Specialist Mathematics marking guide

External assessment

Paper 1: Technology-free (65 marks)

Paper 2: Technology-active (65 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Purpose

This document is an External assessment marking guide (EAMG).

The EAMG:

- Provides a tool for calibrating external assessment markers to ensure reliability of results
- Indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- Informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded. Where no response to a question has been made, a mark of ‘N’ will be recorded.

Allow FT mark(s) – refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working – the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not evident in the student response, but by virtue of subsequent working there is sufficient evidence to award mark(s).

External assessment marking guide

Paper 1

Multiple choice

Question	Response
1	B
2	D
3	C
4	A
5	A
6	B
7	C
8	D
9	C
10	B

Short response

Question 11 (7 marks)

Sample response	The response:
a) $\lvert w\rvert = 1$ $\arg(w) = \frac{2\pi}{3}$ $w = \text{cis}\left(\frac{2\pi}{3}\right)$	correctly expresses w in $r\text{cis}(\theta)$ form **[1 mark]**
b) $w = \cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right)i$	converts w into $r\cos(\theta) + r\cos(\theta)i$ form **[1 mark]**
$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$	uses exact values for $Re(w)$ and $Im(w)$ **[1 mark]**
c) There are 6 roots of unity so the equation is $z^6 = 1$. $n = 6$	correctly states the value of n **[1 mark]**
d) $a = 1$	correctly states the value of a **[1 mark]**
e) Verify that $w^6 = 1$ $\left(\text{cis}\left(\frac{2\pi}{3}\right)\right)^6 = \text{cis}\left(6 \times \frac{2\pi}{3}\right)$	uses De Moivre's theorem in the calculation of w^n for the value of n **[1 mark]**
$= \text{cis}(4\pi)$ $= \text{cis}(0)$ $= 1$ The required result is verified.	verifies the result by showing that the result of the calculation is the value of a **[1 mark]**

Question 12 (8 marks)

Sample response	The response:
a) $A(1,3,0)$, $B(0,3,2)$ and $C(1,0,2)$	correctly states the coordinates of A, B and C **[1 mark]**
b) $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix}0\\3\\2\end{pmatrix} - \begin{pmatrix}1\\3\\0\end{pmatrix} = \begin{pmatrix}-1\\0\\2\end{pmatrix}$ $\overrightarrow{AC} = c - a = \begin{pmatrix}1\\0\\2\end{pmatrix} - \begin{pmatrix}1\\3\\0\end{pmatrix} = \begin{pmatrix}0\\-3\\2\end{pmatrix}$	determines 2 vectors in the plane containing A, B and C **[1 mark]**
$\boldsymbol{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix}-1\\0\\2\end{pmatrix} \times \begin{pmatrix}0\\-3\\2\end{pmatrix}$ $= \begin{pmatrix}0 \times 2 - 2 \times -3\\2 \times 0 - -1 \times 2\\-1 \times -3 - 0 \times 0\end{pmatrix}$ $= \begin{pmatrix}6\\2\\3\end{pmatrix}$	determines a vector normal to the plane **[1 mark]**
$\hat{\boldsymbol{n}} = \frac{1}{\lvert\boldsymbol{n}\rvert}\boldsymbol{n} = \frac{1}{\sqrt{6^2+2^2+3^2}}\boldsymbol{n}$ $= \frac{1}{7}\begin{pmatrix}6\\2\\3\end{pmatrix}$	determines a unit vector normal to the plane **[1 mark]**
c) Verify that $\hat{\boldsymbol{n}} . \overrightarrow{AB} = 0$ $\hat{\boldsymbol{n}} . \overrightarrow{AB} = \frac{1}{7}\begin{pmatrix}6\\2\\3\end{pmatrix} . \begin{pmatrix}-1\\0\\2\end{pmatrix}$	substitutes the results from b) into $\hat{\boldsymbol{n}} . \overrightarrow{AB}$ **[1 mark]**
$= \frac{1}{7}(-6+0+6)$ $= 0$ The required result is verified.	verifies the result by showing that the result of the calculation is 0 **[1 mark]**
d) **Method 1**	

Sample response	The response:
Determining equation of plane: Using $\boldsymbol{n}=\begin{pmatrix}6\\2\\3\end{pmatrix}$ and $\boldsymbol{a}=\begin{pmatrix}1\\3\\0\end{pmatrix}$ Vector equation of plane $\boldsymbol{r}\,.\,\boldsymbol{n}=\boldsymbol{a}\,.\,\boldsymbol{n}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}6\\2\\3\end{pmatrix}=\begin{pmatrix}1\\3\\0\end{pmatrix}.\begin{pmatrix}6\\2\\3\end{pmatrix}$ Cartesian equation of plane $6x+2y+3z=12$	determines the vector equation of the plane **[1 mark]** determines the Cartesian equation of the plane **[1 mark]**
d) **Method 2** Determining equation of plane: Cartesian equation of plane $ax+by+cz+d=0$ where $\boldsymbol{n}=\begin{pmatrix}a\\b\\c\end{pmatrix}$ Using $\boldsymbol{n}=\begin{pmatrix}6\\2\\3\end{pmatrix}$ and A $(1,3,0)$ to determine d $6(1)+2(3)+3(0)+d=0$ $d=-12$ Cartesian equation of plane $6x+2y+3z-12=0$	substitutes a vector normal to the plane and a suitable set of coordinates into the Cartesian equation of plane formula to form an equation in terms of d **[1 mark]** determines the Cartesian equation of the plane **[1 mark]**

Question 13 (4 marks)

Sample response	The response:
$E(X) = \int_0^{\infty} x\,\lambda e^{-\lambda x}\,dx$ $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$	
$u = x \qquad \frac{du}{dx} = 1$ $\frac{dv}{dx} = \lambda e^{-\lambda x} \qquad v = -e^{-\lambda x}$	correctly determines $\frac{du}{dx}$ **and** v **[1 mark]**
$E(X) = -xe^{-\lambda x}\Big\vert_0^{\infty} + \int_0^{\infty} e^{-\lambda x}\,dx$	substitutes into the integration by parts rule **[1 mark]**
$= 0 + \int_0^{\infty} e^{-\lambda x}\,dx$ $= \frac{e^{-\lambda x}}{-\lambda}\Big\vert_0^{\infty}$ $= 0 - \frac{1}{-\lambda}$ $= \frac{1}{\lambda}$	calculates $-xe^{-\lambda x}\Big\vert_0^{\infty}$ to equal 0 **[1 mark]** shows that $E(X) = \frac{1}{\lambda}$ **[1 mark]**

Question 14 (4 marks)

Sample response	The response:
a) $v(x) = \cos^{-1}(2x)$ $a = v\dfrac{dv}{dx}$ $\dfrac{dv}{dx} = \dfrac{-1}{\sqrt{0.25 - x^2}}$	correctly determines $\frac{dv}{dx}$ **[1 mark]**
$a = \dfrac{-\cos^{-1}(2x)}{\sqrt{0.25 - x^2}}$	determines an expression for acceleration as a function of displacement **[1 mark]**
b) When $x = 0$ $a = \dfrac{-\cos^{-1}(0)}{0.5}$ $\cos^{-1}(0) = \dfrac{\pi}{2}$	determines a correct exact value for the inverse trigonometric expression on the numerator **[1 mark]**
$a(0) = \dfrac{-\left(\frac{\pi}{2}\right)}{0.5}$ $= -\pi \quad (\mathrm{m\ s^{-2}})$	determines a reasonable solution for $a(0)$ (based on the given range $(-2\pi \leq a(0) \leq 0)$ **[1 mark]**

Question 15 (6 marks)

Sample response	The response:
a) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AC}$ $= \begin{pmatrix} -6 \\ 2 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix}$ The coordinates of B are $(-3, 3, 0)$.	correctly determines the coordinates of B **[1 mark]**
b) Midpoint of BC is $\left(\frac{-3+3}{2}, \frac{3+1}{2}, \frac{0+2}{2}\right) = (0, 2, 1)$ $\overrightarrow{OM} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$	determines vector $\overrightarrow{OM}$ **[1 mark]**
c) $\overrightarrow{ON} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AM}$ $= \overrightarrow{OA} + \frac{2}{3}(\overrightarrow{OM} - \overrightarrow{OA})$	determines an expression for $\overrightarrow{ON}$ involving $\overrightarrow{OM}$ and the 1:2 ratio **[1 mark]**
$= \frac{1}{3}\overrightarrow{OA} + \frac{2}{3}\overrightarrow{OM}$ $= \frac{1}{3}\begin{pmatrix} -6 \\ 2 \\ -2 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$	determines vector $\overrightarrow{ON}$ **[1 mark]**

Sample response	The response:
d) **Method 1** To show O, B and N lie on a straight line: Using parallel vectors with a common point $\overrightarrow{ON} = k\overrightarrow{OB}$	correctly communicates a requirement that O, B and N lie on a straight line **[1 mark]**
Using the results from a) and c) $\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$ It is shown that O, B and N lie on a straight line.	correctly demonstrates that the requirement has been met **[1 mark]**
Method 2 To show O, B and N lie on a straight line: Using vector product to show vectors are parallel with a common point $\overrightarrow{ON} \times \overrightarrow{OB} = \mathbf{0}$	correctly communicates a requirement that O, B and N lie on a straight line **[1 mark]**
$\overrightarrow{ON} \times \overrightarrow{OB} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} \times \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$ $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ It is shown that O, B and N lie on a straight line.	correctly demonstrates that the requirement has been met **[1 mark]**

Question 16 (6 marks)

Sample response	The response:
$\cos(\theta) - \cos(3\theta) + \cos(5\theta) - \cdots + (-1)^{n+1}\cos((2n-1)\theta) = \frac{1-(-1)^n \cos(2n\theta)}{2\cos(\theta)}$ Let $P(n)$ represent the proposition. R.T.P. $P(1)$ is true. $\text{LHS} = \cos\theta$ $\text{RHS} = \frac{1-(-1)\cos(2\theta)}{2\cos\theta}$ $= \frac{1+\cos(2\theta)}{2\cos\theta}$ $= \frac{1+2\cos^2\theta - 1}{2\cos\theta}$ $= \cos\theta$ $= \text{RHS}$ $\therefore P(1)$ is true.	correctly proves the initial statement **[1 mark]**
Assume $P(k)$ is true. $\cos(\theta) - \cos(3\theta) + \cos(5\theta) - \cdots + (-1)^{k+1}\cos((2k-1)\theta) = \frac{1-(-1)^k\cos(2k\theta)}{2\cos(\theta)}$ R.T.P. $P(k+1)$ is true. $\cos(\theta) - \cos(3\theta) + \cos(5\theta) - \cdots + (-1)^{k+1}\cos\big((2k-1)\theta\big) + (-1)^{(k+1)+1}\cos\big((2(k+1)-1)\theta\big) = \frac{1-(-1)^{k+1}\cos(2(k+1)\theta)}{2\cos(\theta)}$	correctly states the assumption and the proof requirement for the inductive step **[1 mark]**
$\text{LHS} = \frac{1-(-1)^k\cos(2k\theta)}{2\cos(\theta)} + (-1)^{k+2}\cos\big((2k+1)\theta\big)$	uses assumption in the proof of the inductive step **[1 mark]**

Sample response	The response:
$= \dfrac{1-(-1)^k\cos(2k\theta)+(-1)^{k+2}\cos\big((2k+1)\theta\big)\,2\cos(\theta)}{2\cos(\theta)}$	
$= \dfrac{1-(-1)^k\cos(2k\theta)+(-1)^{k+2}(\cos[(2k+1)\theta-\theta]+\cos[(2k+1)\theta+\theta])}{2\cos(\theta)}$	
$= \dfrac{1-(-1)^k\cos(2k\theta)+(-1)^{k+2}\cos(2k\theta)+(-1)^{k+2}\cos\big((2k+2)\theta\big)}{2\cos(\theta)}$	determines a simplified expression based on the use of a common denominator and a suitable trigonometric product identity **[1 mark]**
$= \dfrac{1+(-1)^{k+2}\cos\big((2k+2)\theta\big)}{2\cos(\theta)}$	determines a simplified expression based on the recognition that $(-1)^k = (-1)^{k+2}$ **[1 mark]**
$= \dfrac{1+(-1)^{k+1}(-1)^1\cos(2(k+1)\theta)}{2\cos(\theta)}$	
$= \dfrac{1-(-1)^{k+1}\cos(2(k+1)\theta)}{2\cos(\theta)}$ $=$ RHS So $P(k\ +\ 1)$ is true. By mathematical induction, the formula is true for $n\ =\ 1, 2, \ldots$	completes proof and communicates a suitable conclusion **[1 mark]**

Question 17 (7 marks)

Sample response	The response:
$\int_{-a}^{a} 1+\left(\frac{\sec(2x)+\tan(2x)}{\operatorname{cosec}(2x)+1}\right)^2 dx=1$	
$\int_{-a}^{a} 1+\left(\frac{\frac{1}{\cos(2x)}+\frac{\sin(2x)}{\cos(2x)}}{\frac{1}{\sin(2x)}+1}\right)^2 dx=1$	correctly expresses the expression inside the brackets of the integrand in terms of $\sin(2x)$and $\cos(2x)$ **[1 mark]**
$\int_{-a}^{a} 1+\left(\frac{\frac{1+\sin(2x)}{\cos(2x)}}{\frac{1+\sin(2x)}{\sin(2x)}}\right)^2 dx=1$ $\int_{-a}^{a} 1+\left(\frac{\sin(2x)}{\cos(2x)}\right)^2 dx=1$	
$\int_{-a}^{a} 1+(\tan(2x))^2\,dx=1$	correctly simplifies the expression inside the brackets of the integrand **[1 mark]**
$\int_{-a}^{a} \sec^2(2x)dx=1$	uses a suitable Pythagorean identity to express the integrand as a single trigonometric expression **[1 mark]**
$\frac{1}{2}\tan(2x)\Big\|_{-a}^{a}=1$	determines the definite integral equation [**1 mark]**
$\frac{1}{2}(\tan(2a)-\tan(-2a))=1$	substitutes limits of integration **[1 mark]**
$\frac{1}{2}(\tan(2a)+\tan(2a))=1$	
$\frac{1}{2}(2\tan(2a))=1 \Rightarrow 2a=\tan^{-1}(1)$ $2a=\frac{\pi}{4} \Rightarrow a=\frac{\pi}{8}$	solves equation to determine the smallest positive value of a **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**

Question 18 (6 marks)

Sample response	The response:
$P(z) = 2z^4 + az^3 + 6z^2 + az + b$ where $a, b \in Z^+$ Given $z = -i$ is a root of $P(z)$, then $P(-i) = 0$ $\therefore\ 2(-i)^4 + a(-i)^3 + 6(-i)^2 + a(-i) + b = 0$ $2 + ai - 6 - ai + b = 0$ $-4 + b = 0$ $b = 4$ $\therefore P(z) = 2z^4 + az^3 + 6z^2 + az + 4$	correctly applies the factor theorem to determine b **[1 mark]**
Given that the coefficients of the polynomial are real, another root is $z = i$, another factor of $P(z)$ is $(z - i)$.	correctly uses the conjugate root of the given root to identify another factor of $P(z)$ **[1 mark]**
$(z - i)(z + i) = (z^2 + 1)$ is a factor of $P(z)$	correctly identifies that $(z^2 + 1)$ is a factor of $P(z)$ **[1 mark]**
By inspection, $P(z) = (z^2 + 1)(2z^2 + az + 4)$	determines the remaining quadratic factor in terms of a **[1 mark]**
Given all roots of $P(z)$ have an imaginary component, $2z^2 + az + 4$ must have only complex roots. For complex roots, $b^2 - 4ac < 0$ $a^2 - 4\times2\times4 < 0$	applies the complex root requirement to the remaining quadratic factor **[1 mark]**
$a < \sqrt{32}$ So $a = 1, 2, 3, 4$ or 5 and $b = 4$	determines the possible values for a given $a, b \in Z^+$ **[1 mark]**

Question 19 (7 marks)

Sample response	The response:
Volume of revolution $y = \frac{4}{8-x} - 1 \Rightarrow 8 - x = \frac{4}{y+1}$	
$x = 8 - \frac{4}{y+1}$	correctly expresses x as the subject of the given relationship **[1 mark]**
$V = \pi \int_a^b [f(y)]^2 \, dy = \pi \int_0^h \left(8 - \frac{4}{y+1}\right)^2 dy$	establishes volume of bowl as a definite integral **[1 mark]**
$\frac{dV}{dh} = \pi \left(8 - \frac{4}{h+1}\right)^2$	determines an expression for $\frac{dV}{dh}$ **[1 mark]**
Related rate of change $\frac{dh}{dt} = \frac{dh}{dV}\frac{dV}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{\frac{dV}{dh}} \frac{dV}{dt}$	
$\frac{dh}{dt} = \frac{1}{\pi\left(8 - \frac{4}{h+1}\right)^2} \times 7\pi$ $= \frac{7}{\left(8 - \frac{4}{h+1}\right)^2}$	uses a related rate of change equation to determine a relationship between $\frac{dh}{dt}$ and h **[1 mark]**
Required depth $= \frac{1}{3} \times f(7.6) = \frac{1}{3} \times 9 = 3$ cm	correctly determines the instantaneous depth of liquid for the required calculation **[1 mark]**
$\therefore \left.\frac{dh}{dt}\right\vert_{h=3} = \frac{7}{\left(8 - \frac{4}{3+1}\right)^2} = \frac{1}{7}$ cm s^{-1}	determines the required rate **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**

Paper 2

Multiple choice

Question	Response
1	D
2	B
3	C
4	A
5	C
6	B
7	C
8	A
9	D
10	A

Short response

Question 11 (5 marks)

<table>
<tr><th>Sample response</th><th>The response:</th></tr>
<tr><td>a)

$$\mathbf{N} = \begin{array}{c|c} & \text{Losing teams} \\ \text{Winning teams} & \begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \end{array}$$
</td><td>correctly completes matrix N [1 mark]</td></tr>
<tr><td>b)

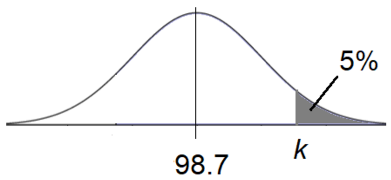
$$\mathbf{N} + \mathbf{N}^2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 3 & 0 & 1 & 1 & 2 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 1 & 2 & 0 & 3 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$
</td><td>calculates $\mathbf{N} + \mathbf{N}^2$ [1 mark]</td></tr>
<tr><td>Ranking teams using the model $\mathbf{N} + \mathbf{N}^2$
<table>
<tr><th>Team</th><th>Model value</th><th>Rank position</th></tr>
<tr><td>A</td><td>4</td><td>3</td></tr>
<tr><td>B</td><td>7</td><td>2</td></tr>
<tr><td>C</td><td>4</td><td>3</td></tr>
<tr><td>D</td><td>9</td><td>1</td></tr>
<tr><td>E</td><td>2</td><td>5</td></tr>
</table>
</td><td>ranks the teams to show that teams A and C are tied [1 mark]</td></tr>
<tr><td>c) The limitation of the ranking model is that it does not provide individual positions from first to fifth.</td><td>identifies a limitation of the organiser’s model based on the model $\mathbf{N} + \mathbf{N}^2$ [1 mark]</td></tr>
<tr><td>d) The ranking model could be improved by including weightings in the calculations (e.g. $\mathbf{N} + \frac{1}{2}\mathbf{N}^2$)</td><td>correctly describes a suitable mathematical refinement [1 mark]</td></tr>
</table>

Question 12 (9 marks)

Sample response	The response:
a) $\frac{dN}{dt} = \frac{1}{1000}N(1000-N)$ $\frac{1000}{N(1000-N)}\frac{dN}{dt} = 1$ $\left(\frac{1}{N} + \frac{1}{1000-N}\right)\frac{dN}{dt} = 1$	correctly uses separation of variables technique and substitutes the given result into the differential equation **[1 mark]**
$\int \frac{1}{N} + \frac{1}{1000-N}\, dN = \int 1\, dt$ $\ln\lvert N\rvert - \ln\lvert 1000-N\rvert = t + c$ $\ln\left\lvert\frac{N}{1000-N}\right\rvert = t + c$	correctly develops the required general solution **[1 mark]**
b) Let t = time after 9:00am (in hours) When $t = 0,\ N = 100$ $\ln\left\lvert\frac{100}{1000-100}\right\rvert = 0 + c$ $c = \ln\left(\frac{1}{9}\right)$	correctly determines c **[1 mark]**
$\ln\left(\frac{N}{1000-N}\right) = t + \ln\left(\frac{1}{9}\right)$ $\frac{N}{1000-N} = e^{t+\ln\left(\frac{1}{9}\right)}$ $\frac{N}{1000-N} = \frac{e^t}{9}$	substitutes the value of c into the general equation and simplifies sufficiently to produce a function that includes the term e^t **[1 mark]**
$9N = 1000e^t - Ne^t$ $N(9+e^t) = 1000e^t$ $N = \frac{1000e^t}{(9+e^t)}$ $N = \frac{1000}{1+9e^{-t}}$	develops the solution for N in the required form **[1 mark]**

Sample response	The response:
c) Given $N = 900$ $900 = \frac{1000}{1+9e^{-t}}$ Using solve facility of GDC $t \approx 4.394$ (hours)	correctly determines the value of t when $N = 900$ **[1 mark]**
The time of day is 1:24 pm.	communicates the time of day **[1 mark]**
d) **Method 1** Given $N = 1200$:	
As $t \to \infty, N \to 1000$ The number of yeast cells has a limit of 1000.	correctly recognises that the number of yeast cells will never exceed 1000 **[1 mark]**
As N will never reach 1200, the scientist's prediction is not reasonable.	comments that the prediction is not reasonable **[1 mark]**
d) **Method 2** Given $N = 1200$: $1200 = \frac{1000}{1+9e^{-t}}$ Using GDC to solve the equation No solution.	correctly recognises that the number of yeast cells will never reach 1200 **[1 mark]**
As N will never reach 1200, the scientist's prediction is not reasonable.	comments that the prediction is not reasonable **[1 mark]**

Question 13 (6 marks)

Sample response	The response:
a) $\mu_X = 98.7$ $\sigma_X = 4.1$ Using normal cdf facility of GDC $P(97.7 \leq X \leq 99.7) \approx 0.193$	correctly calculates required probability **[1 mark]**
Expected number $= 20 \times 0.193$ $= 3.85$ ≈ 4	calculates the expected number **[1 mark]**
b) $\mu_{\overline{X}} = 98.7$ $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{4.1}{\sqrt{20}} \approx 0.917$	correctly calculates $\sigma_{\overline{X}}$ **[1 mark]**
Using normal cdf facility of GDC $P(\overline{X} > 100) \approx 0.08$	calculates required probability **[1 mark]**
c) 	correctly uses appropriate mathematical representation **[1 mark]**
Using inverse normal facility of GDC $k \approx 100.2$ km per hour	correctly determines the value of k **[1 mark]**

Question 14 (5 marks)

Sample response	The response:
a) Using integration facility of GDC $\int_0^\infty 0.16e^{-0.16t}dt = 1$	correctly substitutes the given information into a definite integral to show that $f(t)$ is a probability density function **[1 mark]**
b) 1 year = 12 months $P(0 < x < 12) = \int_0^{12} 0.16e^{-0.16t}dt$	correctly represents the required probability as a definite integral **[1 mark]**
Using integration facility of GDC ≈ 0.85	determines the probability **[1 mark]**
c) $P(1 < x < m) = 0.75$ $\int_1^m 0.16e^{-0.16t}\ dt = 0.75$	correctly establishes definite integral equation **[1 mark]**
Using solve facility of GDC or otherwise $m \approx 14.26$ months	solves equation to determine m **[1 mark]**

Question 15 (4 marks)

Sample response	The response:
a) Determining angle POQ $\cos(\theta) = \dfrac{\overrightarrow{OP} \cdot \overrightarrow{OQ}}{\lvert\overrightarrow{OP}\rvert\lvert\overrightarrow{OQ}\rvert} = \dfrac{-6}{\sqrt{14}\sqrt{24}}$	correctly determines a numerical expression for $\cos(\theta)$ **[1 mark]**
$\theta \approx 1.90$	determines angle POQ **[1 mark]**
b) **Method 1** Finding area of triangle POQ: $A = \dfrac{1}{2}\lvert\overrightarrow{OP} \times \overrightarrow{OQ}\rvert$ Using vector facility of GDC or otherwise $= \frac{1}{2}\lvert 10\hat{\boldsymbol{\imath}} + 10\hat{\boldsymbol{\jmath}} + 10\hat{\boldsymbol{k}}\rvert$	correctly determines $\overrightarrow{OP} \times \overrightarrow{OQ}$ **[1 mark]**
$= 8.66 \quad \text{units}^2$	determines area **[1 mark]**
Method 2 Finding area of triangle POQ: $A = \dfrac{1}{2}\lvert\overrightarrow{OP}\rvert\lvert\overrightarrow{OQ}\rvert \sin(\theta)$ $= \dfrac{1}{2}\sqrt{14}\,\sqrt{24}\sin(1.90)$	correctly substitutes values into suitable rule **[1 mark]**
$= 8.66 \quad \text{units}^2$	determines area **[1 mark]**

Question 16 (6 marks)

Sample response	The response:
a) Method 1 Using De Moivre's theorem: $(\text{cis}(\theta))^4 = \text{cis}(4\theta)$	correctly uses De Moivre's theorem **[1 mark]**
Equating real parts $\boldsymbol{\cos(4\theta) = Re(\cos(\theta) + i\sin(\theta))^4}$ $\boldsymbol{= \cos^4(\theta) - 6\cos^2(\theta)\sin^2(\theta) + \sin^4(\theta)}$	uses binomial expansion with the real parts and simplifies the expression **[1 mark]**
$= \cos^4(\theta) - 6\cos^2(\theta)(1 - \cos^2(\theta)) + (1 - \cos^2(\theta))^2$ $= \cos^4(\theta) - 6\cos^2(\theta) + 6\cos^4(\theta) + 1 - 2\cos^2(\theta) + \cos^4(\theta)$ $\boldsymbol{= 8\cos^4(\theta) - 8\cos^2(\theta) + 1}$	establishes a simplified expression following the use of a suitable Pythagorean identity **[1 mark]**
$\boldsymbol{= 8\cos^4(\theta) - 8(1 - \sin^2(\theta)) + 1}$ $= 8\cos^4(\theta) - 8 + 8\sin^2(\theta) + 1$ $\boldsymbol{= 8\cos^4(\theta) + 8\sin^2(\theta) - 7}$	establishes a simplified expression in the form of $A\cos^4(\theta) + B\sin^2(\theta) + C$ **[1 mark]**
So $\boldsymbol{A = 8}$, $\boldsymbol{B = 8}$, $\boldsymbol{C = -7}$	communicates the values of A, B and C **[1 mark]**

Sample response	The response:
a) Method 2 Using De Moivre's theorem: $\left(\text{cis}(\theta)\right)^4 = \text{cis}(4\theta)$	correctly uses De Moivre's theorem **[1 mark]**
Using the binomial expansion $\left(\text{cis}(\theta)\right)^4 = \left(\cos(\theta) + i\sin(\theta)\right)^4$ $= \boldsymbol{\cos^4(\theta) + 4\cos^3(\theta)\sin(\theta)i - 6\cos^2(\theta)\sin^2(\theta) - 4\cos(\theta)\sin^3(\theta)i + \sin^4(\theta)}$ Equating real parts	uses binomial expansion and simplifies the expression **[1 mark]**
$\cos(4\theta) = \cos^4(\theta) - 6\cos^2(\theta)(1-\cos^2(\theta) + (1-\cos^2(\theta))^2$ $= \boldsymbol{\cos^4(\theta) - 6\cos^2(\theta) + 6\cos^4(\theta) + 1 - 2\cos^2(\theta) + \cos^4(\theta)}$ $= \boldsymbol{8\cos^4(\theta) - 8\cos^2(\theta) + 1}$	establishes a simplified expression following the use of a suitable Pythagorean identity **[1 mark]**
$= \boldsymbol{8\cos^4(\theta) - 8(1-\sin^2(\theta)) + 1}$ $= 8\cos^4(\theta) - 8 + 8\sin^2(\theta) + 1$ $= \boldsymbol{8\cos^4(\theta) + 8\sin^2(\theta) - 7}$	establishes a simplified expression in the form of $A\cos^4(\theta) + B\sin^2(\theta) + C$ **[1 mark]**
So $\boldsymbol{A = 8,\ B = 8,\ C = -7}$	communicates the values of A, B and C **[1 mark]**
b) A verification strategy would be to graph $\boldsymbol{y = \cos(4\theta)}$ and $\boldsymbol{y = 8\cos^4(\theta) + 8\sin^2(\theta) - 7}$ to confirm that the two graphs are the same.	describes an appropriate verification strategy **[1 mark]**

Question 17 (7 marks)

Sample response	The response:
$v\dfrac{dv}{dx} = 9.8 - 0.004v^2\ , v > 0$ $\displaystyle\int \frac{v}{9.8 - 0.004v^2}\,dv = \int dx$	correctly uses separation of variables **[1 mark]**
$\dfrac{-1}{0.008}\displaystyle\int \frac{-0.008v}{9.8 - 0.004v^2}\,dv = \int dx$ $-125\ln\lvert 9.8 - 0.004v^2\rvert = x + c$	correctly develops the general solution of the differential equation **[1 mark]**
Given $v = 0$ when $x = -100$	correctly uses the given position of the origin **[1 mark]**
$-125\ln\lvert 9.8\rvert = -100 + c$ $c \approx -185.298$	uses the given condition to determine value for c **[1 mark]**
$-125\ln\lvert 9.8 - 0.004v^2\rvert = x - 185.298$ Determining v when $x = 0$ $-125\ln\lvert 9.8 - 0.004v^2\rvert = -185.298$	substitutes the displacement at impact to form an equation in terms of v **[1 mark]**
Using graph facility of GDC $v \approx -36.7$ ms^{-1} or $v \approx 36.7$ ms^{-1} As $v > 0$, the negative solution is rejected $\therefore\ v \approx 36.7$ ms^{-1}	determines one reasonable solution of v **[1 mark]** shows logical organisation communicating key steps **[1 mark]**

Question 18 (6 marks)

Sample response	The response:
Sample 1: $n = 1$ $P(X > 83.2) = 0.145$ $P\left(z > \frac{83.2-\mu}{\sigma}\right) = 0.145$ $\frac{83.2-\mu}{\sigma} = 1.058$ $\mu = 83.2 - 1.058\sigma \quad \ldots(1)$	correctly uses the sample of 1 to determine an equation in terms of μ and σ **[1 mark]**
Sample 2: $n = 12$ $P(\bar{X} < 74.1) = 0.079$ $P\left(z < \frac{74.1-\mu}{\frac{\sigma}{\sqrt{12}}}\right) = 0.079$ $\frac{74.1-\mu}{\frac{\sigma}{\sqrt{12}}} = -1.412$ $\mu = 74.1 + \frac{1.412\sigma}{\sqrt{12}} \quad \ldots(2)$	correctly uses the sample of 12 to determine an equation in terms of μ and σ **[1 mark]**
Using graph facility of GDC to solve (1) and (2) $\mu = 76.63$ kg , $\sigma = 6.21$ kg	solves simultaneous equations to determine the values of μ and σ **[1 mark]**
Sample 3: Consider the 90% CI Since $\bar{x} = 79.1$, $\mu = 76.63$ can only lie in an interval below the lower bound of CI. Determining n where the lower bound of $\mathrm{CI} = \mu$ $\bar{x} - z\frac{s}{\sqrt{n}} = 76.63$ $79.1 - 1.64 \times \frac{6.21}{\sqrt{n}} = 76.63$ Using solve facility of GDC, $n \approx 17.1$	determines solution of n **[1 mark]**
As μ must lie in an interval below the lower bound of CI, the range of values is $n \geq 18$ where $n \in Z$.	evaluates the reasonableness of the solution to the equation to determine suitable integer values of n **[1 mark]** shows logical organisation communicating key steps **[1 mark]**

Question 19 (7 marks): Method 1

Sample response	The response:
Finding the angular velocity during the swing: $\omega = \frac{2\pi}{T}$ $= \frac{2\pi}{0.24}$ $= 26.18 \text{ rad } s^{-1}$	
Finding the magnitude of the velocity at release: $v = r\omega$ $= 0.5 \times 26.18$ $= 13.09 \text{ m s}^{-1}$	correctly determines the tangential velocity of object **[1 mark]**
Let the origin in the vertical plane of motion be at the ground directly below the point of release and the angle of release to the horizontal be θ. Assume the unit vectors: 	
Equations of motion for the projectile: $\boldsymbol{a} = -9.8\,\hat{\boldsymbol{j}}$ $\boldsymbol{v} = 13.09\ \cos(\theta)\,\hat{\boldsymbol{i}} + (13.09\ \sin(\theta) - 9.8t)\hat{\boldsymbol{j}}$ $\boldsymbol{r} = 13.09\ \cos(\theta)\,t\,\hat{\boldsymbol{i}}$ $+ (0.3 + 13.09\ \sin(\theta)\,t - 4.9t^2)\hat{\boldsymbol{j}}$	identifies the parametric form of the projectile path **[1 mark]**
The parametric form of the projectile path: $x = 13.09\ \cos(\theta)\,t \quad \ldots(1)$ $y = 0.3 + 13.09\ \sin(\theta)\,t - 4.9t^2 \quad \ldots(2)$ From (1) $t = \frac{x}{13.09\ \cos(\theta)} \quad \ldots(3)$ Substituting into (2) to change into Cartesian form: $y = 0.3 + \frac{13.09\sin(\theta)x}{13.09\cos(\theta)} - \frac{4.9x^2}{(13.09)^2\cos^2(\theta)}$	determines the Cartesian form of the projectile path **[1 mark]**

Sample response	The response:
$$y = 0.3 + \tan(\theta)x - \frac{0.0286x^2}{\cos^2(\theta)}$$ GIVEN THE POINT $(14, 2.05)$ LIES ON THE PATH: $2.05 = 0.3 + 14\,\mathrm{TAN}(\theta) - \frac{0.0286(14)^2}{\cos^2(\theta)}$ SOLVING THE EQUATION USING GDC: $\theta = 0.6444$	determines the angle of release of the object **[1 mark]**
The Cartesian path of the projectile is $y = 0.3 + \tan(0.6444)x - \frac{0.0286x^2}{\cos^2(0.6444)}$ At impact at point A, $y = 0.4$ m $0.4 = 0.3 + \tan(0.6444)x - \frac{0.0286x^2}{\cos^2(0.6444)}$ SOLVING THE QUADRATIC EQUATION USING GDC: $x = 0.134$ or $x = 16.660$ m As $x >$ 14 m, point of impact is $(16.660, 0.4)$	determines the horizontal displacement at impact **[1 mark]**
Time until impact (using (3): $t = \frac{16.660}{13.09\ \cos(0.6444)}$ $= 1.592$ s	determines the time of flight **[1 mark]**
Equations of motion for the cart: $\boldsymbol{a} = a\,\hat{\boldsymbol{\imath}}$ $\boldsymbol{v} = at\,\hat{\boldsymbol{\imath}}$ $\boldsymbol{r} = \left(\frac{at^2}{2} + 14.3\right)\hat{\boldsymbol{\imath}} + 0.4\hat{\boldsymbol{\jmath}}$ Equating $\hat{\boldsymbol{\imath}}$ components when $t = 1.592$ s: $\frac{a(1.592)^2}{2} + 14.3 = 16.660 \Rightarrow a \approx 1.86$ m s^{-2}	determines the acceleration of the trolley **[1 mark]**

Question 19 (7 marks): Method 2

Sample response	The response:
Finding the angular velocity during the swing: $\omega = \frac{2\pi}{T}$ $= \frac{2\pi}{0.24}$ $= 26.18 \text{ rad } s^{-1}$	
Finding the magnitude of the velocity at release: $\boldsymbol{v} = r\omega$ $= 0.5 \times 26.18$ $= 13.09 \text{ m s}^{-1}$	correctly determines the tangential velocity of object **[1 mark]**
Let the origin in the vertical plane of motion be the point of release and the angle of release to the horizontal be θ. Assume the unit  vectors: Equations of motion for the projectile: $\boldsymbol{a} = -9.8\,\hat{\boldsymbol{j}}$ $\boldsymbol{v} = 13.09\ \cos(\theta)\,\hat{\boldsymbol{i}} + (13.09\ \sin(\theta) - 9.8t)\hat{\boldsymbol{j}}$ $\boldsymbol{r} = 13.09\ \cos(\theta)\,t\,\hat{\boldsymbol{i}}$ $+ (13.09\ \sin(\theta)\,t - 4.9t^2)\hat{\boldsymbol{j}}$	
The parametric form of the projectile path: $x = 13.09\ \cos(\theta)\,t \quad \ldots(1)$ $y = 13.09\ \sin(\theta)\,t - 4.9t^2 \quad \ldots(2)$	identifies the parametric form of the projectile path **[1 mark]**
From (1) $t = \frac{x}{13.09\ \cos(\theta)} \quad \ldots(3)$ Substituting into (2) to change into Cartesian form: $y = \frac{13.09 \sin(\theta) x}{13.09 \cos(\theta)} - \frac{4.9x^2}{(13.09)^2 \cos^2(\theta)}$	determines the Cartesian form of the projectile path **[1 mark]**

Sample response	The response:
$$y = \tan(\theta)x - \frac{0.0286x^2}{\cos^2(\theta)}$$ GIVEN THE POINT $(14, 1.75)$ LIES ON THE PATH: $1.75 = 14\,\mathrm{TAN}(\theta) - \frac{0.0286(14)^2}{\cos^2(\theta)}$ SOLVING THE EQUATION USING GDC: $\theta = 0.6444$	determines the angle of release of the object **[1 mark]**
The Cartesian path of the projectile is $y = \tan(0.6444)x - \frac{0.0286x^2}{\cos^2(0.6444)}$ At impact at point A, $y = 0.1$ m $0.1 = \tan(0.6444)x - \frac{0.0286x^2}{\cos^2(0.6444)}$ SOLVING THE QUADRATIC EQUATION USING GDC: $x = 0.134$ or $x = 16.660$ m As $x > 14$ m, point of impact is $(16.660, 0.1)$	determines the horizontal displacement at impact **[1 mark]**
Time until impact (using (3): $t = \frac{16.660}{13.09\ \cos(0.6444)}$ $= 1.592$ s	determines the time of flight **[1 mark]**
Equations of motion for the cart: $\boldsymbol{a} = a\,\hat{\boldsymbol{\imath}}$ $\boldsymbol{v} = at\,\hat{\boldsymbol{\imath}}$ $\boldsymbol{r} = \left(\frac{at^2}{2} + 14.3\right)\hat{\boldsymbol{\imath}} + 0.1\hat{\boldsymbol{\jmath}}$ Equating $\hat{\boldsymbol{\imath}}$ components when $t = 1.592$ s:	determines the acceleration of the trolley **[1 mark]**

Sample response	The response:
$\frac{a(1.592)^2}{2} + 14.3 = 16.660 \Rightarrow a = 1.86 \text{ m s}^{-2}$	