Specialist Mathematics marking guide and response

External assessment 2024

Paper 1: Technology-free (60 marks)

Paper 2: Technology-active (60 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
Queensland Curriculum & Assessment Authority
For all Queensland schools

Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response:

- demonstrates the qualities of a high-level response
- has been annotated using the marking guide.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Where no response to a question has been made, a mark of ‘N’ will be recorded.

Allow FT mark/s — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide: Paper 1

Multiple choice

Question	Response
1	A
2	B
3	A
4	B
5	D
6	B
7	D
8	A
9	C
10	C

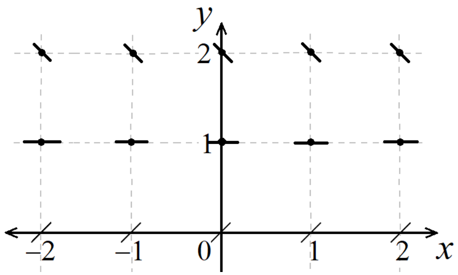
Short response

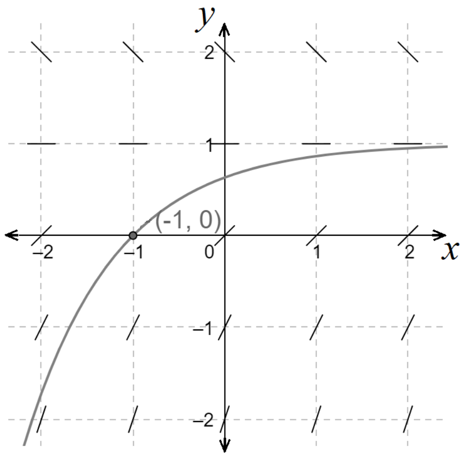
Q	Sample response	The response:
11a)	Given $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + k\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ In parametric form, $x = 2 - k$ $y = 2k$	• correctly express the equation of the line as a pair of parametric equations **[1 mark]**
11b)	$k = 2 - x$ $y = 2(2 - x)$	• expresses equation of line as a Cartesian equation **[1 mark]**
11c)	$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 5\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ 10 \end{pmatrix}$ Required point is $(-3, 10)$.	• determines coordinates of point **[1 mark]**
11d)	At the y-intercept, $x = 0$. $0 = 2 - k$ $k = 2$	• determines value of k **[1 mark]**

Q	Sample response	The response:
12a)	Given A lies on $3x^2+y^2=10$ at $x=\sqrt{2}$ $3\left(\sqrt{2}\right)^2+{y_1}^2=10$ ${y_1}^2=4$ Given A lies in quadrant 4 $y_1=-2$	• correctly substitutes into equation **[1 mark]** • evaluates reasonableness of solution to determine y-coordinate of A **[1 mark]**
12b)	$3x^2+y^2=10$ $6x+2y\dfrac{dy}{dx}=0$ $\dfrac{dy}{dx}=-\dfrac{3x}{y}\ \left(y\neq 0\right)$	• correctly uses implicit differentiation **[1 mark]** • expresses $\dfrac{dy}{dx}$ in terms of x and y **[1 mark]**
12c)	At A, gradient of tangent is $\dfrac{dy}{dx}=\dfrac{3\sqrt{2}}{2}$	• determines gradient of tangent at A **[1 mark]**

Q	Sample response	The response:
12d)	Volume $= \pi\int_{a}^{b}[f(x)]^2\,dx$ $= \pi\int_{0}^{\sqrt{2}}\left(10-3x^2\right)dx$ Volume $= \pi\left(10x-x^3\right)\Big\|_{0}^{\sqrt{2}}$ $= \pi\left(\left(10\sqrt{2}-\left(\sqrt{2}\right)^3\right)-0\right)$ $= \pi\left(10\sqrt{2}-2\sqrt{2}\right)$ $= 8\sqrt{2}\,\pi$ units^3	• correctly determines a definite integral that represents the required volume **[1 mark]** • determines volume in simplest form **[1 mark]**

Q	Sample response	The response:
13	$P(z)$ and $Q(z)$ have the same remainder when $z=i$. $P(i)=a(i)^2-i(i)+1-3i$ $=-a+2-3i$ $Q(i)=(i)^2+3i(i)+2a$ $=-1-3+2a$ $=2a-4$ Using the remainder theorem with the given information, $P(i)=Q(i)$ $-a+2-3i=2a-4$ $3a=6-3i$ $a=2-i$	• correctly uses the remainder theorem **[1 mark]** • determines simplified expression representing $P(i)$ **[1 mark]** • determines simplified expression representing $Q(i)$ **[1 mark]** • determines value of a **[1 mark]**
14a)	$\boldsymbol{r}=\left(2\sec(t)-1\right)\hat{\boldsymbol{i}}+\tan(t)\hat{\boldsymbol{j}}$ $x=2\sec(t)-1$ ... (1) $y=\tan(t)$ ... (2)	• correctly expresses the path of the particle as a pair of parametric equations **[1 mark]**

Q	Sample response	The response:
14b)	From (1), $\sec(t) = \dfrac{x+1}{2}$ ... (3) Using the Pythagorean identity $\tan^2(A)+1=\sec^2(A)$ From (2) and (3), $y^2+1=\left(\dfrac{x+1}{2}\right)^2$ $\dfrac{(x+1)^2}{4}-y^2=1$	• expresses sec(t) as the subject of the formula **[1 mark]** • uses a suitable Pythagorean identity **[1 mark]** • expresses path of the particle in general Cartesian form **[1 mark]**
14c)	Centre is (–1, 0).	• determines the centre of the path of the particle **[1 mark]**
15a)		• correctly sketches the slope as 0 for all five points where $y=1$ **[1 mark]** • correctly sketches the slope as –1 for all five points where $y=2$ **[1 mark]**

Q	Sample response	The response:
15b)		• sketches a solution curve that passes through (–1, 0) and is asymptotic to the line $y=1$ **[1 mark]**
15c)	**Method 1** $\frac{1}{1-y}\frac{dy}{dx}=1$ $\int \frac{1}{1-y}\,dy=\int 1\,dx$ $-\ln\lvert 1-y\rvert = x+c$ At $(-1,0)$, $-\ln(1)=-1+c \Rightarrow c=1$ $\ln(\lvert 1-y\rvert)=-x-1$ $y=1-e^{-x-1}$	• correctly determines a general solution to the equation **[1 mark]** • determines an appropriate constant of integration **[1 mark]** • expresses solution in the form $y=f(x)$ **[1 mark]**

Q	Sample response	The response:
15c)	**Method 2** $\frac{1}{1-y}\frac{dy}{dx}=1$ $\int\frac{1}{1-y}\,dy=\int 1\,dx$ $-\ln\lvert 1-y\rvert = x+c$	• correctly determines a general solution to the equation **[1 mark]**
	$1-y=e^{-x-c} \Rightarrow y=1-Ae^{-x}$ At $(-1, 0)$, $0=1-Ae \Rightarrow A=\frac{1}{e}$	• determines an appropriate constant of integration **[1 mark]**
	$y=1-\frac{1}{e}e^{-x}$	• expresses solution in the form $y=f(x)$ **[1 mark]**

Q	Sample response	The response:
16	**Method 1** Prove $12^n + 2\left(5^{n-1}\right)$ is a multiple of 7, $n \in Z^+$. i.e. $12^n + 2\left(5^{n-1}\right) = 7m,\ m \in Z^+$ Initial statement: Let $n = 1$ $12^1 + 2\left(5^{1-1}\right) = 14$ $= 7 \times 2$ Proposition is true for $n = 1$. Assume proposition is true for $n = k$ $12^k + 2\left(5^{k-1}\right) = 7m$ for some $m \in Z^+ \ldots (1)$ Inductive step: Let $n = k+1$ RTP $12^{k+1} + 2\left(5^k\right)$ is a multiple of 7 $12^{k+1} + 2\left(5^k\right)$	• correctly proves the initial statement **[1 mark]** • correctly formulates an appropriate assumption **[1 mark]** • correctly establishes an appropriate expression representing the LHS of the inductive step **[1 mark]**

Q	Sample response	The response:
	$=12\left(12^k\right)+2\times5\left(5^{k-1}\right)$ $=12\left(12^k\right)+10\left(5^{k-1}\right)$ $=12\left(12^k\right)+24\left(5^{k-1}\right)-14\left(5^{k-1}\right)$ $=12\left(\left(12^k\right)+2\left(5^{k-1}\right)\right)-14\left(5^{k-1}\right)$ $=12(7m)-14\left(5^{k-1}\right)$ ... using (1)	• uses assumption within the inductive step **[1 mark]**
	$=7\left(12m-2\left(5^{k-1}\right)\right)$ $=7p$ where $p\in Z^{+}$	• completes proof by determining an expression with a factor of 7 representing the RHS of the inductive step **[1 mark]**
	The proposition is true for $n=k+1$. By mathematical induction, the proposition is true for $n=1, 2, 3\ ...$	• shows logical organisation, having attempted all steps of the proof, **including** a suitable conclusion **[1 mark]**

Q	Sample response	The response:
16	**Method 2** Prove $12^n + 2\left(5^{n-1}\right)$ is a multiple of 7, $n \in Z^+$ i.e. $12^n + 2\left(5^{n-1}\right) = 7m,\ m \in Z^+$ Initial statement: Let $n = 1$ $12^1 + 2\left(5^{1-1}\right) = 14$ $= 7 \times 2$ Proposition is true for $n = 1$. Assume proposition is true for $n = k$ $12^k + 2\left(5^{k-1}\right) = 7m$ for some $m \in Z^+$ $12^k = 7m - 2\left(5^{k-1}\right)$ for some $m \in Z^+ \ldots (1)$ Inductive step: Let $n = k + 1$ RTP $12^{k+1} + 2\left(5^k\right)$ is a multiple of 7 $12^{k+1} + 2\left(5^k\right)$	 • correctly proves the initial statement **[1 mark]** • correctly formulates an appropriate assumption **[1 mark]** • correctly establishes an appropriate expression representing the LHS of the inductive step **[1 mark]**

Q	Sample response	The response:
	$=12\left(12^{k}\right)+10\left(5^{k-1}\right)$	
	$=12\left(7m-2\left(5^{k-1}\right)\right)+10\left(5^{k-1}\right)$ ... using (1)	• uses assumption within the inductive step **[1 mark]**
	$=84m-24\left(5^{k-1}\right)+10\left(5^{k-1}\right)$	
	$=7\left(12m-2\left(5^{k-1}\right)\right)$	• completes proof by determining an expression with a factor of 7 representing the RHS of the inductive step **[1 mark]**
	$=7p$ where $p \in Z^{+}$	
	The proposition is true for $n=k+1$. By mathematical induction, the proposition is true for $n=1, 2, 3 \ldots$	• shows logical organisation, having attempted all steps of the proof, **including** a suitable conclusion **[1 mark]**

Q	Sample response	The response:
16	**Method 3** Prove $12^n + 2\left(5^{n-1}\right)$ is a multiple of 7, $n \in Z^+$ i.e. $12^n + 2\left(5^{n-1}\right) = 7m,\ m \in Z^+$ Initial statement: Let $n = 1$ $12^1 + 2\left(5^{1-1}\right) = 14$ $= 7 \times 2$ Proposition is true for $n = 1$. Assume proposition is true for $n = k$ $12^k + 2\left(5^{k-1}\right) = 7m$ for some $m \in Z^+$ $2\left(5^{k-1}\right) = 7m - 12^k$ for some $m \in Z^+ \ldots (1)$ Inductive step: Let $n = k + 1$ RTP $12^{k+1} + 2\left(5^k\right)$ is a multiple of 7 $12^{k+1} + 2\left(5^k\right)$	• correctly proves the initial statement **[1 mark]** • correctly formulates an appropriate assumption **[1 mark]** • correctly establishes an appropriate expression representing the LHS of the inductive step **[1 mark]**

Q	Sample response	The response:
	$=12^{k+1}+5\times2\left(5^{k-1}\right)$ $=12^{k+1}+5\left(7m-12^{k}\right)$ ... using (1) $=12^{k+1}+35m-5\times12^{k}$ $=12^{k}(12-5)+35m$ $=7\times12^{k}+35m$ $=7\left(12^{k}+5m\right)$ $=7p$ where $p\in Z^{+}$	• uses assumption within the inductive step **[1 mark]** • completes proof by determining an expression with a factor of 7 representing the RHS of the inductive step **[1 mark]**
	The proposition is true for $n=k+1$. By mathematical induction, the proposition is true for $n=1, 2, 3$ ...	• shows logical organisation, having attempted all steps of the proof, **including** a suitable conclusion **[1 mark]**

Q	Sample response	The response:
17	**Method 1** $a = 2\left(1+v^2\right)$ $\frac{dv}{dt} = 2\left(1+v^2\right)$ $\int \frac{1}{1+v^2}\,dv = \int 2\,dt$ $\tan^{-1}(v) = 2t + c$ At $t=0, v=0: \tan^{-1}(0) = 0 + c$ $\therefore c = 0$ $\therefore \tan^{-1}(v) = 2t$ $v = \tan(2t)$ $x = \int v\,dt$ $= \int \tan(2t)\,dt$ $= -\frac{1}{2}\int \frac{-2\sin(2t)}{\cos(2t)}\,dt$ $= -\frac{1}{2}\ln\left\|\left(\cos(2t)\right)\right\| + c$	• correctly determines a general solution to the differential equation in terms of v and time **[1 mark]** • determines an appropriate constant of integration **[1 mark]** • determines a general solution for displacement in terms of time **[1 mark]**

Q	Sample response	The response:
	Given $x=-\ln\left(\sqrt{2}\right)$ when $t=0$ $-\ln\left(\sqrt{2}\right)=-\frac{1}{2}\ln\left(\cos(0)\right)+c$ $c=-\ln\left(\sqrt{2}\right)$	• determines an appropriate constant of integration **[1 mark]**
	$x=-\frac{1}{2}\ln\left(\cos(2t)\right)-\ln\left(\sqrt{2}\right)$ When $t=\frac{\pi}{6}$ $x=-\frac{1}{2}\ln\left(\cos\left(\frac{\pi}{3}\right)\right)-\ln\left(\sqrt{2}\right)$ $=-\frac{1}{2}\ln\left(\frac{1}{2}\right)-\ln\left(\sqrt{2}\right)$	• determines a result for displacement when $t=\frac{\pi}{6}$ without a trigonometric term **[1 mark]**
	$x=\ln\left(\frac{1}{2}\right)^{-\frac{1}{2}}-\ln\left(\sqrt{2}\right)$ $=\ln\left(\sqrt{2}\right)-\ln\left(\sqrt{2}\right)$ $=0$ So the calculation is reasonable.	• provides an appropriate statement of reasonableness based on mathematical reasoning **[1 mark]**

Q	Sample response	The response:
17	**Method 2** $a = 2\left(1+v^2\right)$ $\frac{dv}{dt} = 2\left(1+v^2\right)$ $\int \frac{1}{1+v^2}\,dv = \int 2\,dt$ $\tan^{-1}(v) = 2t + c$ At $t = 0, v = 0: \tan^{-1}(0) = 0 + c$ $\therefore c = 0$ $\tan^{-1}(v) = 2t$ $v = \tan(2t) \quad \ldots (1)$ $a = 2\left(1+v^2\right)$ $v\frac{dv}{dx} = 2\left(1+v^2\right)$ $\int \frac{v}{1+v^2}\,dv = \int 2\,dx$ $\frac{1}{2}\ln\left(1+v^2\right) = 2x + c$	• correctly determines a general solution to the differential equation in terms of v and time **[1 mark]** • determines an appropriate constant of integration **[1 mark]** • determines a general solution to the differential equation in terms of v and displacement **[1 mark]**

Q	Sample response	The response:
	At $x=-\ln(\sqrt{2}), v=0$ $\frac{1}{2}\ln(1)=-2\ln(\sqrt{2})+c$ $\therefore c=\ln(2)$ $\therefore \frac{1}{2}\ln\left(1+v^2\right)=2x+\ln(2) \quad \dots(2)$	• determines an appropriate constant of integration **[1 mark]**
	When $t=\frac{\pi}{6}$ using (1) $v=\tan\left(2\times\frac{\pi}{6}\right)$ $=\sqrt{3}$	• determines velocity when $t=\frac{\pi}{6}$ **[1 mark]**
	Using (2) $\frac{1}{2}\ln\left(1+v^2\right)=2x+\ln(2)$ $\frac{1}{2}\ln\left(1+\left(\sqrt{3}\right)^2\right)=2x+\ln(2)$ $2x=\frac{1}{2}\ln(4)-\ln(2)$ $x=0$ So the calculation is reasonable.	• provides an appropriate statement of reasonableness based on mathematical reasoning **[1 mark]**

Q	Sample response	The response:
17	**Method 3** $a=2\left(1+v^2\right)$ $\frac{dv}{dt}=2\left(1+v^2\right)$ $\int\frac{1}{1+v^2}\,dv=\int 2\,dt$ $\tan^{-1}(v)=2t+c$ At $t=0, v=0: \tan^{-1}(0)=0+c$ $\therefore c=0$ $\tan^{-1}(v)=2t$ $v=\tan(2t)$ ... (1) $a=2\left(1+v^2\right)$ $v\frac{dv}{dx}=2\left(1+v^2\right)$ $\int\frac{v}{1+v^2}\,dv=\int 2\,dx$ $\frac{1}{2}\ln\left(1+v^2\right)=2x+c$	• correctly determines a general solution to the differential equation in terms of v and time **[1 mark]** • determines an appropriate constant of integration **[1 mark]** • determines a general solution to the differential equation in terms of v and displacement **[1 mark]**

Q	Sample response	The response:
	At $x=-\ln(\sqrt{2}), v=0$ $\frac{1}{2}\ln(1)=-2\ln(\sqrt{2})+c$ $\therefore c=\ln(2)$ $\therefore \frac{1}{2}\ln\left(1+v^2\right)=2x+\ln(2) \quad \ldots(2)$	• determines an appropriate constant of integration **[1 mark]**
	When $t=\frac{\pi}{6}$ (using (1) and (2)) $\frac{1}{2}\ln\left(1+\left(\tan\left(\frac{\pi}{3}\right)\right)^2\right)=2x+\ln(2)$ $\frac{1}{2}\ln\left(1+\left(\sqrt{3}\right)^2\right)=2x+\ln(2)$	• determines a result for the displacement when $t=\frac{\pi}{6}$ without a trigonometric term **[1 mark]**
	$\frac{1}{2}\ln(4)=2x+\ln(2)$ $2x=\ln(2)-\ln(2)$ $x=0$ So the calculation is reasonable.	• provides an appropriate statement of reasonableness based on mathematical reasoning **[1 mark]**

Q	Sample response	The response:
18	**Method 1** Using a property of a pdf $\int_0^1 k\sin^{-1}(x)\ dx = 1 \qquad \ldots(1)$ Consider $\int k\sin^{-1}(x)\ dx$ Using integration by parts $\int u\frac{dv}{dx}\ dx = uv - \int v\frac{du}{dx}\ dx$ Let $u = \sin^{-1}(x) \qquad \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$ Let $\frac{dv}{dx} = k \qquad v = kx$ $\int k\sin^{-1}(x)\ dx = kx\sin^{-1}(x) - \int \frac{kx}{\sqrt{1-x^2}}\ dx$	• correctly uses a suitable pdf property **[1 mark]** • correctly determines both results required to progress integration by parts **[1 mark]** • uses integration by parts **[1 mark]**

Q	Sample response	The response:
	Using substitution for the developed integral $\int \frac{kx}{\sqrt{1-x^2}}\,dx$ Let $u = 1 - x^2$ $\frac{du}{dx} = -2x$ $\int \frac{kx}{\sqrt{1-x^2}}\,dx = -\frac{k}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$ $= -\frac{k}{2}\int \frac{1}{\sqrt{u}}\,du$ $\int \frac{kx}{\sqrt{1-x^2}}\,dx = -\frac{k}{2}\int u^{-\frac{1}{2}}\,du$ $= -ku^{\frac{1}{2}}$ $= -k\sqrt{1-x^2} + c$	• uses a suitable substitution method to progress the developed integral within the use of integration by parts **[1 mark]**
	$\therefore \int k\sin^{-1} x\,dx = kx\sin^{-1}(x) + k\sqrt{1-x^2} + c$	• determines a general result for the required integral **[1 mark]**

Q	Sample response	The response:
	Substituting into (1) $\int_0^1 k\sin^{-1}(x)\,dt\ = 1$ $k\left(x\sin^{-1}(x)+\sqrt{1-x^2}\right)\Big\|_0^1 = 1$ $k\left(\left(\sin^{-1}(1)+\sqrt{1-1}\ \right)-\left(\ 0+\sqrt{1-0}\ \right)\right)=1$ $k\left(\frac{\pi}{2}-1\right)=1$ $k=\frac{2}{\pi-2}$	• determines value of k **[1 mark]**

Q	Sample response	The response:
18	**Method 2** Using a property of a pdf $\int_0^1 k\sin^{-1}(x)\,dx = 1$ $\int_0^1 \sin^{-1}(x)\,dx = \frac{1}{k}$ The area between $y=\sin^{-1}(x)$ and the x-axis for $0 \le x \le 1$ is Area 1: $\int_0^1 \sin^{-1}(x)\,dx = \frac{1}{k}$ Consider the area between $y=\sin^{-1}(x)$ and the y-axis for $0 \le y \le \frac{\pi}{2}$ Area $2 = \int_0^{\frac{\pi}{2}} \sin(y)\,dy$ Area $2 = -\cos(y)\Big\vert_0^{\frac{\pi}{2}}$ $= -\cos\left(\frac{\pi}{2}\right) + \cos(0)$ $= 1$	• correctly uses a suitable pdf property **[1 mark]** • correctly represents the area between $y=\sin^{-1}(x)$ and the x-axis for $0 \le x \le 1$ **[1 mark]** • represents the area between $y=\sin^{-1}(x)$ and the y-axis for $0 \le y \le \frac{\pi}{2}$ **[1 mark]** • determines the area between $y=\sin^{-1}(x)$ and the y-axis for $0 \le y \le \frac{\pi}{2}$ **[1 mark]**

Q	Sample response	The response:
	Area 1 + Area 2 = Area of rectangle $\frac{1}{k}+1=\frac{\pi}{2}\times 1$ $\frac{1}{k}=\frac{\pi-2}{2}$ $k=\frac{2}{\pi-2}$	• determines an equation using the two areas **[1 mark]** • determines value of k **[1 mark]**

Q	Sample response	The response:
19	**Method 1** Given $w = x + i$ Let $w = r\,\text{cis}(\theta)$ $w^7 = r^7\,\text{cis}(7\theta)$ $\arg(w) = \theta$ $= \tan^{-1}\left(\frac{y}{x}\right), x \neq 0$ $= \tan^{-1}\left(\frac{1}{x}\right)$ where $x \in R^+, x \neq 0$ $\arg(w^7) = 7\theta = 7\tan^{-1}\left(\frac{1}{x}\right)$ Given $Re\left(w^7\right) = 0$, then $\arg(w^7) = (2n+1)\frac{\pi}{2}$ where $n \in Z$ $7\tan^{-1}\left(\frac{1}{x}\right) = (2n+1)\frac{\pi}{2}$ $\frac{1}{x} = \tan\left(\frac{(2n+1)\pi}{14}\right)$ $x = \cot\left(\frac{(2n+1)\pi}{14}\right)$	• correctly uses De Moivre's theorem **[1 mark]** • correctly determines an expression representing $\arg(w)$ in terms of x **[1 mark]** • determines a relationship involving $\arg\left(w^7\right)$ using the condition $Re\left(w^7\right) = 0$ **[1 mark]** • determines a general expression representing possible values of x **[1 mark]**

Q	Sample response	The response:
	As $x \in R^{+}$, consider $\frac{(2n+1)\pi}{14}$ for angles that lie in quadrant 1. $n=0: x=\cot\left(\frac{\pi}{14}\right)$ $n=1: x=\cot\left(\frac{3\pi}{14}\right)$ $n=2: x=\cot\left(\frac{5\pi}{14}\right)$	• determines one value of x **[1 mark]** • evaluates the reasonableness of solution by determining the remaining two values of x **[1 mark]**

Q	Sample response	The response:
19	**Method 2** Given $Re\left(w^7\right)=0$, consider solutions of the equation $w^7 = ai$ where $w = x+i$ and $a \in R,\ a \neq 0.$ When $a>0,$ the 7 possible solutions would have arguments that are separated by $\frac{2\pi}{7}$ from the positive imaginary axis. Possible principal arguments for w are $\frac{3\pi}{14},\frac{7\pi}{14},\frac{11\pi}{14},-\frac{\pi}{14},-\frac{5\pi}{14},-\frac{9\pi}{14},-\frac{13\pi}{14}$	• correctly considers $Re\left(w^7\right)=0$ **[1 mark]** • determines distinct arguments of the solution of $w^7 = ai$ where $a>0$ **[1 mark]**
	When $a<0,$ the 7 possible solutions would have arguments that are separated by $\frac{2\pi}{7}$ from the negative imaginary axis. Possible principal arguments for w are $-\frac{3\pi}{14},-\frac{7\pi}{14},-\frac{11\pi}{14},\frac{\pi}{14},\frac{5\pi}{14},\frac{9\pi}{14},\frac{13\pi}{14}$ $\arg(w)=\theta$ $=\tan^{-1}\left(\frac{y}{x}\right),\ x\neq 0$ $=\tan^{-1}\left(\frac{1}{x}\right)$ where $x\in R^+,\ x\neq 0$	• determines distinct arguments of the solution of $w^7 = ai$ where $a<0$ **[1 mark]** • correctly determines an expression representing $\arg(w)$ in terms of x **[1 mark]**

Q	Sample response	The response:
	As $x \in R^+$, the only suitable principal argument for w when $a > 0$ is $\frac{3\pi}{14} = \tan^{-1}\left(\frac{1}{x}\right)$ $\frac{3\pi}{14}.\ \frac{1}{x} = \tan\left(\frac{3\pi}{14}\right)$ $x = \cot\left(\frac{3\pi}{14}\right)$	• determines one possible value of x **[1 mark]**
	As $x \in R^+$, the only suitable principal arguments for w when $a < 0$ are $\frac{\pi}{14}$ and $\frac{5\pi}{14}$. Similarly, $x = \cot\left(\frac{\pi}{14}\right)$ $x = \cot\left(\frac{5\pi}{14}\right)$	• evaluates the reasonableness of solution by determining the remaining two possible values of x **[1 mark]**

Q	Sample response	The response:
19	**Method 3** Given $Re\left(w^7\right)=0$, consider solutions of the equation $z^7=ai$ where $z=r\,\text{cis}(\theta)$ and $a\in R,\ a\neq 0.$ When $a>0,$ the 7 possible solutions would have arguments that are separated by $\frac{2\pi}{7}$ from the positive imaginary axis. So solutions of $z^7=ai$ have the form $z=r\,\text{cis}\left(\frac{\pi}{2}+\frac{2n\pi}{7}\right)$ where $n\in Z.$ When $a<0,$ the 7 possible solutions would have arguments that are separated by $\frac{2\pi}{7}$ from the negative imaginary axis. So solutions of $z^7=ai$ have the form $z=r\,\text{cis}\left(-\frac{\pi}{2}+\frac{2n\pi}{7}\right)$ where $n\in Z.$	• correctly considers $Re\left(w^7\right)=0$ **[1 mark]** • recognises the relationship between the arguments of the solutions of $z^7=ai$ where $a>0$ **[1 mark]** • recognises the relationship between the arguments of the solutions of $z^7=ai$ where $a<0$ **[1 mark]**
	$z=r\cos\left(\pm\frac{\pi}{2}+\frac{2n\pi}{7}\right)+r\sin\left(\pm\frac{\pi}{2}+\frac{2n\pi}{7}\right)i$ Expressing z in the form $w=x+i$ $\therefore w=\dfrac{r\cos\left(\pm\frac{\pi}{2}+\frac{n\pi}{7}\right)}{r\sin\left(\pm\frac{\pi}{2}+\frac{n\pi}{7}\right)}+i$ $x=\cot\left(\pm\frac{\pi}{2}+\frac{n\pi}{7}\right)$	• determines an expression representing possible values of x **[1 mark]**

Q	Sample response	The response:
	As $x \in R^+$, the possible value of x when $a > 0$ is $x = \cot\left(\frac{\pi}{2} - \frac{2\pi}{7}\right)$ $= \cot\left(\frac{3\pi}{14}\right)$	• determines one possible value of x **[1 mark]**
	Similarly, as $x \in R^+$, the possible values of x when $a < 0$ are $x = \cot\left(-\frac{\pi}{2} + \frac{8\pi}{7}\right) = \cot\left(\frac{\pi}{14}\right)$ $x = \cot\left(-\frac{\pi}{2} + \frac{12\pi}{7}\right) = \cot\left(\frac{5\pi}{14}\right)$	• evaluates the reasonableness of solution by determining the remaining two possible values of x **[1 mark]**

Marking guide: Paper 2

Multiple choice

Question	Response
1	A
2	C
3	C
4	D
5	B
6	B
7	B
8	D
9	D
10	C

Short response

Q	Sample response	The response:
11a)	Given $n = 40$, $\overline{x} = 9.31$ and $s = 0.52$ Using GDC $\text{CI}(95\%) = (9.15, 9.47)$ hours	• correctly calculates 95% confidence interval to at least two decimal places **[1 mark]**
11b)	Using GDC $\text{CI}(99\%) = (9.10, 9.52)$ hours	• correctly calculates 99% confidence interval to at least two decimal places **[1 mark]**
11c)	The 95% confidence interval does not include the claimed mean battery life of 9.5 hours, although the 99% CI does. So the comment is not reasonable.	• justifies decision using mathematical reasoning **[1 mark]** • provides appropriate statement of reasonableness **[1 mark]**
12a)	$\begin{bmatrix} 1 & -2 & -2 \\ -3 & -1 & 1 \\ 2 & 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \\ 10 \end{bmatrix}$	• correctly expresses the equations of the form ***AX*** = ***B*** **[1 mark]**
12b)	$X = \boldsymbol{A}^{-1}\boldsymbol{B}$	• correctly expresses ***X*** in terms of ***A*** and ***B*** **[1 mark]**
12c)	Using GDC $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$	• determines solution to equations **[1 mark]**
12d)	Substituting $x = -2, y = 3, z = -1$ into $x - 2y - 2z = -6$ $(-2) - 2(3) - 2(-1) = -6$ So result is verified.	• verifies result by substituting result from 12c) into one of the given linear equations **[1 mark]**

Q	Sample response	The response:
13a)	$z^2 = y^2 + 50^2 \quad \ldots (1)$	• correctly expresses z^2 in terms of y^2 **[1 mark]**
13b)	$\frac{dy}{dt} = 8 \text{ m s}^{-1}$	• correctly states the value of $\frac{dy}{dt}$ **[1 mark]**
13c)	**Method 1** From (1) $z = \left(y^2 + 2500\right)^{\frac{1}{2}} \ (\text{as } z > 0)$ $\frac{dz}{dy} = \frac{1}{2}\left(y^2 + 2500\right)^{\frac{-1}{2}} \times 2y \quad \ldots (2)$ Rate at which z is increasing with respect to time is $\frac{dz}{dt}$ $\frac{dz}{dt} = \frac{dz}{dy} \times \frac{dy}{dt}$ Using (2) When $y = 20$ $\frac{dz}{dt} = \frac{1}{2}\left(20^2 + 2500\right)^{\frac{-1}{2}} \times 2 \times 20 \times 8$ $\approx 2.97 \text{ m s}^{-1}$	• determines a mathematical equation in terms of $\frac{dz}{dy}$ **[1 mark]** • states an equation using related rates **[1 mark]** • determines value of $\frac{dz}{dt}$ when $y = 20$ **[1 mark]**

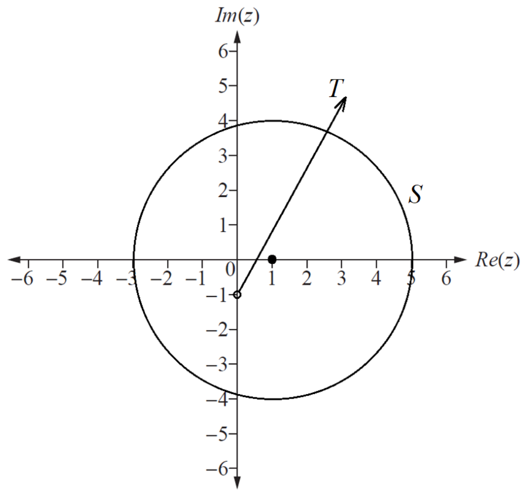
Q	Sample response	The response:
13c)	**Method 2** From (1) When $y=20$ $z^2=20^2+50^2$ $z\approx 53.85\text{ m}\ (\text{as } z>0)$ Rate at which z is increasing with respect to time is $\frac{dz}{dt}$ From (1) $2z\frac{dz}{dt}=2y\frac{dy}{dt}$ When $y=20$ $(53.85)\frac{dz}{dt}\approx(20)(8)$ $\frac{dz}{dt}\approx 2.97\,\text{m}\,\text{s}^{-1}$	• determines value of z when $y=20$ **[1 mark]** • determines the implicit derivative of the result from 13a) **[1 mark]** • determines value of $\frac{dz}{dt}$ when $y=20$ **[1 mark]**

Q	Sample response	The response:
13c)	**Method 3** From 13b) $\frac{dy}{dt} = 8 \Rightarrow y = 8t + c$ At $t = 0, y = 0 \Rightarrow c = 0$ $y = 8t$ When $y = 20, t = 2.5$ s	• determines t when $y = 20$ **[1 mark]**
	From 13b) $z^2 = y^2 + 2500$ $z = \sqrt{y^2 + 2500}$ $= \sqrt{64t^2 + 2500}$	• determines equation representing z in terms of t **[1 mark]**
	Using GDC When $t = 2.5$ $\frac{dz}{dt} \approx 2.97 \text{ m s}^{-1}$	• determines value of $\frac{dz}{dt}$ when $y = 20$ **[1 mark]**

Q	Sample response	The response:
14a)	The distribution of sample means is normally distributed as the population from which a random sample is taken is normally distributed.	• correctly explains the assumption based on the normality of the population distribution **[1 mark]**
14b)	$\mu_{\overline{X}} = 168.6$ cm $\sigma_{\overline{X}} = \frac{12.7}{\sqrt{20}} \approx 2.84$ cm Using GDC $P(\overline{X} > 170) \approx 0.31$	• correctly determines the value for $\sigma_{\overline{X}}$ **[1 mark]** • determines probability **[1 mark]**
14c)	$P\left(\overline{X} \geq 168.6 + h\right) = 0.125$	• correctly determines the probability **[1 mark]**
14d)	Using GDC $168.6 + h \approx 171.867$ $h \approx 3.27$ cm	• determines *h* **[1 mark]**
15a)	**Method 1** If $\boldsymbol{r}_A = (4t-9)\hat{\boldsymbol{i}} - 2(5-t)\hat{\boldsymbol{j}} - 8\hat{\boldsymbol{k}}$ passes through $P(5, -3, -8)$. Consider x_A $4t - 9 = 5$ $t = 3.5$ s Consider y_A $-2(5-t) = -3$ $t = 3.5$ s So particle A passes through point *P*.	• correctly determines time taken for particle A to reach *P* by considering the $\hat{\boldsymbol{i}}$ component of $\mathbf{r}_A$ **[1 mark]** • correctly determines time taken for particle A to reach *P* by considering the $\hat{\boldsymbol{j}}$ component of $\mathbf{r}_A$ **[1 mark]**

Q	Sample response	The response:
15a)	**Method 2** If $\boldsymbol{r}_A = (4t-9)\hat{\boldsymbol{i}} - 2(5-t)\hat{\boldsymbol{j}} - 8\hat{\boldsymbol{k}}$ passes through $P(5,-3,-8)$. Consider x_A $4t-9=5$ $t=3.5$ s $y_A(t=3.5) = -2(5-3.5)$ $= -3$ $\therefore \boldsymbol{r}_A(3.5) = 5\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} - 8\hat{\boldsymbol{k}}$ So particle A passes through point *P*.	• correctly determines time taken for particle *A* to reach *P* by considering a suitable component of $\boldsymbol{r}_A$ **[1 mark]** • correctly uses time taken to show particle *A* passes through *P* **[1 mark]**
15b)	Given $\boldsymbol{r}_B = (t^2+1)\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} + (4-at^2)\hat{\boldsymbol{k}}$ also passes through $P(5,-3,-8)$. Consider x_B $t^2+1=5$ $t^2=4$ $t=2$ s (as $0 \le t \le 10$) Consider z_B $4-a(2)^2=-8$ $4a=12$ $a=3$	• correctly determines time for object *B* to reach *P* **[1 mark]** • determines value of *a* **[1 mark]**

Q	Sample response	The response:
15c)	Displacement vector of particle B relative to particle A is $\boldsymbol{r}_B - \boldsymbol{r}_A$ $= (t^2+1)\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} + (4-3t^2)\hat{\boldsymbol{k}}$ $\quad - \left((4t-9)\hat{\boldsymbol{i}} - 2(5-t)\hat{\boldsymbol{j}} - 8\hat{\boldsymbol{k}}\right)$ $= \left(t^2-4t+10\right)\hat{\boldsymbol{i}} + (7-2t)\hat{\boldsymbol{j}} + (12-3t^2)\hat{\boldsymbol{k}}$ m	• determines the required displacement vector in simplest form **[1 mark]**
15d)	Distance between particles is $\lvert \boldsymbol{r}_B - \boldsymbol{r}_A \rvert$ $= \sqrt{\left(t^2-4t+10\right)^2 + (7-2t)^2 + (12-3t^2)^2}$ m Particles are closest when the distance between the particles is a minimum. Using GDC Shortest distance $= 6.69$ m	• determines an expression that represents the distance between the particles **[1 mark]** • justifies when shortest distance between the particles occurs **[1 mark]** • determines shortest distance between particles **[1 mark]**

Q	Sample response	The response:
16	*S* and *T* can be represented on an Argand diagram as shown. 	• correctly represents subset *S* on the Argand diagram as a circle of radius 4 centred at (1, 0) **[1 mark]** • correctly represents subset *T* on the Argand diagram as a ray starting at (0, –1) and moving through quadrants 4 and 1 **[1 mark]**
	Let $z = x + yi$ represent the points in subsets *S* and *T*. $S : (x-1)^2 + y^2 = 16 \quad \dots(1)$	• determines an equation to represent subset *S* **[1 mark]**
	$T : y = \tan\left(\frac{\pi}{3}\right)x - 1, x > 0 \quad \dots(2)$	• determines an equation to represent subset *T* **[1 mark]**

Q	Sample response	The response:
	Substituting (2) into (1): $(x-1)^2+\left(\sqrt{3}x-1\right)^2=16$ Using GDC $x\approx-1.31$ or $x\approx 2.67$ As the point of intersection of the circle and the ray lies in quadrant 1, the only reasonable solution is $x\approx 2.67$	• determines reasonable solution for *x* **[1 mark]**
	From (2), $y\approx\sqrt{3}\left(2.67\right)-1\approx 3.63$ The complex number where *S* and *T* intersect is $2.67+3.63i$.	• determines where *S* and *T* intersect expressed as a complex number in Cartesian form **[1 mark]**

Q	Sample response	The response:
17	Given $\boldsymbol{r} = r\cos(\omega t)\hat{\boldsymbol{i}} + r\sin(\omega t)\hat{\boldsymbol{j}}$ $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = -r\omega\sin(\omega t)\hat{\boldsymbol{i}} + r\omega\cos(\omega t)\hat{\boldsymbol{j}}$ $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = -r\omega^2\cos(\omega t)\hat{\boldsymbol{i}} - r\omega^2\sin(\omega t)\hat{\boldsymbol{j}}$ $v = \lvert\boldsymbol{v}\rvert$ $= \sqrt{(-r\omega\sin(\omega t))^2 + (r\omega\cos(\omega t))^2}$ $= \sqrt{r^2\omega^2(\sin^2(\omega t) + \cos^2(\omega t))}$ $= r\omega$ $\lvert\boldsymbol{a}\rvert = \sqrt{(-r\omega^2\cos(\omega t))^2 + (-r\omega^2\sin(\omega t))^2}$ $\lvert\boldsymbol{a}\rvert = \sqrt{r^2\omega^4(\cos^2(\omega t) + \sin^2(\omega t))}$ $= r\omega^2 = \frac{r^2\omega^2}{r}$ $= \frac{v^2}{r}$	• correctly determines the velocity vector **[1 mark]** • determines an acceleration vector **[1 mark]** • determines a simplified expression for the modulus of ***v*** **[1 mark]** • determines an expression for the modulus of ***a*** **[1 mark]** • correctly completes the proof based on prior evidence **[1 mark]** • shows logical organisation of a fully attempted proof, communicating key steps **[1 mark]**

Q	Sample response	The response:
18	**Method 1** From Figure 1 Using GDC $\mu - 4 = 0.8416\sigma \quad \ldots(1)$ Let the sample size be n. From Figure 2 Using GDC $6 - \mu = \dfrac{0.8416\sigma}{\sqrt{n}} \quad \ldots(2)$ Given $P\left(4 \le \bar{X} \le 6\right) \approx 0.77$, then $P\left(4 \le \bar{X} \le \mu\right) \approx 0.77 - 0.3 \approx 0.47$ Using GDC $\mu - 4 = \dfrac{1.8808\sigma}{\sqrt{n}} \quad \ldots(3)$ Dividing (2) by (3) $\dfrac{6-\mu}{\mu-4} = \dfrac{0.8416}{1.8808}$ Using GDC $\mu = 5.382$ Using (1): $5.382 - 4 = 0.8416\sigma$ $\sigma = 1.642$ Using GDC $P\left(4 \le X \le 6\right) \approx 0.45$	• correctly determines an equation from Figure 1 in terms of μ and σ **[1 mark]** • correctly determines an equation from Figure 2 in terms of μ, σ and n **[1 mark]** • correctly determines an equation using $P\left(4 \le \bar{X} \le 6\right) \approx 0.77$ in terms of μ, σ and n **[1 mark]** • determines μ **[1 mark]** • determines σ **[1 mark]** • determines required probability **[1 mark]**

Q	Sample response	The response:
18	**Method 2** From Figure 1 Using GDC $\mu - 4 = 0.8416\sigma$ ... (1)	• correctly determines an equation from Figure 1 in terms of μ and σ **[1 mark]**
	Let the sample size be n. From Figure 2 Using GDC $6 - \mu = \dfrac{0.8416\sigma}{\sqrt{n}}$ ... (2)	• correctly determines an equation from Figure 2 in terms of μ, σ and n **[1 mark]**
	Given $P\left(4 \le \overline{X} \le 6\right) \approx 0.77$, then $P\left(4 \le \overline{X} \le \mu\right) \approx 0.77 - 0.3 \approx 0.47$ Using GDC $\mu - 4 = \dfrac{1.8808\sigma}{\sqrt{n}}$ ... (3)	• correctly determines an equation using $P\left(4 \le \overline{X} \le 6\right) \approx 0.77$ in terms of μ, σ and n **[1 mark]**
	Equating (1) and (3) $0.8416\sigma = \dfrac{1.8808\sigma}{\sqrt{n}}$ Using GDC $n = 5$	• determines n **[1 mark]**
	Using 2: $6 - \mu = \dfrac{0.8416\sigma}{\sqrt{5}} \Rightarrow 6 - \mu = 0.376\sigma$ Using GDC $P(\mu \le X \le 6) \approx 0.147$	• determines $P(\mu \le X \le 6)$ **[1 mark]**
	$P(4 \le X \le 6) \approx 0.3 + 0.147 \approx 0.45$	• determines required probability **[1 mark]**

Q	Sample response	The response:
18	**Method 3** From Figure 1 Using GDC $\mu - 4 = 0.8416\sigma$ ... (1) Let the sample size be n. From Figure 2 Using GDC $6 - \mu = \dfrac{0.8416\sigma}{\sqrt{n}}$ ... (2) Given $P\left(4 \le \overline{X} \le 6\right) \approx 0.77$, then $P\left(4 \le \overline{X} \le \mu\right) \approx 0.77 - 0.3 \approx 0.47$ Using GDC $\mu - 4 = \dfrac{1.8808\sigma}{\sqrt{n}}$ ... (3) Equating (1) and (3) $0.8416\sigma = \dfrac{1.8808\sigma}{\sqrt{n}}$ Using GDC $n = 5$	• correctly determines an equation from Figure 1 in terms of μ and σ **[1 mark]** • correctly determines an equation from Figure 2 in terms of μ, σ and n **[1 mark]** • correctly determines an equation using $P\left(4 \le \overline{X} \le 6\right) \approx 0.77$ in terms of μ, σ and n **[1 mark]** • determines n **[1 mark]**

Q	Sample response	The response:
	Using 1: $\dfrac{4-\mu}{\sigma}=-0.8416$ Using 2: $\dfrac{6-\mu}{\sigma}=0.3766$ $P(4 \le X \le 6)=P(-0.8416 \le z \le 0.3766)$	• expresses required probability in terms of z-scores **[1 mark]**
	Using GDC $P(4 \le X \le 6) \approx 0.45$	• determines required probability **[1 mark]**

Q	Sample response	The response:
19	Let the birth rate for Stage 1 females be x and the survival rates for Stage 1 females to Stage 2 females be y. Let $\mathbf{L}$ be the Leslie matrix for this species. $\mathbf{L} = \begin{bmatrix} x & 2x \\ y & 0 \end{bmatrix}$	• correctly determines an appropriate Leslie matrix **[1 mark]**
	Let $\mathbf{P}_n$ represent the population of the species after n weeks. $\mathbf{P}_0 = \begin{bmatrix} 48 \\ 32 \end{bmatrix}$ $\mathbf{P}_2 = \mathbf{L}^2\mathbf{P}_0$ $\begin{bmatrix} 25 \\ 21 \end{bmatrix} = \begin{bmatrix} x & 2x \\ y & 0 \end{bmatrix}^2 \begin{bmatrix} 48 \\ 32 \end{bmatrix}$ $= \begin{bmatrix} x^2+2xy & 2x^2 \\ xy & 2xy \end{bmatrix} \begin{bmatrix} 48 \\ 32 \end{bmatrix}$ $= \begin{bmatrix} 48x^2+96xy+64x^2 \\ 48xy+64xy \end{bmatrix}$	• determines a matrix equation linking the initial population with the population after two weeks **[1 mark]**
	Equating parts $112x^2+96xy=25 \quad ...(1)$ $112xy=21 \quad ...(2)$	• determines two simultaneous equations in terms of the relevant birth and survival rates **[1 mark]**

Q	Sample response	The response:
	From (2) $xy = \dfrac{21}{112} \quad \ldots (3)$ Substituting into (1) $112x^2 + 96\left(\dfrac{21}{112}\right) = 25$ $x = 0.25$ (reject $-$ive solution as $x > 0$) Using 3, $y = 0.75$ $\mathbf{P}_4 = \mathbf{L}^4\mathbf{P}_0$ $= \begin{bmatrix} 0.25 & 0.5 \\ 0.75 & 0 \end{bmatrix}^4 \begin{bmatrix} 48 \\ 32 \end{bmatrix}$ $= \begin{bmatrix} 13.6 \\ 12.6 \end{bmatrix}$ Approximate total number of females at the conclusion of the experiment is 26.	• determines appropriate values of *x* and *y* **[1 mark]** • determines the total number of females at the conclusion of the experiment **[1 mark]** • shows logical organisation of a fully attempted solution, communicating key steps **[1 mark]**