LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book of books used

External assessment 2021

Question and response book

Specialist Mathematics

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 55 marks.

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QUESTION 11 (5 marks)

Let $f(x) = \tan^{-1}\left(\frac{x}{2}\right)$ for suitable values of x where $f(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

a) Determine $f(2)$. *[1 mark]*

b) Determine $f'(2)$. *[2 marks]*

c) Use the results from Questions 11a) and 11b) to determine the equation of the tangent to the graph of $y = f(x)$ at $x = 2$. *[2 marks]*

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QUESTION 12 (8 marks)

Consider the plane $x - y - 2z = 15$.

a) Determine a vector $\boldsymbol{n}$ that is perpendicular to the plane. *[1 mark]*

b) Determine the vector equation of the line l that is perpendicular to the plane and contains the point $A(-2, 1, 3)$. *[1 mark]*

c) Use the result from Question 12b) to express the equation of the line l in parametric form. *[1 mark]*

The line l and the plane intersect at point S.

d) Show that the coordinates of S are $(2, -3, -5)$. *[3 marks]*

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e) Determine $\overrightarrow{AS}$. *[1 mark]*

f) Use a property of parallel vectors to verify that $\overrightarrow{AS}$ and $\boldsymbol{n}$ are parallel. *[1 mark]*

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QUESTION 13 (6 marks)

Use $z = a + bi$ and $w = c + di$, where $a, b, c, d \in R$, to prove

$$|z - w|^2 = |z|^2 + |w|^2 - 2Re(z\overline{w})$$

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QUESTION 14 (6 marks)

An object is projected vertically upwards from ground level. After the object has been in motion for t seconds, its position vector through the air, in metres, is modelled by

$$\boldsymbol{r}(t) = 5t(8-t)\,\hat{\boldsymbol{j}}$$

a) Determine the velocity of the object through the air, $\boldsymbol{v}(t)$, in metres per second. *[2 marks]*

b) Determine the number of seconds until the object reaches its maximum height. *[2 marks]*

c) Determine the maximum height that the object reaches, in metres. *[2 marks]*

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QUESTION 15 (4 marks)

Use partial fractions to determine $\int \frac{4x-17}{x^2-x-6}\,dx$, where $x \in R$, $x \neq -2$, $x \neq 3$.

Express your answer in the form $\ln\left|f(x)\right| + c$.

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QUESTION 16 (6 marks)

Use mathematical induction to prove that $2^{2n} + 3n - 1$ is divisible by 3 $\forall n \in Z^+$.

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QUESTION 17 (7 marks)

The area between the graphs of the functions $y = 4x$ and $y = 2x^2$ is rotated about the y-axis to form a solid of revolution with a volume of V units^3.

Determine the exact value of V.

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QUESTION 18 (6 marks)

This differential equation can be used to determine the current I (amperes) at time t (seconds) with voltage V (volts) in an electric circuit containing a resistance R (ohms):

$$k\frac{dI}{dt} + RI = V$$

where k, R and V are positive constants and $t \geq 0$.

Assuming that there is no current in the electric circuit initially, show that the size of the current can never be greater than $\frac{V}{R}$.

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QUESTION 19 (7 marks)

The velocity vectors of two objects A and B $\left(\text{in m s}^{-1}\right)$ at time t $\left(\text{in s}\right)$ are given respectively by

$$\boldsymbol{v}_{\text{A}} = 6\sin\left(3t\right)\hat{\boldsymbol{i}} + 6\cos\left(3t\right)\hat{\boldsymbol{j}}$$
$$\boldsymbol{v}_{\text{B}} = \cos\left(t\right)\hat{\boldsymbol{i}} - \sin\left(t\right)\hat{\boldsymbol{j}}$$

Objects A and B are initially at $\left(-2, 0, 2\right)$ and $\left(0, 1, -1\right)$ respectively. Determine the position of Object A when it is 4 metres away from Object B for the first time.

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END OF PAPER

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ADDITIONAL PAGE FOR STUDENT RESPONSES

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