Specialist Mathematics marking guide and response

External assessment 2025

Paper 2: Technology-active (60 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
Queensland Curriculum & Assessment Authority

Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response demonstrates the qualities of a high-level response.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Allow FT mark/s — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Multiple choice

Question	Response
1	C
2	D
3	B
4	D
5	B
6	B
7	A
8	D
9	B
10	C

Short response

Q	Sample response	The response:
11a)	$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$ $= \frac{4.2}{\sqrt{16}}$ $= 1.05$ kg	- correctly calculates $\sigma_{\overline{X}}$ **[1 mark]**
	$\mu_{\overline{X}} = 21.3$ Using GDC $P(\overline{X} > 23) = 0.05$	- calculates required probability **[1 mark]**
11b)	$P(21.3 - m \le \overline{X} \le 21.3 + m) = 0.4$	- correctly shows mathematical reasoning that uses the given information **[1 mark]**
	Using GDC $21.3 - m = 20.75$ $m = 0.55$	- calculates value of *m* **[1 mark]**

Q	Sample response	The response:
12a)	$\boldsymbol{F}_{net} = 42\cos(18°)\hat{\boldsymbol{i}} + 42\sin(18°)\hat{\boldsymbol{j}}$ $-60\cos(32°)\hat{\boldsymbol{i}} - 60\sin(32°)\hat{\boldsymbol{j}}\ \text{N}$	• correctly resolves the given vectors **[1 mark]**
	$= -10.94\hat{\boldsymbol{i}} - 18.82\hat{\boldsymbol{j}}\ \text{N}$	• determines a simplified expression for the resultant force in Cartesian form **[1 mark]**
12b)	$\boldsymbol{F}_{net} = m\boldsymbol{a}$ $\boldsymbol{a} = \frac{1}{10}\left(-10.94\hat{\boldsymbol{i}} - 18.82\hat{\boldsymbol{j}}\right)\ \text{m}\,\text{s}^{-2}$ $= -1.09\hat{\boldsymbol{i}} - 1.88\hat{\boldsymbol{j}}$	• determines acceleration in Cartesian form **[1 mark]**
12c)	$\boldsymbol{v} = \int \boldsymbol{a}\ dt$ $= \int\left(-1.09\hat{\boldsymbol{i}} - 1.88\hat{\boldsymbol{j}}\right)dt$ $= -1.09t\,\hat{\boldsymbol{i}} - 1.88t\,\hat{\boldsymbol{j}} + \boldsymbol{c}$	• determines expression for velocity **[1 mark]**
	At $t = 0,\ \boldsymbol{v} = 5\hat{\boldsymbol{j}} \Rightarrow 5\hat{\boldsymbol{j}} = \boldsymbol{c}$ $\boldsymbol{v} = -1.09t\,\hat{\boldsymbol{i}} + \left(-1.88t + 5\right)\hat{\boldsymbol{j}}$	• determines velocity based on initial condition in Cartesian form **[1 mark]**
12d)	When $t = 2$ $\boldsymbol{v} = -2.18\hat{\boldsymbol{i}} + 1.24\hat{\boldsymbol{j}}$	• determines velocity when $t = 2$ **[1 mark]**
	$\lvert\boldsymbol{v}\rvert = \sqrt{(-2.18)^2 + (1.24)^2}$ $\approx 2.51\ \text{m s}^{-1}$	• determines speed when $t = 2$ **[1 mark]**

<table>
<tr><th>Q</th><th>Sample response</th><th>The response:</th></tr>
<tr><td>13a)</td><td>Using GDC: (4, 4)</td><td>• correctly determines A [1 mark]</td></tr>
<tr><td rowspan="3">13b)</td><td>Method 1
Using 4 intervals from $x = 4$ to $12 \Rightarrow w = 2$.
Table of relevant values for $f(x) - g(x)$.
<table>
<tr><td>x</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr>
<tr><td>$f(x)-g(x)$</td><td>0</td><td>7.34</td><td>9.38</td><td>6.86</td><td>0</td></tr>
</table></td><td>• correctly recognises the requirement to determine values of $f(x) - g(x)$ [1 mark]</td></tr>
<tr><td>$$\text{Area} \approx \frac{w}{3}\Big[f(x_0)+4\big[f(x_1)+f(x_3)+\ldots\big]$$ $$+2\big[f(x_2)+f(x_4)+\ldots\big]+f(x_n)\Big]$$ $$\approx \frac{2}{3}\big[0+4(7.34+6.86)+2(9.38)+0\big]$$</td><td>• shows evidence of use of Simpson’s rule [1 mark]</td></tr>
<tr><td>≈ 50.37 units^2</td><td>• determines required approximate area [1 mark]</td></tr>
</table>

<table>
<tr><th>Q</th><th>Sample response</th><th>The response:</th></tr>
<tr><td>13b)</td><td>Method 2

Using 4 intervals from $x = 4$ to 12 $\Rightarrow w = 2$.
Table of relevant values for $f(x)$ and $g(x)$.
<table>
<tr><td>x</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr>
<tr><td>$f(x)$</td><td>4</td><td>9</td><td>10</td><td>7</td><td>0</td></tr>
<tr><td>$g(x)$</td><td>4</td><td>1.66</td><td>0.62</td><td>0.14</td><td>0</td></tr>
</table></td><td>• correctly recognises the requirement to determine values of $f(x)$ and $g(x)$ [1 mark]</td></tr>
<tr><td></td><td>Consider area under $f(x) = -0.5x^2 + 7.5x - 18$
$\text{Area}_1 \approx \frac{w}{3}\left[f(x_0) + 4\left[f(x_1) + f(x_3) + \ldots\right]\right.$
$\left. +2\left[f(x_2) + f(x_4) + \ldots\right] + f(x_n)\right]$ Consider area under
$\approx \frac{2}{3}\left[4 + 4(9 + 7) + 2(10) + 0\right]$
$g(x) = 4\operatorname{cosec}\left(\frac{\pi x}{24}\right) - 4$
$\text{Area}_2 \approx \frac{2}{3}\left[4 + 4(1.66 + 0.14) + 2(0.62) + 0\right]$</td><td>• shows evidence of use of Simpson’s rule [1 mark]</td></tr>
<tr><td></td><td>Area required $\approx 58.667 - 8.293 \approx 50.37$ units^2</td><td>• determines required approximate area [1 mark]</td></tr>
</table>

Q	Sample response	The response:
13c)	Area $= \int_a^b \left(f(x) - g(x) \right) dx$ $= \int_4^{12} \left(-0.5x^2 + 7.5x - 18 - \left(4\operatorname{cosec}\left(\frac{\pi x}{24} \right) - 4 \right) \right) dx$	• correctly determines a definite integral, **including** the *dx* term, that represents the exact area **[1 mark]**
13d)	Area ≈ 50.42 units^2	• determines rounded value of exact area **[1 mark]**
13e)	The accuracy of the result could be improved with the use of more intervals (even number).	• correctly states a suitable strategy **[1 mark]**

Q	Sample response	The response:
14a)	Given $\left\|\overrightarrow{OB}\right\|=5$ Projection of $\boldsymbol{a}$ on $\boldsymbol{b}=\left(\boldsymbol{a}\cdot\hat{\boldsymbol{b}}\right)\hat{\boldsymbol{b}}$ $\overrightarrow{OA}$ on $\overrightarrow{OB}=\left[\begin{pmatrix}1\\2\\5\end{pmatrix}\cdot\frac{1}{5}\begin{pmatrix}-3\\4\\0\end{pmatrix}\right]\frac{1}{5}\begin{pmatrix}-3\\4\\0\end{pmatrix}$	- correctly substitutes into a suitable rule that represents the vector projection **[1 mark]**
	$=\frac{1}{25}\left[-3+8\right]\begin{pmatrix}-3\\4\\0\end{pmatrix}$ $=\frac{1}{5}\begin{pmatrix}-3\\4\\0\end{pmatrix}$	- determines vector projection having demonstrated accurate calculation of the scalar product **[1 mark]**

Q	Sample response	The response:
14b)	Vector projection of $\overrightarrow{OA}$ on $\overrightarrow{OB} = \overrightarrow{OC}$. $\left\|\overrightarrow{OC}\right\| = \frac{1}{5}\times 5$ $= 1$ unit	• determines length of side *OC* **[1 mark]**
14c)	$\left\|\overrightarrow{OA}\right\| = \sqrt{1+4+25}$ $= \sqrt{30}$ units	• correctly determines the length of side *OA* **[1 mark]**
14d)	Using Pythagoras' theorem $\left\|\overrightarrow{OC}\right\|^2 + \left\|\overrightarrow{AC}\right\|^2 = \left\|\overrightarrow{OA}\right\|^2$ $\left\|\overrightarrow{AC}\right\|^2 = 30 - 1$ $\left\|\overrightarrow{AC}\right\| = \sqrt{29}$ units	• determines length of *AC* **[1 mark]**
14e)	Area $\Delta OAB = \frac{1}{2}bh = \frac{1}{2}\times 5\times\sqrt{29}$ $= \frac{5\sqrt{29}}{2}$ units^2	• determines area of triangle *OAB* **[1 mark]**

Q	Sample response	The response:
14f)	$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \times \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} -20 \\ -15 \\ 10 \end{pmatrix}$	• correctly calculates suitable vector product **[1 mark]**
	$\text{Area } \Delta OAB = \frac{1}{2}\left\|\overrightarrow{OA} \times \overrightarrow{OB}\right\|$ $= \frac{1}{2}\sqrt{(-20)^2 + (-15)^2 + 10^2}$ $= \frac{5\sqrt{29}}{2} \text{ units}^2$ So the result is verified.	• verifies result from Question 14e) based on vector product result **[1 mark]**

Q	Sample response	The response:
15	**Method 1** Initial statement Prove the rule is true for $n = 1$. $\text{LHS} = r\left(\cos(\theta) + i\sin(\theta)\right)$ $\text{RHS} = r^1\left(\cos(1\times\theta) + i\sin(1\times\theta)\right)$ $= r\left(\cos(\theta) + i\sin(\theta)\right)$ $= \text{LHS}$	- correctly proves the initial statement **[1 mark]**
	Assume the rule is true for $n = k$. $\left(r\left(\cos(\theta) + i\sin(\theta)\right)\right)^k = r^k\left(\cos(k\theta) + i\sin(k\theta)\right)$	- correctly states a suitable assumption **[1 mark]**
	Inductive step Prove the rule is true for $n = k + 1$. $\left(r\left(\cos(\theta) + i\sin(\theta)\right)\right)^{k+1} = r^{k+1}\left(\cos((k+1)\theta) + i\sin((k+1)\theta)\right)$ $\text{LHS} = \left(r\left(\cos(\theta) + i\sin(\theta)\right)\right)^k . \, r\left(\cos(\theta) + i\sin(\theta)\right)^1$ $= r^k\left(\cos(k\theta) + i\sin(k\theta)\right) r\left(\cos(\theta) + i\sin(\theta)\right)$	- uses the assumption statement **[1 mark]**
	$\text{LHS} = r^{k+1}\left(\cos(k\theta)\cos(\theta) + i\cos(k\theta)\sin(\theta) + i\sin(k\theta)\cos(\theta) - \sin(k\theta)\sin(\theta)\right)$ $= r^{k+1}\left(\cos(k\theta)\cos(\theta) - \sin(k\theta)\sin(\theta) + i\left(\sin(k\theta)\cos(\theta) + \cos(k\theta)\sin(\theta)\right)\right)$	- expresses result in Cartesian form **[1 mark]**
	$\text{LHS} = r^{k+1}\left(\cos(k\theta + \theta) + i\left(\sin(k\theta + \theta)\right)\right)$	- uses angle sum and difference identities **[1 mark]**

Q	Sample response	The response:
	$\text{LHS} = r^{k+1}\left(\cos\left(\left(k+1\right)\theta\right)+i\sin\left(\left(k+1\right)\theta\right)\right)$ $= \text{RHS}$ Conclusion: The rule is proven true for $n = k + 1$. By mathematical induction, the rule is true for $n = 1, 2, \ldots$	- completes proof and states a suitable conclusion **[1 mark]**

Q	Sample response	The response:
15	**Method 2** Initial statement Prove the rule is true for $n = 1$. $\text{LHS} = r\left(\cos(\theta)+i\sin(\theta)\right)$ $\text{RHS} = r^1\left(\cos(1\times\theta)+i\sin(1\times\theta)\right)$ $= r\left(\cos(\theta)+i\sin(\theta)\right)$ $= \text{LHS}$	• correctly proves the initial statement **[1 mark]**
	Assume the rule is true for $n = k$. $\left(r\left(\cos(\theta)+i\sin(\theta)\right)\right)^k = r^k\left(\cos(k\theta)+i\sin(k\theta)\right)$	• correctly states a suitable assumption **[1 mark]**
	Inductive step Prove the rule is true for $n = k + 1$. $\left(r\left(\cos(\theta)+i\sin(\theta)\right)\right)^{k+1} = r^{k+1}\left(\cos((k+1)\theta)+i\sin((k+1)\theta)\right)$ $\text{RHS} = r^{k+1}\left(\cos(k\theta+\theta)+i\sin(k\theta+\theta)\right)$ $= r\times r^k\left(\cos(k\theta)\cos(\theta)-\sin(k\theta)\sin(\theta)+\right.$ $\left. i\left(\sin(k\theta)\cos(\theta)+\cos(k\theta)\sin(\theta)\right)\right)$	• uses angle sum and difference identities **[1 mark]**
	$\text{RHS} = r^k\times r\left(\cos(k\theta)\cos(\theta)+i\sin(k\theta)\cos(\theta)+\right.$ $\left.-\sin(k\theta)\sin(\theta)+i\cos(k\theta)\sin(\theta)\right)$ $= r^k\left(r\cos(\theta)\left(\cos(k\theta)+i\sin(k\theta)\right)+\right.$ $\left. r\,i\sin(\theta)\left(\cos(k\theta)+i\sin(k\theta)\right)\right)$	• regroups expression using a common factor of $\cos(k\theta)+i\sin(k\theta)$ **[1 mark]**

Q	Sample response	The response:
	$\text{RHS} = r^k\left(\cos(k\theta)+i\sin(k\theta)\right)r\left(\cos(\theta)+i\sin(\theta)\right)$ $= \left(r\left(\cos(\theta)+i\sin(\theta)\right)\right)^k r\left(\cos(\theta)+i\sin(\theta)\right)$	• uses the assumption statement **[1 mark]**
	$\text{RHS} = \left(r\left(\cos(\theta)+i\sin(\theta)\right)\right)^{k+1}$ $= \text{LHS}$ Conclusion: The rule is proven true for $n = k + 1$. By mathematical induction, the rule is true for $n = 1, 2, \ldots$	• completes proof and states a suitable conclusion **[1 mark]**

Q	Sample response	The response:
15	**Method 3** Initial statement Prove the rule is true for $n=1$. $\text{LHS}=r\left(\cos(\theta)+i\sin(\theta)\right)$ $\text{RHS}=r^1\left(\cos(1\times\theta)+i\sin(1\times\theta)\right)$ $=r\left(\cos(\theta)+i\sin(\theta)\right)$ $=\text{LHS}$	• correctly proves the initial statement **[1 mark]**
	Assume the rule is true for $n=k$ $\left(r\left(\cos(\theta)+i\sin(\theta)\right)\right)^k=r^k\left(\cos(k\theta)+i\sin(k\theta)\right)$	• correctly states a suitable assumption **[1 mark]**
	Inductive step Prove the rule is true for $n=k+1$ $\left(r\left(\cos(\theta)+i\sin(\theta)\right)\right)^{k+1}=r^{k+1}\left(\cos\left((k+1)\theta\right)+i\sin\left((k+1)\theta\right)\right)$ $\text{LHS}=\left(r\left(\cos(\theta)+i\sin(\theta)\right)\right)^k.\ r\left(\cos(\theta)+i\sin(\theta)\right)^1$ $=r^k\left(\cos(k\theta)+i\sin(k\theta)\right)r\left(\cos(\theta)+i\sin(\theta)\right)$	• uses the assumption statement **[1 mark]**
	$\text{RHS}=r^{k+1}\left(\cos(k\theta+\theta)+i\sin(k\theta+\theta)\right)$ $=r\times r^k\left(\cos(k\theta)\cos(\theta)-\sin(k\theta)\sin(\theta)+\right.$ $\left.i\left(\sin(k\theta)\cos(\theta)+\cos(k\theta)\sin(\theta)\right)\right)$	• uses angle sum and difference identities **[1 mark]**

Q	Sample response	The response:
	$\text{RHS} = r^k \times r\big(\cos(k\theta)\cos(\theta) + i\sin(k\theta)\cos(\theta) - \sin(k\theta)\sin(\theta) + i\cos(k\theta)\sin(\theta)\big)$ $= r^k\big(r\cos(\theta)\big(\cos(k\theta) + i\sin(k\theta)\big) + r\,i\sin(\theta)\big(\cos(k\theta) + i\sin(k\theta)\big)\big)$	- regroups expression using a common factor of $\cos(k\theta) + i\sin(k\theta)$ **[1 mark]**
	$\text{RHS} = r^k\big(\cos(k\theta) + i\sin(k\theta)\big)\big(r\big(\cos(\theta) + i\sin(\theta)\big)\big)$ $= \text{LHS}$ Conclusion: The rule is proven true for *n* = *k* + 1. By mathematical induction, the rule is true for *n* = 1, 2, …	- completes proof and states a suitable conclusion **[1 mark]**

Q	Sample response	The response:
16	**Method 1** Given $\frac{dP}{dt} = 0.5P(1-0.2P)$ $\int \frac{1}{P(1-0.2P)}\ dP = \int 0.5\ dt \quad ...(1)$	- correctly separates the variables **[1 mark]**
	$\frac{1}{P(1-0.2P)} = \frac{A}{P} + \frac{B}{(1-0.2P)} \quad ...(2)$ $A(1-0.2P)+BP=1$ $P=0: A=1$ $P=5: B=0.2$ From (1) and (2) $\int \frac{1}{P} + \frac{0.2}{(1-0.2P)}\ dP = \int 0.5\,dt$	- uses partial fractions **[1 mark]**
	$\ln\lvert P\rvert - \ln\lvert 1-0.2P\rvert = 0.5t + c$	- uses suitable integration methods to determine a solution to the given differential equation **[1 mark]**
	Given $P = 0.3$ when $t = 0$ $\ln(0.3) - \ln(1-0.2\times 0.3) = c$ $c \approx -1.142$	- determines constant of integration **[1 mark]**
	$\ln(P) - \ln(1-0.2P) = 0.5t - 1.142$ When $t = 5$ $\ln(P) - \ln(1-0.2P) = 0.5\times 5 - 1.142$ $\ln(P) - \ln(1-0.2P) = 1.358$	- determines an equation in terms of P whose solution represents the required population **[1 mark]**

Q	Sample response	The response:
	Using GDC $P \approx 2.19$ The estimated population on 1 January 2030 is 2.2 million.	• estimates the required population **[1 mark]**

Q	Sample response	The response:
16	**Method 2** Given $\frac{dP}{dt}=0.5P(1-0.2P)$ $\int\frac{1}{0.5P(1-0.2P)}\ dP=\int dt$ $\int\frac{1}{P(5-P)}\ dP=\int 0.1\ dt\quad ...(1)$	• correctly separates the variables **[1 mark]**
	$\frac{1}{P(5-P)}=\frac{A}{P}+\frac{B}{(5-P)}\quad ...(2)$ $A(5-P)+BP=1$ $P=0: A=0.2$ $P=5: B=0.2$ From (1) and (2) $\int\frac{0.2}{P}+\frac{0.2}{(5-P)}\ dP=\int 0.1dt$	• uses partial fractions **[1 mark]**
	$0.2\ln\lvert P\rvert-0.2\ln\lvert 5-P\rvert=0.1t+c$	• uses suitable integration methods to determine a solution to the given differential equation **[1 mark]**
	Given $P=0.3$ when $t=0$ $0.2\times\ln(0.3)-0.2\times\ln(5-0.3)=c$ $c\approx-0.550$	• determines constant of integration **[1 mark]**
	$0.2\ln(P)-0.2\ln(5-P)=0.1t-0.550$ When $t=5$ $0.2\ln(P)-0.2\ln(5-P)=0.1\times5-0.550$ $0.2\ln(P)-0.2\ln(5-P)=-0.050$	• determines an equation in terms of P whose solution represents the required population **[1 mark]**

Q	Sample response	The response:
	Using GDC $P \approx 2.189$ The estimated population on January 1 2030 is 2.2 million.	• estimates the required population **[1 mark]**

Q	Sample response	The response:
17	Consider first sample. Let $\overline{x}_1$ be the first sample mean and n_1 be the first sample size. Given CI is (23.560, 25.498). $\overline{x}_1 = \frac{25.498 + 23.560}{2}$ $= 24.529$ $\text{Marginal error} = 25.498 - 24.529$ $z\frac{s}{\sqrt{n_1}} = 0.969$	• correctly determines the margin of error for the CI from the first sample **[1 mark]**
	Using $\sigma = 5.102$ and z-score for 90% CI of 1.645 $1.645\left(\frac{5.102}{\sqrt{n_1}}\right) = 0.969$ Using GDC $n_1 \approx 75$ First sample size is 75.	• determines first sample size **[1 mark]**
	Consider second sample. The second sample size is $n_2 = 75 - 60 = 15$ $\text{Margin of error} = z\frac{s}{\sqrt{n_2}}$ $= 1.645\left(\frac{5.102}{\sqrt{15}}\right)$ $= 2.167$	• determines the margin of error for the CI from the second sample **[1 mark]**

Q	Sample response	The response:
	Let $\overline{x}_2$ be the second sample mean. Second CI will overlap first CI when the value of $\overline{x}_2$ lies between $23.560 - 2.167 = 21.393$ and $25.498 + 2.167 = 27.665$	• determines range of values of second sample mean that allows the second CI to overlap the first CI **[1 mark]**
	Consider the distribution of the sample means for sample sizes of 15. Since $X \sim N(24.311, 5.102)$ $\overline{X} \sim N\left(24.311, \frac{5.102}{\sqrt{15}}\right)$ Using GDC $P\left(21.393 \le \overline{X} \le 27.665\right) \approx 0.98$	• determines probability that the CI from the second sample overlaps the CI from the first sample **[1 mark]**

Q	Sample response	The response:
18	**Method 1** Required length, L, has limits of $\theta = 0$ to $\theta = \pi$.	- correctly determines the values of a and b by considering the curve length above the x-axis **[1 mark]**
	Given $r = 1 + \cos(\theta)$ $\frac{dr}{d\theta} = -\sin(\theta)$ $\left(\frac{dr}{d\theta}\right)^2 = \sin^2(\theta)$	- correctly determines expressions for $\frac{dr}{d\theta}$ **[1 mark]**
	$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$ $= \int_0^\pi \sqrt{\left(1+\cos(\theta)\right)^2 + \left(-\sin(\theta)\right)^2}\ d\theta$ $= \int_0^\pi \sqrt{1 + 2\cos(\theta) + \cos^2(\theta) + \sin^2(\theta)}\ d\theta$	- determines an expression for the integrand in expanded form **[1 mark]**
	$= \int_0^\pi \sqrt{2 + 2\cos(\theta)}\ d\theta$	- uses suitable identity to determine a simplified integrand expression **[1 mark]**
	$L = \int_0^\pi \sqrt{2 + 2\left(2\cos^2\left(\frac{\theta}{2}\right) - 1\right)}\ d\theta$	- uses suitable double angle identity within integrand **[1 mark]**

Q	Sample response	The response:
	$L=\int_0^{\pi}\sqrt{4\cos^2\left(\frac{\theta}{2}\right)}\,d\theta$ $=\int_0^{\pi}2\cos\left(\frac{\theta}{2}\right)d\theta$ $=4\sin\left(\frac{\theta}{2}\right)\Big\vert_0^{\pi}$	• uses suitable integration method to determine an expression for L **[1 mark]**
	$=4\left(\sin\left(\frac{\pi}{2}\right)-\sin(0)\right)$ $=4$ units	• determines value for the required length **[1 mark]**

Q	Sample response	The response:
18	**Method 2** Required length, L, has limits of $\theta = 0$ to $\theta = \pi$.	- correctly determines the values of *a* and *b* by considering the curve length above the *x*-axis **[1 mark]**
	Given $r = 1 + \cos(\theta)$ $\frac{dr}{d\theta} = -\sin(\theta)$ $\left(\frac{dr}{d\theta}\right)^2 = \sin^2(\theta)$	- correctly determines expressions for $\frac{dr}{d\theta}$ **[1 mark]**
	$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ $= \int_0^\pi \sqrt{\left(1+\cos(\theta)\right)^2 + \left(-\sin(\theta)\right)^2}\, d\theta$ $= \int_0^\pi \sqrt{1 + 2\cos(\theta) + \cos^2(\theta) + \sin^2(\theta)}\, d\theta$	- determines an expression for the integrand in expanded form **[1 mark]**
	$= \int_0^\pi \sqrt{2 + 2\cos(\theta)}\, d\theta$	- uses suitable identity to determine a simplified integrand expression **[1 mark]**

Q	Sample response	The response:
	$=\int_0^{\pi} \sqrt{2}\sqrt{1+\cos(\theta)}\ d\theta$ Let $u=\cos(\theta) \Rightarrow \sin(\theta)=\sqrt{1-u^2}$ $\theta=0 \Rightarrow u=1$ $\theta=\pi \Rightarrow u=-1$ $du=-\sin(\theta)d\theta \Rightarrow d\theta=\frac{du}{-\sqrt{1-u^2}}$ $L=\sqrt{2}\int_1^{-1} \sqrt{1+u}\ \frac{du}{-\sqrt{1-u^2}}$	• uses suitable substitution within integrand including the *du* term **[1 mark]**
	$L=-\sqrt{2}\int_1^{-1} \sqrt{\frac{1+u}{(1+u)(1-u)}}\ du$ $=-\sqrt{2}\int_1^{-1} (1-u)^{-\frac{1}{2}}\ du$ $=2\sqrt{2}(1-u)^{\frac{1}{2}}\Big\rvert_{1}^{-1}$	• uses suitable integration method to determine an expression for *L* **[1 mark]**
	$=-2\sqrt{2}\left((1-(-1))^{\frac{1}{2}}+(1-1)^{\frac{1}{2}}\right)$ $=4$ units	• determines value for the required length **[1 mark]**