Specialist Mathematics marking guide and response

External assessment 2025

Paper 1: Technology-free (60 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
Queensland Curriculum & Assessment Authority
For all Queensland schools

Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response demonstrates the qualities of a high-level response.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of '0' will be recorded.

Allow FT mark/s — refers to 'follow through', where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Multiple choice

Question	Response
1	A
2	D
3	A
4	D
5	D
6	B
7	C
8	C
9	B
10	C

Short response

Q	Sample response	The response:
11a)	Given $\left[\begin{array}{ccc\|c} 1 & -1 & -1 & -6 \\ -2 & 1 & 1 & 1 \\ 0 & 0 & 4 & 4 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$ The augmented matrix is $\left[\begin{array}{ccc\|c} 1 & -1 & -1 & -6 \\ 0 & -1 & -1 & -11 \\ 0 & 0 & 4 & 4 \end{array}\right] \begin{array}{c} R_1 \\ R_2' = R_2 + 2R_1 \\ R_3 \end{array}$	• correctly determines the new row 2 values in the augmented matrix **[1 mark]**
11b)	**Method 1** From R_3 $4z = 4$ $z = 1$	• correctly determines required value of z **[1 mark]**
	From R_2' $-y - z = -11$ $-y - 1 = -11$ $y = 10$	• determines required value of y **[1 mark]**
	From R_1 $x - y - z = -6$ $x - 10 - 1 = -6$ $x = 5$	• determines required value of x **[1 mark]**

Q	Sample response	The response:
11b)	**Method 2** $\left[\begin{array}{ccc\|c} 1 & -1 & -1 & -6 \\ 0 & -1 & -1 & -11 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 \\ R_2' \\ R_3' = R_3 \div 4 \end{array}$ From R_3' $z = 1$	• correctly determines required value of z **[1 mark]**
	$\left[\begin{array}{ccc\|c} 1 & 0 & 0 & 5 \\ 0 & -1 & -1 & -11 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1' = R_1 - R_2' \\ R_2' \\ R_3' \end{array}$ From R_1' $x = 5$	• determines required value of x **[1 mark]**
	$\left[\begin{array}{ccc\|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1' \\ R_2'' = -R_2 - R_3' \\ R_3' \end{array}$ From R_2'' $y = 10$	• determines required value of y **[1 mark]**
11c)	The planes intersect at a point.	• describes a geometrical interpretation based on prior mathematical reasoning **[1 mark]**

Q	Sample response	The response:
12a)	$a = 2$	• correctly determines the value of *a* **[1 mark]**
12b)	$\boldsymbol{d} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$	• correctly determines a vector in the direction of the line **[1 mark]**
12c)	Given $(b, b, -2b)$ lies on $x - y + 4z = 8$ $b - b + 4(-2b) = 8$ $-8b = 8$ $b = -1$	• correctly determines the value of *b* **[1 mark]**
12d)	$\boldsymbol{n} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$	• correctly determines a vector normal to the plane **[1 mark]**
12e)	$\boldsymbol{d} \cdot \boldsymbol{n} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} = -2 - 2 + 4$ $= 0$	• shows calculation of $\boldsymbol{d} \cdot \boldsymbol{n}$ **[1 mark]**
12f)	The statement is not reasonable as the result from Q12e) demonstrates that the line is perpendicular to the normal to the plane and therefore not perpendicular to the plane.	• comments about the reasonableness of the statement **[1 mark]**

Q	Sample response	The response:
13a)	At $x = 0,\ v = 2\cos^{-1}(0)$ $= 2 \times \frac{\pi}{2}$ $= \pi \text{ m s}^{-1}$	• correctly determines the velocity of the object at the origin **[1 mark]**
	$\text{Momentum} = mv$ $= 3\pi \text{ kg m s}^{-1}$	• determines the momentum of the object at the origin **[1 mark]**

Q	Sample response	The response:
13b)	**Method 1** $v = 2\cos^{-1}\left(\frac{x}{3}\right)$ $\frac{dv}{dx} = \frac{-2}{\sqrt{9-x^2}}$	• correctly determines the expression for $\frac{dv}{dx}$ **[1 mark]**
	$a = v\frac{dv}{dx}$ $= 2\cos^{-1}\left(\frac{x}{3}\right)\frac{-2}{\sqrt{9-x^2}}$	• determines expression for acceleration of the object using $a = v\frac{dv}{dx}$ **[1 mark]**
	At $x = 0$, $a = 2\cos^{-1}(0)\frac{-2}{\sqrt{9}}$ $= -\frac{2\pi}{3}\ \text{m}\,\text{s}^{-2}$	• determines value of the acceleration of the object at the origin **[1 mark]**

Q	Sample response	The response:
13b)	**Method 2** $\frac{d}{dx}\cos^{-1}\left(\frac{x}{3}\right)=\frac{-1}{\sqrt{9-x^2}}$	• correctly determines the expression for $\frac{d}{dx}\cos^{-1}\left(\frac{x}{3}\right)$ **[1 mark]**
	$a=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $=\frac{d}{dx}\left(\frac{1}{2}\times 4\times\left(\cos^{-1}\left(\frac{x}{3}\right)\right)^2\right)$ $=4\cos^{-1}\left(\frac{x}{3}\right)\frac{-1}{\sqrt{9-x^2}}$	• determines expression for acceleration of the object **[1 mark]**
	At $x=0$, $a=\frac{-4\cos^{-1}(0)}{\sqrt{9}}$ $=-\frac{2\pi}{3}\ \text{m s}^{-2}$	• determines value of the acceleration of the object at the origin **[1 mark]**

Q	Sample response	The response:
14a)	Given $n = 100,\ \bar{x} = 498.9,\ s = 4.0,\ z = 2$	• correctly translates given information into relevant variables **[1 mark]**
	Confidence interval is $\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$ $= \left(498.9 - 2\frac{4}{\sqrt{100}},\ 498.9 + 2\frac{4}{\sqrt{100}}\right)$	• substitutes into confidence interval formula **[1 mark]**
	= (498.9 – 0.8, 498.9 + 0.8) = (498.1, 499.7) mL	• determines required confidence interval **[1 mark]**
14b)	The labelled juice volume of 500 mL lies above the upper value of the confidence interval.	• discusses the labelled juice volume in relation to the confidence interval values **[1 mark]**
	Current production does not meet the regulation.	• comments on the reasonableness of the current production **[1 mark]**

Q	Sample response	The response:
15a)	**Method 1** Consider $\int x^2 e^{-x}\, dx = \int u \frac{dv}{dx}\, dx$ $\int u \frac{dv}{dx}\, dx = uv - \int v \frac{du}{dx}\, dx$ $u = x^2 \Rightarrow \frac{du}{dx} = 2x$ $\frac{dv}{dx} = e^{-x} \Rightarrow v = -e^{-x}$	• correctly determines suitable expressions to prepare for a first application of the integration by parts formula **[1 mark]**
	$\int x^2 e^{-x}\, dx = -x^2 e^{-x} + \int 2x e^{-x}\, dx$ $u = 2x \Rightarrow \frac{du}{dx} = 2$ $\frac{dv}{dx} = e^{-x} \Rightarrow v = -e^{-x}$	• determines suitable expressions to prepare for a second application of the integration by parts formula **[1 mark]**
	$\int x^2 e^{-x}\, dx = -x^2 e^{-x} - 2x e^{-x} - \int -2 e^{-x}\, dx$ $= -x^2 e^{-x} - 2x e^{-x} - 2 e^{-x} + c$ $= -e^{-x}\left(x^2 + 2x + 2\right) + c$	• determines the required result based on evidence of prior mathematical reasoning **[1 mark]**

<table>
<tr><th>Q</th><th>Sample response</th><th>The response:</th></tr>
<tr><td>15a)</td><td>Method 2
Consider $\int x^2 e^{-x}\ dx = \int u \frac{dv}{dx}\ dx$
where $u = x^2$ and $\frac{dv}{dx} = e^{-x}$
$\int u \frac{dv}{dx}\ dx = uv - \int v \frac{du}{dx}\ dx$
<table>
<tr><th>Signs</th><th>Derivative</th><th>Integral</th></tr>
<tr><td>+</td><td>x^2</td><td>e^{-x}</td></tr>
<tr><td>–</td><td>$2x$</td><td>$-e^{-x}$</td></tr>
</table></td><td>• correctly determines suitable table expressions under derivative and integral headings to prepare for a first application of the integration by parts formula [1 mark]</td></tr>
<tr><td></td><td><table>
<tr><th>Signs</th><th>Derivative</th><th>Integral</th></tr>
<tr><td>+</td><td>x^2</td><td>e^{-x}</td></tr>
<tr><td>–</td><td>$2x$</td><td>$-e^{-x}$</td></tr>
<tr><td>+</td><td>2</td><td>e^{-x}</td></tr>
</table></td><td>• determines suitable subsequent table expressions under derivative and integral headings to prepare for a second application of the integration by parts formula [1 mark]</td></tr>
<tr><td></td><td><table>
<tr><th>Signs</th><th>Derivative</th><th>Integral</th></tr>
<tr><td>+</td><td>x^2</td><td>e^{-x}</td></tr>
<tr><td>–</td><td>$2x$</td><td>$-e^{-x}$</td></tr>
<tr><td>+</td><td>2</td><td>e^{-x}</td></tr>
<tr><td>–</td><td>0</td><td>$-e^{-x}$</td></tr>
</table>
$\int x^2 e^{-x}\ dx = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c$
$= -e^{-x}\left(x^2 + 2x + 2\right) + c$</td><td>• determines the required result based on evidence of prior mathematical reasoning [1 mark]</td></tr>
</table>

Q	Sample response	The response:
15b)	Area $= \int_0^1 \left(x^2e^{-x} - x^2e^{-1}\right)\, dx$ $= \left(-e^{-x}\left(x^2+2x+2\right) - \frac{x^3}{3}e^{-1}\right)\Big\|_0^1$	• correctly determines an expression for the integral **[1 mark]**
	$= \left(-e^{-1}(1+2+2) - \frac{1}{3}e^{-1}\right) - (-2)$ $= 2 - \frac{16}{3}e^{-1}$ units^2	• uses mathematical reasoning to determine the area expressed in simplified form **[1 mark]**

Q	Sample response	The response:
16	**Method 1**	
	$\overrightarrow{PR} = \boldsymbol{b} + \boldsymbol{c} - \boldsymbol{a}$	• correctly represents $\overrightarrow{PR}$ in terms of ***a***, ***b*** and ***c*** **[1 mark]**
	$\overrightarrow{QS} = \boldsymbol{c} - \boldsymbol{a} - \boldsymbol{b}$	• correctly represents $\overrightarrow{QS}$ in terms of ***a***, ***b*** and ***c*** **[1 mark]**
	Vector representing the midpoint of $\overrightarrow{PR}$ is $\boldsymbol{a} + \frac{1}{2}\overrightarrow{PR} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$	• determines a vector representing the midpoint of $\overrightarrow{PR}$ **[1 mark]**
	Vector representing the midpoint of $\overrightarrow{QS}$ is $\boldsymbol{a} + \boldsymbol{b} + \frac{1}{2}\overrightarrow{QS} = \boldsymbol{a} + \boldsymbol{b} + \frac{1}{2}(\boldsymbol{c} - \boldsymbol{a} - \boldsymbol{b})$ $= \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$	• determines a vector representing the midpoint of $\overrightarrow{QS}$ in terms of ***a***, ***b*** and ***c*** **[1 mark]**
	As the position vectors representing the midpoints of $\overrightarrow{PR}$ and $\overrightarrow{QS}$ coincide, the diagonal from *P* to *R* and the diagonal from *Q* to *S* bisect each other.	• completes proof with appropriate justification **[1 mark]**

Q	Sample response	The response:
16	**Method 2** $\overrightarrow{PR} = \boldsymbol{b} + \boldsymbol{c} - \boldsymbol{a}$	• correctly represents $\overrightarrow{PR}$ in terms of ***a***, ***b*** and ***c*** **[1 mark]**
	$\overrightarrow{QS} = \boldsymbol{c} - \boldsymbol{a} - \boldsymbol{b}$	• correctly represents $\overrightarrow{QS}$ in terms of ***a***, ***b*** and ***c*** **[1 mark]**
	Line *PR* can be expressed as $\boldsymbol{a} + t_1\overrightarrow{PR} = \boldsymbol{a} + t_1(\boldsymbol{b} + \boldsymbol{c} - \boldsymbol{a})$ where $t_1 \in R$ Line *QS* can be expressed as $\boldsymbol{a} + \boldsymbol{b} + t_2\overrightarrow{QS}$ $= \boldsymbol{b} + \boldsymbol{c} + t_2(\boldsymbol{c} - \boldsymbol{a} - \boldsymbol{b})$ where $t_2 \in R$	• determines equations of lines representing $\overrightarrow{PR}$ and $\overrightarrow{QS}$ **[1 mark]**
	The diagonals intersect when $\boldsymbol{a} + t_1(\boldsymbol{b} + \boldsymbol{c} - \boldsymbol{a}) = \boldsymbol{a} + \boldsymbol{b} + t_2(\boldsymbol{c} - \boldsymbol{a} - \boldsymbol{b})$ $t_1(\boldsymbol{b} + \boldsymbol{c} - \boldsymbol{a}) = \boldsymbol{b} + t_2(\boldsymbol{c} - \boldsymbol{a} - \boldsymbol{b})$ Equating vectors ***a*** and ***b***: $\boldsymbol{a}: -t_1 = -t_2$ $\boldsymbol{b}: t_1 = 1 - t_2$	• determines two independent simultaneous equations **[1 mark]**
	So $t_1 = t_2 = 0.5$ As the lines intersect at their midpoints, the diagonal from *P* to *R* and the diagonal from *Q* to *S* bisect each other.	• completes proof with appropriate justification **[1 mark]**

Q	Sample response	The response:
16	**Method 3**	
	$\overrightarrow{PR} = \boldsymbol{b} + \boldsymbol{c} - \boldsymbol{a}$	• correctly represents $\overrightarrow{PR}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ **[1 mark]**
	$\overrightarrow{QS} = \boldsymbol{c} - \boldsymbol{a} - \boldsymbol{b}$	• correctly represents $\overrightarrow{QS}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ **[1 mark]**
	Let M be the midpoint of PR $\overrightarrow{QM} = -\boldsymbol{b} + \frac{1}{2}\overrightarrow{PR}$ $= -\boldsymbol{b} + \frac{1}{2}(\boldsymbol{b} + \boldsymbol{c} - \boldsymbol{a})$ $= \frac{1}{2}(\boldsymbol{c} - \boldsymbol{a} - \boldsymbol{b})$	• determines a vector representing $\overrightarrow{QM}$ **[1 mark]**
	$\therefore \overrightarrow{QM} = \frac{1}{2}\overrightarrow{QS}$	• determines relationship between $\overrightarrow{QM}$ and $\overrightarrow{QS}$ **[1 mark]**
	M is also the midpoint of RS, so the diagonal from P to R and the diagonal from Q to S bisect each other.	• completes proof with appropriate justification **[1 mark]**

Q	Sample response	The response:
17	Given $V = \pi r^2 h$ $V = 4\pi r^2$	• correctly determines the rule for the volume of cylinder in terms of r **[1 mark]**
	Given $\frac{dr}{dt} = -0.5$ m s^{-1}	• correctly expresses the rate of change of the radius as a mathematical expression **[1 mark]**
	$V = 4\pi r^2$ $\frac{dV}{dt} = 8\pi r \frac{dr}{dt}$	• determines a general expression for $\frac{dV}{dt}$ **[1 mark]**
	At $t = 0$, $V = 100\pi$ (given) $100\pi = 4\pi r^2$ $r = 5$ m (as $r > 0$)	• determines initial value of r **[1 mark]**
	At $t = 4$, $r = 5 - 4 \times 0.5$ $= 3$ m	• determines value of r after 4 seconds **[1 mark]**
	When $r = 3$ $\frac{dV}{dt} = 8\pi \times 3 \times (-0.5)$ $= -12\pi$ m^3 s^{-1}	• determines value of $\frac{dV}{dt}$ when $t = 4$ **[1 mark]**
		• shows logical organisation, having fully attempted the question **[1 mark]**

Q	Sample response	The response:
18	**Method 1** Let $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$ be the horizontal and vertical unit vectors and assume downwards as the positive direction. Let t represent the time (s) after projection. $\boldsymbol{a}(t) = 10\hat{\boldsymbol{j}}$ $\boldsymbol{v}(t) = \int \boldsymbol{a}(t)\, dt = 10t\,\hat{\boldsymbol{j}} + c$ Given $\boldsymbol{v}(0) = 30\cos(\theta)\hat{\boldsymbol{i}} + 30\sin(\theta)\hat{\boldsymbol{j}}$ $\boldsymbol{v}(t) = 30\cos(\theta)\hat{\boldsymbol{i}} + \left(30\sin(\theta) + 10t\right)\hat{\boldsymbol{j}}$ $\boldsymbol{r}(t) = \int \boldsymbol{v}(t)\, dt$ $= 30\cos(\theta)t\,\hat{\boldsymbol{i}} + \left(30\sin(\theta)t + 5t^2\right)\hat{\boldsymbol{j}} + c$ Let origin be at the release point: $\boldsymbol{r}(0) = 0\hat{\boldsymbol{i}} - 0\hat{\boldsymbol{j}}$ $\boldsymbol{r}(t) = 30\cos(\theta)t\,\hat{\boldsymbol{i}} + \left(30\sin(\theta)t + 5t^2\right)\hat{\boldsymbol{j}}$	• correctly determines the position vector of the object showing evidence of the use of vector calculus **[1 mark]**
	Consider the object's position as it hits the ground $30\cos(\theta)t = 90$ ... (1) $30\sin(\theta)t + 5t^2 = 90$ ... (2)	• determines two simultaneous equations in terms of θ and t **[1 mark]**

Q	Sample response	The response:
	From (1), $t = \dfrac{3}{\cos(\theta)}$ Substituting into (2) $30\sin(\theta)\left(\dfrac{3}{\cos(\theta)}\right)+5\left(\dfrac{3}{\cos(\theta)}\right)^2 = 90$	• uses simultaneous equations to determine equation in terms of θ **[1 mark]**
	$90\tan(\theta)+45\sec^2(\theta)=90$ $2\tan(\theta)+\left(1+\tan^2(\theta)\right)=2$ $\tan^2(\theta)+2\tan(\theta)-1=0$	• determines equation in terms of tan(θ) **[1 mark]**
	Using the quadratic formula $\tan(\theta)=\dfrac{-2\pm\sqrt{4-4\times1\times-1}}{2}$ $=\dfrac{-2\pm2\sqrt{2}}{2}$ $=-1\pm\sqrt{2}$	• determines both possible values of tan(θ) **[1 mark]**
	As θ is acute $\theta=\tan^{-1}\left(-1+\sqrt{2}\right)$	• determines θ by evaluating the reasonableness of the solution **[1 mark]**

Q	Sample response	The response:
18	**Method 2** Let $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$ be the horizontal and vertical unit vectors and assume upwards as the positive direction. Let t represent the time (s) after projection. $\boldsymbol{a}(t) = -10\hat{\boldsymbol{j}}$ $\boldsymbol{v}(t) = \int \boldsymbol{a}(t)\, dt = -10t\,\hat{\boldsymbol{j}} + c$ Given $\boldsymbol{v}(0) = 30\cos(\theta)\hat{\boldsymbol{i}} - 30\sin(\theta)\hat{\boldsymbol{j}}$ $\boldsymbol{v}(t) = 30\cos(\theta)\hat{\boldsymbol{i}} - \left(30\sin(\theta) + 10t\right)\hat{\boldsymbol{j}}$ $\boldsymbol{r}(t) = \int \boldsymbol{v}(t)\, dt$ $= 30\cos(\theta)t\,\hat{\boldsymbol{i}} - \left(30\sin(\theta)t + 5t^2\right)\hat{\boldsymbol{j}} + c$ Let origin be at the release point: $\boldsymbol{r}(0) = 0\hat{\boldsymbol{i}} + 0\hat{\boldsymbol{j}}$ $\boldsymbol{r} = 30\cos(\theta)t\,\hat{\boldsymbol{i}} - \left(30\sin(\theta)t + 5t^2\right)\hat{\boldsymbol{j}}$	• correctly determines the position vector of the object showing evidence of the use of vector calculus **[1 mark]**
	Consider the object's position as it hits the ground $30\cos(\theta)t = 90$ ... (1) $30\sin(\theta)t + 5t^2 = 90$ ... (2)	• determines two simultaneous equations in terms of θ and t **[1 mark]**

Q	Sample response	The response:
	From (1), $t = \frac{3}{\cos(\theta)}$ Substituting into (2) $30\sin(\theta)\left(\frac{3}{\cos(\theta)}\right) + 5\left(\frac{3}{\cos(\theta)}\right)^2 = 90$	• uses simultaneous equations to determine equation in terms of θ **[1 mark]**
	$\left(\frac{90\sin(\theta)}{\cos(\theta)} + \frac{45}{\cos^2(\theta)}\right) \times \cos^2(\theta) = 90 \times \cos^2(\theta)$ $2\sin(\theta)\cos(\theta) = 2\cos^2(\theta) - 1$ $\sin(2\theta) = \cos(2\theta)$ $\tan(2\theta) = 1$ (where $\cos(2\theta) \neq 0$)	• determines equation in terms of 2θ **[1 mark]**
	$2\theta = \frac{\pi}{4} \pm \pi$	• determines a value for 2θ in terms of π **[1 mark]**
	As θ is acute $\theta = \frac{\pi}{8}$	• determines θ by evaluating the reasonableness of the solution **[1 mark]**

Q	Sample response	The response:
18	**Method 3** Let $\hat{i}$ and $\hat{j}$ be the horizontal and vertical unit vectors and assume downwards as the positive direction. Let t represent the time (s) after projection. $\boldsymbol{a}(t)=10\hat{\boldsymbol{j}}$ $\boldsymbol{v}(t)=\int \boldsymbol{a}(t)\,dt=10t\,\hat{\boldsymbol{j}}+c$ Given $\boldsymbol{v}(0)=30\cos(\theta)\hat{\boldsymbol{i}}+30\sin(\theta)\hat{\boldsymbol{j}}$ $\boldsymbol{v}(t)=30\cos(\theta)\hat{\boldsymbol{i}}+\left(30\sin(\theta)+10t\right)\hat{\boldsymbol{j}}$ $\boldsymbol{r}(t)=\int \boldsymbol{v}(t)\,dt$ $=30\cos(\theta)t\,\hat{\boldsymbol{i}}+\left(30\sin(\theta)t+5t^2\right)\hat{\boldsymbol{j}}+c$ Let origin be at ground level below the release point: $\boldsymbol{r}(0)=0\hat{\boldsymbol{i}}-90\hat{\boldsymbol{j}}$ $\boldsymbol{r}=30\cos(\theta)t\,\hat{\boldsymbol{i}}+\left(-90+30\sin(\theta)t+5t^2\right)\hat{\boldsymbol{j}}$	• correctly determines the position vector of the object showing evidence of the use of vector calculus **[1 mark]**
	Consider the object's position as it hits the ground $30\cos(\theta)t=90 \quad \text{... (1)}$ $-90+30\sin(\theta)t+5t^2=0 \quad \text{... (2)}$	• determines two simultaneous equations in terms of θ and t **[1 mark]**
	From (1), $t=\frac{3}{\cos(\theta)}$ Substituting into (2) $-90+30\sin(\theta)\left(\frac{3}{\cos(\theta)}\right)+5\left(\frac{3}{\cos(\theta)}\right)^2=0$	• uses simultaneous equations to determine equation in terms of θ **[1 mark]**

Q	Sample response	The response:
	$90\tan(\theta)+45\sec^2(\theta)-90=0$ $2\tan(\theta)+\left(1+\tan^2(\theta)\right)-2=0$ $\tan^2(\theta)+2\tan(\theta)-1=0$	• determines equation in terms of tan (θ) **[1 mark]**
	Using the quadratic formula $\tan(\theta)=\frac{-2\pm\sqrt{4-4\times1\times-1}}{2}$ $=\frac{-2\pm2\sqrt{2}}{2}$ $=-1\pm\sqrt{2}$	• determines both possible values of tan (θ) **[1 mark]**
	As θ is acute $\theta=\tan^{-1}\left(-1+\sqrt{2}\right)$	• determines θ by evaluating the reasonableness of the solution **[1 mark]**

Q	Sample response	The response:
19	**Method 1** w is a root of $w^5 = 1$, $w \notin R$	• correctly uses the given information that w is a fifth root of unity **[1 mark]**
	Consider the solutions of $w^5 - 1 = 0$ ... (1) The real solution is $w = 1$. $\therefore w^5 - 1 = (w-1)Q(w)$	• recognises that a factor of $w^5 - 1$ is $w - 1$ **[1 mark]**
	The remaining four non-real solutions meet the condition $Q(w) = 0$ where $(w-1)\left(w^4 + aw^3 + bw^2 + cw + 1\right) = w^5 - 1$ $w : 1 - c = 0 \Rightarrow c = 1$, $w^2 : c - b = 0 \Rightarrow b = 1$, $w^3 : b - a = 0 \Rightarrow a = 1$ $(w-1)\left(w^4 + w^3 + w^2 + w + 1\right) = w^5 - 1$	• determines the quartic polynomial factor **[1 mark]**
	From (1) $(w-1)\left(w^4 + w^3 + w^2 + w + 1\right) = 0$ So the remaining four non-real solutions meet the condition $w^4 + w^3 + w^2 + w + 1 = 0$ ... (2)	• determines a polynomial relationship between the four non-real solutions **[1 mark]**
	Consider the given relationship $w^3(1+w)\left(1+w^3\right)$ $= \left(w^3 + w^4\right)\left(1 + w^3\right)$ $= w^7 + w^6 + w^4 + w^3$ $= w^2 + w^1 + w^4 + w^3$	• recognises the equivalence of w^7 and w^6 with w^2 and w^1 respectively **[1 mark]**
	From (2) $w^4 + w^3 + w^2 + w = -1$	

Q	Sample response	The response:
	$\therefore w^3(1+w)\left(1+w^3\right)=-1$ $\in Z^-$	• shows the required result based on evidence of prior mathematical reasoning **[1 mark]**

Q	Sample response	The response:
19	**Method 2** w is a root of $w^5 = 1$, $w \notin R$ ... (1)	• correctly uses the given information that w is a fifth root of unity **[1 mark]**
	The remaining four non-real solutions meet the condition $Q(w) = 0$ where $Q(w) = \dfrac{w^5 - 1}{w - 1}$	• recognises that a factor of $w^5 - 1$ is $w - 1$ **[1 mark]**
	$$\begin{array}{r} w^4 + w^3 + w^2 + w + 1 \\ w - 1 \overline{)\, w^5 - 1} \\ -\ \underline{w^5 - w^4} \\ w^4 - 1 \\ -\ \underline{w^4 - w^3} \\ w^3 - 1 \\ -\ \underline{w^3 - w^2} \\ w^2 - 1 \\ -\ \underline{w^2 - w} \\ w - 1 \\ -\ \underline{w - 1} \\ 0 \end{array}$$ $(w-1)\left(w^4 + w^3 + w^2 + w + 1\right) = w^5 - 1$	• determines the quartic polynomial factor **[1 mark]**
	From (1) $(w-1)\left(w^4 + w^3 + w^2 + w + 1\right) = 0$ So the remaining four non-real solutions meet the condition $w^4 + w^3 + w^2 + w + 1 = 0$ ... (2)	• determines a polynomial relationship between the four non-real solutions **[1 mark]**

Q	Sample response	The response:
	Considering the given relationship $w^3(1+w)(1+w^3)$ $=(w^3+w^4)(1+w^3)$ $=w^7+w^6+w^4+w^3$ $=w^7+w^6+w^5+w^4+w^3-w^5$	• recognises to include w^5 terms in the expansion of $w^3(1+w)(1+w^3)$ **[1 mark]**
	$=w^3(w^4+w^3+w^2+w+1)-w^5$ $=w^3\times 0-1$ ... from (1) and (2) $=-1$ $\in Z^-$	• shows the required result based on evidence of prior mathematical reasoning **[1 mark]**

Q	Sample response	The response:
19	**Method 3** Considering the given relationship $w^3(1+w)(1+w^3)$ $=(w^3+w^4)(1+w^3)$ $=w^7+w^6+w^4+w^3$ $=w^2+w^1+w^4+w^3$	• correctly recognises the equivalence of w^7 and w^6 with w^2 and w^1 respectively **[1 mark]**
	$=w^1+w^2+w^3+w^4$ This represents the sum of a geometric sequence where	• recognises that the given relationship can be expressed as the sum of a geometric sequence **[1 mark]**
	$n=4$ $t_1=w$ $r=\frac{t_2}{t_1}=w$	• determines appropriate values related to the geometric sequence **[1 mark]**
	$S_n=t_1\frac{(r^n-1)}{r-1}$ $=w\frac{(w^4-1)}{w-1} \quad (w\notin\mathbb{R})$ $\therefore w^3(1+w)(1+w^3)=\frac{w^5-w}{w-1}$	• expresses the given relationship as a fraction with the numerator in expanded form **[1 mark]**

Q	Sample response	The response:
	Given w is a root of $w^5 = 1$, then $w^3(1+w)(1+w^3) = \dfrac{1-w}{w-1}$	• uses the given information that w is a fifth root of unity **[1 mark]**
	$\therefore w^3(1+w)(1+w^3) = -1$ $\in Z^-$	• shows the required result based on evidence of prior mathematical reasoning **[1 mark]**

Q	Sample response	The response:
19	**Method 4** w is a root of $w^5 = 1$, $w \notin R$	• correctly uses the given information that w is a fifth root of unity **[1 mark]**
	Consider the solutions of $w^5 - 1 = 0$. The real solution of $w = 1$ is rejected. The remaining four unique non-real solutions are $w = \text{cis}\left(\pm\frac{2\pi}{5}\right)$ and $\text{cis}\left(\pm\frac{4\pi}{5}\right)$	• determines the four non-real solutions of $w^5 = 1$ **[1 mark]**
	Considering the given relationship $w^3(1+w)(1+w^3) = (w^3+w^4)(1+w^3)$ $= w^7 + w^6 + w^4 + w^3$ $= w^2 + w^1 + w^4 + w^3 \quad \ldots (1)$	• recognises the equivalence of w^7 and w^6 with w^2 and w^1 respectively **[1 mark]**
	Substituting $w = \text{cis}\left(\frac{2\pi}{5}\right)$ into (1) $w^1 + w^2 + w^3 + w^4$ $= \text{cis}\left(\frac{2\pi}{5}\right) + \text{cis}\left(\frac{4\pi}{5}\right) + \text{cis}\left(\frac{6\pi}{5}\right) + \text{cis}\left(\frac{8\pi}{5}\right)$ $= \cos\left(\frac{2\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right) + \cos\left(\frac{2\pi}{5}\right)$ $+ i\sin\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{\pi}{5}\right) - i\sin\left(\frac{\pi}{5}\right) - i\sin\left(\frac{2\pi}{5}\right)$ $= 2\cos\left(\frac{2\pi}{5}\right) - 2\cos\left(\frac{\pi}{5}\right)$	• determines a simplified expression by substituting a non-real solution into the given condition **[1 mark]**

Q	Sample response	The response:
	Let $\theta = \frac{\pi}{10} \Rightarrow 5\theta = \frac{\pi}{2}$ $\sin(2\theta) = \sin\left(\frac{\pi}{2} - 3\theta\right) = \cos(3\theta)$ $\quad 2\sin(\theta)\cos(\theta) = 4\cos^3(\theta) - 3\cos(\theta)$ $\cos(\theta)\left(4\cos^2(\theta) - 2\sin(\theta) - 3\right) = 0$ $4\cos^2(\theta) - 2\sin(\theta) - 3 = 0 \quad \text{... as } \cos(\theta) \neq 0$ $4\sin^2(\theta) + 2\sin(\theta) - 1 = 0$ Using the quadratic formula $\sin(\theta) = \frac{-1 \pm \sqrt{5}}{4}$	
	$\therefore \sin\left(\frac{\pi}{10}\right) = \frac{-1+\sqrt{5}}{4} \quad \text{... as } \theta \text{ is acute}$ Using $\cos(2\theta) = 1 - 2\sin^2(\theta)$ $\cos\left(\frac{\pi}{5}\right) = \frac{1+\sqrt{5}}{4}$ Using $\cos(2\theta) = 2\cos^2(\theta) - 1$ $\cos\left(\frac{2\pi}{5}\right) = \frac{\sqrt{5}-1}{4}$	• determines result for $\cos\left(\frac{\pi}{5}\right)$ and $\cos\left(\frac{2\pi}{5}\right)$ **[1 mark]**
	For any of the four non-real solutions, $w^1 + w^2 + w^3 + w^4 = 2\cos\left(\frac{2\pi}{5}\right) - 2\cos\left(\frac{\pi}{5}\right)$ $w^3(1+w)(1+w^3) = 2\left(\frac{\sqrt{5}-1}{4}\right) - 2\left(\frac{1+\sqrt{5}}{4}\right)$ $= -1$ $\in Z^-$	• shows the required result based on evidence of prior mathematical reasoning **[1 mark]**

Q	Sample response	The response:
19	**Method 5** w is a root of $w^5 = 1$, $w \notin R$	• correctly uses the given information that w is a fifth root of unity **[1 mark]**
	Considering the given relationship $w^3(1+w)(1+w^3)$ $= (w^3 + w^4)(1+w^3)$ $= w^7 + w^6 + w^4 + w^3$	• correctly expands the given relationship **[1 mark]**
	$= w^2 + w^1 + w^4 + w^3$	• recognises the equivalence of w^7 and w^6 with w^2 and w^1 respectively **[1 mark]**
	The sum of the roots of unity is 0. $w^5 + w^4 + w^3 + w^2 + w = 0$	• correctly states the relationship between the roots of unity **[1 mark]**
	Using $w^5 = 1$ $w^4 + w^3 + w^2 + w + 1 = 0$ $w^4 + w^3 + w^2 + w = -1$	• determines a polynomial relationship between the four non-real solutions **[1 mark]**
	Considering the given relationship $w^3(1+w)(1+w^3)$ $= w^4 + w^3 + w^2 + w$ $= -1$ $\in Z^-$	• shows the required result based on evidence of prior mathematical reasoning **[1 mark]**