LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your
barcode ID label here

Book of books used

External assessment

Question and response book

Specialist Mathematics

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 55 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

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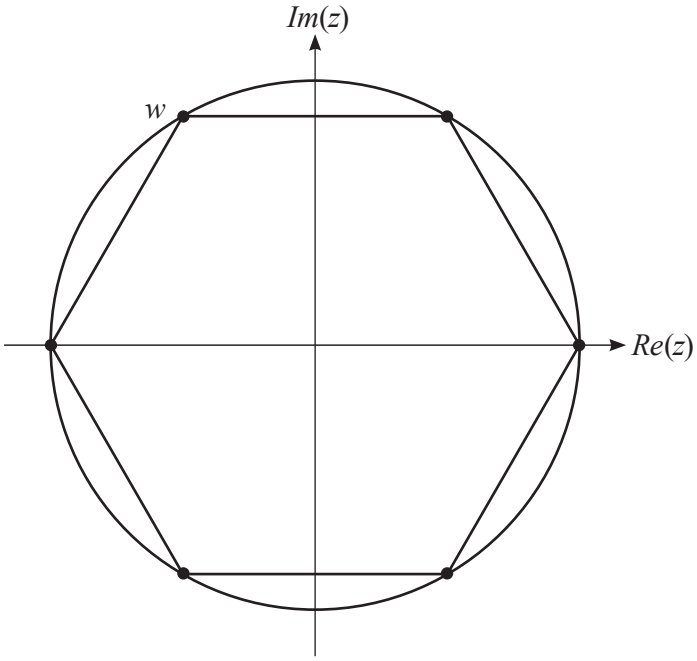
QUESTION 11 (7 marks)

The vertices of a regular hexagon are positioned on the circumference of a unit circle as shown on the Argand plane.



Consider the complex number w, as shown on the plane.

a) Determine w, expressing your answer in the form $r\,\text{cis}(\theta)$. *[1 mark]*

b) Convert w into Cartesian form. *[2 marks]*

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Each vertex of the hexagon is a solution of an equation of the form $z^n = a$ where $z \in C$.

c) State the value of n. *[1 mark]*

d) State the value of a. *[1 mark]*

e) Verify that w satisfies the equation $z^n = a$ using the results from 11c) and 11d). *[2 marks]*

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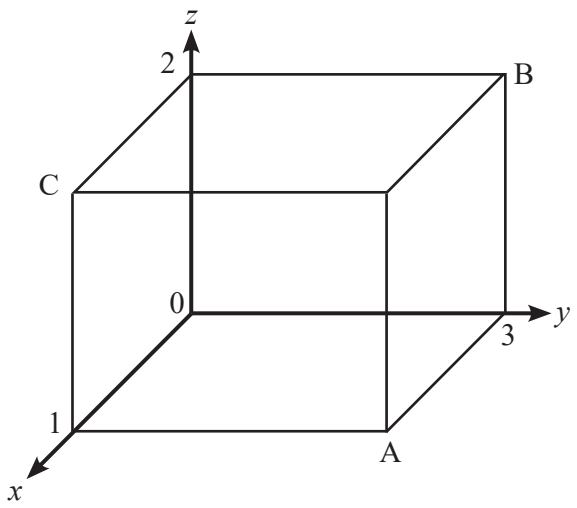
QUESTION 12 (8 marks)

Consider the vertices A, B and C of the rectangular prism as shown.



a) State the coordinates of A, B and C. *[1 mark]*

b) Determine a unit vector, $\hat{\boldsymbol{n}}$, that is normal to the plane containing A, B and C. *[3 marks]*

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c) Verify that $\hat{\boldsymbol{n}}$ is perpendicular to $\overrightarrow{AB}$. *[2 marks]*

d) Determine the Cartesian equation of the plane that contains A, B and C. *[2 marks]*

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QUESTION 13 (4 marks)

The expected value of an exponential random variable X with parameter $\lambda > 0$ can be determined using the rule

$$E(X) = \int_0^\infty x\lambda e^{-\lambda x}\, dx$$

Use integration by parts to determine $E(X)$.

Express your answer in simplest form.

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QUESTION 14 (4 marks)

The motion of an object that moves in a straight line is given by $v(x) = \cos^{-1}(2x)$ where v is the velocity (m s^{-1}) and x is the displacement (m) from the origin.

a) Determine $a(x)$ where a is the acceleration (m s^{-2}) of the object. *[2 marks]*

b) Use the result from 14a) to determine $a(0)$, given $-2\pi \leq a(0) \leq 0$. Express your answer in simplest form. *[2 marks]*

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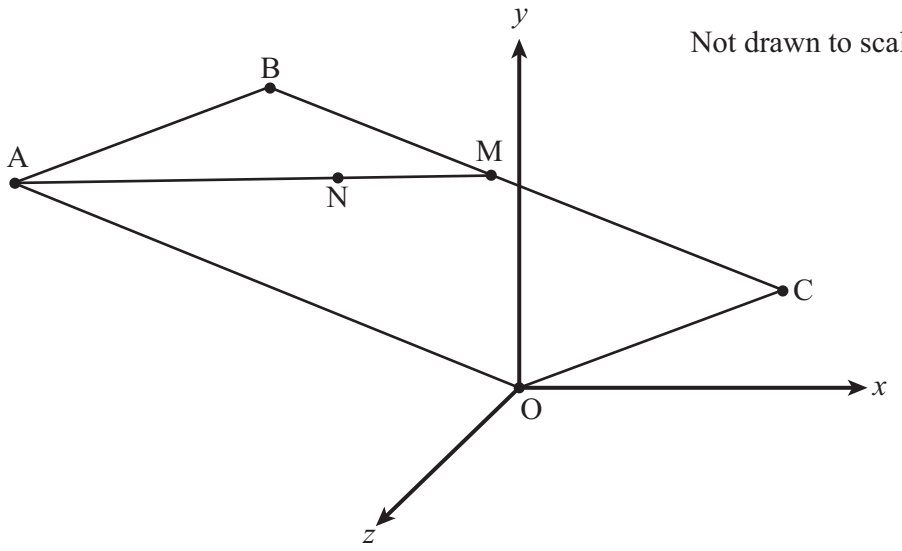
QUESTION 15 (6 marks)

The points O(0, 0, 0), A(–6, 2, –2) and C(3, 1, 2) are represented in three-dimensional space in the diagram.



OABC forms a parallelogram in three-dimensional space.

a) Determine the coordinates of B. *[1 mark]*

M is the midpoint of BC.

b) Determine the vector that represents $\overrightarrow{OM}$. *[1 mark]*

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N divides AM in the ratio 2:1.

c) Determine the vector that represents $\overrightarrow{ON}$. *[2 marks]*

d) Use a vector method to show that O, B and N lie on a straight line. *[2 marks]*

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QUESTION 16 (6 marks)

Given $\cos(\theta) \neq 0 \; \forall \; n \in Z^+$, use mathematical induction to prove

$$\cos(\theta) - \cos(3\theta) + \cos(5\theta) - \ldots + (-1)^{n+1} \cos\big((2n-1)\theta\big) = \frac{1-(-1)^n \cos(2n\theta)}{2\cos(\theta)}$$

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QUESTION 17 (7 marks)

Determine the smallest positive value of a given

$$\int_{-a}^{a} 1+\left(\frac{\sec(2x)+\tan(2x)}{\operatorname{cosec}(2x)+1}\right)^2 dx = 1$$

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Do not write outside this box.

QUESTION 18 (6 marks)

Consider the function $P(z) = 2z^4 + az^3 + 6z^2 + az + b$ where $a, b \in Z^+$

One of the roots of $P(z)$ is $z = -i$

Determine the possible value/s for a and b such that all remaining roots of $P(z)$ have an imaginary component.

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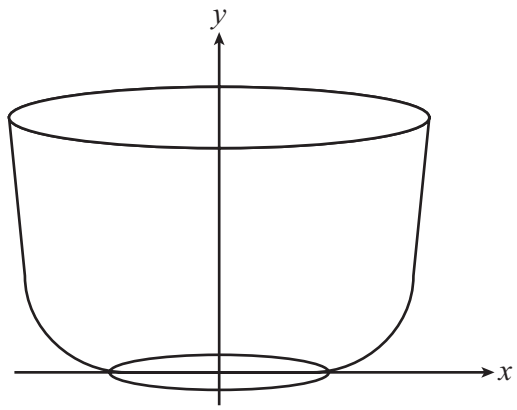
QUESTION 19 (7 marks)

A circular-based bowl has been positioned symmetrically on a Cartesian plane as shown in the diagram.



The bowl has a shape that can be generated by rotating the curve $y = \frac{4}{8-x} - 1$ about the y-axis for $4 \leq x \leq 7.6$ cm.

The bowl is being filled with a liquid at the rate of 7π cm^3 s^{-1}.

Determine the rate at which the depth of liquid is increasing when the depth of liquid reaches one-third of the height of the bowl.

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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