– Public use –

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Sample assessment 2020

Question and response book

Specialist Mathematics

Paper 2 — Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted**.
- QCAA formula sheet provided.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (60 marks)

- 10 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 60 marks.

QUESTION 11 (6 marks)

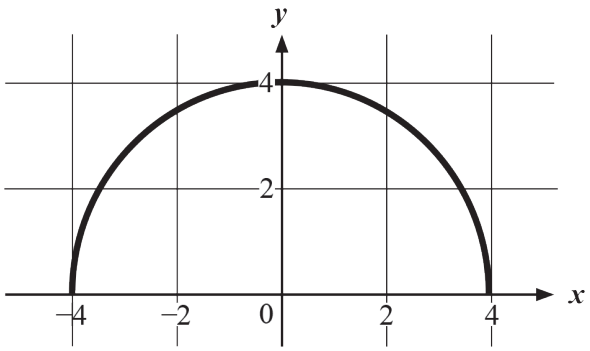
a) Use the binomial theorem to expand $(\cos(\theta) + i\sin(\theta))^3$. Express your result in simplified form. *[2 marks]*

b) Use De Moivre's theorem to expand $(\cos(\theta) + i\sin(\theta))^3$. *[1 mark]*

c) Use your results from 11a) and 11b) to prove $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$. *[3 marks]*

QUESTION 12 (6 marks)

Consider the semicircle shown below.



a) Determine the equation of the semicircle in the form $y = f(x)$. *[2 marks]*

b) Determine the volume formed by rotating the semicircle about the x-axis. *[2 marks]*

c) Evaluate the reasonableness of your answer to 12b). *[2 marks]*

QUESTION 13 (5 marks)

The heights of a sample of male and female students in a secondary school are summarised below.

	Sample size	Mean height (cm)	Standard deviation (cm)
Female	78	167.4	4.3
Male	62	176.3	5.1

a) Determine the approximate 95% confidence interval for the population mean of the heights of the female students. *[2 marks]*

The confidence interval (175.377 cm, 177.223 cm) was calculated for the population mean of the heights of the male students.

b) Determine the confidence level that was used to calculate this interval. Give your answer correct to 0.1%. *[3 marks]*

QUESTION 14 (5 marks)

An exponential random variable X with parameter $\lambda > 0$ has a probability density function given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The mean of X is $\frac{1}{\lambda}$.

The length of time that car owners keep their first car is an exponentially distributed random variable with a mean of 4 years.

a) Determine the probability that an individual keeps their first car for between 3 and 5 years. *[3 marks]*

50% of all individuals keep their first car for at most k years.

b) Determine the value of k. *[2 marks]*

QUESTION 15 (6 marks)

A research student has collected fertility and survival rates for a certain endangered species over a number of years. A small group of this species has been moved into a secure property. The Leslie matrix for the survival rates of the species is

$$\mathbf{L} = \begin{pmatrix} 0 & 0.1 & 0.8 & 0.5 \\ 0.95 & 0 & 0 & 0 \\ 0 & 0.83 & 0 & 0 \\ 0 & 0 & 0.64 & 0 \end{pmatrix}$$

The female population of the species moved into the property at the start of Year 1 is

$$\mathbf{N}_1 = \begin{pmatrix} 5 \\ 50 \\ 25 \\ 20 \end{pmatrix}$$

a) Explain the meaning of the number 0.8 in the Leslie matrix. *[1 mark]*

b) Calculate the expected total female population in the property at the start of Year 2. *[2 marks]*

c) Determine the expected total population in the property at the start of Year 11, based on 55% of the population being female. *[3 marks]*

QUESTION 16 (4 marks)

A plane passes through the points $A(0, 0, 1)$, $B(1, 2, 3)$ and $C(-1, 0, -2)$.

a) Determine a vector normal to the plane. *[2 marks]*

b) Use your result from 16a) to determine the Cartesian equation of the plane. *[2 marks]*

QUESTION 17 (7 marks)

At a honey bottling factory, the honey dispensing machine is designed to fill each bottle with masses that are normally distributed with a mean of 503.5 grams and a standard deviation of 17.2 grams.

The company randomly samples bottles of honey so that the probability of the mean mass being greater than 500 grams is 0.9.

a) Determine the sample size that the company used. *[5 marks]*

A worker at the factory claims that the results of the sample from 17a) would not be valid as the mass of honey dispensed into each bottle cannot be assumed to be normally distributed.

b) Comment on the reasonableness of this claim. *[2 marks]*

QUESTION 18 (7 marks)

An object of mass 5 kg moves in a straight line while being acted on by a net force F (N) in terms of its position from a reference point, x (5 m $\leq x \leq$ 10 m), such that

$$F = \frac{60 + 5x}{12x - 7x^2 + x^3}$$

Given the object was at rest when $x = 5$ m, determine the position of the object from the reference point when its speed is 0.5 m s^{-1}.

QUESTION 19 (7 marks)

Two objects, A and B, move in three-dimensional space such that their positions over time ($t \geq 0$) are described by the following vectors until they collide.

$$\boldsymbol{r_A}(t) = \cos(at)\,\hat{\boldsymbol{\imath}} + 2\sin(at)\,\hat{\boldsymbol{\jmath}} - \sqrt{3}\cos(at)\,\hat{\boldsymbol{k}}$$
$$\boldsymbol{r_B}(t) = \frac{bt}{2}\hat{\boldsymbol{\imath}} + \sqrt{3}bt\hat{\boldsymbol{\jmath}} + \frac{\sqrt{3}t}{2}\hat{\boldsymbol{k}}$$

a) Given $-\pi \leq a \leq 0$, determine **when** and **where** the objects collide. *[5 marks]*

b) Evaluate the reasonableness of your result from 19a). *[2 marks]*

QUESTION 20 (7 marks)

A cubic function of the form $y = ax^3 + bx^2 + cx$ passes through the points $(2, 2)$, $(3, 1)$ and $(5, 5)$.

a) Use matrix algebra to determine the values of a, b and c. *[5 marks]*

b) Evaluate the reasonableness of your result from 20a). *[2 marks]*

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Write the question number you are responding to.

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