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Sample assessment 2020

Question and response book

Specialist Mathematics

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula sheet provided.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (60 marks)

- 8 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

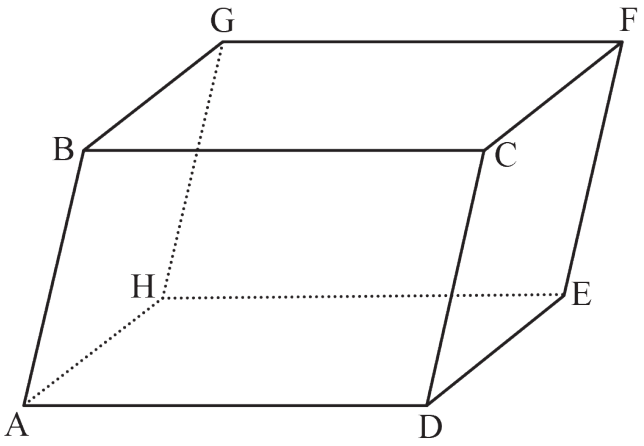
Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has eight questions and is worth 60 marks.

QUESTION 11 (9 marks)

A parallelepiped is a three-dimensional figure defined by three vectors that form six faces that are parallelograms.



Let $\overrightarrow{AD} = \boldsymbol{p}$, $\overrightarrow{AH} = \boldsymbol{q}$, $\overrightarrow{AB} = \boldsymbol{r}$.

a) Express the area of parallelogram ABGH in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and/or $\boldsymbol{r}$. *[1 mark]*

b) Assuming A is the origin, express the vector equation of the line passing through points B and D in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and/or $\boldsymbol{r}$. *[2 marks]*

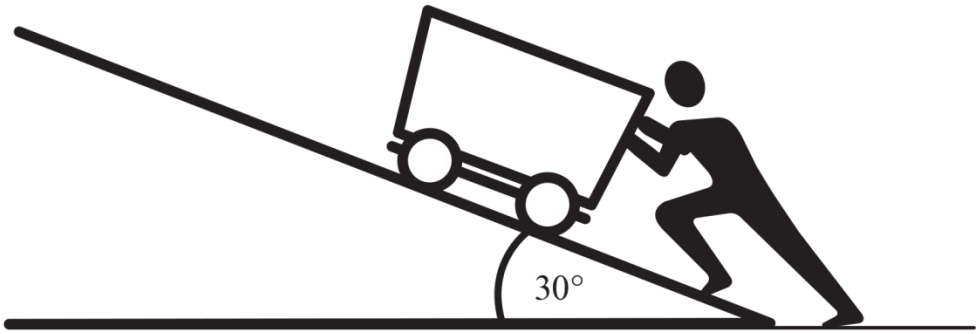
Two internal diagonals of the parallelepiped, AF and BE, are contained by the parallelogram ABFE.

c) Express both $\overrightarrow{AF}$ and $\overrightarrow{BE}$ in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$. *[2 marks]*

d) Use the results from 11c) to prove that internal diagonals AF and BE bisect each other. *[4 marks]*

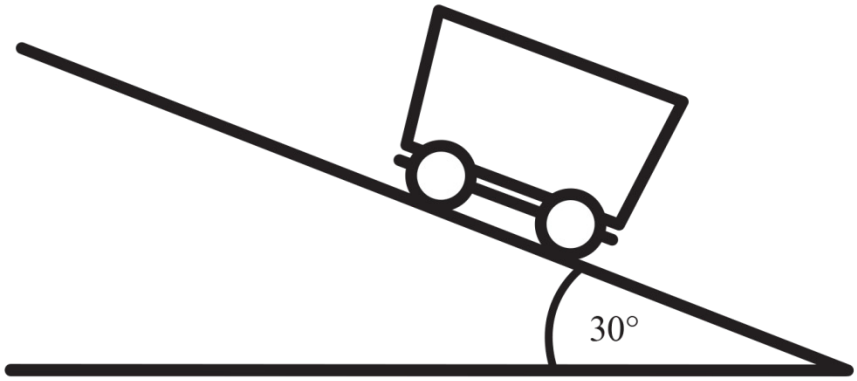
QUESTION 12 (10 marks)

The diagram below shows a ramp inclined at an angle of 30° to the horizontal. A trolley of mass 20 kg is being pushed up the ramp by a worker, with a net force parallel to the plane.



Not drawn to scale

a) Sketch and appropriately label all forces acting on the trolley in this situation on the diagram below. Assume there are negligible frictional forces and no air resistance. *[2 marks]*



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Note: If you make a mistake in the diagram, cancel it by ruling a single diagonal line through your work and use the additional diagram on page 18 of this question and response book.

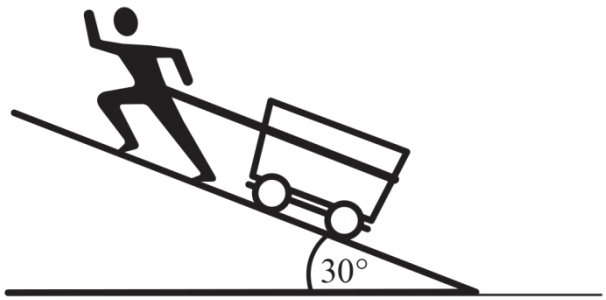
b) Determine the minimum force required by the worker to prevent the trolley from moving down the ramp. *[2 marks]*

c) If the worker exerts a force of 178 N on the trolley, determine the acceleration of the trolley up the ramp. *[3 marks]*

A worker **pulls** the trolley up the same ramp with a net force parallel to the plane, as shown in the diagram below.

The velocity v (m s^{-1}) of the trolley at time t (s) is given by $v = \frac{t}{2}$ m s^{-1} $(0 \leq t \leq 5$ s).

After 2 seconds, the trolley is positioned 5 m from the bottom of the ramp.



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d) Determine the position of the trolley from the bottom of the ramp after 4 seconds. *[3 marks]*

QUESTION 13 (7 marks)

Consider the following system of equations.

$$\begin{aligned} x + 2y + 3z &= 4 \\ 3x - 2y + 3z &= 5 \\ 2x + 4y + 6z &= k \end{aligned}$$

a) Use a Gaussian technique to determine the value of k for which there are infinitely many solutions. *[3 marks]*

Consider the original system of equations when $k = 3$.

b) Determine the number of solutions of this system of equations. Justify your conclusion. *[2 marks]*

c) Use your result from 13b) to describe a geometric interpretation of this system of equations. *[2 marks]*

QUESTION 14 (6 marks)

a) Determine the values of A and B in the following equation.

$$\frac{4x-1}{(2x-1)^2} = \frac{A}{(2x-1)} + \frac{B}{(2x-1)^2}$$

[3 marks]

b) Use the results from 14a) to determine the value of the following definite integral.

$$\int_1^2 \frac{4x-1}{(2x-1)^2}\, dx$$

[3 marks]

QUESTION 15 (6 marks)

Use mathematical induction to prove that $2^{2n} - 3n - 1$ is divisible by 3 for $n \in Z^+$.

QUESTION 16 (8 marks)

The general form of an ellipse in Cartesian form is

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

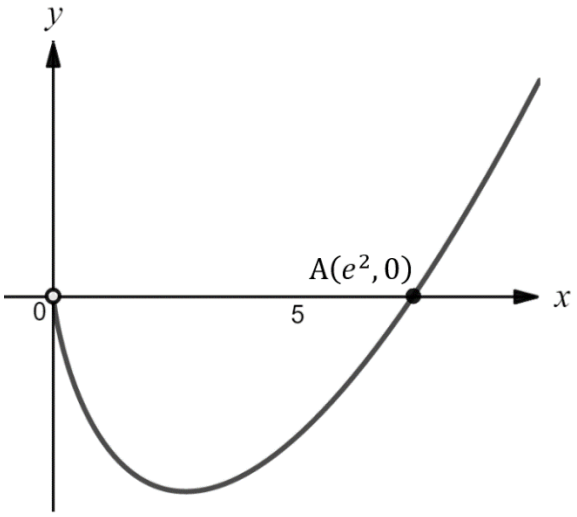
The path of an object with a position function of $\boldsymbol{r}(t)=\cos^2(t)\,\hat{\boldsymbol{\imath}}+\sin(2t)\,\hat{\boldsymbol{\jmath}}$ is stated to be elliptical.

Evaluate the reasonableness of this statement.

QUESTION 17 (7 marks)

The curve shown below represents the solution to the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + 1, x > 0$$



Use $y = wx$ (where w is a function of x) to determine the equation of the curve in simplest form.

QUESTION 18 (7 marks)

The following complex equations share at least one common solution.

$$z^3 - 8 = 0$$
$$kz^4 + 2kz^3 - 2kz^2 - 6z - 24k = 0$$

Given $k \in R$, determine the value of k.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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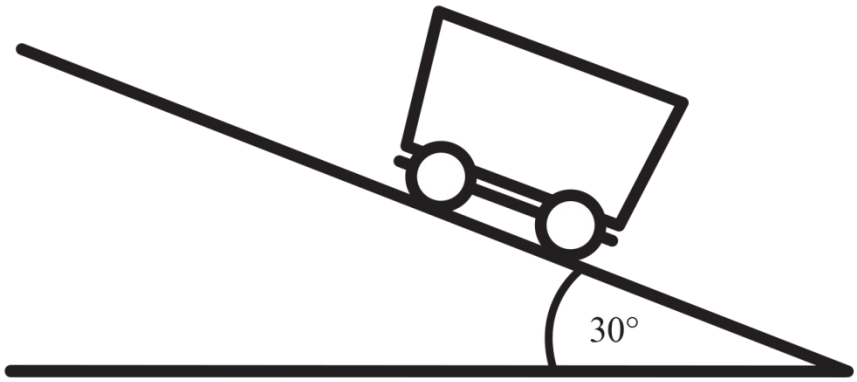
Write the question number you are responding to.

ADDITIONAL RESPONSE SPACE FOR QUESTION 12

If you want this diagram to be marked, rule a diagonal line through the diagram on page 4.



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