# Specialist Mathematics marking guide and response

External assessment 2023

## Paper 2: Technology-active (60 marks)

### Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

# Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response:

- demonstrates the qualities of a high-level response
- has been annotated using the marking guide.

# Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Where no response to a question has been made, a mark of ‘N’ will be recorded.

*Allow FT mark/s* — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

*This mark may be implied by subsequent working* — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

# Marking guide

## Multiple choice

| Question | Response |
|---|---|
| 1 | D |
| 2 | B |
| 3 | C |
| 4 | D |
| 5 | A |
| 6 | D |
| 7 | A |
| 8 | A |
| 9 | B |
| 10 | C |

## Short response

| Q | Sample response | The response: |
|---|---|---|
| 11a) | $(7,\ 4.9)$ | • correctly determines point *A* **[1 mark]** |
| 11b) | Using GDC<br>Area = $6.14\ \text{units}^2$ | • calculates area **[1 mark]** |
| 11c) | $\left\lvert \pi\int_a^b\left(\left[f(x)\right]^2-\left[g(x)\right]^2\right)dx \right\rvert$<br>$=\pi\int_0^7\left(\left[0.1x^2\right]^2-\left[\left(-1+\sec\left(\frac{x}{5}\right)\right)\right]^2\right)dx$ | • determines a definite integral representing a value related to the volume **[1 mark]** |
| | Using GDC<br>Volume $=69.76\,\text{units}^3$ | • calculates volume **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 12a) | Using binomial theorem:<br>$z^3 = (-3+2i)^3$<br>$= (-3)^3 + 3(-3)^2(2i) + 3(-3)(2i)^2 + (2i)^3$<br>$= -27 + 54i + 36 - 8i$<br>$= 9 + 46i$ | • correctly uses the binomial theorem **[1 mark]**<br>• expresses $z^3$ in the form of $a + bi$ **[1 mark]** |
| 12b) | $z = 3.61\,\text{cis}(2.55)$ | • correctly converts $z$ into $r\,\text{cis}(\theta)$ form where $-\pi < \theta \le \pi$ **[1 mark]** |
| 12c) | Using De Moivre's theorem<br>$z^3 = (3.61\,\text{cis}(2.55))^3$<br>$= 3.61^3\,\text{cis}(3 \times 2.55)$<br>$= 46.87\,\text{cis}(7.66)$<br>$= 46.87\,\text{cis}(1.38)$ | • uses De Moivre's theorem to calculate $z^3$ **[1 mark]**<br>• determines $z^3$ in the form of $r\,\text{cis}(\theta)$ where $-\pi < \theta \le \pi$ **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 12d) | $46.87\operatorname{cis}(1.38) = 46.87\left(\cos(1.38) + i\sin(1.38)\right)$<br>$= 8.89 + 46.02i$<br>$\approx 9 + 46i$ | • shows mathematical reasoning to convert from $r\operatorname{cis}(\theta)$ form to $a+bi$ form **[1 mark]** |
| | The two methods produce approximately the same $z^3$ value. The small variation is a result of rounding used in earlier calculations. | • states a decision regarding the reasonableness **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 13a) | Using GDC<br>$\bar{x} = 9.23$ minutes | • correctly determines the mean of the data **[1 mark]** |
| 13b) | Using result fom 13a) and given $s = 2.384$ and $n = 12$.<br>Using GDC<br>$\text{CI}(95\%) = (7.88, 10.58)$ minutes | • calculates 95% confidence interval **[1 mark]** |
| | The company's claim is not reasonable. | • states an appropriate evaluation of the reasonableness of the claim **[1 mark]** |
| | This decision is justified as the claimed population mean wait time lies outside of the confidence interval. | • uses mathematical reasoning to justify the decision **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 14 | Use Simpson's rule to estimate the cross-sectional area of the river.<br><br>Using the given data: $w = 2$.<br><br>$\text{Area} \approx \frac{w}{3}\left(y_0 + 4\left(y_1 + y_3 + ...\right) + 2\left(y_2 + y_4 + ...\right) + y_n\right)$<br>$\approx \frac{2}{3}(0.52 + 4(2.15 + 4.27 + 1.28) + ...$<br>$2(3.70 + 3.32) + 0.59)$<br><br>$\text{Area} \approx 30.63\,\text{m}^2$<br><br>The estimate of the area is less than half of the value obtained using Simpson's rule, so it is not reasonable. | • correctly identifies the interval width **[1 mark]**<br><br>• justifies the area calculation by substituting the depth values of the data into Simpson's rule **[1 mark]**<br><br>• calculates area **[1 mark]**<br><br>• states and justifies a decision regarding the reasonableness of the estimation using mathematical reasoning **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 15a) | Given $\mu_{\overline{X}} = 25.2$<br>$\sigma_{\overline{X}_1} = \frac{\sigma}{\sqrt{n}} = \frac{4.7}{\sqrt{120}}$<br>$= 0.429$ minutes<br><br>Using GDC<br>$P(\overline{X}_1 \le 25) = 0.32$ | • correctly calculates $\sigma_{\overline{X}}$ for the first sample **[1 mark]**<br><br>• calculates required probability **[1 mark]** |
| 15b) | $P(\overline{X}_1 > k) = 0.9$<br>Using GDC<br>$k = 24.65$ minutes | • calculates $k$ **[1 mark]** |
| 15c) | $P(z \le z_1) \approx 0.4 \Rightarrow z_1 = -0.253$<br>$z = \frac{\overline{X}_2 - \mu}{\frac{\sigma}{\sqrt{n}}}$<br>$-0.253 = \frac{25 - 25.2}{\frac{4.7}{\sqrt{n}}}$<br>Using GDC<br>$n \approx 35.3$<br><br>The sample size is 35. | • correctly calculates the $z$-value based on given probability **[1 mark]**<br><br>• determines an equation in terms of the sample size $(n)$ **[1 mark]**<br><br>• determines an approximate value of $n$ **[1 mark]**<br><br>• evaluates the reasonableness of the solution by rounding $n$ to an integer value **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 16 | Given $xy^2 - y + \cos^{-1}(2x) = 1$<br>Determining $y$-coordinate of $A$<br>$0 - y + \cos^{-1}(0) = 1 \Rightarrow y = 0.57$ | • correctly determines $y$-intercept **[1 mark]** |
| | $\frac{d}{dx}\cos^{-1}(2x) = \frac{-1}{\sqrt{0.25 - x^2}}$ | • correctly determines $\frac{d}{dx}\cos^{-1}(2x)$ **[1 mark]** |
| | $\frac{d}{dx}(xy^2) = y^2 + 2xy\frac{dy}{dx}$ | • correctly determines $\frac{d}{dx}xy^2$ **[1 mark]** |
| | Determining $\frac{dy}{dx}$<br>$\frac{d}{dx}\left(xy^2 - y + \cos^{-1}(2x)\right) = \frac{d}{dx}(1)$<br>$y^2 + 2xy\frac{dy}{dx} - \frac{dy}{dx} + \frac{-1}{\sqrt{0.25 - x^2}} = 0$ | |
| | $\frac{dy}{dx}(2xy - 1) = \frac{1}{\sqrt{0.25 - x^2}} - y^2$<br>$\frac{dy}{dx} = \frac{\frac{1}{\sqrt{0.25 - x^2}} - y^2}{2xy - 1}$ | • determines an expression for $\frac{dy}{dx}$ using a common factor **[1 mark]** |
| | Determining $\frac{dy}{dx}$ at $A$.<br>$\frac{dy}{dx} = -\left(\frac{1}{\sqrt{0.25}} - (0.571)^2\right) = -1.67$ | • determines a value for $\frac{dy}{dx}$ at $A$ **[1 mark]** |
| | Determining equation of tangent at $A$<br>$y = mx + c$<br>$y = -1.67x + 0.57$ | • determines equation of the tangent at $A$ **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 17 | **Method 1**<br>Let $\hat{i}$ and $\hat{j}$ be the horizontal and vertical unit vectors respectively. Let $t$ represent the time in seconds after the projection of the object.<br>$\boldsymbol{a}(t) = -9.8\hat{\boldsymbol{j}}$<br>$\boldsymbol{v}(t) = \int \boldsymbol{a}(t)\,dt = -9.8t\,\hat{\boldsymbol{j}} + c$<br>Given $\boldsymbol{v}(0) = 15\cos\left(54^\circ\right)\hat{\boldsymbol{i}} + 15\sin\left(54^\circ\right)\hat{\boldsymbol{j}}$<br>$\boldsymbol{v}(t) = 15\cos\left(54^\circ\right)\hat{\boldsymbol{i}} + \left(15\sin\left(54^\circ\right) - 9.8t\right)\hat{\boldsymbol{j}}$ | • correctly determines the velocity function of the object using vector calculus **[1 mark]** |
| | $\boldsymbol{r}(t) = \int \boldsymbol{v}(t)\,dt$<br>$= 15\cos\left(54^\circ\right)t\,\hat{\boldsymbol{i}} + \left(15\sin\left(54^\circ\right)t - 4.9t^2\right)\hat{\boldsymbol{j}} + c$<br>Let origin be at the release point: $\boldsymbol{r}(0) = 0\hat{\boldsymbol{i}} + 0\hat{\boldsymbol{j}}$<br>$\boldsymbol{r}(t) = 15\cos\left(54^\circ\right)t\,\hat{\boldsymbol{i}} + \left(15\sin\left(54^\circ\right)t - 4.9t^2\right)\hat{\boldsymbol{j}}$ | • determines displacement function of the object **[1 mark]** |
| | When $r_x = 20 \Rightarrow 15\cos\left(54^\circ\right)t = 20$<br>Time object just passes drone: $t = 2.27\text{ s}$ | • determines time when the object just passes drone **[1 mark]** |
| | Finding maximum value of $r_y : 15\sin\left(54^\circ\right)t - 4.9t^2$<br>Using GDC<br>Time object reaches maximum height: $t = 1.24\text{ s}$ | • determines time when the object reaches maximum height **[1 mark]** |
| | While the estimation of the time taken for the object to reach the drone is reasonable, the comment regarding the direction of the object as it passed the drone is not reasonable as it would have been moving in a downward direction at that time. | • uses mathematical justification to evaluate the reasonableness of both comments based on prior mathematical reasoning **[1 mark]**<br>• shows logical organisation, communicating key steps **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 17 | **Method 2**<br>Let $\hat{i}$ and $\hat{j}$ be the horizontal and vertical unit vectors respectively. Let $t$ represent the time in seconds after the projection of the object.<br>$\boldsymbol{a}(t) = -9.8\hat{\boldsymbol{j}}$<br>$\boldsymbol{v}(t) = \int \boldsymbol{a}(t)\,dt$<br>$= -9.8t\,\hat{\boldsymbol{j}} + c$<br>Given $\boldsymbol{v}(0) = 15\cos\left(54^\circ\right)\hat{\boldsymbol{i}} + 15\sin\left(54^\circ\right)\hat{\boldsymbol{j}}$ | |
| | $\boldsymbol{v}(t) = 15\cos\left(54^\circ\right)\hat{\boldsymbol{i}} + \left(15\sin\left(54^\circ\right) - 9.8t\right)\hat{\boldsymbol{j}}$ | • correctly determines the velocity function of the object using vector calculus **[1 mark]** |
| | Considering horizontal component of velocity:<br>$v_x = 15\cos\left(54^\circ\right) = 8.82$<br>At $t = 2$, $x = vt = 2 \times 8.82 = 17.64$ m | • determines horizontal distance travelled after 2 seconds **[1 mark]** |
| | At $t = 2.5$, $x = 2.5 \times 8.82 = 22.05$ m | • determines horizontal distance travelled after 2.5 seconds **[1 mark]** |
| | Considering vertical component of velocity:<br>At $t = 2, v_y = 12.14 - 9.8 \times 2 = -7.46$ $\text{ms}^{-1}$ | • determines vertical velocity after 2 seconds **[1 mark]** |
| | While the estimation of the time taken for the object to reach the drone is reasonable, the comment regarding the direction of the object as it passed the drone is not reasonable as it would have been moving in a downward direction at that time. | • uses mathematical justification to evaluate the reasonableness of both comments based on prior mathematical reasoning **[1 mark]**<br>• shows logical organisation, communicating key steps **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 17 | **Method 3**<br>Let $\hat{i}$ and $\hat{j}$ be the horizontal and vertical unit vectors respectively. Let $t$ represent the time in seconds after the projection of the object.<br>$\boldsymbol{a}(t) = -9.8\hat{\boldsymbol{j}}$<br>$\boldsymbol{v}(t) = \int \boldsymbol{a}(t)\ dt = -9.8t\,\hat{\boldsymbol{j}} + c$<br>Given $\boldsymbol{v}(0) = 15\cos\left(54^\circ\right)\hat{\boldsymbol{i}} + 15\sin\left(54^\circ\right)\hat{\boldsymbol{j}}$ | |
| | $\boldsymbol{v}(t) = 15\cos\left(54^\circ\right)\hat{\boldsymbol{i}} + \left(15\sin\left(54^\circ\right) - 9.8t\right)\hat{\boldsymbol{j}}$ | • correctly determines the velocity function of the object using vector calculus **[1 mark]** |
| | $\boldsymbol{r}(t) = \int \boldsymbol{v}(t)\ dt$<br>$= 15\cos\left(54^\circ\right)t\,\hat{\boldsymbol{i}} + \left(15\sin\left(54^\circ\right)t - 4.9t^2\right)\hat{\boldsymbol{j}} + c$<br>Let origin be at the release point: $\boldsymbol{r}(0) = 0\hat{\boldsymbol{i}} + 0\hat{\boldsymbol{j}}$ | |
| | $\boldsymbol{r}(t) = 15\cos\left(54^\circ\right)t\,\hat{\boldsymbol{i}} + \left(15\sin\left(54^\circ\right)t - 4.9t^2\right)\hat{\boldsymbol{j}}$ | • determines displacement function of the object **[1 mark]** |
| | At $t = 2$, $x = 17.64$ m and $y = 4.67$ m | • determines horizontal and vertical distances travelled after 2 seconds **[1 mark]** |
| | At $t = 2.5$, $x = 22.05$ m and $y = -0.29$ m | • determines horizontal and vertical distances travelled after 2.5 seconds **[1 mark]** |
| | While the estimation of the time taken for the object to reach the drone is reasonable, the comment regarding the direction of the object as it passed the drone is not reasonable as it would have been moving in a downward direction at that time. | • uses mathematical justification to evaluate the reasonableness of both comments based on prior mathematical reasoning **[1 mark]**<br>• shows logical organisation, communicating key steps **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 18 | $(z+1)(z^{14}-z^{13}+z^{12}-z^{11}+\ldots+z^4-z^3+z^2-z)=1-z$<br>$z^{15}+z^{14}-z^{14}+z^{13}-z^{13}+\ldots$<br>$+z^4-z^4-z^3+z^3+z^2-z^2-z=1-z$<br>$z^{15}=1$<br><br>The solutions are $z=\text{cis}\left(\frac{2n\pi}{15}\right)$ where $n \in Z$ and $0<arg(z)<\pi$.<br><br>Solution with the maximum possible real part has its argument closest to 0.<br>$w_1=\text{cis}\left(\frac{2\pi}{15}\right)$<br>Solution with the maximum possible imaginary part has its argument closest to $\frac{\pi}{2}$.<br>$w_2=\text{cis}\left(\frac{8\pi}{15}\right)$<br><br>$\frac{w_1{}^4}{w_2}=\frac{\text{cis}\left(4\times\frac{2\pi}{15}\right)}{\text{cis}\left(\frac{8\pi}{15}\right)}=1\in Z$ | • correctly simplifies the original equation **[1 mark]**<br><br>• describes the location of the solutions **[1 mark]**<br><br>• determines $w_1$ **[1 mark]**<br><br>• determines $w_2$ **[1 mark]**<br><br>• shows that $\frac{w_1{}^4}{w_2}$ is an integer **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| 19 | **Method 1**<br>**Situation 1**<br>$z$-score for $95\%\text{CI} = 1.96$<br>Teacher's class:<br>$\bar{x}_1 = \frac{166.9 + 163.7}{2} = 165.3$<br>Other class:<br>$\bar{x}_2 = \frac{172.4 + 167.8}{2} = 170.1$<br>Teacher's class:<br>Let $s_1$ be the sample standard deviation for Class 1 of sample size $n$.<br>$165.3 + \frac{1.96 s_1}{\sqrt{n}} = 166.9$<br>$\frac{s_1}{\sqrt{n}} = 0.816$<br>Other class:<br>$\bar{x}_2 = \frac{172.4 + 167.8}{2} = 170.1$<br>Let $s_2$ be the sample standard deviation for Class 2 of sample size $n$.<br>$170.1 + \frac{1.96 s_2}{\sqrt{n}} = 172.4$<br>$\frac{s_2}{\sqrt{n}} = 1.173$ | • correctly determines the $z$-score associated with a $95\%$ CI **[1 mark]**<br>• correctly determines the sample means for both classes **[1 mark]**<br>• determines a relationship between the sample standard deviation and the sample size for the teacher's class **[1 mark]**<br>• determines a relationship between the sample standard deviation and the sample size for the other class **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| | **Situation 2**<br>Let the $z$-score for the CI using a confidence level of $x\%$ be $z_x$.<br>New upper limit of CI for teacher's class equals new lower limit of CI for other class.<br>$\overline{x}_1 + \frac{z_x s_1}{\sqrt{n}} = \overline{x}_2 - \frac{z_x s_2}{\sqrt{n}}$<br>$z_x\left(\frac{s_1}{\sqrt{n}} + \frac{s_2}{\sqrt{n}}\right) = \overline{x}_2 - \overline{x}_1$ | • determines an equation in terms of the $z$-score associated with the new CIs using the data from the two classes **[1 mark]** |
| | Using earlier results<br>$z_x(0.816 + 1.173) = 170.1 - 165.3$<br>$z_x = 2.413$ | • determines the $z$-score associated with the new CI calculations **[1 mark]** |
| | Using GDC<br>$x = 98.4$ | • determines the confidence level for the new CI calculations, rounded to one decimal place **[1 mark]** |

| Q | Sample response | The response: |
| --- | --- | --- |
| 19 | **Method 2**<br><br>**Situation 1**<br><br>$z$-score for $95\%\ \text{CI} = 1.96$<br><br>Teacher's class:<br><br>$\bar{x}_1 = \frac{166.9 + 163.7}{2} = 165.3$<br><br>Other class:<br><br>$\bar{x}_2 = \frac{172.4 + 167.8}{2} = 170.1$<br><br>Teacher's class:<br><br>Let $s_1$ be the sample standard deviation for Class 1 of sample size $n$.<br><br>$165.3 + \frac{1.96 s_1}{\sqrt{n}} = 166.9$<br><br>$\frac{s_1}{\sqrt{n}} = 0.816$<br><br>Other class:<br><br>$\bar{x}_2 = \frac{172.4 + 167.8}{2} = 170.1$<br><br>Let $s_2$ be the sample standard deviation for Class 2 of sample size $n$.<br><br>$170.1 + \frac{1.96 s_2}{\sqrt{n}} = 172.4$<br><br>$\frac{s_2}{\sqrt{n}} = 1.173$ | • correctly determines the $z$-score associated with a $95\%$ CI **[1 mark]**<br><br>• correctly determines the sample means for each class **[1 mark]**<br><br>• determines a relationship between the sample standard deviation and the sample size for the teacher's class **[1 mark]**<br><br>• determines a relationship between the sample standard deviation and the sample size for the other class **[1 mark]** |

| Q | Sample response | The response: |
|---|---|---|
| | **Situation 2**<br>Let the $z$-score for the CI using a confidence level of $x\%$ be $z_x$.<br>New upper limit of CI for teacher's class is<br>$\overline{x}_1 + \frac{z_x s_1}{\sqrt{n}} = 165.3 + 0.816 z_x \quad \ldots (1)$<br>New lower limit of CI for other class is<br>$\overline{x}_2 - \frac{z_x s_2}{\sqrt{n}} = 170.1 - 1.173 z_x \quad \ldots (2)$ | • determines two expressions in terms of the $z$-score associated with the new CIs using the data from the two classes **[1 mark]** |
| | Equating CI results and solving:<br>$165.3 + 0.816 z_x = 170.1 - 1.173 z_x$<br>$z_x (1.173 + 0.816) = 170.1 - 165.3$<br>$z_x = 2.413$ | • determines the $z$-score associated with the new CI calculations **[1 mark]** |
| | Using GDC<br>$x = 98.4$ | • determines the confidence level for the new CI calculations, rounded to one decimal place **[1 mark]** |