Specialist Mathematics marking guide

Sample external assessment 2020

Paper 1: Technology-free (70 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
Queensland Curriculum & Assessment Authority
For all Queensland schools

Introduction

The Queensland Curriculum and Assessment Authority (QCAA) has developed mock external assessments for each General senior syllabus subject to support the introduction of external assessment in Queensland.

An external assessment marking guide (EAMG) has been created specifically for each mock external assessment.

The mock external assessments and their marking guides were:

- developed in close consultation with subject matter experts drawn from schools, subject associations and universities
- aligned to the external assessment conditions and specifications in General senior syllabuses
- developed under secure conditions.

Purpose

This document consists of an EAMG and an annotated response.

The EAMG:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of '0' will be recorded.

Where no response to a question has been made, a mark of 'N' will be recorded.

External assessment marking guide (EAMG)

Multiple choice

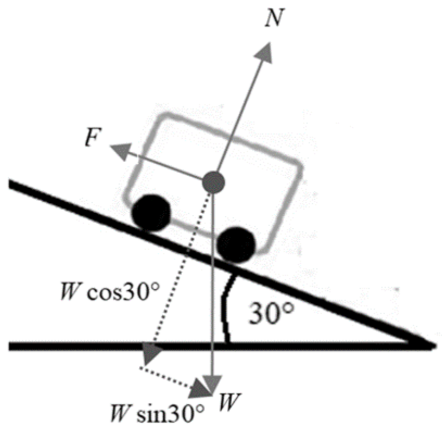
Question	Response
1	D
2	D
3	D
4	A
5	C
6	D
7	D
8	C
9	A
10	B

Short response

Question 11 (9 marks)

Sample response	The response
a) Area = $\lvert \boldsymbol{q} \times \boldsymbol{r} \rvert$	correctly establishes the expression for the area **[1 mark]**
b) $\overrightarrow{BD} = \boldsymbol{p} - \boldsymbol{r}$	correctly establishes the expression for the direction vector **[1 mark]**
Line BD is $\boldsymbol{r} + k(\boldsymbol{p} - \boldsymbol{r})$ where k is a parameter.	correctly expresses the vector equation **[1 mark]**
c) $\overrightarrow{AF} = \boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r}$	correctly expresses $\overrightarrow{AF}$ **[1 mark]**
$\overrightarrow{BE} = \boldsymbol{p} + \boldsymbol{q} - \boldsymbol{r}$	correctly expresses $\overrightarrow{BE}$ **[1 mark]**
d) Consider A as the origin. Required to Prove: the position vectors of the midpoint of $\overrightarrow{AF}$ and $\overrightarrow{BE}$ are the same.	correctly communicates a worded statement of the result to be proven **[1 mark]**
Mid-point of $\overrightarrow{AF} = \frac{1}{2}\overrightarrow{AF} = \frac{1}{2}(\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$	expresses midpoint vector of $\overrightarrow{AF}$ in terms of $\boldsymbol{p}, \boldsymbol{q}$ and $\boldsymbol{r}$ in simplified form **[1 mark]**
Mid-point of $\overrightarrow{BE} = \boldsymbol{r} + \frac{1}{2}\overrightarrow{BE} = \boldsymbol{r} + \frac{1}{2}(\boldsymbol{p} + \boldsymbol{q} - \boldsymbol{r})$ $= \frac{1}{2}(\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$	expresses midpoint vector of $\overrightarrow{BE}$ in terms of $\boldsymbol{p}, \boldsymbol{q}$ and $\boldsymbol{r}$ in simplified form **[1 mark]**
Since the mid-point of each diagonal have the same position vector, they must bisect each other.	provides an explanation to complete the proof **[1 mark]**

Question 12 (10 marks)

Sample response	The response
a)  	correctly sketches the three forces (indicated by solid lines) **[1 mark]** appropriately labels the three forces (F, N and W) **[1 mark]**
b) $F = 20g\sin 30°$	correctly establishes the expression for the minimum force **[1 mark]**
$= 98\text{ N}$	correctly determines the minimum force **[1 mark]**
c) $F_{NET} = ma$ $178 - 98 = 20a$	establishes equation of motion in terms of the acceleration a **[1 mark]**
$a = \frac{80}{20} = 4\ \text{m s}^{-2}$	determines a **[1 mark]** uses correct units for a **[1 mark]**

Sample response	The response
d) $s = \int \frac{t}{2} dt$ $s = \frac{t^2}{4} + c$	correctly determines the general displacement function **[1 mark]**
Given $s = 5$ when $t = 2$ $5 = \frac{2^2}{4} + c$ $c = 4$ $s = \frac{t^2}{4} + 4$	determines displacement function **[1 mark]**
When $t = 4$, $s = \frac{4^2}{4} + 4$ $= 8$ m	determines position at the given time **[1 mark]**

Question 13 (7 marks)

Sample response	The response
a) In matrix form: $\left[\begin{array}{ccc\|c} 1 & 2 & 3 & 4 \\ 3 & -2 & 3 & 5 \\ 2 & 4 & 6 & k \end{array}\right]$	correctly establishes a row of zeros using a Gaussian technique **[1 mark]**
$\left[\begin{array}{ccc\|c} 1 & 2 & 3 & 4 \\ 0 & -8 & -6 & -7 \\ 0 & 0 & 0 & k-8 \end{array}\right] \begin{array}{l} \boldsymbol{R_1} \\ \boldsymbol{R_2' = R_2 - 3R_1} \\ \boldsymbol{R_3' = R_3 - 2R_1} \end{array}$	correctly communicates the corresponding row operations **[1 mark]**
If $k = 8$, there are infinitely many solutions.	determines value of k **[1 mark]**
b) For $k = 3$, equation 3 becomes $0 = -5$	generates inconsistent equation **[1 mark]**
The system has no solutions.	determines number of solutions **1 mark]**
c) This system of equations can be geometrically interpreted as two parallel planes and one intersecting plane.	describes two planes as parallel **[1 mark]** describes remaining plane as an intersecting (non-parallel) plane **[1 mark]**

Question 14 (6 marks)

Sample response	The response
a) $\frac{4x-1}{(2x-1)^2} = \frac{A}{(2x-1)} + \frac{B}{(2x-1)^2}$ $= \frac{A(2x-1)+B}{(2x-1)^2}$	
Equating parts: $4x-1 = A(2x-1)+B$	correctly equates numerators to generate an equation in A and B **[1 mark]**
Let $x = \frac{1}{2}$: $4 \times \frac{1}{2} - 1 = A(0) + B$ $1 = B$	correctly determines B **[1 mark]**
Let $x = 0$: $-1 = -A + B$ $A = 2$	correctly determines A **[1 mark]**
b) $\int_1^2 \frac{4x-1}{(2x-1)^2}\,dx$	
$= \int_1^2 \frac{2}{(2x-1)} + \frac{1}{(2x-1)^2}$ $= \left(\ln(2x-1) + \frac{-1}{2(2x-1)}\right)\Big\|_1^2$	determines $\int \frac{2}{(2x-1)}\,dx$ **[1 mark]** determines $\int \frac{1}{(2x-1)^2}\,dx$ **[1 mark]**
$= \left(\ln(3) + \frac{-1}{6}\right) - \left(0 + \frac{-1}{2}\right)$ $= \ln(3) + \frac{1}{3}$	determines value of the definite integral **[1 mark]**

Question 15 (6 marks)

Sample response	The response
$P(n) = 2^{2n} - 3n - 1$ is divisible by 3	
Initial statement: Show that $P(1)$ is true: $2^2 - 3 - 1 = 0$ which is divisible by 3	correctly proves the *initial statement* **[1 mark]**
Assume $P(k)$ is true $2^{2k} - 3k - 1 = 3A$ for $A \in Z^+$ ...(1) *Inductive step:* R.T.P $P(k+1)$ is true: $2^{2(k+1)} - 3(k+1) - 1 = 3B$ for $B \in Z^+$	correctly states the assumption and the proof requirement for the *inductive step* **[1 mark]**
LHS $= 2^{2k}2^2 - 3k - 4$ $= 4(2^{2k} - 1) - 3k$ $= 4(3A + 3k) - 3k$ ...using (1)	generates linear expression using simplification and substitution techniques **[1 mark]**
$= 12A + 12k - 3k$ $= 12A + 9k$	shows $P(k+1)$ has a factor of 3 **[1 mark]**
$= 3(4A + 3k)$	provides evidence of reasoning used to identify result as a multiple of 3 **[1 mark]**
$= 3B$ as $4A + 3k \in Z^+$ $=$ RHS	
So $P(k+1)$ is true. By mathematical induction, the formula is true for $n = 1, 2, \ldots$	communicates using LHS = RHS format and suitable conclusion **[1 mark]**

Question 16 (8 marks)

Sample response	The response
$\boldsymbol{r}(t) = \cos^2(t)\,\hat{\boldsymbol{\imath}} + \sin(2t)\,\hat{\boldsymbol{\jmath}}$. In parametric form, $x = \cos^2(t) \quad \ldots (1)$ $y = \sin(2t)$	correctly expresses the given equation in parametric form **[1 mark]**
$y = 2\sin(t)\cos(t)$	correctly uses the double-angle identity **[1 mark]**
$y^2 = 4\sin^2(t)\cos^2(t)$	correctly determines y^2 **[1 mark]**
$y^2 = 4(1-\cos^2(t))\cos^2(t)$	expresses y^2 in terms of $\cos^2(t)$ **[1 mark]**
Using (1): $y^2 = 4(1-x)x$	determines Cartesian equation **[1 mark]**
$4(x^2 - x) + y^2 = 0$ $4\left(x^2 - x + \frac{1}{4} - \frac{1}{4}\right) + y^2 = 0$ $4\left(x - \frac{1}{2}\right)^2 + y^2 = 1$	uses appropriate procedure (completing the square) **[1 mark]**
$\dfrac{\left(x-\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^2} + \dfrac{y^2}{1^2} = 1$	derives elliptical form of the equation **[1 mark]**
As the position of the object can be expressed in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, the path is elliptical.	evaluates the reasonableness of the statement **[1 mark]**

Question 17 (7 marks)

Sample response	The response
Given $y = wx$ Using product rule $\frac{dy}{dx} = w + x\frac{dw}{dx}$	correctly determines $\frac{dy}{dx}$ **[1 mark]**
Equating $\frac{dy}{dx}$ $\frac{y}{x} + 1 = w + x\frac{dw}{dx}$	correctly generates the equation independent of $\frac{dy}{dx}$ **[1 mark]**
$\frac{y}{x} + 1 = \frac{y}{x} + x\frac{dw}{dx}$ $\left(\text{given } w = \frac{y}{x}\right)$	correctly generates the equation in terms of $\frac{dw}{dx}$, x and y **[1 mark]**
$1 = x\frac{dw}{dx}$ $\int 1\,dw = \int \frac{dx}{x}$ $w = \ln\|x\| + c$ $\frac{y}{x} = \ln\|x\| + c$	correctly determines the general solution for w **[1 mark]**
Given $y = 0$ when $x = e^2$ $c = -\ln(e^2) = -2$	determines c **[1 mark]**
The equation of the curve is $y = x\ln\|x\| - 2x$	determines equation in simplest form **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**

Question 18 (7 marks): Method 1

Sample response	The response
Consider $z^3 - 8 = 0$ By inspection, one solution is $z_1 = 2$ Checking if z_1 is a solution of equation 2: $k(2)^4 + 2k(2)^3 - 2k(2)^2 - 6(2) - 24k$ $= 16k + 16k - 8k - 12 - 24k$ $= -12$ $\neq 0$ So, $z_1 = 2$ is not a common solution.	correctly identifies 2 is not the common solution **[1 mark]**
Since the coefficients of both equations are real, both complex conjugate solutions from the first equation must be common solutions. One complex conjugate solution of equation 1 is $z_2 = 2\text{cis}\left(\frac{2\pi}{3}\right)$	correctly identifies the complex conjugate solution from the first equation **[1 mark]**
Substituting z_2 into the equation 2: $k\left(2\text{cis}\left(\frac{2\pi}{3}\right)\right)^4 + 2k\left(2\text{cis}\left(\frac{2\pi}{3}\right)\right)^3 -$ $2k\left(2\text{cis}\left(\frac{2\pi}{3}\right)\right)^2 - 6\left(2\text{cis}\left(\frac{2\pi}{3}\right)\right) - 24k = 0$	correctly applies the remainder theorem **[1 mark]**

Sample response	The response
$k\ 16\text{cis}\left(\frac{8\pi}{3}\right) + k\ 16\text{cis}\left(\frac{6\pi}{3}\right) - k\ 8\text{cis}\left(\frac{4\pi}{3}\right) - 12\text{cis}\left(\frac{2\pi}{3}\right) - 24k = 0$	uses de Moivre's theorem **[1 mark]**
Equating real parts: $\left(16\cos\left(\frac{8\pi}{3}\right) + 16\cos(2\pi) - 8\cos\left(\frac{4\pi}{3}\right) - 24\right)k = 12\cos\left(\frac{2\pi}{3}\right)$ $(-8 + 16 + 4 - 24)k = -6$	uses appropriate procedure **[1 mark]**
$k = \frac{1}{2}$	solves for k **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**

Question 18 (7 marks): Method 2

Sample response	The response
Consider $z^3 - 8 = 0$ By inspection, one solution is $z_1 = 2$ Checking if z_1 is a solution of equation 2: $k(2)^4 + 2k(2)^3 - 2k(2)^2 - 6(2) - 24k$ $= 16k + 16k - 8k - 12 - 24k$ $= -12$ $\neq 0$ So, $z_1 = 2$ is not a common solution.	correctly identifies 2 is not the common solution **[1 mark]**
Since the coefficients of both equations are real, both complex conjugate solutions from the equation 1 must be common solutions. $(z-2)$ is a factor of $z^3 - 8 = 0$ $\begin{array}{r} z^2+2z+4 \\ z-2\ \overline{)\,z^3+0z^2+0z-8} \\ \underline{z^3-2z^2} \\ 2z^2+0z \\ -\ \underline{2z^2-4z} \\ 4z-8 \\ -\ \underline{4z-8} \\ \underline{0} \end{array}$	correctly uses the division procedure **[1 mark]**
So $z^2 + 2z + 4$ must be a common factor of equation 2.	correctly determines the quadratic factor **[1 mark]**
$\begin{array}{r} kz^2 \qquad\quad - 6k \\ z^2+2+4\ \overline{)\,kz^4+2kz^3-2kz^2-6z-24k} \\ -\ \underline{kz^4+2kz^3+4kz^2} \qquad\qquad\quad \end{array}$	uses division procedure **[1 mark]** determines remaining quadratic factor **[1 mark]**

Sample response	The response
$$\begin{array}{r} -6kz^2 - 6z - 24k \\ - \underline{6kz^2 - 12kz - 24k} \\ \underline{-6z + 12kz} \end{array}$$	applies remainder theorem and solves for k **[1 mark]**
$-6z + 12kz = 0$ $k = \frac{1}{2}$	shows logical organisation communicating key steps **[1 mark]**

Question 18 (7 marks): Method 3

Sample response	The response
Consider $z^3 - 8 = 0$ By inspection, one solution is $z_1 = 2$ Checking if z_1 is a solution of the other equation: $k(2)^4 + 2k(2)^3 - 2k(2)^2 - 6(2) - 24k$ $= 16k + 16k - 8k - 12 - 24k$ $= -12$ $\neq 0$ So, $z_1 = 2$ is not a common solution.	correctly identifies that 2 is not a common solution **[1 mark]**
The other solutions are: $z_2 = 2\text{cis}\left(\frac{2\pi}{3}\right), z_3 = 2\text{cis}\left(-\frac{2\pi}{3}\right)$	correctly identifies at least one complex conjugate solution **[1 mark]**
$= -1 + \sqrt{3}i, -1 - \sqrt{3}i$	correctly converts at least one solution to Cartesian form **[1 mark]**
Since the coefficients of both equations are real, both complex conjugate solutions from the first equation must be common solutions.	
Substituting z_2 into equation 2: $k\left(-1+\sqrt{3}i\right)^4 + 2k\left(-1+\sqrt{3}i\right)^3 - 2k\left(-1+\sqrt{3}i\right)^2 - 6\left(-1+\sqrt{3}i\right) - 24k = 0$ $\left(-8+8\sqrt{3}i\right)k + 2k \times 8 - 2k\left(-2-2\sqrt{3}i\right) + 6 - 6\sqrt{3}i - 24k = 0$	applies remainder theorem **[1 mark]**

Sample response	The response
$-12k + 12\sqrt{3}ik + 6 - 6\sqrt{3}i = 0$ Equating real parts: $-12k + 6 = 0$	equates corresponding parts **[1 mark]**
$k = \frac{1}{2}$	solves for k **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**