Formula book

Specialist Mathematics v1.2

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

Mensuration

circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus

$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x}dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x)dx = -\cos(x) + c$

Calculus (continued)			
$\frac{d}{dx}\cos(x) = -\sin(x)$		$\int \cos(x)\,dx = \sin(x) + c$	
$\frac{d}{dx}\tan(x) = \sec^2(x)$		$\int \sec^2(x)\,dx = \tan(x) + c$	
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$		$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$	
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$		$\int \frac{-1}{\sqrt{a^2 - x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c$	
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$		$\int \frac{a}{a^2 + x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$	
chain rule	If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	

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Calculus (continued)			
integration by parts	$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$		$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
volume of a solid of revolution	about the x-axis		$V = \pi\int_a^b \left[f(x)\right]^2 dx$
	about the y-axis		$V = \pi\int_a^b \left[f(y)\right]^2 dy$
Simpson's rule	$\int_a^b f(x)\,dx \approx \frac{w}{3}\left[f(x_0) + 4\left[f(x_1) + f(x_3) + \cdots\right] + 2\left[f(x_2) + f(x_4) + \cdots\right] + f(x_n)\right]$		
simple harmonic motion	If $\frac{d^2x}{dt^2} = -\omega^2 x$ then $x = A\sin(\omega t + \alpha)$ or $x = A\cos(\omega t + \beta)$		
	$v^2 = \omega^2\left(A^2 - x^2\right)$	$T = \frac{2\pi}{\omega}$	$f = \frac{1}{T}$
acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$		

Real and complex numbers	
complex number forms	$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\ \mathrm{cis}(\theta)$
modulus	$\lvert z\rvert = r = \sqrt{x^2 + y^2}$
argument	$\arg(z) = \theta,\ \tan(\theta) = \frac{y}{x},\ -\pi < \theta \le \pi$
product	$z_1 z_2 = r_1 r_2\ \mathrm{cis}(\theta_1 + \theta_2)$
quotient	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\ \mathrm{cis}(\theta_1 - \theta_2)$
De Moivre's theorem	$z^n = r^n\,\mathrm{cis}(n\theta)$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
permutation	${}^nP_r = \dfrac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$	
combination	${}^nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\dfrac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}}, \bar{x} + z\dfrac{s}{\sqrt{n}}\right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \operatorname{cosec}^2(A)$
angle sum and difference identities	$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

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Trigonometry (continued)	
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$ $\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$ $\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$ $\cos(A)\sin(B) = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$

Vectors and matrices	
magnitude	$\lvert \mathbf{a} \rvert = \left\lvert \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right\rvert = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$
scalar (dot) product	$\mathbf{a} \cdot \mathbf{b} = \lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert \cos(\theta)$
	$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$
vector equation of a line	$\mathbf{r} = \mathbf{a} + k\mathbf{d}$

Vectors and matrices (continued)		
Cartesian equation of a line	$\frac{x-a_1}{d_1}=\frac{y-a_2}{d_2}=\frac{z-a_3}{d_3}$	
vector (cross) product	$a\times b=\lvert a\rvert\lvert b\rvert\sin(\theta)\hat{n}$	
	$a\times b=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$	
vector projection	a on $b=\lvert a\rvert\cos(\theta)\hat{b}=(a\cdot\hat{b})\hat{b}$	
vector equation of a plane	$r\cdot n=a\cdot n$	
Cartesian equation of a plane	$ax+by+cz+d=0$	
determinant	If $\mathbf{A}=\begin{bmatrix}a&b\\c&d\end{bmatrix}$ then $\det(\mathbf{A})=ad-bc$	
multiplicative inverse matrix	$\begin{bmatrix}a&b\\c&d\end{bmatrix}^{-1}=\frac{1}{\det(A)}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$, $\det(A)\neq 0$	
linear transformations	dilation	$\begin{bmatrix}a&0\\0&b\end{bmatrix}$
	rotation	$\begin{bmatrix}\cos(\theta)&-\sin(\theta)\\\sin(\theta)&\cos(\theta)\end{bmatrix}$
	reflection (in the line $y=x\tan(\theta)$)	$\begin{bmatrix}\cos(2\theta)&\sin(2\theta)\\\sin(2\theta)&-\cos(2\theta)\end{bmatrix}$

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g=9.8\ \text{m s}^{-2}$