Specialist Mathematics 2025

IA2 sample marking scheme

July 2025

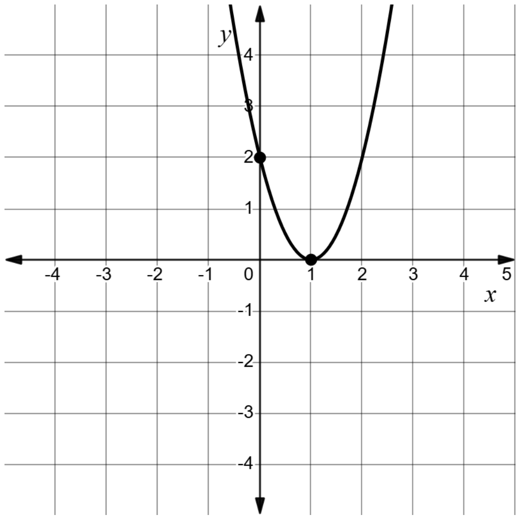
This sample has been compiled by the QCAA to model one possible approach to allocating marks in an examination. It matches the examination mark allocations as specified in the syllabus (~ 60% simple familiar, ~ 20% complex familiar and ~ 20% complex unfamiliar) and ensures that a balance of the objectives are assessed.

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Marking scheme

Q	Sample response	The response	Notes
1a)	$\boldsymbol{r}=(t+1)\hat{\boldsymbol{i}}+2t^2\hat{\boldsymbol{j}}=x\hat{\boldsymbol{i}}+y\hat{\boldsymbol{j}}$ $x=t+1 \quad ...(1)$ $y=2t^2 \quad ...(2)$	• correctly expresses the x and y components of the vector in terms of the parameter **[1 mark]**	Half marks apply. One for each correct component.
	From (1) $\Rightarrow t=x-1$	• uses a suitable procedure to eliminate the parameter **[1 mark]**	Allow FT mark for errors in prior working. Accept alternative method, e.g. expresses t in terms of y.
	Substituting into (2) $y=2(x-1)^2$	• uses a suitable procedure to determine a Cartesian equation **[1 mark]**	Accept equivalent equation, e.g. $y=2\left(x^2+2x+1\right)$.
1b)		• sketches parabolic path of object indicating turning point **[1 mark]**	Accept equivalent indication, e.g. states turning point.
		• indicates y-intercept **[1 mark]**	Accept equivalent indication, e.g. states y-intercept.

Q	Sample response	The response	Notes
2a)	$\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$ $= \begin{pmatrix} 3 \\ 4 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ -6 \end{pmatrix}$	• correctly determines $\overrightarrow{AB}$ **[1 mark]**	
2b)	Vector equation of line l is $\boldsymbol{r} = \boldsymbol{a} + k\overrightarrow{AB}\ (k \in R)$ $= \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} + k\begin{pmatrix} 5 \\ 7 \\ -6 \end{pmatrix}$	• expresses l in vector form **[1 mark]**	Accept alternative method, e.g. uses a different parameter. Half marks apply. One for suitable direction and one for suitable point.
2c)	Cartesian equation of line l is $\frac{x+2}{5} = \frac{y+3}{7} = \frac{z-3}{-6}$	• expresses l in Cartesian form **[1 mark]**	Accept alternative result, e.g. $\frac{x-3}{5} = \frac{y-4}{7} = \frac{z+3}{-6}$.
2d)	$\text{Area} = \frac{1}{2}\left\|\overrightarrow{OA} \times \overrightarrow{OB}\right\|$	• correctly identifies correct formula for area of a triangle **[1 mark]**	This mark may be implied by subsequent working.
	$= \frac{1}{2}\left\|\begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 4 \\ -3 \end{pmatrix}\right\|$	• correctly substitutes into a suitable vector rule for area of a triangle **[1 mark]**	Accept alternative method, e.g. $\frac{1}{2}\left\|\begin{pmatrix} 3 \\ 4 \\ -3 \end{pmatrix} \times \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix}\right\|$.
	$= \frac{1}{2}\left\|\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}\right\|$	• calculates vector product **[1 mark]**	Accept alternative result, e.g. $\begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix}$.
	$= \frac{1}{2}\sqrt{19}$ units^2	• calculates vector magnitude **[1 mark]**	Accept appropriate rounding of result, e.g. 2.18 .

Q	Sample response	The response	Notes
3a)	$\frac{z_1}{z_2} = \frac{2\operatorname{cis}\left(\frac{\pi}{3}\right)}{4\operatorname{cis}\left(\frac{\pi}{4}\right)}$ $= \frac{2}{4}\operatorname{cis}\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$	• correctly uses a suitable analytic procedure to represent the modulus and argument of $\frac{z_1}{z_2}$ **[1 mark]**	Half marks apply. One for the modulus step and one for the argument step.
	$= \frac{1}{2}\operatorname{cis}\left(\frac{\pi}{12}\right)$	• correctly expresses result in simplified polar form **[1 mark]**	
3b)	$\left(\frac{z_1}{z_2}\right)^6 = \left(\frac{1}{2}\operatorname{cis}\left(\frac{\pi}{12}\right)\right)^6$ $= \left(\frac{1}{2}\right)^6 \operatorname{cis}\left(6 \times \frac{\pi}{12}\right)$	• correctly uses De Moivre's theorem to represent the modulus of $\left(\frac{z_1}{z_2}\right)^6$ **[1 mark]** • correctly uses De Moivre's theorem to represent the argument of $\left(\frac{z_1}{z_2}\right)^6$ **[1 mark]**	These marks may be implied by subsequent working.
	$= \frac{1}{64}\operatorname{cis}\left(\frac{\pi}{2}\right)$ $= \frac{1}{64} i$	• expresses $\left(\frac{z_1}{z_2}\right)^6$ in Cartesian form **[1 mark]**	Allow FT marks for errors in prior working. It is not necessary to simplify $\frac{1}{2^6}$.
	This result has no real component, so the comment is reasonable	• evaluates the reasonableness of the comment providing a suitable mathematical explanation for the decision **[1 mark]**	

Q	Sample response	The response	Notes
4a)	$\det(\boldsymbol{B}) = 13$	• correctly determines $\det(\boldsymbol{B})$ **[1 mark]**	
4b)	$\boldsymbol{B}$ is a non-singular matrix as $\det(\boldsymbol{B}) \neq 0$.	• provides a suitable mathematical reason for why $\boldsymbol{B}$ is non-singular **[1 mark]**	Allow FT mark for errors in prior working.
4c)	$A^{-1} = \begin{bmatrix} -1 & -0.5 & 0.5 \\ 3 & 1.5 & -0.5 \\ -2 & -0.5 & 0.5 \end{bmatrix}$	• correctly determines A^{-1} **[1 mark]**	
4d)	$\boldsymbol{X} = \boldsymbol{B}A^{-1}$	• correctly demonstrates the use of matrix algebra **[1 mark]**	This mark may be implied by subsequent working.
	$= \begin{bmatrix} 1 & 0.5 & 0.5 \\ 5 & 3 & -1 \\ -3 & 0 & 1 \end{bmatrix}$	• solves for $\boldsymbol{X}$ **[1 mark]**	Allow FT marks for errors in prior working.
4e)	$\text{LHS} = \boldsymbol{XA}$ $= \begin{bmatrix} 1 & 0.5 & 0.5 \\ 5 & 3 & -1 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$ $= \boldsymbol{B}$ $= \text{RHS}$	• verifies the result from Question 4d) **[1 mark]**	

Q	Sample response	The response	Notes
5a)	$P(z)=z^3+(2-6i)z^2-(9+12i)z-18$ $P(-2)=(-2)^3+(2-6i)(-2)^2-(9+12i)(-2)-18=0$	• correctly uses the factor theorem **[1 mark]**	
5b)	$P(z)=(z+2)Q(z)$ $=(z+2)(z^2+bz+c)$ $=z^3+(b+2)z^2+(2b+c)+2c$	• expresses $P(z)$ as an expanded cubic polynomial **[1 mark]**	Allow FT marks for errors in prior working. This mark may be awarded for valid alternative method, e.g. polynomial long division.
	$2c=-18$ and $b+2=2-6i$ $c=-9$ $b=-6i$	• determines the values of *b* and *c* **[1 mark]**	Half marks apply. One for the value of *b* and one for the value of *c*.
5c)	$\therefore Q(z)=z^2-6iz-9$ Consider $z^2-6iz-9=0$ $z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{6i\pm\sqrt{(-6i)^2-4\times1\times(-9)}}{2}$	• substitutes correctly into the quadratic formula in order to solve $Q(z)=0$ **[1 mark]**	Accept alternative method, e.g. factorises $Q(z)$.
	$=\frac{6i\pm\sqrt{-36+36}}{2}$ $=3i$	• determines the identical roots of $Q(z)$ **[1 mark]**	
5d)	$P(z)=(z+2)(z-3i)^2$	• fully factorises $P(z)$ **[1 mark]**	

Q	Sample response	The response	Notes
6	$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ $\overrightarrow{BC}=\boldsymbol{q}-\boldsymbol{p}$	• correctly expresses $\overrightarrow{AM}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$ **[1 mark]**	
	$\overrightarrow{AM}=\overrightarrow{AB}+\frac{1}{2}\overrightarrow{BC}\Rightarrow \boldsymbol{p}+\frac{1}{2}(\boldsymbol{q}-\boldsymbol{p})$ $=\frac{1}{2}(\boldsymbol{p}+\boldsymbol{q})$	• expresses $\overrightarrow{BC}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$ **[1 mark]**	Allow FT marks for errors in prior working.
	$\overrightarrow{AM}\cdot\overrightarrow{BC}=\frac{1}{2}(\boldsymbol{p}+\boldsymbol{q})\cdot(\boldsymbol{q}-\boldsymbol{p})$ $=\frac{1}{2}(\boldsymbol{p}\cdot\boldsymbol{q}-\boldsymbol{p}\cdot\boldsymbol{p}+\boldsymbol{q}\cdot\boldsymbol{q}-\boldsymbol{q}\cdot\boldsymbol{p})$	• expresses $\overrightarrow{AM}\cdot\overrightarrow{BC}$ as scalar products in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$ in expanded form **[1 mark]**	
	$=\frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{q}-\boldsymbol{p}\cdot\boldsymbol{p})$ $=\frac{1}{2}\left(\lvert\boldsymbol{q}\rvert^2-\lvert\boldsymbol{p}\rvert^2\right)$	• expresses $\overrightarrow{AM}\cdot\overrightarrow{BC}$ in terms of the magnitudes of $\boldsymbol{p}$ and $\boldsymbol{q}$ **[1 mark]**	
	$=0$ as triangle ABC is isosceles So it is proven that $\overrightarrow{AM}$ is perpendicular to $\overrightarrow{BC}$.	• correctly completes the proof providing suitable mathematical reasoning **[1 mark]**	

Q	Sample response	The response	Notes
7	Expressing the 3 equations as an augmented matrix: $\left[\begin{array}{ccc\|c} 1 & -2 & 2\lambda & -5 \\ -1 & 1 & \lambda^2 & 2 \\ 2 & -3 & \lambda^3 & 5 \end{array}\right]$ $=\left[\begin{array}{ccc\|c} 1 & -2 & 2\lambda & -5 \\ 0 & -1 & \lambda^2+2\lambda & -3 \\ 0 & -1 & \lambda^3+2\lambda^2 & 9 \end{array}\right]$ $R_2' = R_2 + R_1$ $R_3' = R_3 + 2R_2$	• correctly produces an augmented matrix and uses Gaussian elimination to modify one row **[1 mark]**	Half marks apply. One for producing an augmented matrix and one for the modification of any row. Accept alternative method, e.g. uses a different row calculation.
	$=\left(\begin{array}{ccc\|c} 1 & -2 & 2\lambda & -5 \\ 0 & -1 & \lambda^2+2\lambda & -3 \\ 0 & 0 & \lambda^3+\lambda^2-2\lambda & 12 \end{array}\right)$ $R_3' = R_3 - R_2$	• uses Gaussian elimination to produce an augmented matrix with a row containing two elements equal to 0 **[1 mark]**	Allow FT marks for errors in prior working. Accept alternative method, e.g. modifies row 2 to contain two elements equal to 0. This mark may be implied by subsequent working
	The system has no solutions when $\lambda^3+\lambda^2-2\lambda=0$	• determines an equation that considers the case when the system of equations has no solutions **[1 mark]**	Accept alternative method, e.g. produces an equation based on row 2.
	Solving the equation $\lambda = -2, 0, 1$	• determines the solutions of the cubic equation formed **[1 mark]**	This mark may be implied by subsequent working.
	Given $\lambda \neq 0$, the only reasonable solutions are $\lambda = -2$ and $\lambda = 1$.	• evaluates the reasonableness of solution to determine all suitable values of λ **[1 mark]**	

Q	Sample response	The response	Notes
8	Determining $\boldsymbol{v}\times\boldsymbol{B}$ $\boldsymbol{v}\times\boldsymbol{B}=\begin{pmatrix}a\\b\\c\end{pmatrix}\times\begin{pmatrix}3\\-2\\1\end{pmatrix}$ $=(b+c)\hat{\boldsymbol{i}}+(-a+3c)\hat{\boldsymbol{j}}+(-2a-3b)\hat{\boldsymbol{k}}$ $=\begin{pmatrix}b+2c\\-a+3c\\-2a-3b\end{pmatrix}$	• correctly determines an expression for $\boldsymbol{v}\times\boldsymbol{B}$ in terms of a, b and c **[1 mark]**	Accept equivalent vector, e.g. $(b+c)\hat{\boldsymbol{i}}+(-a+3c)\hat{\boldsymbol{j}}+(-2a-3b)\hat{\boldsymbol{k}}$.
	Determining the force vector: $\boldsymbol{F}=q(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})$ $=1\left(\begin{pmatrix}2a\\b\\3c\end{pmatrix}+\begin{pmatrix}b+2c\\-a+3c\\-2a-3b\end{pmatrix}\right)$ $=\begin{pmatrix}2a+b+2c\\-a+b+3c\\-2a-3b+3c\end{pmatrix}$	• correctly determines an expression for $\boldsymbol{F}$ in terms of a, b and c **[1 mark]**	
	When $\boldsymbol{F}=\begin{pmatrix}-2\\-6\\1\end{pmatrix}$ $2a+b+2c=-2$ $-a+b+3c=-6$ $-2a-3b+3c=1$	• translates information regarding $\boldsymbol{F}$ into a matrix equation **[1 mark]**	Allow FT marks for errors in prior working.

Q	Sample response	The response	Notes
	Rewriting in matrix form $\begin{pmatrix} 2 & 1 & 2 \\ -1 & 1 & 3 \\ -2 & -3 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \\ 1 \end{pmatrix}$ Using matrix algebra $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 & 1 & 2 \\ -1 & 1 & 3 \\ -2 & -3 & 3 \end{pmatrix}^{-1}\begin{pmatrix} -2 \\ -6 \\ 1 \end{pmatrix}$ $\therefore \boldsymbol{v} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$	• determines required velocity of the particle **[1 mark]**	This could be determined using $\boldsymbol{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$
	Speed of particle is $\lvert \boldsymbol{v} \rvert = \sqrt{6}\ \text{m s}^{-1}$	• determines required speed of the particle **[1 mark]**	Accept appropriate rounding of result, e.g. 2.4 .

Q	Sample response	The response	Notes
9	The 3 roots of unity of $w^3=1$ are: $w_1=1$ $w_2=\text{cis}\left(\frac{2\pi}{3}\right)=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ $w_3=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ As $\text{Im}(w)>0$, the required root is w_2.	• correctly identifies the complex number that satisfies the given conditions **[1 mark]**	Half marks apply. One for determining the required root based on the first condition and one for identifying this root based on the second condition.
	Given $P(z)=z^3+az^2+bz+c$ has roots at $1,\ -w$ and $-\overline{w}$, the roots are $1,\ \frac{1}{2}-\frac{\sqrt{3}}{2}i,\ \frac{1}{2}+\frac{\sqrt{3}}{2}i$	• determines the two non-real roots of $P(z)$ **[1 mark]**	Half marks apply. One for determining $-w$ and one for determining $-\overline{w}$.
	In factorised form $P(z)=(z-1)\left(z-\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)\right)\left(z-\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right)$	• expresses $P(z)$ in factorised form involving real and imaginary numbers **[1 mark]**	Accept equivalent result, e.g. $P(z)=(z-1)\left(z-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(z-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$
	$=(z-1)\left(\left(z-\frac{1}{2}\right)+\frac{\sqrt{3}}{2}i\right)\left(\left(z-\frac{1}{2}\right)-\frac{\sqrt{3}}{2}i\right)$ $=(z-1)\left(\left(z-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2\right)$	• expresses $P(z)$ in factorised form using only real numbers **[1 mark]**	This mark may be implied by subsequent working.
	$=(z-1)(z^2-z+1)$ $=z^3-2z^2+2z-1$ $\therefore a=-2,\ b=2,\ c=-1$	• determines the values of a, b and c **[1 mark]**	