by hand and by using software packages [complex].

Sub-topic: Right-angled triangles (5 hours)

- Apply Pythagoras' theorem to solve problems for all side lengths where c is length of hypotenuse and a and b are lengths of the perpendicular sides, using the formulas:
  - $c^2 = a^2 + b^2$
  - $a^2 = c^2 - b^2$
  - $b^2 = c^2 - a^2$
- Apply the cosine, sine and tangent ratios to find unknown angles (θ) and sides [complex].
  - $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
  - $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
  - $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
- Use the concepts of angle of elevation and angle of depression to solve practical problems [complex].

Topic 3: Probability and relative frequencies

Sub-topic: Simulations (5 hours)

- Express probabilities formally using fractions, decimals, ratios and percentages.
- Perform simulations of probability experiments using technology.
- Recognise that the repetition of chance events is likely to produce different results.
- Identify relative frequency as probability.
- Identify factors that could complicate the simulation of real-world events [complex].

Sub-topic: Simple probabilities (8 hours)

- Construct a sample space for an experiment.
- Use a sample space to determine the probability of outcomes for an experiment.
- Use arrays and tree diagrams to determine the outcomes and the probabilities for experiments.
- Model and solve problems involving probability experiments.

Unit 4: Graphs, data and loans

In Unit 4, students will develop the mathematical understandings and skills to solve problems relating to:

- Fundamental topic: Calculations
- Topic 1: Bivariate graphs
- Topic 2: Summarising and comparing data
- Topic 3: Loans and compound interest.

The subject matter of the topics in this unit should be applied in a context that is meaningful and of interest to students. Two possible contexts that may be used in this unit are ‘Mathematics of health’ and ‘Mathematics of home loans’. However, these contexts may not be relevant to all students. Suitable contexts relevant to the particular student cohort should be chosen.

Unit objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Subject matter

Fundamental topic: Calculations

Calculations should be integrated throughout Unit 4.

Sub-topic: Calculations

- Solve practical problems requiring basic number operations.
- Apply arithmetic operations according to their correct order.
- Ascertain the reasonableness of answers to arithmetic calculations.
- Use leading-digit approximation to obtain estimates of calculations.
- Use a calculator for multi-step calculations.
- Check results of calculations for accuracy.
- Recognise the significance of place value after the decimal point.
- Evaluate decimal fractions to the required number of decimal places.
- Round up or round down numbers to the required number of decimal places.
- Apply approximation strategies for calculations.

Topic 1: Bivariate graphs

Sub-topic: Cartesian plane (6 hours)

- Demonstrate familiarity with Cartesian coordinates in two dimensions by identifying and plotting points on the Cartesian plane.
- Generate a table of values for a given linear function, including for negative values of x.
- Graph a linear function from a table of values with pencil and paper and with graphing software.

Sub-topic: Bivariate scatterplots (4 hours)

- Construct a scatterplot using a given dataset.
- Describe the patterns and features of bivariate data.
- Describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak).

Sub-topic: Line of best fit (8 hours)

- Identify dependent and independent variables.
- Draw a line of best fit by eye.
- Use technology to determine the equation of the line of best fit in the form $y = mx + c$ where m is slope (gradient) and c is *y*-intercept [complex].
- Interpret the effect of the parameters m and c from the equation of the line of best fit in the form $y = mx + c$ [complex].

- Use technology to calculate the correlation coefficient (an indicator of the strength of linear association) [complex].
- Use the line of best fit to make predictions, both by interpolation and extrapolation [complex].
- Recognise the dangers of extrapolation [complex].
- Distinguish between causality and correlation through examples [complex].

Topic 2: Summarising and comparing data

Sub-topic: Summarising and interpreting data (10 hours)

- Identify mode from a dataset.
- Calculate measures of central tendency: median and mean ($\bar{x}$) from a dataset of n values
  - mean: $\bar{x} = \dfrac{\sum x}{n}$ where $\sum x = \text{sum of all data values}$
- Investigate the suitability of measures of central tendency in various real-world contexts [complex].
- Investigate the effect of outliers on the mean and the median [complex].
- Calculate quartiles from a dataset.
- Interpret quartiles, deciles and percentiles from a graph [complex].
- Use everyday language to describe spread, including spread out, dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers.
- Calculate and interpret statistical measures of spread, including the range, interquartile range and standard deviation [complex].
  - $\text{range} = \text{highest score} - \text{lowest score}$
  - $\text{IQR} = Q_3 - Q_1$
- Investigate real-world examples from the media illustrating inappropriate uses of measures of central tendency and spread [complex].

Sub-topic: Comparing datasets (9 hours)

- Complete a five-number summary for different datasets.
- Construct a box plot using a five-number summary.
- Compare parallel box plots and back-to-back stem plots for different datasets [complex].
- Compare the characteristics of the shape of histograms using symmetry, skewness and bimodality, where applicable [complex].

Topic 3: Loans and compound interest

Sub-topic: Compound interest (10 hours)

- Understand the concept of simple interest where P is principal, i is interest rate per period and n is total number of periods to find unknown values using the formulas:
  - $I = Pin$
  - $P = \frac{I}{in}$
  - $i = \frac{I}{Pn}$
  - $n = \frac{I}{Pi}$
- Understand the concept of compound interest as a recurrence relation.
- Use an online calculator to determine the future value of a compound interest loan or investment.
- Calculate the future value (A) of a compound interest loan or investment with annual periods using the formula $A = P(1+i)^n$ where P is principal, i is interest rate per annum and n is total number of years.
- Calculate the total interest paid or earned for compound interest loan or investment.
- Use a spreadsheet to determine the future value of a compound interest loan or investment and the total interest paid or earned [complex].
- Compare, numerically and graphically, the growth of simple interest and compound interest loans and investments [complex].
- Investigate the effect of the principal amount, the interest rate and the number of compounding periods on the future value of a loan or investment [complex].

Sub-topic: Reducing balance loans (8 hours)

- Understand that reducing balance loans are compound interest loans with periodic repayments.
- Use a calculator or an online calculator to model a reducing balance loan with annual repayments.
- Use a spreadsheet to model a reducing balance loan with non-annual repayments [complex].
- Investigate the effect of the repayment amount, the interest rate and the number of compounding periods on the time taken to repay a loan [complex].

Assessment

Internal assessment 1: Problem-solving and modelling task

Students provide a written response to a specific mathematical investigative scenario or context using subject matter from Fundamental topic: Calculations, and at least one topic in Unit 3. While students may undertake some research, it is not the focus of this task.

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

This task requires students to:

- independently respond to a specific mathematical investigative scenario or context that highlights a real-life application of mathematics
- use relevant stimulus material involving the selected subject matter
- address all the stages of the problem-solving and mathematical modelling approach
- respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Conditions

- Students will use 8 hours of class time and their own time out of class to develop their response.
- This is an individual task.
- Data may be provided or collected individually or collected in groups.
- Appendixes can include raw data, repeated calculations, evidence of authentication and student notes (appendixes are not marked).
- Students must use technology, e.g. scientific calculator, graphics calculator, spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation or word processing.

Response requirements

Written: up to 8 A4 pages, up to 1000 words

Instrument-specific standards

Formulate	Solve	Evaluate	Communicate	Grade
The student response has the following characteristics:				
• justified statements of important assumptions • justified statements of important observations • justified mathematical translation of important simple and important complex aspects of the task	• accurate use of simple and complex mathematical knowledge for important aspects of the task • efficient use of technology • a complete solution	• verified results • justified statements about the reasonableness of the solution by considering the assumptions • justified statements about the reasonableness of the solution by considering the observations • justified statements of relevant strengths of the solution • justified statements of relevant limitations of the solution	• correct use of appropriate mathematical language • logical organisation of the response, which can be read independently of the task sheet • justification of decisions using mathematical reasoning	A
• statements of important assumptions • statements of important observations • mathematical translation of important simple and important complex aspects of the task	• use of simple and complex mathematical knowledge for an important aspect of the task • use of technology • substantial progress towards a solution	• a verified result • statements about the reasonableness of the solution by considering the assumptions • statements about the reasonableness of the solution by considering the observations • statements of relevant strengths of the solution • statements of relevant limitations of the solution	• use of appropriate mathematical language • logical organisation of the response • statements of decisions using mathematical reasoning	B
• statement of a relevant assumption • statement of a relevant observation • mathematical translation of a simple or complex aspect of the task	• use of simple mathematical knowledge relevant to the task • simplistic use of technology • progress towards a solution	• progress towards a verified result • statement about the reasonableness of the solution by considering an assumption or observation • statement of a relevant strength or relevant limitation of the solution	• use of some appropriate mathematical language • adequate organisation of the response • statement of a decision using mathematical reasoning	C
• statement of an assumption or observation • mathematical translation of an aspect of the task.	• simplistic use of mathematical knowledge • inappropriate use of technology.	• statement about the reasonableness of a result or the solution • statement of a strength or limitation.	• use of everyday language • basic organisation of the response.	D
The student response does not match any of the descriptors above.	The student response does not match any of the descriptors above.	The student response does not match any of the descriptors above.	The student response does not match any of the descriptors above.	E

Internal assessment 2: Common internal assessment

The common internal assessment (CIA) is provided by the QCAA and assists in strengthening reliability and validity in Essential Mathematics. The CIA is:

- developed by the QCAA
- common to all schools
- marked internally by the school.

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

This examination:

- asks students to respond to a number of unseen short response questions relating to the Fundamental topic: Calculations and all Unit 3 topics
- may ask students to respond using single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams.

Part A contains short response simple familiar questions.

Part B contains short response complex familiar and complex unfamiliar questions.

Question specifications

The examination will be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	80%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • knowledge of simple subject matter is required to solve the problem; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	10%	These questions can focus on any of the objectives.	respond to situations where: • knowledge of [complex] subject matter is required to solve the problem; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	10%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • knowledge of [complex] subject matter is required to solve the problem; and • all the information to solve the problem is not immediately identifiable; that is – the required procedure is not clear from the way the problem is posed, and – in a context in which students have had limited prior experience.

Conditions

- This is an individual supervised task delivered in one continuous session.
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 60 minutes
- The teacher must provide the QCAA Essential Mathematics formula book.
- Students
  - are required to use technology
  - must not bring notes into the examination.

Internal assessment 3: Problem-solving and modelling task

Students respond to a specific mathematical investigative scenario or context using subject matter from Fundamental topic: Calculations, and at least one topic in Unit 4. While students may undertake some research, it is not the focus of this task.

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

This task requires students to:

- independently respond to a specific mathematical investigative scenario or context that highlights a real-life application of mathematics
- use relevant stimulus material involving the selected subject matter
- address all the stages of the problem-solving and mathematical modelling approach
- respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Conditions

- Students will use 8 hours of class time and their own time out of class to develop their response.
- This is an individual task.
- Data may be provided or collected individually or collected in groups.
- Appendixes can include raw data, repeated calculations, evidence of authentication and student notes (appendixes are not marked).
- Students must use technology, e.g. scientific calculator, graphics calculator, spreadsheet program and/or other mathematical software; use of technology must go beyond simple computation or word processing.

Response requirements

Written: up to 8 A4 pages, up to 1000 words

Instrument-specific standards

<table>
<tr><th>Formulate</th><th>Solve</th><th>Evaluate</th><th>Communicate</th><th>Grade</th></tr>
<tr><td colspan="5">The student response has the following characteristics:</td></tr>
<tr><td>• justified statements of important assumptions
• justified statements of important observations
• justified mathematical translation of important simple and important complex aspects of the task</td><td>• accurate use of simple and complex mathematical knowledge for important aspects of the task
• efficient use of technology
• a complete solution</td><td>• verified results
• justified statements about the reasonableness of the solution by considering the assumptions
• justified statements about the reasonableness of the solution by considering the observations
• justified statements of relevant strengths of the solution
• justified statements of relevant limitations of the solution</td><td>• correct use of appropriate mathematical language
• logical organisation of the response, which can be read independently of the task sheet
• justification of decisions using mathematical reasoning</td><td>A</td></tr>
<tr><td>• statements of important assumptions
• statements of important observations
• mathematical translation of important simple and important complex aspects of the task</td><td>• use of simple and complex mathematical knowledge for an important aspect of the task
• use of technology
• substantial progress towards a solution</td><td>• a verified result
• statements about the reasonableness of the solution by considering the assumptions
• statements about the reasonableness of the solution by considering the observations
• statements of relevant strengths of the solution
• statements of relevant limitations of the solution</td><td>• use of appropriate mathematical language
• logical organisation of the response
• statements of decisions using mathematical reasoning</td><td>B</td></tr>
<tr><td>• statement of a relevant assumption
• statement of a relevant observation
• mathematical translation of a simple or complex aspect of the task</td><td>• use of simple mathematical knowledge relevant to the task
• simplistic use of technology
• progress towards a solution</td><td>• progress towards a verified result
• statement about the reasonableness of the solution by considering an assumption or observation
• statement of a relevant strength or relevant limitation of the solution</td><td>• use of some appropriate mathematical language
• adequate organisation of the response
• statement of a decision using mathematical reasoning</td><td>C</td></tr>
<tr><td>• statement of an assumption or observation
• mathematical translation of an aspect of the task.</td><td>• simplistic use of mathematical knowledge
• inappropriate use of technology.</td><td>• statement about the reasonableness of a result or the solution
• statement of a strength or limitation.</td><td>• use of everyday language
• basic organisation of the response.</td><td>D</td></tr>
<tr><td>The student response does not match any of the descriptors above.</td><td>The student response does not match any of the descriptors above.</td><td>The student response does not match any of the descriptors above.</td><td>The student response does not match any of the descriptors above.</td><td>E</td></tr>
</table>

Internal assessment 4: Examination — short response

Assessment objectives

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Specifications

The teacher provides an examination that:

- asks students to respond to a number of unseen short response questions
- representatively samples subject matter from the Fundamental topic: Calculations and all Unit 4 topics
- may ask students to respond using single words, sentences or paragraphs
- may ask students to
  - interpret unseen stimulus
  - calculate using algorithms
  - draw or label graphs, tables or diagrams.

Question specifications

The examination must be aligned to the specifications provided in the table below.

Degree of difficulty	Mark allocation (± 2%)	Objectives	In these questions, students:
Simple familiar	80%	Typically, these questions focus on Objectives 1, 2 and 3.	respond to situations where: • knowledge of simple subject matter is required to solve the problem; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex familiar	10%	These questions can focus on any of the objectives.	respond to situations where: • knowledge of [complex] subject matter is required to solve the problem; and • all of the information to solve the problem is identifiable; that is – the required procedure is clear from the way the problem is posed, or – in a context that has been a focus of prior learning
Complex unfamiliar	10%	Typically, these questions focus on Objectives 4, 5 and 6.	respond to situations where: • knowledge of [complex] subject matter is required to solve the problem; and • all the information to solve the problem is not immediately identifiable; that is – the required procedure is not clear from the way the problem is posed, and – in a context in which students have had limited prior experience.

Conditions

- This is an individual supervised task.
- The task may be delivered in two consecutive sessions only if
  - questions in each session are unseen
  - teaching or feedback is not provided between the sessions.
- Time allowed
  - Perusal time: 5 minutes
  - Working time: 60 minutes
- The teacher must provide the QCAA Essential Mathematics formula book.
- Students
  - are required to use technology
  - must not bring notes into the examination.

Instrument-specific standards

Foundational knowledge and problem-solving	Cut-off	Grade
The student response has the following characteristics:		
• comprehensive recall and use of simple and complex mathematical knowledge; clear communication of simple and complex mathematical knowledge; considered evaluation of the reasonableness of solutions; use of mathematical reasoning to justify procedures and decisions; and proficient application of simple and complex mathematical knowledge to solve problems	> 80%	A
• thorough recall and use of simple and some complex mathematical knowledge; communication of simple and some complex mathematical knowledge; evaluation of the reasonableness of some solutions; use of mathematical reasoning to justify procedures and decisions; and application of simple and some complex mathematical knowledge to solve problems	> 60%	B
• recall and use of simple mathematical knowledge; communication of simple mathematical knowledge; discussion of the reasonableness of solutions; use of some mathematical reasoning; and some application of simple mathematical knowledge to make progress towards solving problems	> 40%	C
• infrequent recall and use of mathematical knowledge; and basic communication of mathematical knowledge	> 20%	D
• isolated and inaccurate recall and use of mathematical knowledge; and disjointed and unclear communication of mathematical knowledge.	≥ 0%	E

Glossary

The syllabus glossary is available at www.qcaa.qld.edu.au/downloads/senior-qce/common/snr_glossary_cognitive_verbs.pdf.

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Version history

Version	Date of change	Information
1.0	January 2024	Released for familiarisation and planning (with implementation starting in 2025)