Specialist Mathematics marking guide

External assessment 2022

Paper 1: Technology-free (60 marks)

Paper 2: Technology-active (60 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
Queensland Curriculum & Assessment Authority

Purpose

This marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Where no response to a question has been made, a mark of ‘N’ will be recorded.

Allow FT mark/s — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Multiple choice
Paper 1: Technology-free (60 marks)

Question	Response
1	B
2	C
3	B
4	D
5	A
6	D
7	A
8	B
9	D
10	C

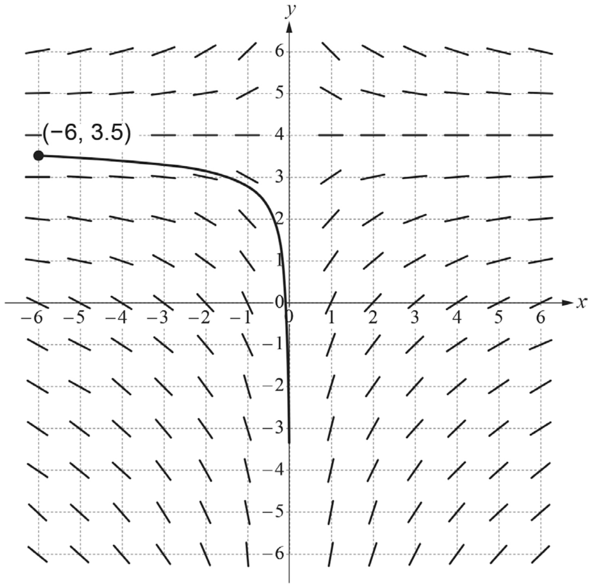
Short response

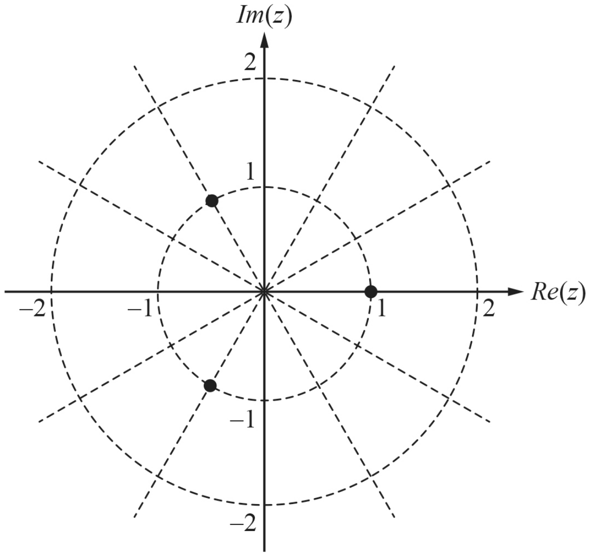
Q	Sample response	The response:
11a)	$\boldsymbol{v_1}(t) = 2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} - 2\hat{\boldsymbol{k}}$	• correctly determines the velocity vector **[1 mark]**
11b)	Require $\boldsymbol{r_1}.\boldsymbol{v_1} = 0$ $\begin{pmatrix} 2t+1 \\ t+3 \\ -(2t-3) \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 0$ $2(2t+1) + 1(t+3) + 2(2t-3) = 0$ $9t - 1 = 0$ $t = \frac{1}{9}$ s	• establishes an equation in the form $x_1x_2 + y_1y_2 + z_1z_2 = 0$ **[1 mark]** • solves equation to determine the time **[1 mark]**
11c)	Let $\boldsymbol{r_1}(t) = \boldsymbol{r_2}(t)$ Equating $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ components $2t + 1 = 16 - 4t$ ... (1) $t + 3 = -3t + 13$ ... (2) $-2t + 3 = 2$ ... (3) From (1), $t = 2.5$ s From (2), $t = 2.5$ s From (3), $t = 0.5$ s The particles do not collide as the solutions are not consistent.	• correctly equates 2 pairs of components for the particles, one of which must be the $\hat{\boldsymbol{k}}$ component, to form 2 equations **[1 mark]** • solves equations to determine two different time values **[1 mark]** • uses mathematical reasoning to make an appropriate conclusion whether the two particles collide **[1 mark]**

Q	Sample response	The response:
12	**Method 1** Prove $\left\|\frac{z_1}{z_2}\right\| = \frac{\|z_1\|}{\|z_2\|}$, given $z_1 = a + bi$, $z_2 = c + di$, $z_2 \neq 0$ $\text{LHS} = \left\|\frac{a+bi}{c+di}\right\|$ $= \left\|\frac{(a+bi)}{(c+di)} \cdot \frac{(c-di)}{(c-di)}\right\|$ $= \left\|\frac{ac - adi + bci - bdi^2}{c^2+d^2}\right\|$ $= \left\|\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}\right\|$ $= \sqrt{\frac{(ac+bd)^2+(bc-ad)^2}{(c^2+d^2)^2}}$ $= \sqrt{\frac{(ac)^2 + 2abcd + (bd)^2 + (bc)^2 - 2abcd + (ad)^2}{(c^2+d^2)^2}}$ $= \sqrt{\frac{(ac)^2+(bd)^2+(bc)^2+(ad)^2}{(c^2+d^2)^2}}$ $= \sqrt{\frac{a^2(c^2+d^2)+b^2(c^2+d^2)}{(c^2+d^2)^2}}$ $= \sqrt{\frac{(a^2+b^2)(c^2+d^2)}{(c^2+d^2)^2}}$ $= \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$ $= \frac{\|z_1\|}{\|z_2\|}$ = RHS QED	• correctly multiplies the numerator and denominator by the complex conjugate of z_2 **[1 mark]** • realises denominator (in simplest form) and expands numerator **[1 mark]** • determines modulus of the expression **[1 mark]** • simplifies numerator **[1 mark]** • factorises numerator **[1 mark]** • completes the proof using mathematical reasoning **[1 mark]**

Q	Sample response	The response:
	Method 2 $z_1 = a + bi = \sqrt{a^2+b^2}\operatorname{cis}(\theta_1)$, $Arg(z_1) = \theta_1$ $z_2 = c + di = \sqrt{c^2+d^2}\operatorname{cis}(\theta_2)$, $z_2 \neq 0$, $Arg(z_2) = \theta_2$	• correctly determines the magnitude of z_1 and z_2 **[1 mark]** • correctly expresses the argument of z_1 and z_2 **[1 mark]**
	$\text{LHS} = \left\lvert \dfrac{\sqrt{a^2+b^2}\operatorname{cis}(\theta_1)}{\sqrt{c^2+d^2}\operatorname{cis}(\theta_2)} \right\rvert$ $= \left\lvert \dfrac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}\operatorname{cis}(\theta_1-\theta_2) \right\rvert$ $= \left\lvert \dfrac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} \right\rvert$ $= \dfrac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$	• uses De Moivre's formula to express quotient of modulus **[1 mark]** • uses De Moivre's formula to express difference of arguments **[1 mark]** • determines magnitude of quotient **[1 mark]**
	$\text{RHS} = \dfrac{\left\lvert \sqrt{a^2+b^2}\operatorname{Arg}(z_1) \right\rvert}{\left\lvert \sqrt{c^2+d^2}\operatorname{Arg}(z_2) \right\rvert}$ $= \dfrac{\left\lvert \sqrt{a^2+b^2} \right\rvert}{\left\lvert \sqrt{c^2+d^2} \right\rvert}$ $= \dfrac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$ $= \text{LHS}$	• completes the proof using mathematical reasoning **[1 mark]**

Q	Sample response	The response:
13a)	Using partial fractions $\frac{22}{(2x-3)(x+4)} = \frac{A}{2x-3} + \frac{B}{x+4}$ $\therefore 22 = A(x+4) + B(2x-3)$ Let $x = \frac{3}{2}$: $A\left(\frac{3}{2}+4\right) = 2 \Rightarrow A = 4$ Let $x = -4$: $B(2 \times -4 - 3) = 22 \Rightarrow B = -2$ $\int \frac{22}{2x^2+5x-12}dx = \int \frac{4}{2x-3}dx + \int \frac{-2}{x+4}dx$ $= 2\int \frac{2}{2x-3}dx - 2\int \frac{1}{x+4}dx$ $= 2\ln\|2x-3\| - 2\ln\|x+4\| + c$	• correctly sets up the partial fractions **[1 mark]** • determines value of A **[1 mark]** • determines value of B **[1 mark]** • determines an expression for the indefinite integral **[1 mark]**
13b)	$\int_{-3}^{0} \frac{22}{(2x-3)(x+4)}dx$ $= (2\ln\|2x-3\| - 2\ln\|x+4\|)\Big\|_{-3}^{0}$ $= ((2\ln\|-3\| - 2\ln\|4\|) - (2\ln\|-9\| - 2\ln\|1\|))$ $= 2(\ln(3) - \ln(4) - \ln(9)) = 2\ln\left(\frac{3}{9\times4}\right)$ $= 2\ln\left(\frac{1}{12}\right)$	• substitutes limits of integration into the result from 13a) **[1 mark]** • expresses a definite integral value in simplest form **[1 mark]**

Q	Sample response	The response:
14a)	Substituting $A(-4,-2)$ $\frac{dy}{dx}=\frac{-0.5(-2-4)}{-4}$ $=-\frac{3}{4}$	• correctly substitutes the coordinates of A into the differential equation **[1 mark]** • determines a value of the slope at A **[1 mark]**
14b)		• correctly shows a solution curve that starts at (–6, 3.5) and has a negative slope in the 2nd quadrant **[1 mark]** • correctly shows a solution curve with an asymptotic nature in the 3rd quadrant as $x \to 0^-$ and there is no solution curve in the 1st or 4th quadrant **[1 mark]**

Q	Sample response	The response
15a)		• correctly sketches 3 solutions, each with a modulus of 1 **[1 mark]** • correctly sketches 3 solutions with arguments of 0 and $\pm\frac{2\pi}{3}$ **[1 mark]**
15b)	The only positive product of $\boldsymbol{a}$ and $\boldsymbol{b}$ occurs for the solution $\boldsymbol{z} = \text{cis}\left(\frac{-2\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$ Determining the largest value of ab $ab = \frac{-1}{2} \times \frac{-\sqrt{3}}{2}$ $= \frac{\sqrt{3}}{4}$	• recognises a solution from which its a and b values produce the largest possible positive product **[1 mark]** • determines a positive product of ab **[1 mark]**

Q	Sample response	The response
16a)	Rearranging equations: $x+5y-2z=1$ $x-3y+\ z=3$ $8y-3z=\lambda$ Expressing in matrix form: $\left[\begin{array}{ccc\|c} 1 & 5 & -2 & 1 \\ 1 & -3 & 1 & 3 \\ 0 & 8 & -3 & \lambda \end{array}\right]$ $\left[\begin{array}{ccc\|c} 1 & 5 & -2 & 1 \\ 0 & -8 & 3 & 2 \\ 0 & 8 & -3 & \lambda \end{array}\right]$ $\quad R_2'=R_2-R_1$ $\left[\begin{array}{ccc\|c} 1 & 5 & -2 & 1 \\ 0 & -8 & 3 & 2 \\ 0 & 0 & 0 & \lambda+2 \end{array}\right]$ $\quad R_3'=R_3+R_2$ For $\lambda=-2$ there are infinitely many solutions.	• correctly rearranges the three equations **[1 mark]** • establishes an augmented matrix **[1 mark]** • establishes a row of zeros in the row containing λ **[1 mark]** • determines a value of λ **[1 mark]**
16b)	**Method 1** using $\left[\begin{array}{ccc\|c} 1 & 5 & -2 & 1 \\ 0 & -8 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ Letting $z=k\ (k\in R)$ Row 2: $-8y+3z=2$ $8y-3k=-2 \Rightarrow y=\dfrac{3k-2}{8}$ Row 1: $x+5y-2z=1$ $x+5\left(\dfrac{3k-2}{8}\right)-2k=1$ $x+\dfrac{15k-10}{8}-\dfrac{16k}{8}=1 \Rightarrow x=\dfrac{k}{8}+\dfrac{9}{4}$	• expresses y in terms of a parameter **[1 mark]** • expresses x in terms of a parameter **[1 mark]**

Q	Sample response	The response
	The solutions in vector form are: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{8} \\ \frac{3}{8} \\ 1 \end{bmatrix} k + \begin{bmatrix} \frac{9}{4} \\ -\frac{1}{4} \\ 0 \end{bmatrix}$	• determines the infinite solutions expressed in the form of a vector equation of a line **[1 mark]**
	Method 2 using $\left[\begin{array}{ccc\|c} 1 & 5 & -2 & 1 \\ 0 & -8 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$	
	$\left[\begin{array}{ccc\|c} 1 & 5 & -2 & 1 \\ 0 & 1 & -\frac{3}{8} & -\frac{1}{4} \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \boldsymbol{R_2' = \frac{R_2}{-8}}$	
	$\left[\begin{array}{ccc\|c} 1 & 0 & -\frac{1}{8} & \frac{9}{4} \\ 0 & 1 & -\frac{3}{8} & -\frac{1}{4} \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \boldsymbol{R_1' = R_1 - 5R_2}$	
	Letting $z = k$ $(k \in R)$	
	$x = \frac{k}{8} + \frac{9}{4}$	• expresses x in terms of a parameter **[1 mark]**
	$y = \frac{3k}{8} - \frac{1}{4}$	• expresses y in terms of a parameter **[1 mark]**
	The solutions in vector form are: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{8} \\ \frac{3}{8} \\ 1 \end{bmatrix} k + \begin{bmatrix} \frac{9}{4} \\ -\frac{1}{4} \\ 0 \end{bmatrix}$	• determines the infinite solutions expressed in the form of a vector equation of a line **[1 mark]**

Q	Sample response	The response
	Method 3 using $\left[\begin{array}{ccc\|c} 1 & 5 & -2 & 1 \\ 1 & -3 & 1 & 3 \\ 0 & 8 & -3 & -2 \end{array}\right]$ Letting $y = k$ $(k \in R)$ Row 3: $-8y + 3z = 2$ $-8k + 3z = 2$ $z = \frac{8k}{3} + \frac{2}{3}$ Row 2: $x - 3y + z = 3$ $x - 3k + \frac{8k}{3} + \frac{2}{3} = 3$ $x = \frac{k}{3} + \frac{7}{3}$ The solutions in vector form are: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ 1 \\ \frac{8}{3} \end{bmatrix} k + \begin{bmatrix} \frac{7}{3} \\ 0 \\ \frac{2}{3} \end{bmatrix}$	• expresses z in terms of a parameter **[1 mark]** • expresses x in terms of a parameter **[1 mark]** • determines the infinite solutions expressed in the form of a vector equation of a line **[1 mark]**

Q	Sample response	The response
17	Volume can be found using $V = \pi \int_a^b y^2 dx$ $= \pi \int_0^{\frac{\pi}{2}} (1 + \sin(2x))^2 \, dx$	• correctly substitutes into the appropriate volume of a solid of revolution rule **[1 mark]**
	$= \pi \int_0^{\frac{\pi}{2}} 1 + 2\sin(2x) + \sin^2(2x) \, dx$	• expands an integrand **[1 mark]**
	$= \pi \int_0^{\frac{\pi}{2}} 1 + 2\sin(2x) + \frac{1}{2}(1 - \cos(4x)) \, dx$	• uses a suitable double-angle identity to enable an integration process to be completed **[1 mark]**
	$= \pi \int_0^{\frac{\pi}{2}} \left(\frac{3}{2} + 2\sin(2x) - \frac{1}{2}\cos(4x)\right) dx$	
	$V = \pi \left(\frac{3}{2}x - \cos(2x) - \frac{1}{8}\sin(4x)\right)\Big\vert_0^{\frac{\pi}{2}}$	• integrates an expression **[1 mark]**
	$= \pi \left[\left(\frac{3\pi}{4} - \cos(\pi) - \frac{1}{8}\sin(2\pi)\right) - (0 - \cos(0) - 0)\right]$	
	$= \pi\left(\frac{3\pi}{4} + 2\right)$ units^3	• determines volume in simplest form **[1 mark]**

Q	Sample response	The response
18	Mathematical induction can be used to prove the proposition $\sum_{r=0}^{2n+1} \text{cis}(r\theta)$ is divisible by $\left(1+\text{cis}\,(\theta)\right)$ for $n \in Z^+$ Let $n = 1$ $\sum_{r=0}^{3} \text{cis}(r\theta) = \text{cis}(0) + \text{cis}(\theta) + \text{cis}(2\theta) + \text{cis}(3\theta)$ $= 1 + \text{cis}(\theta) + \left(\text{cis}(\theta)\right)^2 + \left(\text{cis}(\theta)\right)^3$ $= \left(1+\text{cis}(\theta)\right)\left(1+(\text{cis}\theta)^2\right)$ This expression is divisible by $\left(1+\text{cis}\,(\theta)\right)$, so the proposition is true for $n = 1$.	• correctly proves the initial statement **[1 mark]**
	Assume $n = k$ is true for $k \in Z^+$: $\sum_{r=0}^{2k+1} \text{cis}(r\theta) = \left(1+\text{cis}(\theta)\right)Q(\theta)$ where $Q(\theta)$ is a function of θ	• correctly establishes an appropriate assumption for $n = k$ **[1 mark]**
	Let $n = k+1$ $\sum_{r=0}^{2k+3} \text{cis}(r\theta) = \sum_{r=0}^{2k+1} \text{cis}(r\theta) + \text{cis}\left((2k+2)\theta\right) + \text{cis}\left((2k+3)\theta\right)$ $= \left(1+\text{cis}(\theta)\right)Q(\theta) + \left(\text{cis}(\theta)\right)^{2k+2} + \left(\text{cis}(\theta)\right)^{2k+3}$	• expresses the sum based on $n = k+1$ in terms of the assumption **[1 mark]**
	$= \left(1+\text{cis}(\theta)\right)Q(\theta) + \left(\text{cis}(\theta)\right)^{2k+2}\left(1+\text{cis}(\theta)\right)$ $= \left(1+\text{cis}(\theta)\right)\left(Q(\theta) + \left(\text{cis}(\theta)\right)^{2k+2}\right)$	• expresses a result using a common factor of $\left(1+\text{cis}(\theta)\right)$ **[1 mark]**
	$= \left(1+\text{cis}(\theta)\right)R(\theta)$ where $R(\theta)$ is a function of θ The proposition is true for $n = k+1$. By mathematical induction, the formula is true for $n = 1, 2, \ldots$	• proves the inductive step **[1 mark]**

Q	Sample response	The response
19	**Method 1** $f(x) = \int e^x \sin^{-1}(e^x)\, dx$ Using integration by parts: $\int u\frac{dv}{dx}\, dx = uv - \int v\frac{du}{dx}\, dx$ Let $u = \sin^{-1}(e^x) \Rightarrow \frac{du}{dx} = \frac{e^x}{\sqrt{1-e^{2x}}}$ Let $\frac{dv}{dx} = e^x \Rightarrow v = e^x$ $f(x) = e^x \sin^{-1}(e^x) - \int \frac{e^x}{\sqrt{1-e^{2x}}}.e^x\, dx$ $= e^x \sin^{-1}(e^x) - \int \frac{e^{2x}}{\sqrt{1-e^{2x}}}\, dx$ Using the substitution $u = 1 - e^{2x}$ $du = -2e^{2x}dx \Rightarrow dx = \frac{du}{-2e^{2x}}$ $f(x) = e^x\sin^{-1}(e^x) - \int \frac{e^{2x}}{\sqrt{u}}\frac{du}{-2e^{2x}}$ $= e^x\sin^{-1}(e^x) + \frac{1}{2}\int \frac{1}{\sqrt{u}}du$ $= e^x\sin^{-1}(e^x) + \frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$ $= e^x\sin^{-1}(e^x) + \sqrt{1-e^{2x}} + c$ Substituting (0, 0) to determine the c value $f(0) = e^0\sin^{-1}(e^0) + \sqrt{1-e^0} + c$ $0 = \frac{\pi}{2} + c \Rightarrow c = -\frac{\pi}{2}$ $f(x) = e^x\sin^{-1}(e^x) + \sqrt{1-e^{2x}} - \frac{\pi}{2}$	• correctly determines the expressions for $\frac{du}{dx}$ and v in preparation for the use of the integration by parts rule **[1 mark]** • applies the integration by parts rule and simplifies an integrand **[1 mark]** • determines a suitable substitution variable in preparation for using a substitution method of integration **[1 mark]** • expresses an integrand in terms of the substitution variable **[1 mark]** • determines a general solution for $f(x)$ **[1 mark]** • determines a value of the constant of integration and communicates a solution **[1 mark]** • shows logical organisation communicating key steps up to the stage where an integration method using substitution is considered **[1 mark]**

Q	Sample response	The response
	Method 2 $f(x) = \int e^x \sin^{-1}(e^x)\, dx$	
	Using the substitution $w = e^x$ $dw = e^x\, dx \Rightarrow dx = \dfrac{dw}{e^x}$ $f(w) = \int \sin^{-1}(w)\, dw$	• correctly determines the equivalent substitution variable in preparation for using a substitution method of integration **[1 mark]** • expresses an integrand in terms of the substitution variable **[1 mark]**
	Using integration by parts: $\int u \dfrac{dv}{dx}\, dx = uv - \int v \dfrac{du}{dx}\, dx$ Let $u = \sin^{-1}(w) \Rightarrow \dfrac{du}{dw} = \dfrac{1}{\sqrt{1-w^2}}$ Let $\dfrac{dv}{dw} = 1 \Rightarrow v = w$	• determines expressions for u' and v in preparation for using the integration by parts rule **[1 mark]**
	$f(w) = w\sin^{-1}(w) - \int \dfrac{w}{\sqrt{1-w^2}}\, dw$ $= w\sin^{-1}(w) - \int w(1-w^2)^{\frac{-1}{2}}\, dw$ $= w\sin^{-1}(w) - \left(-\dfrac{1}{2}\right)(2)(1-w^2)^{\frac{1}{2}}$ $= w\sin^{-1}(w) + \sqrt{1-w^2} + c$	• applies the integration by parts rule **[1 mark]**
	Using the substitution $w = e^x$ $f(x) = e^x\sin^{-1}(e^x) + \sqrt{1-e^{2x}} + c$	• determines a general solution for $f(x)$ **[1 mark]**
	Substituting (0, 0) to determine the c value $f(0) = e^0\sin^{-1}(e^0) + \sqrt{1-e^0} + c$ $0 = \frac{\pi}{2} + c \Rightarrow c = -\frac{\pi}{2}$ $f(x) = e^x\sin^{-1}(e^x) + \sqrt{1-e^{2x}} - \dfrac{\pi}{2}$	• determines a value of the constant of integration and communicates a solution **[1 mark]** • shows logical organisation communicating key steps up to the stage where an integration method using integration by parts is considered **[1 mark]**

Marking guide

Multiple choice
Paper 2: Technology-active (60 marks)

Question	Response
1	C
2	A
3	C
4	D
5	C
6	C
7	B
8	B
9	A
10	A

Short response

Q	Sample response	The response
11a)	$w = \frac{b-a}{n} = \frac{6}{4} = \mathbf{1.5}$	• correctly determines the width of each interval **[1 mark]**
	Considering values in Quadrant 1: $\frac{(x-2)^2}{16} + \frac{y^2}{9} = 1 \Rightarrow y = \sqrt{9\left(1 - \frac{(x-2)^2}{16}\right)}$ **Table of values**: <table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>2.598</td></tr><tr><td>1.5</td><td>2.976</td></tr><tr><td>3</td><td>2.905</td></tr><tr><td>4.5</td><td>2.342</td></tr><tr><td>6</td><td>0</td></tr></table>	• correctly determines the 5 required y-values **[1 mark]**
	Determining the area in the upper half of the ellipse $A \approx \frac{w}{3}[f(x_0) + 4[f(x_1) + f(x_3) + \cdots] + 2[f(x_2) + f(x_4) + \cdots] + f(x_n)]$ $\approx \frac{1.5}{3}(2.598 + 0 + 4(2.976 + 2.342) + 2 \times 2.905)$ $\approx 14.84 \text{ km}^2$	• substitutes y-values into Simpson's rule **[1 mark]**
	Using symmetry of the ellipse Required area $\approx \mathbf{2 \times 14.84}$ $\approx 29.68 \text{ km}^2$	• determines an approximate area **[1 mark]**

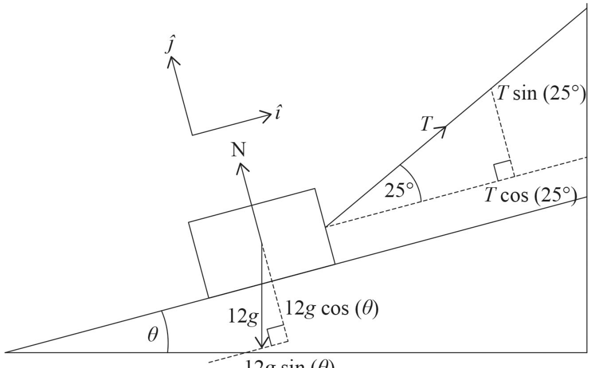
Q	Sample response	The response
11b)	Area $= 2\int_0^6 \sqrt{9\left(1-\frac{(x-2)^2}{16}\right)}\,dx$ Using integration facility of GDC, Area $\approx$ $30.32\ \text{km}^2$ The approximation is reasonable as the error is only around 2%.	• uses a suitable mathematical representation to communicate the approach **[1 mark]** • determines area and evaluates the reasonableness of the approximation **[1 mark]**

Q	Sample response	The response
12a)	Initial population matrix = $\mathbf{N_1} = \begin{bmatrix} 150 \\ 101 \\ 84 \\ 62 \end{bmatrix}$	• correctly states the initial population matrix **[1 mark]**
12b)	Leslie matrix = $\mathbf{L} = \begin{bmatrix} 0.4 & 0.7 & 0.5 & 0.1 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \end{bmatrix}$	• correctly determines the Leslie matrix **[1 mark]**
12c)	Consider the population in Year 20 Using matrix facility of GDC $\mathbf{N_{20}} = \mathbf{L}^{19}\mathbf{N_1}$ $\approx \begin{bmatrix} \mathbf{62.7} \\ \mathbf{39.7} \\ \mathbf{12.6} \\ \mathbf{2.7} \end{bmatrix}$	• calculates a matrix representing the female population within a 20-year period **[1 mark]**
	Female population in Year 20 $\approx \mathbf{119}$	• calculates female population for a year within a 20-year period **[1 mark]**
	The female population is less than 125 within the 20-year period so the species is considered to be endangered.	• makes a suitable decision whether the species is considered endangered **[1 mark]**

Q	Sample response	The response
13a)	As $n \geq 30$, the sample mean distribution can be assumed to be normal. $\mu_{\overline{X}} = 64\ 800, \sigma_{\overline{X}} = \frac{4500}{\sqrt{360}}$ $P(64\ 000 \leq \overline{X} \leq 65\ 000) = 0.8$	• correctly calculates $\sigma_{\overline{X}}$ **[1 mark]** • calculates required probability **[1 mark]**
13b)	$\bar{x} = \frac{65\ 811 + 64589}{2} = \$65\ 200$	• correctly determines the value of $\bar{x}$ **[1 mark]**
13c)	μ lies within the confidence interval. The website's claim is reasonable.	• correctly recognises that the population mean lies within the confidence interval **[1 mark]** • correctly evaluates the reasonableness of the claim using a suitable comment **[1 mark]**

Q	Sample response	The response
14a)	$\frac{dv}{dt} = -(4+v^2)$ $\frac{1}{4+v^2}\frac{dv}{dt} = -1$ $\int \frac{1}{4+v^2}\,dv = \int -1\,dt$ $\frac{1}{2}\int \frac{2}{4+v^2}\,dv = \int -1\,dt$ $\frac{1}{2}\tan^{-1}\left(\frac{v}{2}\right)$ $= -t + c$	• correctly establishes a suitable integration result based on the separation of variables technique **[1 mark]** • determines one side of the general solution in terms of *v* **[1 mark]** • determines the other side of the general solution in terms of *t* **[1 mark]**
14b)	Given $v(0) = 1.5$ $\frac{1}{2}\tan^{-1}\left(\frac{1.5}{2}\right) = c$ $c \approx 0.32$ When $v = 0$: $\frac{1}{2}\tan^{-1}(0) \approx -t + 0.32$ $t \approx 0.32$ s The particle comes to rest after 0.32 s.	• determines an expression that represents the integration constant **[1 mark]** • determines the time when the particle is at rest **[1 mark]**

Q	Sample response	The response
15a)	$\overrightarrow{AB} = \begin{pmatrix}1\\1\\6\end{pmatrix} - \begin{pmatrix}3\\-1\\3\end{pmatrix} = \begin{pmatrix}-2\\2\\3\end{pmatrix}$	• correctly determines $\overrightarrow{AB}$ **[1 mark]**
15b)	Cartesian equation of the line is $\frac{x-3}{-2} = \frac{y+1}{2} = \frac{z-3}{3}$	• correctly substitutes the coordinates of A into the numerator of the equation **[1 mark]** • substitutes direction vector of the line into the denominator **[1 mark]**
15c)	From above, normal of the plane is $\begin{pmatrix}-2\\2\\3\end{pmatrix}$ Equation of plane using point A $\begin{pmatrix}x\\y\\z\end{pmatrix} . \begin{pmatrix}-2\\2\\3\end{pmatrix} = \begin{pmatrix}3\\-1\\3\end{pmatrix} . \begin{pmatrix}-2\\2\\3\end{pmatrix}$ $-2x + 2y + 3z = 1$	• substitutes values into a suitable mathematical rule that represents an equation of the plane **[1 mark]** • determines a Cartesian equation of the plane **[1 mark]**

Q	Sample response	The response
16		
	Assume the direction of the unit vectors as shown.	
	Consider rope attached: $\lvert \boldsymbol{F}_{net} \rvert = \boldsymbol{0}$	
	Consider the $\hat{\boldsymbol{\imath}}$ components	
	$\lvert \boldsymbol{T} \rvert \cos(25°) = 12\mathbf{g}\sin(\boldsymbol{\theta})$ ...[1]	• correctly determines an equation in terms of $\lvert \boldsymbol{T} \rvert$ (or T) and θ when the rope is attached **[1 mark]**
	Consider rope detached:	
	$\lvert \boldsymbol{F}_{\text{net}} \rvert = ma = 12 \times 5.6 = \mathbf{67.2}$	• correctly calculates the magnitude of the net force acting when the rope is detached **[1 mark]**
	Consider the $\hat{\boldsymbol{\imath}}$ components	
	$-12g\sin(\boldsymbol{\theta}) = 12 \times (-5.6)$	• determines an equation in terms of θ when the rope is detached **[1 mark]**
	$\boldsymbol{\theta} = 34.85°$	• determines a value representing θ **[1 mark]**
	Substituting into [1]	
	$\lvert \boldsymbol{T} \rvert \cos(25°) = 12\mathbf{g}\sin(34.85°)$	• determines an equation in terms of $\lvert \boldsymbol{T} \rvert$ (or T) **[1 mark]**
	$\lvert T \rvert \approx 74.15\ \text{N}$	• determines the magnitude of $\boldsymbol{T}$ **[1 mark]**

Q	Sample response	The response
17	Consider the sample: $\bar{x} = 5206 \text{ kg}$ $s = 356 \text{ kg}$ For a 90% CI, $z = 1.645$ Given the 90% CI is $(5159.1, 5252.9)$ kg Lower value of the CI $= \bar{x} + z\frac{s}{\sqrt{n}}$ $5159.1 = 5206 - 1.645 \times \frac{356}{\sqrt{n}}$ Using solve facility of GDC $n \approx 155.9$ Sample size is 156 Determining a 99% CI for this sample Using statistics facility of GDC CI is (5132.6, 5279.4) kg	• correctly determines the required value of z for a 90% CI **[1 mark]** • establishes an equation in terms of n **[1 mark]** • solves the equation to determine n **[1 mark]** • rounds n to an integer value **[1 mark]** • determines a 99% confidence interval **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response
18	$P(z) = z^3 + (i-a)z^2 - 2biz + 3i$ Using the remainder theorem $P(2i) = a - bi$	• correctly determines an expression for $P(2i)$ using the remainder theorem **[1 mark]**
	By substitution $P(2i) = (2i)^3 + (i-a)(2i)^2 - 2bi(2i) + 3i$ $= -8i - 4(i-a) + 4b + 3i$ $= -8i - 4i + 4a + 4b + 3i$ $= 4a + 4b - 9i$	• correctly determines an expression for $P(2i)$ using substitution into $P(z)$ **[1 mark]**
	Equating parts: $a = 4a + 4b$ $-b = -9$	• forms two simultaneous equations by equating parts **[1 mark]**
	So $a = -12$ and $b = 9$ $\therefore P(z) = z^3 + (i+12)z^2 - 18iz + 3i$ $P(a-bi) = P(-12-9i) \approx 1566 - 285i$	• determines $P(a-bi)$ using the values for a and b **[1 mark]**
	Since $P(-12-9i) \neq 0$, it is not reasonable that $(z-(a-bi))$ is a factor of $P(z)$.	• evaluates the reasonableness of the statement using mathematical reasoning **[1 mark]**

Q	Sample response	The response
19	**Method 1** – Considers a differential equation in terms of v and x and assumes the origin is at the point of release. Assume downwards as the positive direction $a = 9.8 - 0.1v$ $v\frac{dv}{dx} = 9.8 - 0.1v$ $\int \frac{v}{9.8 - 0.1v} dv = \int dx$ $\int \frac{v}{v - 98} dv = \int \frac{-1}{10} dx$ $\int 1 + \frac{98}{v - 98} dv = \int \frac{-1}{10} dx$ $v + 98\ln\|v - 98\| = \frac{-x}{10} + c$	• correctly establishes a differential equation in terms of v and x **[1 mark]** • determines general solution of the differential equation **[1 mark]**
	Assume the origin is at the point of release. Given $v(0) = 0 \Rightarrow c = 98\ln(98)$	• determines value for the constant **[1 mark]**
	$v + 98\ln\|v - 98\| = \frac{-x}{10} + 98\ln(98)$ Let the distance to the ocean surface be h metres. Consider time of drop for each required velocity When $v = 20\ \mathrm{m\,s^{-1}}, x = h$ $20 + 98\ln\|-78\| = \frac{-h}{10} + 98\ln(98)$ $h = 23.7\ \mathrm{m}$	• determines model for the velocity in terms of displacement **[1 mark]** • determines displacement of the drop for the minimum acceptable speed **[1 mark]**
	When $v = 50\ \mathrm{m\,s^{-1}}, x = h$ $50 + 98\ln\|-48\| = 98\ln(98) - \frac{h}{10}$ $h = 199.5\ \mathrm{m}$	• determines displacement of the drop for the maximum acceptable speed **[1 mark]**
	The range of the drone's flying height above the ocean surface should be between 23.7 m and 199.5 m.	• communicates range of the drone's flying height including units **[1 mark]**

Q	Sample response	The response
19	**Method 2** – Considers a differential equation in terms of v and x and assumes the origin is at the ocean surface. Assume downwards as the positive direction $a = 9.8 - 0.1v$ $v\frac{dv}{dx} = 9.8 - 0.1v$ $\int \frac{v}{9.8 - 0.1v} dv = \int 1\, dx$ $\int \frac{v}{v-98} dv = \int \frac{-1}{10} dx$ $\int 1 + \frac{98}{v-98} dv = \int \frac{-1}{10} dx$ $v + 98\ln\lvert v - 98\rvert = \frac{-x}{10} + c$ Assume the origin is the point directly below the drone on the ocean surface and let the height of release be h metres above the origin. Given $v(h) = 0 \Rightarrow\ c = 98\ln(98) + \frac{h}{10}$ $v + 98\ln\lvert v - 98\rvert = \frac{-x}{10} + 98\ln(98) + \frac{h}{10}$ Consider time of drop for each required velocity When $v = 20\ \mathrm{m\ s^{-1}}, x = 0$ $20 + 98\ln\lvert -78\rvert = 98\ln(98) + \frac{h}{10}$ $h = -23.7\ \mathrm{m}$ When $v = 50\ \mathrm{m\ s^{-1}}, x = 0$ $50 + 98\ln\lvert -48\rvert = 98\ln(98) + \frac{h}{10}$ $h = -199.5\ \mathrm{m}$ The range of the drone's flying height above the ocean surface should be between 23.7 m and 199.5 m.	• correctly establishes a differential equation in terms of v and x [1 mark] • determines a general solution of the differential equation **[1 mark]** • determines a value for the constant **[1 mark]** • determines a model for the velocity in terms of its displacement and initial height **[1 mark]** • determines displacement of the drop for the minimum acceptable speed **[1 mark]** • determines displacement of the drop for the maximum acceptable speed **[1 mark]** • communicates range of the drone's flying height including units **[1 mark]**

Q	Sample response	The response
19	**Method 3** – Considers a differential equation in terms of v and t and assumes release time at $t = 0$. Assume downwards as the positive direction $a = 9.8 - 0.1v$ $\frac{dv}{dt} = 9.8 - 0.1v$ $\int \frac{dv}{9.8 - 0.1v} = \int dt$	• correctly establishes a differential equation in terms of v and t **[1 mark]**
	$\frac{\ln(9.8 - 0.1v)}{-0.1} = t + c$	• determines a general solution of the differential equation **[1 mark]**
	$9.8 - 0.1v = Ae^{-0.1t}$ $v = 10(9.8 - Ae^{0.1t})$ Given $v(0) = 0 \Rightarrow A = 9.8$ $\therefore\ v = 98(1 - e^{-0.1t})$	• determines a model for the velocity using the initial boundary condition **[1 mark]**
	Consider time of drop for each required velocity When $v = 20\ \mathrm{m\ s^{-1}}$ and $v = 50\ \mathrm{m\ s^{-1}}$ $20 = 98(1 - e^{-0.1t})$ and $50 = 98(1 - e^{-0.1t})$ Using solve facility of GDC $t = 2.283$ s and $t = 7.138$ s	• uses this model to determine the time of the drop for both the minimum and maximum acceptable speeds **[1 mark]**
	Determining the change in displacement of drop using calculus facility of GDC: $v = 20\ \mathrm{m\ s^{-1}}$, $\Delta x = \int_0^{2.283} 98(1 - e^{-0.1t})\,dt \approx 23.7\ \mathrm{m}$	• determines displacement of the drop for the minimum acceptable speed **[1 mark]**
	$v = 50\ \mathrm{m\ s^{-1}}$, $\Delta x = \int_0^{7.138} 98(1 - e^{-0.1t})\,dt \approx 199.5\ \mathrm{m}$	• determines displacement of the drop for the maximum acceptable speed **[1 mark]**
	The range of the drone's flying height above the ocean surface should be between 23.7 m and 199.5 m.	• communicates range of the drone's flying height including units **[1 mark]**