LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book of books used

External assessment 2021

Question and response book

Specialist Mathematics

Paper 2 — Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted**.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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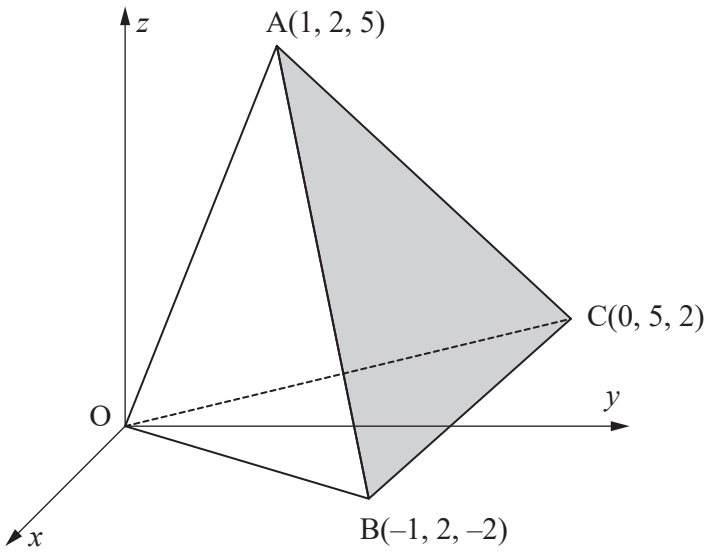
Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 55 marks.

QUESTION 11 (4 marks)

OABC is a triangular-based pyramid, as shown.



Not to scale

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Use a vector method to determine the area of the shaded face of the pyramid.

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QUESTION 12 (6 marks)

The life span of batteries manufactured by a company is assumed to be normally distributed with an unknown mean and standard deviation.

A supervisor at the company randomly selects n batteries and uses the life spans from this sample to calculate an approximate 95% confidence interval for the population mean of $(2321.4, 2423.6)$ hours.

a) Determine the mean life span for this sample of batteries. *[1 mark]*

The standard deviation of the life spans of batteries in this sample is 125 hours.

b) Determine n. *[3 marks]*

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c) Use the result from Question 12b) to explain whether the assumption that the life span of batteries is normally distributed is required to support the supervisor's calculations. *[2 marks]*

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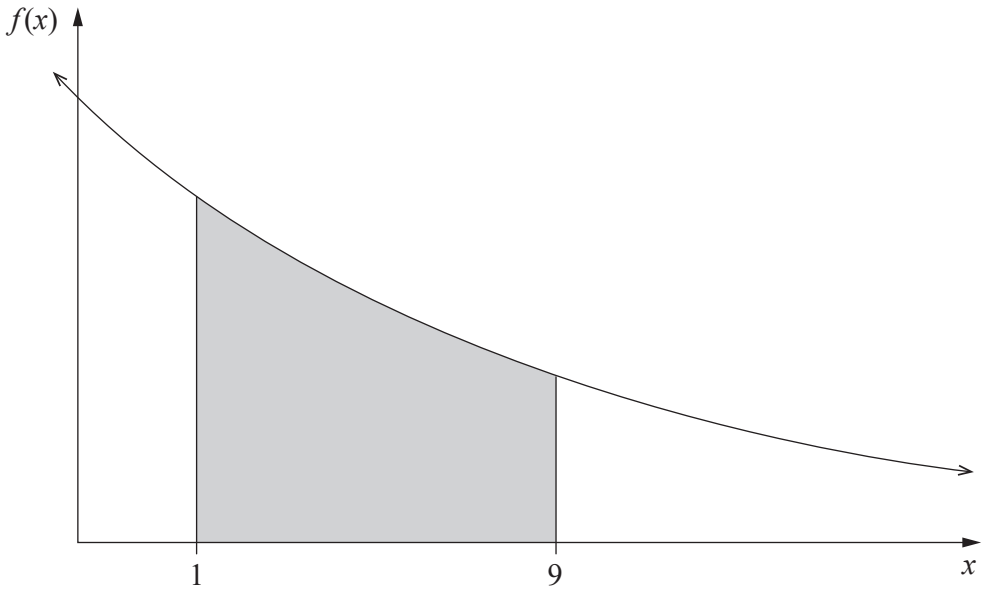
QUESTION 13 (6 marks)

The area under the graph of the function $f(x) = 0.2e^{-0.2x}$ for $1 \leq x \leq 9$ is shaded.



Not to scale

a) Use Simpson's rule with four intervals to determine an approximation for this area. *[4 marks]*

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b) Use a calculus approach to evaluate the reasonableness of your area approximation from Question 13a). *[2 marks]*

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QUESTION 14 (5 marks)

The Tasmanian thornbill is a species of bird that has an average life span of three years. Female thornbills do not reproduce in their first year, but produce an average of four female offspring in each of their second and third years. The survival rate of each age group is estimated as 25% in their first year and 30% in their second year.

A Leslie matrix, **L**, modelling the population distribution of the Tasmanian thornbill, has been partially completed.

$$\mathbf{L} = \begin{bmatrix} 0 & 4 & 4 \\ x & 0 & 0 \\ 0 & y & 0 \end{bmatrix}$$

a) State the values of x and y. *[1 mark]*

At the start of 2021, a study began into the population of Tasmanian thornbills. The study:

- estimated that the initial female population was 510 in their first year, 480 in their second year and 420 in their third year
- found that the ratio of male to female was approximately 1:2.

b) Estimate the total population of Tasmanian thornbills at the start of 2025. *[4 marks]*

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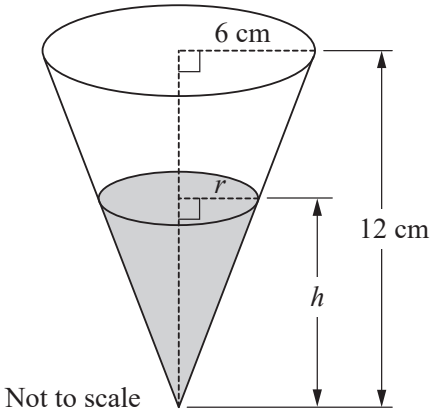
QUESTION 15 (8 marks)

Water is poured into a cone-shaped cup at a rate of 2 $\text{cm}^3\ \text{s}^{-1}$. The cup has a height of 12 cm and a radius of 6 cm, as shown.



As the cup fills, the ratio of the height of the water h to the surface radius of the water r remains constant.

a) Given that $h = 2r$, determine a function for the volume of water in the cup, V, in terms of h. Express your answer in simplified form. *[1 mark]*

b) Use the results from Question 15a) to show that the rate at which the height of water in the cup is increasing with respect to time is given by $\frac{8}{\pi h^2}$. *[3 marks]*

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c) Determine the rate at which the height of water in the cup is increasing with respect to time when the volume of water in the cup reaches half of the total capacity of the cup. *[4 marks]*

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QUESTION 16 (6 marks)

Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

Given $\mathbf{A}^4 = p\mathbf{A}^3 + q\mathbf{A}^2 + r\mathbf{A} + 3\mathbf{I}$, use matrix algebra to determine the value of the scalars p, q and r.

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QUESTION 17 (7 marks)

An object with a mass of 2 kg is released from rest at the top of a 1 metre long frictionless plane inclined at 30° to the horizontal.

A force of $\boldsymbol{P}$ newtons acting parallel to the plane opposes the motion of the object as it travels down the plane.

When the object is x metres from the top of the plane, its velocity is v m s^{-1}.

Given $\left|\boldsymbol{P}\right| = \dfrac{4}{\sqrt{4 - x^2}}$, determine x when $v = 2$.

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QUESTION 18 (6 marks)

Consider the polynomial $P(z) = z^3 + az^2 + bz + c$, where $a, b, c \in R$ and $z \in C$.

Two of the roots of $P(z)$ are also roots of $z^4 + z^3 + z^2 + z + 1$. The remaining root of $P(z)$ is $z = 2$.

Given $z^5 - 1 = (z-1)\left(z^4 + z^3 + z^2 + z + 1\right)$, determine a possible expression for $P(z)$.

Leave your answer in expanded form.

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QUESTION 19 (7 marks)

Consider the following information.

Continuous random variable X	mean	$E(X) = \mu = \int_{-\infty}^{\infty} x\, p(x)\, dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2\, p(x)\, dx$

The waiting time (minutes) until workers at a certain call centre receive their nth phone call, where $n \in Z^+$, is a random variable T with probability density function

$$f(t) = \begin{cases} \dfrac{k^n t^{n-1}}{(n-1)!} e^{-\frac{t}{3}}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where k is a positive constant.

The waiting time until workers receive their 5th call is collected from a random sample of 80 workers. Determine the probability that the mean waiting time from this sample is more than 16 minutes.

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END OF PAPER

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Write the question number you are responding to.

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