LUI | | | | | | | | | | |

School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book of books used

External assessment

**Question and response book**

# Specialist Mathematics

## Paper 2 — Technology-active

### Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

### General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted**.
- QCAA formula book provided.
- Planning paper will not be marked.

### Section 1 (10 marks)

- 10 multiple choice questions

### Section 2 (55 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

# Section 1

## Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

| | A | B | C | D |
|---|---|---|---|---|
| Example: | ● | ○ | ○ | ○ |

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| | A | B | C | D |
|---|---|---|---|---|
| **1.** | ○ | ○ | ○ | ○ |
| **2.** | ○ | ○ | ○ | ○ |
| **3.** | ○ | ○ | ○ | ○ |
| **4.** | ○ | ○ | ○ | ○ |
| **5.** | ○ | ○ | ○ | ○ |
| **6.** | ○ | ○ | ○ | ○ |
| **7.** | ○ | ○ | ○ | ○ |
| **8.** | ○ | ○ | ○ | ○ |
| **9.** | ○ | ○ | ○ | ○ |
| **10.** | ○ | ○ | ○ | ○ |

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## Section 2

### Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 55 marks.

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**THIS PAGE WILL NOT BE MARKED**

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## QUESTION 11 (5 marks)

Teams A, B, C, D and E participated in a competition with the following results:

- A defeated D.
- B defeated A, C and E.
- C defeated A and E.
- D defeated B, C and E.
- E defeated A.

To rank the teams at the end of the competition, the organisers constructed a dominance matrix, **N**, that is partially completed.

a) By allocating 1 to represent 'defeated' and 0 to represent either 'was defeated by' or 'no result', complete matrix **N**. *[1 mark]*

$\mathbf{N} =$

| Winning teams \ Losing teams | A | B | C | D | E |
|---|---|---|---|---|---|
| A | | | | | |
| B | | | | | |
| C | | | | | |
| D | | | | | |
| E | | | | | |

**Note:** If you make a mistake in the matrix, cancel it by ruling a single diagonal line through your work and use the additional matrix on page 26 of this question and response book.

The organisers need to rank the teams into individual places from first to fifth place.

They decide to use the ranking model $\mathbf{N} + \mathbf{N}^2$ to achieve this.

b) Use the model $\mathbf{N} + \mathbf{N}^2$ to rank the teams. *[2 marks]*

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c) Use the result from 11b) to identify a limitation of the organisers' ranking model. *[1 mark]*

d) State a mathematical refinement the organisers could consider to overcome the limitation of the ranking model identified in 11c). *[1 mark]*

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## QUESTION 12 (9 marks)

For a certain experiment, the number of yeast cells, $N$, after $t$ hours in a test tube can be modelled by the differential equation

$$\frac{dN}{dt} = \frac{1}{1000} N(1000 - N) \text{ for } t \geq 0$$

a) Given $\frac{1000}{N(1000-N)} = \frac{1}{N} + \frac{1}{1000-N}$, show that the general solution of the differential equation can be expressed as

$$\ln\left|\frac{N}{1000-N}\right| = t + c$$

*[2 marks]*

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A scientist commenced this experiment at 9:00 am on a certain day and observed that 100 yeast cells were present at this time.

b) Show that the solution of the differential equation can be expressed as

$$N = \frac{1000}{1+9e^{-t}}$$

*[3 marks]*

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c) Determine the time of day when 900 yeast cells were present. *[2 marks]*

The scientist predicted that the number of yeast cells would eventually exceed 1200.

d) Evaluate the reasonableness of the scientist's prediction. *[2 marks]*

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## QUESTION 13 (6 marks)

Data records show that the speeds of cars at a particular location on a highway are normally distributed with a mean of 98.7 km $h^{-1}$ and a standard deviation of 4.1 km $h^{-1}$. The speed limit at this location is 100 km $h^{-1}$.

A police officer plans to record the speeds of 20 randomly selected cars at this location.

a) Determine the expected number of cars in the sample that will be travelling within $\pm 1$ km $h^{-1}$ of the population mean. *[2 marks]*

b) Determine the probability that the mean speed of this sample will exceed the speed limit. *[2 marks]*

There is a 5% probability that the mean speed of this sample will exceed $k$.

c) Determine the value of $k$. *[2 marks]*

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## QUESTION 14 (5 marks)

The time, $t$, (months) that it takes before a phone owner cracks the screen on their phone can be modelled by an exponentially distributed random variable

$$f(t) = \begin{cases} 0.16e^{-0.16t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

a) Show that $f(t)$ is a probability density function. *[1 mark]*

b) Determine the probability that a phone owner cracks the screen on their phone within 1 year. *[2 marks]*

Three-quarters of phone owners take between 1 and $m$ months before they crack the screen on their phone.

c) Determine the value of $m$. *[2 marks]*

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## QUESTION 15 (4 marks)

The position vectors of points P and Q are $2\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}}$ and $2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} - 4\hat{\boldsymbol{k}}$ respectively.

Let O be the origin.

a) Determine the angle POQ. *[2 marks]*

Points O, P and Q are joined to form a triangle.

b) Determine the area of triangle POQ. *[2 marks]*

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## QUESTION 16 (6 marks)

Consider the identity

$$\cos(4\theta) = A\cos^4(\theta) + B\sin^2(\theta) + C \text{ where } A, B \text{ and } C \in Z$$

a) Determine the values of $A$, $B$ and $C$ using De Moivre's theorem. *[5 marks]*

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b) State an appropriate method of verifying your results from 16a). *[1 mark]*

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## QUESTION 17 (7 marks)

An object is released from rest at a height of 100 m above the ground.

The motion of the vertical descent of the object is modelled by

$$v\frac{dv}{dx} = 9.8 - 0.004v^2 \quad (v \geq 0)$$

where $v$ is the velocity (m s$^{-1}$) and $x$ is the displacement from the ground (m).

Determine the velocity of the object when it strikes the ground.

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## QUESTION 18 (6 marks)

The mass of a certain species of kangaroo is known to be normally distributed with a mean mass of $\mu$ kg and standard deviation of $\sigma$ kg.

When one of the kangaroos is randomly selected, the probability that its mass is greater than 83.2 kg is 0.145.

When a sample of 12 kangaroos is randomly selected, the probability that the sample mean mass is less than 74.1 kg is 0.079.

A 90% approximate confidence interval for $\mu$ is calculated using a random sample of $n$ of the kangaroos that has a sample mean mass of 79.1 kg and a sample standard deviation equal to $\sigma$.

Determine the possible range of values that $n$ could have been, given that the confidence interval did not contain $\mu$.

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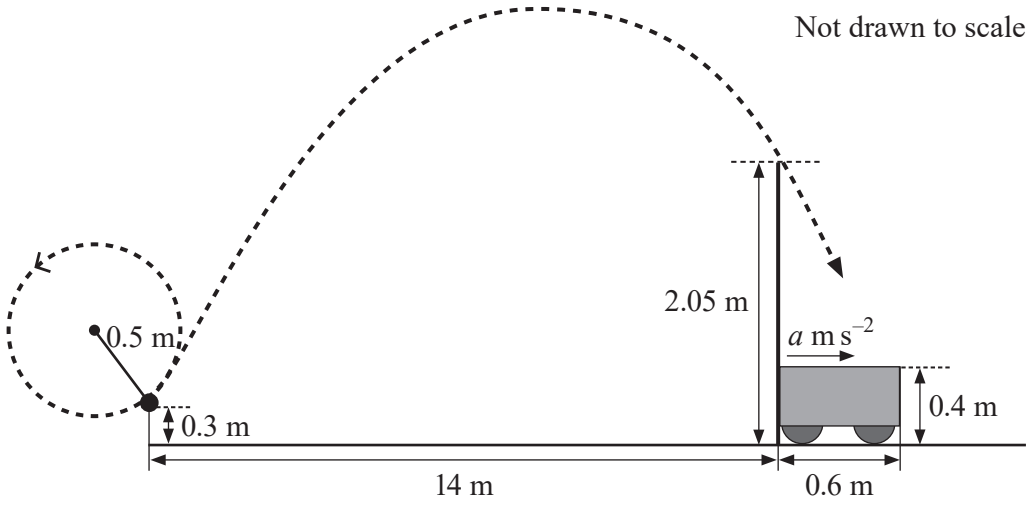
## QUESTION 19 (7 marks)

An object is swinging at the end of a 0.5 m length of string in a vertical circular path with a constant angular speed, completing each revolution in 0.24 seconds.

The object is projected from a height of 0.3 m above the ground in a vertical plane and **just** passes over a narrow pole as shown in the diagram. The pole is 2.05 m high and its base is 14 m horizontally from where the object was projected.




A flat-topped vehicle of length 0.6 m and height 0.4 m is initially at rest against the pole as shown in the diagram. At the instant that the object is projected, the vehicle moves in a horizontal direction away from the pole in the same vertical plane with an acceleration of magnitude of $a$ m s$^{-2}$. The object strikes the middle of the top of the vehicle.

Assuming that air resistance is negligible, use vector calculus to model the motion of the projectile in order to determine the value of $a$.

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**END OF PAPER**

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## ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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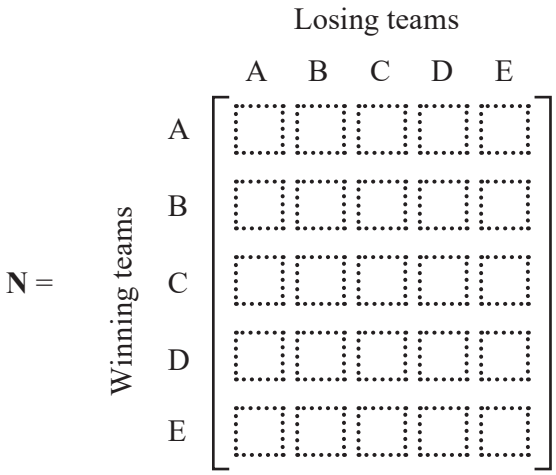
## ADDITIONAL RESPONSE SPACE FOR QUESTION 11a)

If you want this matrix to be marked, rule a single diagonal line through the matrix on page 3.

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