Specialist Mathematics marking guide

Sample external assessment 2020

Paper 2: Technology-active (70 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
Queensland Curriculum & Assessment Authority
For all Queensland schools

Introduction

The Queensland Curriculum and Assessment Authority (QCAA) has developed mock external assessments for each General senior syllabus subject to support the introduction of external assessment in Queensland.

An external assessment marking guide (EAMG) has been created specifically for each mock external assessment.

The mock external assessments and their marking guides were:

- developed in close consultation with subject matter experts drawn from schools, subject associations and universities
- aligned to the external assessment conditions and specifications in General senior syllabuses
- developed under secure conditions.

Purpose

This document consists of an EAMG and an annotated response.

The EAMG:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Where no response to a question has been made, a mark of ‘N’ will be recorded.

External assessment marking guide (EAMG)

Multiple choice

Question	Response
1	A
2	B
3	C
4	D
5	D
6	C
7	D
8	C
9	D
10	B

Short response

Question 11 (6 marks)

Sample response	The response
a) $(\cos(\theta) + i\sin(\theta))^3$ $= \cos^3(\theta) + 3i\cos^2(\theta)\sin(\theta) + 3i^2\cos(\theta)\sin^2(\theta) + i^3\sin^3(\theta)$	correctly uses binomial expansion **[1 mark]**
$= \cos^3(\theta) + 3i\cos^2(\theta)\sin(\theta) - 3\cos(\theta)\sin^2(\theta) - i\sin^3(\theta)$	correctly simplifies the expression **[1 mark]**
b) $(\cos(\theta) + i\sin(\theta))^3 = \cos(3\theta) + i\sin(3\theta)$	correctly uses De Moivre's theorem **[1 mark]**
c) Equating real parts: $\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\sin^2(\theta)$	correctly establishes the equation **[1 mark]**
$= \cos^3(\theta) - 3\cos(\theta)(1 - \cos^2(\theta))$ $= \cos^3(\theta) - 3\cos(\theta) + 3\cos^3(\theta)$ $= 4\cos^3(\theta) - 3\cos(\theta)$	correctly simplifies to complete the proof **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**

Question 12 (6 marks)

Sample response	The response
a) The equation of the circle is $x^2 + y^2 = 4^2$	correctly determines the equation of the circle **[1 mark]**
The equation of the semicircle is $y = \sqrt{16 - x^2}$	determines equation of semicircle **[1 mark]**
b) $V = \pi \int_{-4}^{4} \left(\sqrt{16 - x^2}\right)^2 dx$	establishes volume as definite integral **[1 mark]**
Using the integration facility of GDC $V \approx 268.08$ units³	determines volume **[1 mark]**
c) This result should equate to the volume of a sphere.	communicates rotation will form a sphere **[1 mark]**
$V = \frac{4}{3}\pi(4)^3 \approx 268.08$ units³ The answer is reasonable.	evaluates the reasonableness of the result **[1 mark]**

Question 13 (5 marks)

Sample response	The response
a) Female heights: 95% CI: $\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$ $\left(167.4 - z\frac{4.3}{\sqrt{78}}, 167.4 + z\frac{4.3}{\sqrt{78}}\right)$	correctly establishes the interval using the given information and the confidence interval definition **[1 mark]**
Using confidence interval facility of GDC $\text{CI} = (166.45, 168.35)$ cm	correctly determines the 95% CI **[1 mark]**
b) Male heights: Margin of error: $z\frac{s}{\sqrt{n}} = \frac{177.223 - 175.377}{2}$ $z\frac{5.1}{\sqrt{62}} = 0.923$ $z \approx 1.425$	correctly calculates the z value **[1 mark]**
Using normal cdf facility of GDC $P(-1.425 \leq z \leq 1.425) \approx 84.59\%$	determines confidence level **[1 mark]**
Confidence level used was 84.6%.	determines answer to required degree of accuracy (nearest 0.1%) **[1 mark]**

Question 14 (5 marks)

Sample response	The response
a) μ = 4 years $\Rightarrow$ $\lambda = \frac{1}{4}$	correctly determines the parameter λ **[1 mark]**
$P(3 \leq x \leq 5) = \int_3^5 0.25e^{-0.25x}dx$	establishes probability as definite integral **[1 mark]**
Using the integration facility of GDC $P(3 \leq x \leq 5) \approx 0.19$	determines probability **[1 mark]**
b) $P(0 < x < k) = 0.5$ $\int_0^k 0.25\, e^{-0.25x}dx = 0.5$	establishes definite integral equation **[1 mark]**
Using the solve facility of GDC Number of years $\approx$ 2.77	determines k **[1 mark]**

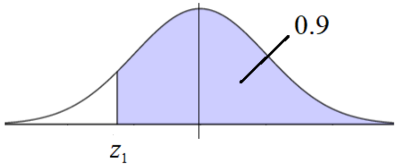
Question 15 (6 marks)

Sample response	The response
a) The number 0.8 refers to number of females born on average to a 3-year-old female.	correctly explains the meaning of the value **[1 mark]**
b) Year 2: Using matrix facility of GDC $\mathbf{N}_2 = \mathbf{L}^1 \times \mathbf{N}_1 = \begin{bmatrix} 35 \\ 4.75 \\ 41.5 \\ 16 \end{bmatrix}$	correctly generates the population matrix **[1 mark]**
Total female population ≈ 97	correctly calculates the total female population in Year 2, rounded to the nearest whole number **[1 mark]**
c) Year 11: Using matrix facility of GDC $\mathbf{N}_{11} = \mathbf{L}^{10} \times \mathbf{N}_1$	correctly establishes the formula to generate the population matrix **[1 mark]**
Total female population ≈ 91	correctly calculates the total female population at the start of Year 11 **[1 mark]**
Total population $\approx 91 \div 0.55$	
≈ 165	determines total population at the start of Year 11, rounded to the nearest whole number **[1 mark]**

Question 16 (4 marks)

Sample response	The response
a) Vectors in the plane: $\overrightarrow{AB} = b - a = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ $\overrightarrow{AC} = c - a = \begin{pmatrix} -1 \\ 0 \\ -3 \end{pmatrix}$	correctly determines 2 vectors in the plane **[1 mark]**
Normal to plane: Using vector facility of GDC $\boldsymbol{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix}$	determines normal to plane **[1 mark]**
b) Equation of plane: $\boldsymbol{r}.\boldsymbol{n} = \boldsymbol{a}.\boldsymbol{n}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix}.\begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix}. = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.\begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix}$	establishes equation using appropriate substitution **[1 mark]**
$-6x + y + 2z = 2$	determines Cartesian equation of plane **[1 mark]**

Question 17 (7 marks)

Sample response	Mark
a) $P(X > 500) = 0.9$ 	uses appropriate mathematical representation **[1 mark]**
Using inverse normal facility of GDC $z_1 \approx -1.282$	correctly determines the z value **[1 mark]**
$P(x > X) = \frac{X-\mu}{\frac{\sigma}{\sqrt{n}}}$ $P(x > 500) = \frac{500-503.5}{\frac{17.2}{\sqrt{n}}}$ $\frac{500-503.5}{\frac{17.2}{\sqrt{n}}} = -1.282$	generates equation in n **[1 mark]**
Using solve facility of GDC $n \approx 39.69$	solves for n **[1 mark]**
Smallest sample size is 40.	provides an authentic solution **[1 mark]**
b) Since $n \geq 30$, the sample mean distribution can be assumed to be normal (even if the amount dispensed into each bottle is not).	identifies normality for $n \geq 30$ **[1 mark]**
The claim is not reasonable.	evaluates the reasonableness of the claim **[1 mark]**

Question 18 (7 marks)

Sample response	The response
$F = ma$ $\frac{60+5x}{12x-7x^2+x^3} = 5a$ $a = \frac{12+x}{12x-7x^2+x^3}$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{12+x}{12x-7x^2+x^3}$	correctly determines a relationship between v and x **[1 mark]**
$v^2 = \int \frac{24+2x}{x(x-3)(x-4)}\,dx$ Using partial fractions: $\frac{24+2x}{x(x-3)(x-4)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x-4}$ $= \frac{A(x-3)(x-4) + Bx(x-4) + Cx(x-3)}{x(x-3)(x-4)}$ Equating parts: $24 + 2x = A(x-3)(x-4) + Bx(x-4) + Cx(x-3)$	correctly establishes the equation required to determine numerators A, B and C **[1 mark]**
Let $x = 0$: $24 = 12A \rightarrow A = 2$ Let $x = 3$: $30 = -3B \rightarrow B = -10$ Let $x = 4$: $32 = 4C \rightarrow C = 8$	correctly determines A, B and C **[1 mark]**

Sample response	The response
$v^2 = \int \frac{2}{x} - \frac{10}{x-3} + \frac{8}{x-4}\, dt$ $= 2\ln(x) - 10\ln(x-3) + 8\ln(x-4) + k \quad (5 \le x \le 10)$	determines general solution for v^2 **[1 mark]**
Given $v = 0$ when $x = 5$: $0 = 2\ln\lvert 5\rvert - 10\ln\lvert 5-3\rvert + 8\ln\lvert 5-4\rvert + k$ $k = 10\ln(2) - 2\ln(5)$ $v^2 = 2\ln(x) - 10\ln(x-3) + 8\ln(x-4) + 10\ln(2) - 2\ln(5)$	determines v^2 **[1 mark]**
When speed = 0.5, $v^2 = 0.25$ $2\ln(x) - 10\ln(x-3) + 8\ln(x-4) + 10\ln(2) - 2\ln(5) = 0.25$	establishes required value of velocity for equation **[1 mark]**
Using solve facility of GDC $x = 5.08$ m	solves for position of object **[1 mark]**

Question 19 (7 marks)

Sample response	The response
a) If the objects are to collide, they must have the same position at the same time, t_1	provides reasons for equating components of the two position vectors **[1 mark]**
$\cos(at_1) = \frac{bt_1}{2}$ ... [1] $2\sin(at_1) = \sqrt{3}bt_1$ $\sin(at_1) = \frac{\sqrt{3}bt_1}{2}$ ... [2] $-\sqrt{3}\cos(at_1) = \frac{\sqrt{3}t_1}{2}$ $\cos(at_1) = \frac{-t_1}{2}$ ... [3]	correctly establishes three simultaneous equations **[1 mark]**
Equating [1] and [3]: $b = -1$	solves for b **[1 mark]**
$[1]^2 + [2]^2$: $\cos^2(at_1) + \sin^2(at_1) = \frac{t_1^2}{4} + \frac{3t_1^2}{4}$ $1 = t_1^2$ $t_1 = 1 \quad (\text{as } t \geq 0)$	
collision occurs after 1 s.	determines when objects collide **[1 mark]**

Sample response	The response
At $t = 1$: $\boldsymbol{r_B}(t) = \frac{-1\times 1}{2}\hat{\boldsymbol{\imath}} + \sqrt{3}\times -1\times 1\,\hat{\boldsymbol{\jmath}} + \frac{\sqrt{3}\times 1}{2}\widehat{\boldsymbol{k}}$ collision occurs at $\frac{-1}{2}\hat{\boldsymbol{\imath}} - \sqrt{3}\,\hat{\boldsymbol{\jmath}} + \frac{\sqrt{3}}{2}\widehat{\boldsymbol{k}}$	determines position vector of point of collision **[1 mark]**
b) substituting $b = -1, t = 1$ into [1] $\cos(a) = \frac{-1}{2}$ $a = \cos^{-1}\left(\frac{-1}{2}\right)$ $= \frac{4\pi}{3}$ (given $-\pi \le a \le 0$)	determines value of a **[1 mark]**
$\boldsymbol{r_A}(t) = \cos\left(\frac{4\pi}{3}\right)\hat{\boldsymbol{\imath}} + 2\sin\left(\frac{4\pi}{3}\right)\hat{\boldsymbol{\jmath}} - \sqrt{3}\cos\left(\frac{4\pi}{3}\right)\widehat{\boldsymbol{k}}$ $= \frac{-1}{2}\hat{\boldsymbol{\imath}} - \sqrt{3}\,\hat{\boldsymbol{\jmath}} + \frac{\sqrt{3}}{2}\widehat{\boldsymbol{k}}$ Since $r_A = r_B$, the solution is reasonable.	evaluates the reasonableness of the solution **[1 mark]**

Question 20 (7 marks)

Sample response	The response
a) Substituting each of the points into $y = ax^3 + bx^2 + cx$ $(2, 2) \rightarrow \quad 2 = 8a + 4b + 2c$ $(3, 1) \rightarrow \quad 1 = 27a + 9b + 3c$ $(5, 5) \rightarrow \quad 5 = 125a + 25b + 5c$	correctly establishes three equations in a, b and c **[1 mark]**
Using matrix form: $\begin{bmatrix} 8 & 4 & 2 \\ 27 & 9 & 3 \\ 125 & 25 & 5 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$	translates three equations into a matrix equation **[1 mark]**
$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8 & 4 & 2 \\ 27 & 9 & 3 \\ 125 & 25 & 5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$	uses multiplicative inverse **[1 mark]**
Using matrix facility of GDC $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ -\frac{7}{3} \\ \frac{13}{3} \end{bmatrix}$ $a = \dfrac{1}{3}, b = -\dfrac{7}{3}, c = \dfrac{13}{3}$	determine a, b and c **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**
b) Verifying by substituting $(2, 2)$ into $y = \frac{1}{3}x^3 - \frac{7}{3}x^2 + \frac{13}{3}x$	substitutes given point into cubic function **[1 mark]**
$\text{RHS} = \frac{1}{3}(2)^3 - \frac{7}{3}(2)^2 + \frac{13}{3}(2)$ $= 2$ $= \text{LHS}$	evaluates the reasonableness of the solution **[1 mark]**