Specialist Mathematics marking guide and response

External assessment 2023

Paper 1: Technology-free (60 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
Queensland Curriculum & Assessment Authority

Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response:

- demonstrates the qualities of a high-level response
- has been annotated using the marking guide.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Where no response to a question has been made, a mark of ‘N’ will be recorded.

Allow FT mark/s — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Multiple choice

Question	Response
1	C
2	C
3	A
4	D
5	A
6	B
7	D
8	A
9	D
10	B

Short response

Q	Sample response	The response:
11a)	$\int_{0}^{1} \frac{1}{1+x^2}\,dx$ $= \tan^{-1}(x)\Big\|_0^1$ $= \tan^{-1}(1) - \tan^{-1}(0)$ $= \frac{\pi}{4}$	• correctly uses the required integration rule **[1 mark]** • calculates value of the definite integral **[1 mark]**

Q	Sample response	The response:
11b)	$\int_0^{\frac{\pi}{4}} 2\sin^2(x)\,dx$	
	$=\int_0^{\frac{\pi}{4}} 2\times\frac{1}{2}(1-\cos(2x))\,dx$	• correctly uses the required trigonometric substitution **[1 mark]**
	$=\int_0^{\frac{\pi}{4}} 1-\cos(2x)\,dx$	
	$=x-\frac{\sin(2x)}{2}\Big\|_0^{\frac{\pi}{4}}$	• uses suitable integration rules **[1 mark]**
	$=\left(\frac{\pi}{4}-\frac{\sin\left(\frac{\pi}{2}\right)}{2}\right)-\left(0-\frac{\sin(0)}{2}\right)$	
	$=\frac{\pi}{4}-\frac{1}{2}$	• calculates value of the definite integral **[1 mark]**

Q	Sample response	The response:
12	$XA - XC = B$ $X(A - C) = B$	• correctly recognises the need to use X as a common factor **[1 mark]**
	$X = B(A - C)^{-1}$	• expresses X as the subject of the equation **[1 mark]**
	$X = \begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix} \left[\begin{pmatrix} 1 & -2 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} -1 & -1 \\ 0 & 3 \end{pmatrix} \right]^{-1}$ $= \begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix}^{-1}$	• represents X in terms of two matrices **[1 mark]**
	$= \begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix} \frac{1}{-1} \begin{pmatrix} -1 & 1 \\ -1 & 2 \end{pmatrix}$	• calculates the inverse of an appropriate matrix **[1 mark]**
	$= \begin{pmatrix} 2 & -4 \\ 4 & -7 \end{pmatrix}$	• determines X **[1 mark]**

Q	Sample response	The response:
13	**Method 1** Prove $\frac{\lvert z \rvert}{z\bar{z}} = \lvert z^{-1} \rvert$ Let $z = a + bi$, where $a, b \in R$ $\text{LHS} = \frac{\lvert z \rvert}{z\bar{z}}$ $= \frac{\lvert a+bi \rvert}{(a+bi)(a-bi)}$ $= \frac{\sqrt{a^2+b^2}}{a^2+b^2}$ $= \frac{1}{\sqrt{a^2+b^2}}$ $= \frac{1}{\lvert a+bi \rvert}$ $= \left\lvert \frac{1}{a+bi} \right\rvert$ $= \lvert z^{-1} \rvert$ $= \text{RHS}$	• correctly represents $\bar{z}$ in terms of a and b **[1 mark]** • correctly represents $\lvert z \rvert$ as a radical expression in terms of a and b **[1 mark]** • correctly simplifies $z\bar{z}$ in terms of a and b **[1 mark]** • determines an expression without an index that represents $\lvert z \rvert^{-1}$ **[1 mark]** • correctly shows mathematical reasoning to complete proof **[1 mark]**

Q	Sample response	The response:
13	**Method 2** Prove $\dfrac{\lvert z\rvert}{z\bar{z}} = \lvert z^{-1}\rvert$ Let $z = a + bi$ where $a, b \in R$ $\text{LHS} = \dfrac{\lvert z\rvert}{z\bar{z}}$ $= \dfrac{\lvert a+bi\rvert}{(a+bi)(a-bi)}$	• correctly represents $\bar{z}$ in terms of a and b **[1 mark]**
	$= \dfrac{\sqrt{a^2+b^2}}{a^2+b^2}$	• correctly represents $\lvert z\rvert$ as a radical expression in terms of a and b **[1 mark]** • correctly simplifies $z\bar{z}$ in terms of a and b **[1 mark]**
	$\text{RHS} = \lvert z^{-1}\rvert$ $= \lvert (a+bi)^{-1}\rvert$ $= \left\lvert \dfrac{1}{a+bi}\right\rvert$ $= \dfrac{1}{\sqrt{a^2+b^2}}$	• determines an expression without an index that represents $\lvert z\rvert^{-1}$ **[1 mark]**
	$= \dfrac{\sqrt{a^2+b^2}}{a^2+b^2}$ $= \text{LHS}$	• correctly shows mathematical reasoning to establish the intermediate position to complete proof **[1 mark]**

Q	Sample response	The response:
13	**Method 3** Prove $\frac{\lvert z\rvert}{z\overline{z}}=\lvert z^{-1}\rvert$ $\text{LHS}=\frac{\lvert z\rvert}{z\overline{z}}$ $=\frac{\lvert z\rvert}{\lvert z\rvert^{2}}$ $=\frac{1}{\lvert z\rvert}$	• correctly simplifies $z\overline{z}$ **[1 mark]** • correctly simplifies LHS **[1 mark]**
	$\text{RHS}=\lvert z^{-1}\rvert$ $=\frac{\lvert z\rvert\lvert z^{-1}\rvert}{\lvert z\rvert}$ $=\frac{\lvert z z^{-1}\rvert}{\lvert z\rvert}$ $=\frac{1}{\lvert z\rvert}$ $=\text{LHS}$	• correctly uses a suitable method to form the product $\lvert z\rvert\lvert z^{-1}\rvert$ **[1 mark]** • recognises that the product of the moduli equals the modulus of the product **[1 mark]** • correctly shows mathematical reasoning to establish the intermediate position to complete proof **[1 mark]**

Q	Sample response	The response:
13	**Method 4** Let $z = r\operatorname{cis}(\theta)$ Prove $\frac{\lvert z\rvert}{z\bar{z}} = \lvert z^{-1}\rvert$ $\text{LHS} = \frac{\lvert z\rvert}{z\bar{z}}$ $= \frac{\lvert a+bi\rvert}{(a+bi)(a-bi)}$	• correctly represents $\bar{z}$ **[1 mark]**
	$= \frac{r}{r\operatorname{cis}(\theta)r\operatorname{cis}(-\theta)}$ $= \frac{r}{r^2}$ $= \frac{1}{r}$	• correctly expresses the numerator in terms of r **[1 mark]** • simplifies the denominator **[1 mark]**
	$\text{RHS} = \lvert z^{-1}\rvert$ $= \lvert r^{-1}\operatorname{cis}(-\theta)\rvert$	• correctly uses De Moivre's theorem **[1 mark]**
	$= r^{-1}$ $= \frac{1}{r}$ $= \text{LHS}$	• correctly shows mathematical reasoning to establish the intermediate position to complete proof **[1 mark]**

Q	Sample response	The response:
14a)	$\overrightarrow{OB} = 2\hat{\boldsymbol{j}} + 2\hat{\boldsymbol{k}}$	• correctly states $\overrightarrow{OB}$ **[1 mark]**
14b)	$\overrightarrow{OA} \times \overrightarrow{OB} = 2\hat{\boldsymbol{i}} \times \left(2\hat{\boldsymbol{j}} + 2\hat{\boldsymbol{k}}\right)$ $= -4\hat{\boldsymbol{j}} + 4\hat{\boldsymbol{k}}$	• calculates $\overrightarrow{OA} \times \overrightarrow{OB}$ **[1 mark]**
14c)	$\text{Area} = \frac{1}{2}\left\|\overrightarrow{OA} \times \overrightarrow{OB}\right\|$ $= \frac{1}{2}\sqrt{(-4)^2 + (4)^2}$ $= 2\sqrt{2} \text{ units}^2$	• states an expression representing area of ΔOAB **[1 mark]** • calculates area **[1 mark]**

Q	Sample response	The response:
14d)	$m = \frac{1}{2}\overrightarrow{OA} = \hat{i}$ $n = \frac{1}{2}\overrightarrow{OB} = \hat{j} + \hat{k}$ $\overrightarrow{MN} = -\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$	• determines $\overrightarrow{MN}$ **[1 mark]**
14e)	$\overrightarrow{AB} = -2\hat{i} + 2\hat{j} + 2\hat{k}$ $= 2\left(-\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right)$ $= 2\overrightarrow{MN}$ So, the length of AB is twice the length of MN.	• shows $\overrightarrow{AB} = 2\overrightarrow{MN}$ **[1 mark]**

Q	Sample response	The response:
15a)	Initial statement Prove the rule is true for $n = 1$. $\text{LHS} = 1$ $\text{RHS} = \dfrac{r-1}{r-1}$ $= 1 = \text{LHS}$	• correctly proves the initial statement **[1 mark]**
15b)	Given assumption $1 + r + r^2 + r^3 + \ldots + r^{k-1} = \dfrac{r^k - 1}{r-1} \quad (r \neq 1)$ Inductive step Prove the rule is true for $n = k+1$ for $r \neq 1$ $1 + r + r^2 + r^3 + \ldots + r^{k-1} + r^k = \dfrac{r^{k+1} - 1}{r-1}$ $\text{LHS} = \dfrac{r^k - 1}{r-1} + r^k$ $= \dfrac{r^k - 1 + r^{k+1} - r^k}{r-1}$ $= \dfrac{r^{k+1} - 1}{r-1}$ $= \text{RHS}$	• correctly establishes an expression representing the left-hand side requirement of the inductive step proof **[1 mark]** • uses the given assumption within the inductive step proof **[1 mark]** • shows mathematical reasoning to complete the inductive step proof **[1 mark]**
15c)	Conclusion The rule is proven true for $n = k+1$. By mathematical induction, the rule is true for $n = 1, 2, \ldots$	• states a suitable conclusion to the proof **[1 mark]**

Q	Sample response	The response:
16	$x = 2\tan(\theta)$ $\frac{dx}{d\theta} = 2\sec^2(\theta)$ $y = 3\sin(2\theta)$ $\frac{dy}{d\theta} = 6\cos(2\theta)$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \text{ or } \frac{dy}{d\theta} \times \frac{d\theta}{dx}$ $= \frac{6\cos(2\theta)}{2\sec^2(\theta)}$ $= 3\left(2\cos^2(\theta) - 1\right)\cos^2(\theta)$ $= 6\cos^4(\theta) - 3\cos^2(\theta)$ $\therefore a = 6,\ b = -3$	• correctly determines $\frac{dx}{d\theta}$ **[1 mark]** • correctly determines $\frac{dy}{d\theta}$ **[1 mark]** • determines an expression for $\frac{dy}{dx}$ **[1 mark]** • determines an expression for $\frac{dy}{dx}$ in the required form **[1 mark]** • states the values of a and b **[1 mark]**

Q	Sample response	The response:
17	**Method 1** $\boldsymbol{F}_{net} = F_1 + F_2 = (5t\,\hat{j} - 3\hat{k}) + (-t\,\hat{j} + \hat{k})$ $= 4t\,\hat{\boldsymbol{j}} - 2\hat{\boldsymbol{k}},\ 0 \le t \le 2$ $= m\boldsymbol{a}$	• correctly determines an expression for the net force acting on the object **[1 mark]**
	$4t\,\hat{\boldsymbol{j}} - 2\hat{\boldsymbol{k}} = 2\boldsymbol{a}$ $\boldsymbol{a} = 2t\,\hat{j} - \hat{k}$	• determines an expression involving the object's acceleration after the forces act **[1 mark]**
	$\boldsymbol{v} = \int \boldsymbol{a}\,dt$ $= t^2\hat{\boldsymbol{j}} - t\,\hat{\boldsymbol{k}} + c$	• determines a general solution representing the object's velocity after the forces act **[1 mark]**
	At $t = 0$, $\boldsymbol{v} = 3\hat{i} + \hat{k}$ $\therefore c = 3\hat{i} + \hat{k}$ $\boldsymbol{v} = t^2\hat{\boldsymbol{j}} - t\,\hat{\boldsymbol{k}} + 3\hat{\boldsymbol{i}} + \hat{\boldsymbol{k}}$ $= 3\hat{i} + t^2\hat{j} + (1-t)\hat{k}$	• determines a particular solution representing the velocity of the object after the forces act **[1 mark]**
	Momentum $= m\boldsymbol{v}$ $= 2\left(3\hat{i} + t^2\hat{j} + (1-t)\hat{k}\right)$ At $t = 1$, $m\boldsymbol{v} = 2\left(3\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}}\right)$	• determines an expression for the momentum of the object at $t = 1$ **[1 mark]**
	$\lvert m\boldsymbol{v}\rvert = 2\sqrt{3^2 + 1^2} = 2\sqrt{10}\ \text{kg}\,\text{m}\,\text{s}^{-1}$	• determines a magnitude of the momentum of the object at $t = 1$ **[1 mark]**
		• shows logical organisation, communicating key steps to at least the start of determining the momentum of the object **[1 mark]**

Q	Sample response	The response:
17	**Method 2** $\boldsymbol{F}_{net} = F_1 + F_2 = (5t\,\hat{\boldsymbol{j}} - 3\hat{\boldsymbol{k}}) + (-t\,\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}})$ $= 4t\,\hat{\boldsymbol{j}} - 2\hat{\boldsymbol{k}},\ 0 \le t \le 2$ $= m\boldsymbol{a} = m\dfrac{d\boldsymbol{v}}{dt}$ $4t\,\hat{\boldsymbol{j}} - 2\hat{\boldsymbol{k}} = 2\dfrac{d\boldsymbol{v}}{dt}$ $2t\,\hat{\boldsymbol{j}} - \hat{\boldsymbol{k}} = \dfrac{dv_x}{dt} + \dfrac{dv_y}{dt} + \dfrac{dv_z}{dt}$ $\dfrac{dv_x}{dt} = 0 \Rightarrow v_x = c_1$ $\dfrac{dv_y}{dt} = 2t \Rightarrow v_y = t^2 + c_2$ $\dfrac{dv_z}{dt} = -1 \Rightarrow v_z = -t + c_3$ At $t = 0,\ \boldsymbol{v} = 3\hat{\boldsymbol{i}} + \hat{\boldsymbol{k}}$ $\therefore c_1 = 3,\ c_2 = 0,\ c_3 = 1$ $v_x = 3,\ v_y = t^2,\ v_z = 1 - t$ At $t = 1$: $\boldsymbol{v} = 3\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}}$ $\text{Momentum} = m\boldsymbol{v} = 2\left(3\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}}\right)$ $\lvert m\boldsymbol{v}\rvert = \sqrt{6^2 + 2^2} = 2\sqrt{10}\ \text{kg}\,\text{m}\,\text{s}^{-1}$	• correctly determines an expression for the net force acting on the object **[1 mark]** • determines an expression involving the object's acceleration after the forces act **[1 mark]** • determines a general solution representing the object's velocity for each component after the forces act **[1 mark]** • determines a particular solution representing the velocity of the object after the forces act in component form **[1 mark]** • determines an expression for the momentum of the object at $t = 1$ **[1 mark]** • determines a magnitude of the momentum of the object at $t = 1$ **[1 mark]** • shows logical organisation, communicating key steps to at least the start of determining the momentum of the object **[1 mark]**

Q	Sample response	The response:
18	$\frac{dy}{dx}=\frac{x}{\left(x^2+1\right)\tan(y)}$	
	$\int\tan(y)\,dy=\int\frac{x}{x^2+1}\,dx$	• correctly separates the variables **[1 mark]**
	$-\int\frac{-\sin(y)}{\cos(y)}dy=\frac{1}{2}\int\frac{2x}{x^2+1}dx$	
	$-\ln\lvert\cos(y)\rvert=\frac{1}{2}\ln\lvert x^2+1\rvert+c$	• applies suitable integration methods **[1 mark]**
	Given $y=0$ when $x=0$,	
	$-\ln\lvert\cos(0)\rvert=\frac{1}{2}\ln\lvert 1\rvert+c\Rightarrow c=0$	• determines a value for the constant of integration **[1 mark]**
	$\therefore\frac{1}{2}\ln\lvert x^2+1\rvert=-\ln\lvert\cos(y)\rvert$	
	$\ln\sqrt{x^2+1}=\ln\left\lvert\frac{1}{\cos(y)}\right\rvert$	
	$\sqrt{x^2+1}=\lvert\sec(y)\rvert$	• determines an expression for a solution that does not contain logarithms **[1 mark]**
	$x^2+1=\sec^2(y)$	
	$x^2=\tan^2(y)$	
	$x=\pm\tan(y)$	• expresses x in terms of y **[1 mark]**
	As $x\geq 0$, $-\frac{\pi}{2}<y\leq 0$	
	$x=-\tan(y)$	• evaluates the reasonableness of the results and expresses the solution in the form of $x=f(y)$ in simplified form **[1 mark]**

Q	Sample response	The response:
19	**Method 1** $r_A = 2\sqrt{3}t\,\hat{i} + 3t\hat{j} + 2t\hat{k}, t \geq 0$ $v_A = 2\sqrt{3}\,\hat{i} + 3\hat{j} + 2\hat{k}$ $\lvert v_A \rvert = \sqrt{\left(2\sqrt{3}\right)^2 + 3^2 + 2^2} = 5$ $\therefore \lvert v_B \rvert = 5\sqrt{2}\ \text{ms}^{-1}$	• correctly determines the value of $\lvert v_B \rvert$ **[1 mark]**
	Given v_B is constant, the position of object B from the origin as it moves can be represented along the line $r_B = b + ld, l \in R$, where $b = 3\sqrt{3}\,\hat{i} + 6\hat{j}$ $d = 5\sqrt{3}\,\hat{i} + 8\hat{j} + 4\hat{k} - \left(3\sqrt{3}\,\hat{i} + 6\hat{j}\right)$ $= 2\sqrt{3}\,\hat{i} + 2\hat{j} + 4\hat{k}$ $r_B = \left(3\sqrt{3} + 2\sqrt{3}\,l\right)\hat{i} + (6 + 2l)\,\hat{j} + 4l\,\hat{k}$	• determines a vector in terms of a parameter representing the position of object B from the origin as it moves **[1 mark]**
	Let the objects collide when $t = t_1$ $r_A(t_1) = 2\sqrt{3}t_1\,\hat{i} + 3t_1\hat{j} + 2t_1\hat{k}$ Collision occurs when $r_A = r_B$ Equating $\hat{i}$, $\hat{j}$ and $\hat{k}$ components: $3\sqrt{3} + 2\sqrt{3}\,l = 2\sqrt{3}\,t_1 \quad \ldots(1)$ $6 + 2l = 3t_1 \quad \ldots(2)$ $4l = 2t_1 \quad \ldots(3)$	• determines at least two simultaneous equations based on the collision of the objects **[1 mark]**

Q	Sample response	The response:
	From (3), $l = \frac{t_1}{2}$ Substituting into (1): $3\sqrt{3} + 2\sqrt{3}\left(\frac{t_1}{2}\right) = 2\sqrt{3}\,t_1$ $t_1 = 3 \text{ s}$ The collision occurs 3 seconds after object A is released.	• determines time from release to collision for object A **[1 mark]**
	Collision point is $2\sqrt{3}(3)\hat{\boldsymbol{i}} + 3(3)\hat{\boldsymbol{j}} + 2(3)\hat{\boldsymbol{k}}$ $= 6\sqrt{3}\,\hat{\boldsymbol{i}} + 9\hat{\boldsymbol{j}} + 6\hat{\boldsymbol{k}}$ Distance object B travels from P to collision point is $\sqrt{\left(6\sqrt{3} - 3\sqrt{3}\right)^2 + (6-9)^2 + 6^2} = 6\sqrt{2}$	• determines distance that object B travels to reach collision point **[1 mark]**
	Time taken for object B to reach collision point is $\frac{d}{s} = \frac{6\sqrt{2}}{5\sqrt{2}}$ $= 1.2 \text{ s}$ Time between the release of the two objects is $3 - 1.2 = 1.8 \text{ s}$	• determines time between the release of the two objects **[1 mark]**

Q	Sample response	The response:
19	**Method 2** $r_A = 2\sqrt{3}t\hat{i} + 3t\hat{j} + 2t\hat{k}, t \geq 0$ $v_A = 2\sqrt{3}\hat{i} + 3\hat{j} + 2\hat{k}$ $\lvert v_A \rvert = \sqrt{(2\sqrt{3})^2 + 3^2 + 2^2} = 5$ $\therefore \lvert \boldsymbol{v_B} \rvert = 5\sqrt{2}\ \text{ms}^{-1}$ $\overrightarrow{PQ} = \boldsymbol{q} - \boldsymbol{p} = 2\sqrt{3}\,\hat{\boldsymbol{i}} + 2\,\hat{\boldsymbol{j}} + 4\,\hat{\boldsymbol{k}}\boldsymbol{a}$ $\lvert \overrightarrow{PQ} \rvert = \sqrt{(2\sqrt{3})^2 + 2^2 + 4^2}$ $= \sqrt{32}$ $\widehat{\overrightarrow{PQ}} = \frac{\overrightarrow{PQ}}{\lvert \overrightarrow{PQ} \rvert} = \frac{1}{4\sqrt{2}}\left(2\sqrt{3}\,\hat{\boldsymbol{i}} + 2\,\hat{\boldsymbol{j}} + 4\,\hat{\boldsymbol{k}}\right)$ Given $\boldsymbol{v_B}$ is in the direction of $\overrightarrow{PQ}$, $\boldsymbol{v_B} = 5\sqrt{2}\,\widehat{\overrightarrow{PQ}}$ $= \frac{5\sqrt{3}}{2}\hat{\boldsymbol{i}} + \frac{5}{2}\hat{\boldsymbol{j}} + 5\hat{\boldsymbol{k}}$ Given $\boldsymbol{v_A}$ and $\boldsymbol{v_B}$ are constant, the position of object B from the origin as it moves can be represented along the line $\boldsymbol{r_B} = \boldsymbol{b} + l\boldsymbol{v_B}, l \in R$, where l is the time of motion $\boldsymbol{b} = 3\sqrt{3}\,\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{j}}$.	• correctly determines the value of $\lvert v_B \rvert$ **[1 mark]** • determines the velocity vector of object B **[1 mark]**

Q	Sample response	The response:
	$\boldsymbol{r_B} = \left(3\sqrt{3} + \frac{5\sqrt{3}}{2}l\right)\hat{\boldsymbol{i}} + \left(6 + \frac{5}{2}l\right)\hat{\boldsymbol{j}} + 5l\,\hat{\boldsymbol{k}}$	• determines a vector in terms of a parameter representing the position of B from the origin as it moves **[1 mark]**
	Let the objects collide when $t = t_1$. $\boldsymbol{r_A}(t_1) = 2\sqrt{3}t_1\,\hat{\boldsymbol{i}} + 3t_1\hat{\boldsymbol{j}} + 2t_1\hat{\boldsymbol{k}}$ Collision occurs when $\boldsymbol{r_A} = \boldsymbol{r_B}$ Equating $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$ and $\hat{\boldsymbol{k}}$ components: $3\sqrt{3} + \frac{5\sqrt{3}}{2}l = 2\sqrt{3}\,t_1 \quad \ldots(1)$ $6 + \frac{5}{2}l = 3t_1 \quad \ldots(2)$ $5l = 2t_1 \quad \ldots(3)$	• determines at least two simultaneous equations based on the collision of the objects **[1 mark]**
	From (3), $l = \frac{2t_1}{5}$ Substituting into (2): $6 + \frac{5}{2}\left(\frac{2t_1}{5}\right) = 3t_1$ $t_1 = 3\text{ s}$	• determines time from release to collision for object A **[1 mark]**
	$\therefore l = 1.2\text{ s}$ Time between the release of the two objects is $3 - 1.2 = 1.8\text{ s}$.	• determines time between the release of the two objects **[1 mark]**

Q	Sample response	The response:
19	**Method 3** $r_A = 2\sqrt{3}t\hat{i} + 3t\hat{j} + 2t\hat{k}, t \geq 0$ $v_A = 2\sqrt{3}\hat{i} + 3\hat{j} + 2\hat{k}$ $\lvert v_A \rvert = \sqrt{(2\sqrt{3})^2 + 3^2 + 2^2} = 5$ $\therefore \lvert \mathbf{v}_B \rvert = 5\sqrt{2}\ \mathrm{ms}^{-1}$ $\overrightarrow{PQ} = q - p = 2\sqrt{3}\hat{i} + 2\hat{j} + 4\hat{k}$ $\lvert \overrightarrow{PQ} \rvert = \sqrt{(2\sqrt{3})^2 + 2^2 + 4^2} = \sqrt{32}$ $\widehat{PQ} = \frac{\overrightarrow{PQ}}{\lvert \overrightarrow{PQ} \rvert} = \frac{1}{4\sqrt{2}}(2\sqrt{3}\hat{i} + 2\hat{j} + 4\hat{k})$ Given that v_B is in the direction of $\overrightarrow{PQ}$ $v_B = 5\sqrt{2}\,\widehat{PQ}$ $= \frac{5\sqrt{3}}{2}\hat{i} + \frac{5}{2}\hat{j} + 5\hat{k}$ Given r_B is in the direction of $\overrightarrow{PQ}$ and the object is released from B. Let the time between the time of release of the two objects be t_0. Displacement of object B from P to collision point is: $d = v_B t = 5\sqrt{2}\,\widehat{PQ}(t - t_0)$ $= \frac{5\sqrt{2}}{4\sqrt{2}}(2\sqrt{3}\hat{i} + 2\hat{j} + 4\hat{k})(t - t_0)$	• correctly determines the value of $\lvert \mathbf{v}_B \rvert$ **[1 mark]** • determines the velocity vector of object B **[1 mark]** • determines displacement of object B from P to the collision point in terms of time between release of the two objects **[1 mark]**

Q	Sample response	The response:
	The position of object B from the origin as it moves can be represented by $\boldsymbol{r}_B = \boldsymbol{b} + 5\sqrt{2}\,\widehat{PQ}(t - t_0)$ $= 3\sqrt{3}\,\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{j}} + \frac{5}{4}\left(2\sqrt{3}\,\hat{\boldsymbol{i}} + 2\,\hat{\boldsymbol{j}} + 4\hat{\boldsymbol{k}}\right)(t - t_0)$ $= 3\sqrt{3}\,\hat{\boldsymbol{i}} + 6\hat{\boldsymbol{j}} + \frac{5(t - t_0)}{4}\left(2\sqrt{3}\,\hat{\boldsymbol{i}} + 2\,\hat{\boldsymbol{j}} + 4\hat{\boldsymbol{k}}\right)$	• determines a vector representing the position of B from the origin as it moves **[1 mark]**
	Collision occurs when $\boldsymbol{r}_A = \boldsymbol{r}_B$ Equating $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$ and $\hat{\boldsymbol{k}}$ components: $2\sqrt{3}t = 3\sqrt{3} + \frac{5\sqrt{3}}{2}t - \frac{5\sqrt{3}}{2}t_0 \quad \ldots(1)$ $3t = 6 + \frac{5}{2}t - \frac{5}{2}t_0 \quad \ldots(2)$ $2t = 5t - 5t_0 \quad \ldots(3)$ From (3), $t = \frac{5}{3}t_0$ Substituting into (2) $\frac{5}{6}t_0 = 6 - \frac{5}{2}t_0$ $t_0 = 1.8$	• determines at least two simultaneous equations based on the time between release of the two objects and original position of object B **[1 mark]**
	Time between the release of the two objects is 1.8 s.	• determines time between the release of the two objects **[1 mark]**

Q	Sample response	The response:
19	**Method 4** $\mathbf{r}_A = 2\sqrt{3}t\,\hat{i} + 3t\,\hat{j} + 2t\hat{k}, t \geq 0$ $v_A = 2\sqrt{3}\,\hat{i} + 3\hat{j} + 2\hat{k}$ $\lvert \mathbf{v}_A \rvert = \sqrt{\left(2\sqrt{3}\right)^2 + 3^2 + 2^2} = 5$ $\therefore \lvert \mathbf{v}_B \rvert = 5\sqrt{2}\ \text{ms}^{-1}$ $\overrightarrow{PQ} = q - p = 2\sqrt{3}\,\hat{i} + 2\,\hat{j} + 4\,\hat{k}a$ $\lvert \overrightarrow{PQ} \rvert = \sqrt{\left(2\sqrt{3}\right)^2 + 2^2 + 4^2}$ $= \sqrt{32}$ $\widehat{PQ} = \frac{\overrightarrow{PQ}}{\lvert \overrightarrow{PQ} \rvert} = \frac{1}{4\sqrt{2}}\left(2\sqrt{3}\,\hat{i} + 2\,\hat{j} + 4\,\hat{k}\right)$ Given v_B is in the direction of $\overrightarrow{PQ}$ $\boldsymbol{v}_B = 5\sqrt{2}\,\widehat{PQ}$ $= \frac{5\sqrt{3}}{2}\hat{\boldsymbol{i}} + \frac{5}{2}\hat{\boldsymbol{j}} + 5\hat{\boldsymbol{k}}$ $\boldsymbol{r}_B = \int v_B\, dt$ $= \left(\frac{5\sqrt{3}}{2}t + c_1\right)\hat{\boldsymbol{i}} + \left(\frac{5}{2}t + c_2\right)\hat{\boldsymbol{j}} + \left(5t + c_3\right)\hat{\boldsymbol{k}}$ Let object B be released from $\left(3\sqrt{3}, 6, 0\right)$ at b seconds after object A is released.	• correctly determines the value of $\lvert \mathbf{v}_B \rvert$ **[1 mark]** • determines velocity vector of object B **[1 mark]** • determines a general expression for the position of object B **[1 mark]**

Q	Sample response	The response:
	$\mathbf{r}_B(b) = \left(\frac{5\sqrt{3}}{2}b + c_1\right)\hat{\mathbf{i}} + \left(\frac{5}{2}b + c_2\right)\hat{\mathbf{j}} + (5b + c_3)\hat{\mathbf{k}}$ $\frac{5\sqrt{3}}{2}b + c_1 = 3\sqrt{3} \Rightarrow c_1 = 3\sqrt{3} - \frac{5\sqrt{3}}{2}b \quad ...(1)$ $\frac{5}{2}b + c_2 = 6 \Rightarrow c_2 = 6 - \frac{5}{2}b \quad ...(2)$ $5b + c_3 = 0 \Rightarrow c_3 = -5b \quad ...(3)$	• determines at least two simultaneous equations based on the time between release of the two objects and original position of object B **[1 mark]**
	Let the objects collide a seconds after object A is released. $\mathbf{r}_A(a) = \mathbf{r}_B(a)$ $2\sqrt{3}a = \frac{5\sqrt{3}}{2}a + c_1 \Rightarrow c_1 = -\frac{\sqrt{3}a}{2}$ $3a = \frac{5}{2}a + c_2 \Rightarrow c_2 = \frac{a}{2}$ $2a = 5a + c_3 \Rightarrow c_3 = -3a$ Equating parts: $3\sqrt{3} - \frac{5\sqrt{3}}{2}b = -\frac{\sqrt{3}a}{2} \quad ...(4)$ $6 - \frac{5}{2}b = \frac{a}{2} \quad ...(5)$ $-5b = -3a \quad ...(6)$	• determines at least two simultaneous equations based on time between release of the two objects and the time that the two objects collide **[1 mark]**
	Using $6 \times (5) + (6)$ $36 - 20b = 0$ $b = 1.8$ Time between the release of the two objects is 1.8 s.	• determines time between the release of the two objects **[1 mark]**